DEPOLARIZATION OF METRIC RADIO SIGNALS AND THE SPATIAL SPECTRUM OF SCATTERED RADIATION BY MAGNETIZED TURBULENT PLASMA SLAB

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Abstract—The mutual correlation function of the phase fluctuations of scattered ordinary and extraordinary waves by the magnetized plasma slab with electron density fluctuations and the variance of the Faraday angle is calculated by the perturbation method. Analytical expression of broadening of the spatial spectrum of scattered radiation is obtained for arbitrary fluctuation spectrum. Numerical calculations are carried out for the anisotropic Gaussian fluctuation spectrum at different anisotropy factor and the angle of inclination of prolate irregularities with respect to the external magnetic field. Isolines of the normalized root mean square deviation of the Faraday angle nonlinearly depends on the angle of inclination of prolate irregularities and increases in proportion to the anisotropy factor; two receiving antennas are located in orthogonal planes. It is shown that the broadening of the spatial spectrum of scattered electromagnetic waves

by turbulent magnetized plasma slab in the principle plane (location of an external magnetic field) is less than in the perpendicular one.

1. INTRODUCTION

At the present time the features of light propagation in random media have been studied quite well [1]. Many articles and reviews are related to statistical characteristics of scattered radiation and observations in the ionosphere [2, 3]. The analysis of the statistical properties of small-amplitude electromagnetic waves that have passed through a plane turbulent plasma slab is very important in many practical applications associated with both natural and laboratory plasmas [4, 5]. In most papers isotropic irregularities have been considered. However, in reality, irregularities in the ionosphere are anisotropic and mainly elongated along the geomagnetic field. Investigation of statistical characteristics of scattered radiation in randomly inhomogeneous magnetized plasma is of a great practical importance. Statistical characteristics of the angular power spectrum (broadening and displacement of its maximum), scintillation effects and the angle-of-arrival of scattered electromagnetic waves by turbulent anisotropic magnetized ionospheric plasma slab for both power-law and anisotropic Gaussian correlation functions of electron density fluctuations were investigated analytically in the complex geometrical optics approximation and numerically by statistical simulation using the Monte Carlo method [6–9]. Statistical characteristics of the amplitude and phase of scattered electromagnetic waves by turbulent magnetized plasma slab with electron density and magnetic field fluctuations (both in magnitude and direction) were considered in [10] via the perturbation method taking into account boundary conditions. The phase portraits of correlation functions of the amplitude and phase fluctuations were constructed.

The geomagnetic field plays a key role in the dynamics of the plasma in the ionosphere. In the ionosphere electron gyrofrequency $\sim 1.4\,\mathrm{MHz}$ is much smaller than the typical frequencies used for space communications. For this frequency band the main effect connected with the presence of magnetic field is the Faraday rotation. Faraday effect is quasi-longitudinal propagation of electromagnetic waves and covers almost the entire ray path. Linearly polarized wave in the earth's anisotropic ionosphere with electron density fluctuations generates the ordinary and extraordinary waves and in the presence of an external magnetic field the plane of polarization undergoes a rotation. The characteristic magneto-ionic components are circularly polarized waves with opposite senses of rotation, traveling with slightly

differing phase velocities [5] and recording in the two orthogonally polarized receiving channels. Phase difference is proportional to the rotation angle (Faraday angle) of the polarization plane. Polarization characteristics of scattered radio signals provide important information about physical conditions in the area of localization of the sources and about the medium parameters on the path of wave propagation.

In the approximation of the smooth perturbation method scattering of linearly polarized incident wave radiated by a geostationary satellite at the frequency 136 MHz on random irregularities of electron density fluctuations using the Gaussian and power law spectral models of ionospheric irregularities was considered in [11]. Analytical expression for the variance of the Faraday angle θ_F, caused by the phase shift of scattered ordinary and extraordinary waves at transionospheric propagation has been obtained. A strong dependence of $\langle \theta_F^2 \rangle$ on the spectral index of fluctuating geostationary satellites signals near the humps of equatorial anomaly has been observed. Substantial fluctuations of the Faraday angle θ_F, rotation of the polarization plane were registered in received signals from the radio beacons of the low-orbit Earth's satellites in the frequency band 20–50 MHz.

Variances of the of metric ordinary and extraordinary waves scattered by magnetized plasma slab at different orientation of the receiving antennas, variance of the Faraday angle $\langle \theta_F^2 \rangle$ and also the features of broadening of the spatial spectrum of the scattered radiation are investigated in this paper analytically by the perturbation method and numerically using anisotropic Gaussian fluctuation spectrum.

2. CALCULATION OF THE STATISTICAL CHARACTERISTICS OF SCATTERED RADIATION BY MAGNETIZED PLASMA SLAB

Let's ions frequency satisfies the condition $\omega \gg \Omega_i = eH_0/Mc$, where Ω_i is the ion gyrofrequency, H_0 is the strength of the external magnetic field, M is the mass of an ion, c is the speed of light in vacuum. If $\omega \gg \nu_{eff}$, ν_{eff} is the effective electron collision frequency with the ions and molecules, then conduction current can be neglected and the total current in the medium equals to the displacement current $\mathbf{j} = -eN\mathbf{w}$, $\mathbf{w}$ — velocity of electrons. If the fields have time-harmonic dependence, electric field strength satisfies the equation [10]:

$$
\begin{aligned}
&\operatorname{grad} \operatorname{div} \mathbf{E} - \Delta \mathbf{E} - k_0^2 \mathbf{E} \\
&= -\frac{\tilde{v} k_0^2}{1 - \tilde{u}} \left\{ \mathbf{E} - \frac{ie}{mc\omega}[\mathbf{E} \cdot \mathbf{H}_0] - \left(\frac{e}{mc\omega}\right)^2 (\mathbf{E} \cdot \mathbf{H}_0)\mathbf{H}_0 \right\},
\end{aligned} \quad (1)
$$

where: $u = \Omega_H^2/\omega^2$, $\Omega_H = eH_0/mc$ is the electron gyrofrequency, $\tilde{v} = \omega_p^2/\omega^2$, $\omega_p = (4\pi Ne^2/m)^{1/2}$ is the plasma frequency, sign "$\sim$" means that the electron density is a random variable. Vector of electric induction $\mathbf{D} = \mathbf{E} - 4\pi ieN_0\mathbf{w}/\omega$ is described by the expression:

$$\mathbf{D} = \left(1 - \frac{\tilde{v}}{1-u}\right)\mathbf{E} + i\frac{\tilde{v}}{1-u}\left[\mathbf{E} \cdot \frac{\mathbf{\Omega_H}}{\omega}\right] + \frac{\tilde{v}}{1-u}\left(\mathbf{E} \cdot \frac{\mathbf{\Omega_H}}{\omega}\right)\frac{\mathbf{\Omega_H}}{\omega}. \quad (2)$$

Now we consider second order statistical moments of scattered radiation by turbulent magnetized plasma slab with electron density fluctuations if wave propagates along the external magnetic field. When a wave passes through a region containing irregularities of refractive index both amplitude and phase fluctuations arise in the wave front. Each of the magnitudes in the Equation (1) can be presented as the sum of the mean value and small fluctuating terms. Using the small perturbation method: $\mathbf{E} = \langle\mathbf{E}\rangle + \mathbf{e}$, $N = N_0 + n_1$ (N_0 is constant, fluctuating terms are random functions of the spatial coordinates; the angular brackets indicate the statistical average) we obtain linearized set of stochastic differential equations for fluctuating electric field [10]:

$$\hat{M}_{ij}e_j(\mathbf{r}) = j_i, \quad \hat{M}_{ij} = \nabla \times \nabla \times -\Delta\delta_{ij} - k_0^2\varepsilon_{ij},$$
$$\mathbf{j} = -k_0^2 v(1-u)^{-1}n_1\{\langle\mathbf{E}\rangle - i\sqrt{u}[\langle\mathbf{E}\rangle\tau] - u(\langle\mathbf{E}\rangle\tau)\tau\}.$$

Current density contains electron density fluctuations: $n_1 = n/N_0$, $\tau = \langle\mathbf{H}_0\rangle/|\langle\mathbf{H}_0\rangle|$, and the equation of the scattered field satisfies the boundary conditions: at $z \geq L$ the waves propagating in the negative direction must be absent, and at $z \leq 0$ — in the positive direction. Free space is under and above the plasma slab. If an incident linearly polarized electromagnetic wave propagates along the z axis and the vector τ lies in the yz plane ($\mathbf{k} \parallel z$, $\langle\mathbf{H}_0\rangle \in yz$), using the results of [10] the scattered fields of the ordinary and extraordinary waves in the direction of the y axes causing by the fluctuations of the electron density inside the magnetized plasma slab have the following form:

$$\tilde{e}_{yD}^{(o,e)}(\text{æ}, L) = -\frac{2k_0\Upsilon_0\Upsilon_1}{\delta_1\varepsilon_{zz}}E_{x(o,e)}\exp[i\,k_{(o,e)}z](a_1\gamma_x\gamma_y + ic_1\gamma_x^2 + id_1\gamma_y^2)$$

$$\int_0^L dz' n_1(\text{æ}, z')\sin[(L-z')k_{(o,e)}x_1] - \frac{2k_{(o,e)}\Upsilon_0\Upsilon_1}{\delta_2\varepsilon_{zz}}$$

$$E_{x(o,e)}\exp[ik_{(o,e)}z](a_2\gamma_x\gamma_y + ib_2 + ic_2\,\gamma_x^2 + id_2\gamma_y^2)$$

$$\int_0^L dz' n_1(\text{æ}, z')\sin[(L-z')k_{(o,e)}x_2], \quad (3)$$

where: $\ae = \{k_x, k_y\}$ — transverse wave vector relative to the external magnetic field, $v = \omega_p^2/\omega^2$, $\omega_p = (4\pi N_0 e^2/m)^{1/2}$, $\Upsilon_0 = -v/(1-u)$, $\Upsilon_1 = 1+\sqrt{u}$, $\varepsilon_{zz} = 1-v$, $\varepsilon_{xx} = \varepsilon_{yy} = 1-v/(1-u)$, $\tilde{\varepsilon}_{xy} = v\sqrt{u}/(1-u)$, $a_1 = \zeta_1^2 - \varepsilon_{zz}$, $c_1 = 2\zeta_1\zeta_2\varepsilon_{zz} - \varepsilon_{xx} - \tilde{\varepsilon}_{xy}$, $a_2 = \zeta_3^2 - \varepsilon_{zz}$, $c_2 = 2\zeta_3\zeta_4\varepsilon_{zz} - \varepsilon_{xx} - \tilde{\varepsilon}_{xy}$, $b_2 = 2\varepsilon_{zz}\tilde{\varepsilon}_{xy}$, $d_1 = 2\zeta_1\zeta_2\varepsilon_{zz} - \varepsilon_{zz}$, $d_2 = 2\zeta_3\zeta_4\varepsilon_{zz} - \varepsilon_{zz} - 2\tilde{\varepsilon}_{xy}$, $\delta_2 = -4\zeta_3\tilde{\varepsilon}_{xy}$, $\delta_1 = 4\zeta_1\tilde{\varepsilon}_{xy}$, $\zeta_1 \approx \sqrt{\varepsilon_{xx}}(1 + \tilde{\varepsilon}_{xy}/2\varepsilon_{xx})$, $\zeta_3 \approx \sqrt{\varepsilon_{xx}}(1 - \tilde{\varepsilon}_{xy}/2\varepsilon_{xx})$, $\zeta_2 \approx \frac{\varepsilon_{xx}+\varepsilon_{zz}}{4\varepsilon_{zz}\sqrt{\varepsilon_{xx}}}[1 + \frac{\tilde{\varepsilon}_{xy}}{2(\varepsilon_{xx}+\varepsilon_{zz})}]$, $\zeta_4 \approx \frac{\varepsilon_{xx}+\varepsilon_{zz}}{4\varepsilon_{zz}\sqrt{\varepsilon_{xx}}}[1 - \frac{\tilde{\varepsilon}_{xy}}{2(\varepsilon_{xx}+\varepsilon_{zz})}]$, $x_1 = \zeta_1 - \zeta_2(\gamma_x^2 + \gamma_y^2)$, $x_2 = \zeta_3 - \zeta_4(\gamma_x^2 + \gamma_y^2)$, E_{xo} and E_{xe} are the amplitudes of the linearly polarized ordinary and extraordinary waves (we assume $E_{xo} = E_{xe} \equiv E_x$), L — thickness of a turbulent slab. The plane wave corresponds to the source remote at infinity, which is a good approximation at calculation of the statistical characteristics of fluctuating radio signals radiated from geostationary satellites.

The spectrum and energy of electromagnetic waves propagating in randomly inhomogeneous media are transformed and polarization characteristics of radio waves provide important information of scattered radiation. Therefore it is of interest to consider the effect of rotation of polarization plane. The Faraday angle is defined as the difference of phase fluctuations of the scattered ordinary and extraordinary waves [11]: $\theta_F = (\varphi_1^{(o)} - \varphi_1^{(e)})/2$. For the frequency band of waves radiated by the source exceeding ion gyrofrequency, scattering on irregularities does not lead to the interaction of modes [12].

Phase fluctuation φ_1 of a scattered radiation is determined by the imaginary part of a scattered fluctuating field (3) [8, 10]. Hence, correlation function of scattered ordinary and the extraordinary waves for arbitrary correlation function of electron density fluctuations in general case has the following form

$$\left\langle \varphi_1^{(o)} \varphi_1^{(e)*} \right\rangle_{yD}$$

$$= \frac{v^2(1 + \sqrt{u})^2}{(1-u)^2} E_x^2 \frac{b_2}{\delta_2} \frac{k_0^2 L}{\varepsilon_{zz}^2} \exp(ik_- z) \int\limits_{-\infty}^{\infty} dk_x \int\limits_{-\infty}^{\infty} dk_y$$

$$\exp(ik_x\rho_x + ik_y\rho_y) \int\limits_{-\infty}^{\infty} d\rho_z W_D(k_x, k_y, \rho_z) \cdot \exp(ik_x\rho_x + ik_y\rho_y)$$

$$\left(\frac{1}{\delta_2}(b_2 + c_2\gamma_x^2 + d_2\gamma_y^2) \frac{1}{2x_2k_+L}\{[1 - \cos(2x_2k_+L)] \right.$$

$$\sin(x_2k_-\rho_z) + \sin(2x_2k_+L)\cos(x_2k_-\rho_z)\}$$

$$-\frac{1}{\delta_2}(b_2 + c_2\gamma_x^2 + d_2\gamma_y^2) \cdot \frac{2\sin(x_2k_-L)}{x_2k_-L}[\cos(2x_2k_-L)$$

$$\cos(x_2 k_+ \rho_z) + \sin(2x_2 k_- L)\sin(x_2 k_+ \rho_z)] + \frac{2}{\delta_1}(c_1\gamma_x^2 + d_1\gamma_y^2)$$

$$\cdot\frac{1}{yk_+ L}\left\{[1 - \cos(tk_- L)\cos(yk_+ L)]\cos\left(\frac{t}{2}k_+ \rho_z\right)\sin\left(\frac{y}{2}k_- \rho_z\right)\right.$$

$$+ \cos(tk_- L)\sin(yk_+ L)\cos\left(\frac{t}{2}k_+ \rho_z\right)\cos\left(\frac{y}{2}k_- \rho_z\right)$$

$$+ \sin(tk_- L)\sin(yk_+ L)\sin\left(\frac{t}{2}k_+ \rho_z\right)\cos\left(\frac{y}{2}k_- \rho_z\right)$$

$$\left. - \sin(tk_- L)\cos(yk_+ L)\sin\left(\frac{t}{2}k_+ \rho_z\right)\sin\left(\frac{y}{2}k_- \rho_z\right)\right\}$$

$$- \frac{2}{\delta_1}(c_1\gamma_x^2 + d_1\gamma_y^2)\frac{1}{tk_+ L}$$

$$\left\{[1 - \cos(yk_- L)\cos(tk_+ L)]\cos\left(\frac{y}{2}k_+ \rho_z\right)\sin\left(\frac{t}{2}k_- \rho_z\right)\right.$$

$$+ \cos(yk_- L)\sin(tk_+ L)\cdot\cos\left(\frac{y}{2}k_+ \rho_z\right)\cos\left(\frac{t}{2}k_- \rho_z\right)$$

$$+ \sin(yk_- L)\sin(tk_+ L)\sin\left(\frac{y}{2}k_+ \rho_z\right)\cos\left(\frac{t}{2}k_- \rho_z\right)$$

$$\left.\left.\sin(yk_- L)\cos(tk_+ L)\cdot\sin\left(\frac{y}{2}k_+ \rho_z\right)\sin\left(\frac{t}{2}k_- \rho_z\right)\right\}\right\}, \tag{4}$$

where: $k_\pm = k_0(N_o \pm N_e)/2 = (k_o \pm k_e)/2$, k_o and k_e are the phase constants of the ordinary and extraordinary waves, respectively; $\gamma^2 = \gamma_x^2 + \gamma_y^2$, N_o and N_e are refractive indices of these waves: $N_o^2 = 1 - v(1 - \sqrt{u})^{-1}$, $N_e^2 = 1 - v(1 + \sqrt{u})^{-1}$, $t = x_1 - x_2 = t_1 - t_2\gamma^2$, $q = x_1 + x_2 = q_1 - q_2\gamma^2$.

The variance of the Faraday angle θ_F is determined by the expression

$$\langle\theta_F^2\rangle = \frac{1}{4}\left(\left\langle\phi_1^{(o)2}\right\rangle + \left\langle\varphi_1^{(e)2}\right\rangle - 2\left\langle\varphi_1^{(o)}\varphi_1^{(e)*}\right\rangle\right), \tag{5}$$

which is inversely proportional to the frequency square. This means that in the ionospheric investigations depolarization effect — rotation angle of the polarization plane of scattered radio waves caused by the Faraday Effect is strongly manifested in the long-wave region. Here $\langle\varphi_1^{(o)2}\rangle$ and $\langle\varphi_1^{(e)2}\rangle$ are the phase variances of the ordinary and extraordinary waves which can be easily obtained from the Equation (4).

If $N_o = N_e = 1$ correlation function of the phase (4) is simplified:

$$W_\varphi(\rho_x, \rho_y, L)_D$$
$$\equiv \langle \varphi_1(X + \rho_x, Y + \rho_y, L)\varphi_1^*(X, Y, L)\rangle_{yD}$$
$$= -2L\frac{k_0^2 \Upsilon_0^2 \Upsilon_1^2}{\varepsilon_{zz}^2}\frac{b_2}{\delta_2}\langle E_x\rangle^2 \int\limits_{-\infty}^{\infty} dk_x \int\limits_{-\infty}^{\infty} dk_y \exp(ik_x\rho_x + ik_y\rho_y)$$

$$\cdot \int\limits_{-\infty}^{\infty} d\rho_z W_D(k_x, k_y, \rho_z)$$

$$\left\{\frac{1}{\delta_2}\left[b_2 + \frac{2}{k_0^2}(c_2 k_x^2 + d_2 k_y^2)\right]\left[\frac{\sin(2x_2 k_0 L)}{2x_2 k_0 L} - \cos(x_2 k_0 \rho_z)\right]\right.$$
$$+ \frac{2}{\delta_1 k_0^2}(c_1 k_x^2 + d_1 k_y^2) \cdot \left[\frac{\sin(yk_0 L)}{yk_0 L}\cos\left(\frac{t}{2}k_0\rho_z\right)\right.$$
$$\left.\left. - \frac{\sin(tk_0 L)}{tk_0 L}\cos\left(\frac{y}{2}k_0\rho_z\right)\right]\right\}, \tag{6}$$

where: $R = (L/k_0 l_\parallel^2)$ — wave parameter, $X = \rho_x/l_\parallel$ and $Y = \rho_y/l_\parallel$ are normalized distances between the observation points in the xoz and yoz planes, respectively (two receiving antennas located in the perpendicular planes registering phase difference between the ordinary and extraordinary waves are at different distances from the third antenna, located at the origin, that is $X = Y = 0$). The expression (6) is valid in both near $R \ll 1$ and far $R \gg 1$ zones. Knowledge of the phase correlation function allows us to calculate the broadening of the spatial spectrum in the principle and perpendicular planes [6–8]:

$$\langle k_y^2\rangle = -\partial^2 W_\varphi/\partial\rho_y^2|_{\rho_x=\rho_y=0}, \quad \langle k_x^2\rangle == -\partial^2 W_\varphi/\partial\rho_x^2|_{\rho_x=\rho_y=0}. \tag{7}$$

Spatial spectrum has a great practical importance and is the Fourier transformed correlation function of the scattered field. This statistical characteristic of the scattered radiation is equivalent to the ray intensity (brightness), which usually enters in the radiation transport equation.

3. CORRELATION FUNCTIONS OF ELECTRON DENSITY FLUCTUATION AND THE PHASE

It is shown [4] that small-scale (< 200 m) irregularities with the Gaussian spectrum are responsible for the polarization fluctuations at the frequencies of 20–50 MHz. Two-dimensional spectral density function which describes the irregularities in random medium depends

on the particular case and may differ from medium to medium. The most widely used spectral density function is the Gaussian, which has certain mathematical advantages. In case of assumption of forward scattering, $\langle n_1^2 \rangle k_0 L \ll 1 \ll k_0 l_D$, if the single scattering condition is fulfilled $\langle n_1^2 \rangle k_0^2 l_D L \ll 1$ a medium is characterized by the Gaussian irregularity spectrum. On the lower boundary of inhomogeneous slab having thickness $100\,\mathrm{km}$, locating at the heights from 300 up to $500\,\mathrm{km}$ it is easy to show that the these conditions are satisfied for the electromagnetic waves with a frequency of several tens of MHz and higher. Therefore in the analytical calculations we use anisotropic Gaussian correlation function of electron density fluctuation [13]:

$$W_D(k_x, k_y, \rho_z) = \frac{\sigma_N^2}{4\pi} \frac{l_\parallel^2}{\chi \Gamma_0} \exp\left(-\frac{m^2}{l_\parallel^2}\rho_z^2 + \in k_x\rho_z\right)\exp\left(-\frac{k_x^2 l_\parallel^2}{4\Gamma_0^2} - \frac{k_y^2 l_\parallel^2}{4\chi^2}\right), \quad (8)$$

where: $m^2 = \chi^2/\Gamma_0^2$, $\Gamma_0^2 = \sin^2\gamma_0 + \chi^2\cos^2\gamma_0$, $n = (\chi^2 - 1)\sin\gamma_0\cos\gamma_0/\Gamma_0^2$, σ_N^2 — variance of the electron density fluctuations. The average shape of electron density irregularities has the form of elongate ellipsoid of rotation. The ellipsoid is characterized with two parameters: the anisotropy factor $\chi = l_\parallel/l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities with respect to the external magnetic field) and orientation equaling to the inclination angle of the rotation axis with respect to the magnetic field (sometimes with respect to horizon) γ_0. Anisotropy of the shape of irregularities is connected with the difference of diffusion coefficients in the field align and field perpendicular directions.

Using Equations (4), (5) and (8) we can obtain the variance of the Faraday angle $\langle \theta_F^2 \rangle$, and also investigate the behavior of the correlation function of phase fluctuation and broadening of the spatial spectrum of scattered electromagnetic waves.

4. NUMERICAL CALCULATIONS

Numerical calculations of statistical characteristics of scattered radiation are carried out for anisotropic Gaussian correlation function (8) of electron density fluctuation. Frequency of a linearly-polarized incident electromagnetic wave is $0.1\,\mathrm{MHz}$. The mean ionospheric height is taken as $300\,\mathrm{km}$, plasma parameters $u = 0.22$ and $v = 0.28$.

For example, spaced-receiver experiments were undertaken during 1968 and 1969 at Battersea ($41°09'\,\mathrm{S}, 184°35'\,\mathrm{W}$) and Campbell Island ($52°339\,\mathrm{S}, 190°535\,\mathrm{W}$). The $40\,\mathrm{MHz}$ transmission from ionospheric beacon satellite BE-B were received at each station on two circularly-polarized turnstile aerials separated by about $0.7\,\mathrm{km}$ along a baseline

parallel to the sub-satellite track [14]. Values of χ have been calculated. A mid-latitude axial ratio of approximately 2 and a sub-auroral axial ratio of 10 are typical. The Battersea size histogram peaks between 0.4 and 0.5 km with $\chi = 2$ and most irregularity sizes occur in the range 0.2–0.7 km. The Campbell Island size histogram peaks between 0.1 and 0.2 km with $\chi = 10$ and most irregularity sizes occur in the range of 0.05–0.35 km. The satellite perigee was 300 km.
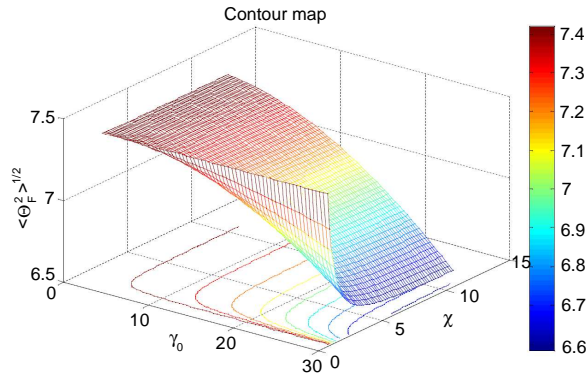
Figure 1. Three-dimensional pattern of the variance of the Faraday angle normalized on the dispersion of electron density fluctuations versus anisotropy factor χ and the angle of inclination γ_0 of prolate irregularities with respect to the external magnetic field.

Figure 1 illustrates spatial pattern of the root mean square deviation of the Faraday angle normalizing on $\sigma_N = (\langle N_1^2 \rangle / N_0)^{1/2}$ versus anisotropy factor χ and γ_0 at $X = Y = 0$ (one receiving antenna, located at the origin). Characteristic linear scale of irregularities of electron density fluctuations equals to $l_\parallel = 1$ km and $l_\parallel = 2$ km. The dependence of the function $\langle \theta_F^2 \rangle^{1/2} / \sigma_N$ versus anisotropy factor is nonlinear at big angles γ_0 and exponentially decreases in proportion to the parameter χ. At $\gamma_0 = 20°$, $\chi = 1$ and $\chi = 10$, this function equals to $7.42°$ and $6.56°$, respectively, which is in agreement with [11]. According to this paper, for small-scale irregularities with Gaussian spectrum responsible for the polarization fluctuations at the frequencies of 20–50 MHz, the value $\langle \theta_F^2 \rangle^{1/2} / \sigma_N$ varies in the range $10^{-1} - 10^2$ radians for linear scales of irregularities $0.2 < l < 1$ km. The curves were constructed [11] in accordance to the observations on the Ascension Island near the humps of equatorial anomaly at the frequency of the received signal from the geostationary satellite 136 MHz, using also power law spectral model of the spectrum of
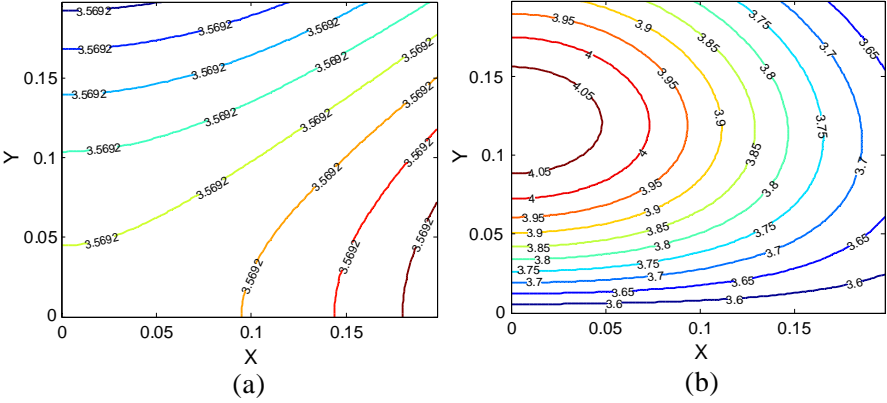
Figure 2. Isolines of the normalized root mean square deviation of the Faraday angle $\langle\theta_F^2\rangle^{1/2}/\sigma_N$ versus normalized distance between the receiving equipments locating in the orthogonal planes at $l_\parallel = 1\,\text{km}$; (a) $\chi = 1$, $\gamma_0 = 0°$. (b) $\chi = 5$, $\gamma_0 = 5°$.

ionospheric irregularities. Thickness and height of the inhomogeneous slab were equal to $L = 200\,\text{km}$, $z_0 = 350\,\text{km}$, respectively. Analysis showed that the moderate fluctuations of the Faraday angle $\langle\theta_F^2\rangle^{1/2} \approx 0.05$ radian can be caused by both small-scale ($< 200\,\text{m}$) and larger $l \approx 1\,\text{km}$ irregularities.

Figures 2(a) and (b). Isolines show nonlinear dependence of $\langle\theta_F^2\rangle^{1/2}/\sigma_N$ on the angle γ_0 when two antennas are located in the orthogonal planes. This function increases from $\sim 4°$ up to $9°$–$10°$ in proportion of the anisotropy factor χ. If characteristic spatial scale of electron density fluctuation is $l_\parallel \approx 1\,\text{km}$, thickness of the turbulent magnetized plasma slab is about $100\,\text{km}$, $\xi = 2.8$, $k_0 L = 280$, $R = 36$ ($0.1\,\text{MHz}$) (Figure 4), the normalized correlation function oscillates at small distances ($Y < 1$) and asymptotically tends to zero.

Figure 5 shows the broadening of the spatial spectrum of scattered electromagnetic waves in the principle plane for the far zone ($R \gg 1$, $\xi = 2.8$), using the perturbation method applying Equations (6)–(8). Analyses show, that the broadening of the spatial spectrum in the main plane (the location of the external magnetic field) is less than in perpendicular plane. This is in agreement with the conclusions [8], where similar calculations were carried out in the complex geometrical optics approximation. In the near zone ($R \ll 1$) at $X = Y = 0$ and in isotropic case ($\chi = 1$) from (6) we obtain the well-known expression for the phase variance: $\langle\varphi_1^2\rangle = \sqrt{\pi}\sigma_N^2 k_0^2 L l v^2/4$ [4].
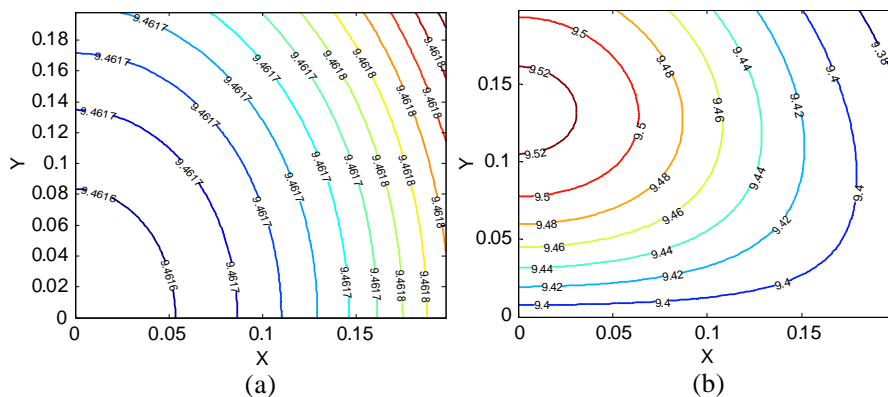
Figure 3. The as in Figure 2, at $l_\parallel = 2\,\mathrm{km}$; (a) $\chi = 1$, $\gamma_0 = 0°$. (b) $\chi = 5$, $\gamma_0 = 5°$.
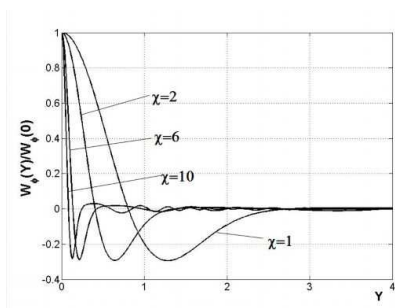

Figure 4. Normalized correlation function of the phase fluctuations as a function of parameter $Y = \rho_y/l_\parallel$ for different anisotropy factor $l \leq \chi \leq 10$ and $\gamma_0 = 0°$.
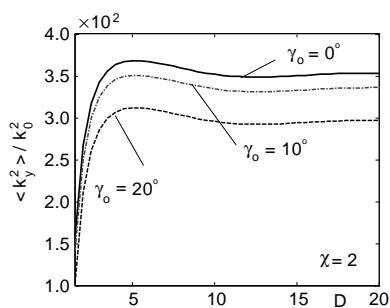
Figure 5. Normalized broadening of the spatial spectrum of scattered radiation in the principle plane $(X = 0)$ as a function of nondimensional parameter $D = L/l_\parallel$; $\chi = 2$ and $0° \leq \gamma_0 \leq 20°$.

5. CONCLUSION

Statistical characteristics of scattered ordinary and extraordinary waves by the magnetized plasma slab with electron density fluctuations are investigated by the perturbation method. Variances of the phase fluctuation of these waves and the mutual correlation function of phase fluctuations of the ordinary and extraordinary waves for the arbitrary correlation function of electron density fluctuations are derived using the perturbation method. The obtained results are valid for the near and far zones from turbulent plasma slab boundaries. The

3D pattern of the normalized root mean square deviation of the Faraday angle versus the anisotropy factor and the inclination angle of elongated irregularities with respect to the external geomagnetic field for the anisotropic Gaussian fluctuation spectrum of the electron density fluctuation is constructed. Isolines of the normalized root mean square deviation of the Faraday angle nonlinearly depends on the angle of inclination of prolate irregularities and increases in proportion to the anisotropy factor; two receiving antennas are located in orthogonal planes. Analytical expressions of the correlation function of phase fluctuation of a scattered radiation and the broadening of the spatial spectrum of scattered electromagnetic waves for the anisotropic Gaussian fluctuation spectrum are obtained. It is shown that the broadening of the spatial spectrum in the principle plane (the location of an external magnetic field) is less than the broadening of the spectrum in the perpendicular plane.

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