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Derivation of Correlation Coefficient Formula for Determination of Doppler Angle Using Time Domain Correlation Ultrasonic Flowmeter

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Calculation of the Doppler angle from the correlation coefficient of two aligned windowed data sets of two echoes in the time domain correlation method has been demonstrated. By assuming ideal sound and velocity fields, it is shown that the Doppler angle can be obtained from the statistics of the reflected ultrasonic signals in time domain correlation ultrasonic flowmeter. The resolution cell is chosen to be much longer than the impulse response of the system in order that the signals reflected from the scatterers preceding and following the resolution cell will be sufficiently small that they do not contribute to the resolution cell. Hence, the truncation effects of the resolution cell on the nearby scatterers may be ignored. The mathematical derivation of correlation coefficient is presented clearly. The measurement method of Doppler angle is given from the position of two aligned resolution cells.

KEY WORDS: Time domain correlation; doppler angle; correlation coefficient; ultrasonic flowmeter; resolution cell.

INTRODUCTION

The measurement of blood flow velocity by ultrasonic techniques provides valuable information for clinical diagnosis of vascular disease. Two ultrasonic Doppler techniques that are most widely used for blood flow velocity measurements are continuous wave Doppler and pulsed wave Doppler.(1) Since the Doppler shift of high frequency ultrasound is difficult to assess with useful accuracy, Doppler based ultrasonic blood flow measurement techniques intrinsically have some practical as well as theoretical difficulties in yielding accurate and precise flow velocity estimates.(2,3) Time domain processing of Doppler signal can overcome the above problems.

A third flow velocity measurement technique, which performs a time domain operation on samples of the reflected ultrasonic signal for estimating the flow velocity has been introduced.(4-7) The technique is called *time domain correlation*

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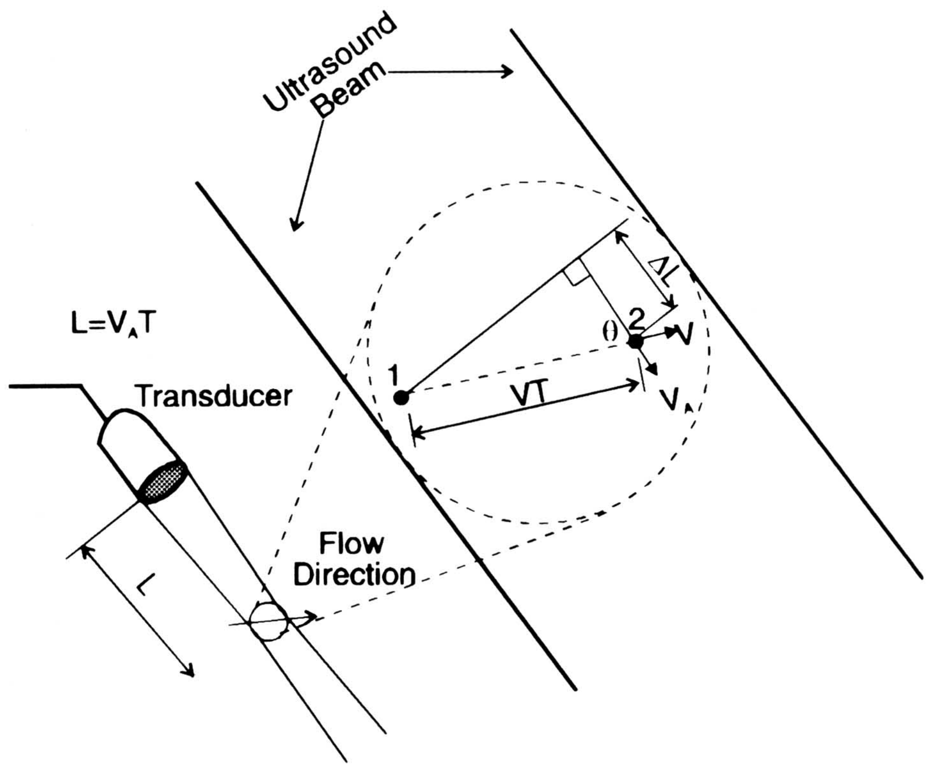

Fig. 1. A schematic representation of the essential principle of the time domain correlation ultrasonic flow velocity measurement method.

method since it correlates two successive echoes and uses interpulse time interval for flow velocity estimation. The technique was discussed in detail in previous papers[4,8] therefore, only basic properties and formulae will be presented here.

The essential principle of the time domain correlation flow velocity measurement technique is to track scatterers in a resolution cell within a pulsed ultrasound beam and assess the distance travelled by the time shift between two echoes. When performing time domain correlation method, two sequential ultrasonic pulses separated by a time T are transmitted and two reflected echoes are received. The two echoes are amplified with a radio frequency (rf) amplifier and digitized for computer processing. The signal processing is carried out at rf frequencies, so there is no demodulation. A windowed part of the first echo is shifted over the second echo so as to obtain maximum similarity between the two echoes. The degree of similarity is assessed from the correlation of the windowed sections of the echoes. The amount of time shift at which the maximum correlation is obtained is directly proportional to the axial velocity of the scatterers moving in the resolution cell which lies within the ultrasonic beam.

Shown in Fig. 1 is the schematic representation of the time domain flow velocity measurement technique. The time domain correlation method may be considered in the following way for a point scatterer: An ultrasonic pulse is transmitted at time $t = t_0$ and the echo from the scatterer at position 1, $e_1(t)$, is received at time $t = t_0 + t_1$. After a time amount of T, in which the scatterer at position 1 has moved to the position 2, a second ultrasonic pulse is transmitted at $t = t_0 + T$, and the second echo $e_2(t)$ is received at time $t = t_0 + T + t_2$, where T is the time difference between the transmission of the two ultrasonic pulses. The axial distance that the scatterer has moved in time T is ΔL where $\Delta L = V_A T = VT \cos\theta$. The time shift between the two echoes, t_s, where $t_s = t_2 - t_1$ is also the value which maximizes the correlation between the two reflected ultrasonic echoes. The axial component of the flow velocity is then found as

$$V_A = V\cos\theta = \frac{c(t_2 - t_1)}{2T} = \frac{ct_s}{2T}, \tag{1}$$

where c represents the speed of the ultrasonic signal in the medium, t_s represents the amount of time shift with maximum correlation and θ represents the Doppler angle. If the Doppler angle is known, then

$$V_A = \frac{V_A}{\cos\theta} = \frac{ct_s}{2T\cos\theta}. \tag{2}$$

Next, the flow velocity for multiple scatterers is found in the same way:

$$V_A = \frac{cs_{max}}{2T\cos\theta}, \tag{3}$$

where s_{max} represents the time shift which corresponds to the maximum correlation between the resolution cells of the two ultrasonic echoes. The correlation function between the transmitted pulses is defined as

$$R(u) = \int_{-\infty}^{\infty} e_1(t)e_2(t+u)dt. \tag{4}$$

Readers interested in details are referred to the references.[4,5,8,9]

In this work, the determination of the Doppler angle using the time domain correlation ultrasonic flowmeter is presented. The correlation coefficient is derived and a measurement model is introduced.

DEFINITION OF THE MEASUREMENT ANGLE

One of the data parameters that can be obtained by performing the time domain correlation method is the correlation coefficient of the aligned resolution cells. Since the Doppler angle can be calculated from this correlation coefficient, it can be obtained from the statistics of the reflected ultrasonic echoes by assuming ideal sound and velocity fields. An ideal sound beam is one in which the lateral intensity distribution at all po-

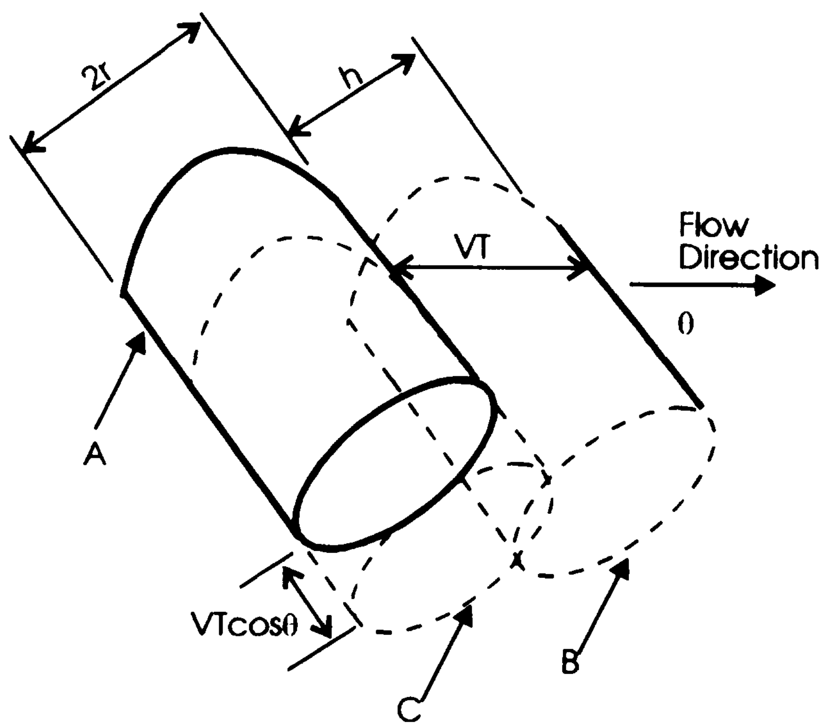

Fig. 2. Alignment geometry of the resolution cells from two ultrasound echo signals.

sitions in the focal region is a uniform intensity cylinder within the beam width and zero intensity elsewhere. It is assumed that the phase fronts inside of the beamwidth are planar and equally spaced. An ideal velocity field is one in which the velocity throughout the entire ultrasound field is uniform and the same. The resolution cell is chosen to be much longer than the impulse response of the system in order that the signals reflected from the scatterers preceding and following the resolution cell will be sufficiently small that they do not contribute to the resolution cell signal. Hence, the truncation effects of the resolution cell on the nearby scatterers may be ignored.[4,5]

In order to calculate the Doppler angle from the correlation coefficient, a detailed analysis of the alignment procedure is needed. The scatterers of the first resolution cell (cylinder A in Fig. 2) have moved the distance VT (cylinder B in Fig. 2) during time T. The first resolution cell (cylinder A) is aligned with the second resolution cell (cylinder C) by the correlation method. The first resolution cell is shifted along the beam axis until the scatterers of the second resolution cell align in range with the scatterers of the first resolution cell. The scatterers that are not common to the two resolution cells have no effect on the alignment procedure since they are uncorrelated.

For two cylindrical resolution cells each containing M scatterers and sharing N scatterers, the correlation coefficient is directly proportional to N, the number of scatterers that are common to both cells. A convenient form of the correlation coefficient is then found by scaling it with the maximum value it can achieve ($N = M$). The correlation coefficient is then found as

$$\rho = \frac{N}{M} \tag{5}$$

The above equation sets the correlation coefficient equal to the fraction of the scatterers remaining in the second echo that were in the first echo. Hence, for two cylindrical resolution cells each having a radius of r and unit height, the correlation coefficient is equal to the intersectional volume that is common to the two cylinders, which is given by[4]

$$\rho\left(\frac{h}{2r}\right) = \frac{2}{\pi}\left[\arctan\frac{\sqrt{1-\frac{h^2}{4r^2}}}{\frac{h}{2r}} - \frac{h}{2r}\sqrt{1-\frac{h^2}{4r^2}}\right] \tag{6}$$

This equation can be simplified by letting $z = (h/2r)$

$$\rho(z) = \frac{2}{\pi}\left[\arctan\frac{\sqrt{1-z^2}}{z} - z\sqrt{1-z^2}\right] \tag{7}$$

The derivation of Eq. (5) is given in Appendix.

Figure 3 shows the graphical representation of the correlation coefficient as a function of the distance between the axes of the two cylindrical resolution cells. An ideal circular field pattern is assumed in this figure.

PRACTICAL METHOD

Two data parameters are utilized when performing the time domain correlation method: (1) The distance that the aligned resolution cell of the second echo has moved relative to the resolution cell of the first echo, (2) the correlation coefficient of the two resolution cells. From Fig. 3, the equivalent lateral shift of the scatterers of the resolution cell of the first echo with the location of the same scatterers at a later time T can be extrapolated based on the correlation coefficient yielding $VT\sin\theta$. Since $V\cos\theta$ is known from Eq. 2, an expression for $\tan\theta$ may be derived as follows:

$$V\cos\theta = \frac{cs_{max}}{2T} \tag{8}$$

$$\frac{VT\sin\theta}{2r} = \frac{h}{2r} = z \tag{9}$$

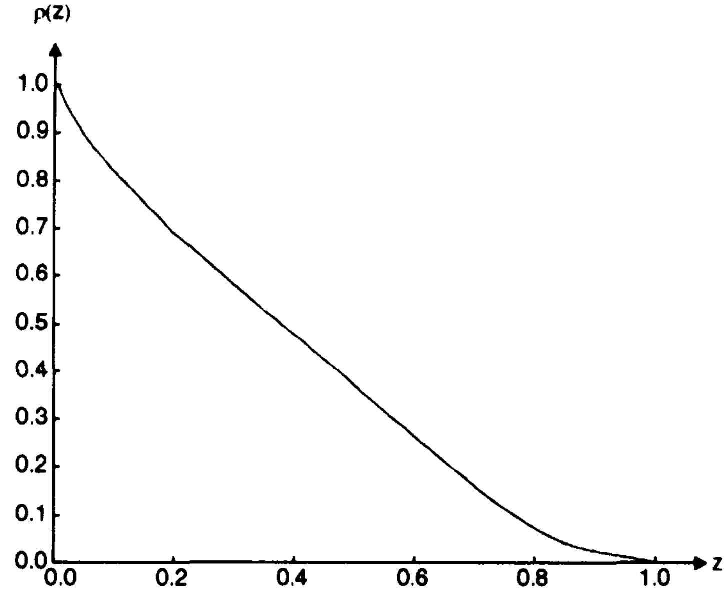

Fig. 3. Graph showing the fractional common volume r of two overlapping cylinders of radius r and axial separation h where $z = h/2r$.

$$\tan\theta = \frac{2h}{cs_{max}} = \frac{4zr}{cs_{max}}, \quad (10)$$

where h is the distance between the axes of the two resolution cells, r is the radius of the ultrasonic beam.

For example, if a correlation coefficient r of 0.5 is measured, a value of 0.39 for z may be found by using Fig. 3. Then, the Doppler angle may be calculated by using Eq. 8. Since, in practice, transducers usually have noncylindrical resolution cells and different beam patterns from the ideal one, Fig. 3 will differ for each transducer. Therefore, in order to implement this method in practice, a lookup table similar to the one shown in Fig. 3 should be built for a particular transducer.

DISCUSSION AND CONCLUSIONS

The calculation of the Doppler angle from the correlation coefficient of two ultrasonic resolution cells and a practical method for determining the Doppler angle are presented. Since the method discussed here is developed under the assumptions that the sound and velocity fields are ideal, there is still a little uncertainty in the determination of the Doppler angle.

Since there are some limitations of the correlation method, the accurate and precise measurement of Doppler angle is not possible exactly. These limitations are of windowing of the echoes, of the variation in velocity of the scattering medium within the resolution cell, and of ultrasound transducer beamwidth. All of the above

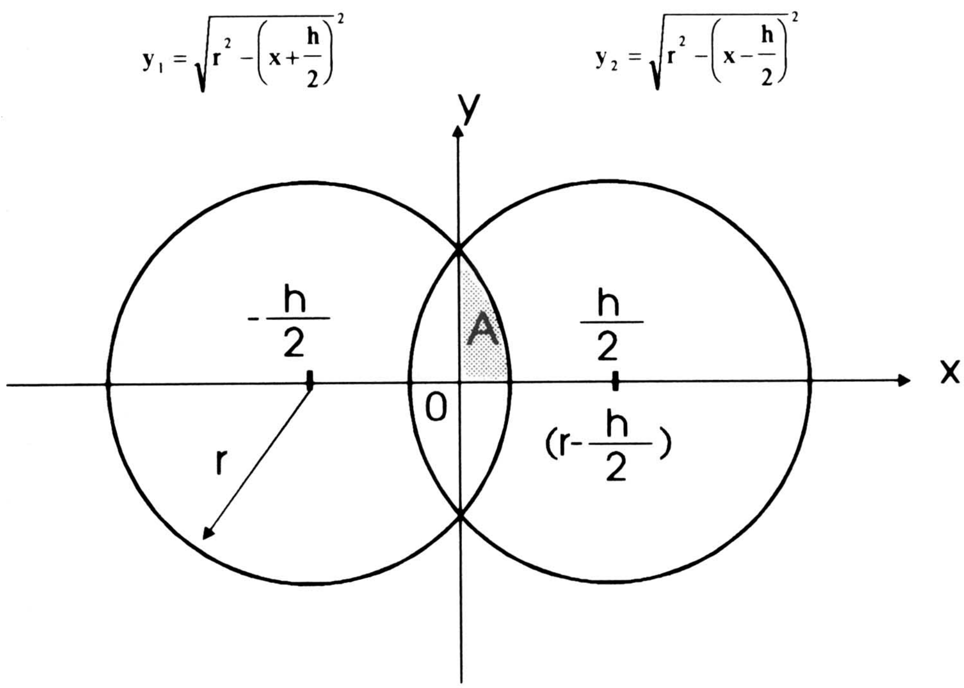

Fig. 4. Top view of two aligned cylindrical resolution cells of radius *r* and axial separation *h*.

processes contribute to the measurement of Doppler angle. In the windowing process, it is not possible to find the exact amount of shift, so this arises an error even if the two echoes are identical. On the other hand, since the scatterers within a resolution cell are at different velocities, this introduces another error. In the case of turbulent flow, this is a serious problem that prevents the correct measurement of the Doppler angle. The beamwidth of ultrasound transducer is not necessarily uniform at the measurement site, so this causes an error. All of these errors can be reduced by the optimal system design.

Additional research should be directed to finding optimal values for transducer beamwidth and correlation length for a given vessel. Since the correlation coefficient is the key factor in determining the Doppler angle, further research should be done on the dependence of the correlation coefficient on the vessel diameters approaching the resolution cell size. On the other hand, the resolution of the system should be increased while the quantization errors should be decreased.

APPENDIX: DERIVATION OF CORRELATION COEFFICIENT FORMULA

Figure 4 shows the two aligned cylindrical resolution cells of radius *r* and axial separation *h*. The correlation coefficient is directly proportional to the intersectional volume shared by the two cylinders. Since the two cylinders are of unit height, the intersectional volume, in other words the correlation coefficient, is equal to 4A where A is the shaded area in Fig. 4.

From Fig. 4, a formula for A can be obtained as follows:

$$A = \int_0^{r-\frac{h}{2}} y_1 dx = \int_0^{r-\frac{h}{2}} \sqrt{r^2 - \left(x + \frac{h}{2}\right)^2} dx$$

$$x + \frac{h}{2} = u \Rightarrow dx = du$$

$$x = 0 \Rightarrow u = \frac{h}{2}; x = r - \frac{h}{2} \Rightarrow u = r$$

$$A = \int_{\frac{h}{2}}^{2} \sqrt{r^2 - u^2} dx$$

By letting

$$u = r \sin\theta \Rightarrow du = r \cos t \cdot dt$$

$$u = \frac{h}{2} \Rightarrow t = \arcsin\frac{h}{2r}; u = r \Rightarrow t = \frac{\pi}{2}.$$

Hence,

$$A = \int_{\arcsin\frac{h}{2r}}^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cdot r \cos t \cdot dt$$

$$A = r^2 \int_{\arcsin\frac{h}{2r}}^{\frac{\pi}{2}} \cos^2 t \cdot dt$$

$$A = \frac{r^2}{2} \int_{\arcsin\frac{h}{2r}}^{\frac{\pi}{2}} (1 + \cos t) \cdot dt$$

$$A\frac{r^2}{2}\left[t+\frac{\sin 2t}{2}\right]_{\arcsin\frac{h}{2r}}^{\frac{\pi}{2}}$$

$$A=\frac{r^2}{2}[t+\sin t\cdot\cos t]_{\arcsin\frac{h}{2r}}^{\frac{\pi}{2}}$$

$$A=\frac{r^2}{2}\left[\frac{\pi}{2}+\arcsin\left(\frac{h}{2r}\right)-\sin\left(\arcsin\left(\frac{h}{2r}\right)\right)\cos\left(\arcsin\left(\frac{h}{2r}\right)\right)\right]$$

$$A=\frac{r^2}{2}\left[\frac{\pi}{2}+\arcsin\left(\frac{h}{2r}\right)-\frac{h}{2r}\cos\left(\arcsin\left(\frac{h}{2r}\right)\right)\right] \quad (A.1)$$

Suppose that

$$w=\arcsin\left(\frac{h}{2r}\right),\text{ then }\sin w=\frac{h}{2r}$$

From the trigonometric identity,

$$\cos w=\sqrt{1-\sin^2 w}\;=\;\sqrt{1-\frac{h^2}{4r^2}} \quad (A.2)$$

$$A=\frac{r^2}{2}\left[\frac{\pi}{2}-\arcsin\left(\frac{h}{2r}\right)-\frac{h}{2r}\sqrt{1-\frac{h^2}{4r^2}}\right]$$

ρ = 4A (From Fig. 4)

$$\rho=2r^2\left[\frac{\pi}{2}-\arcsin\left(\frac{h}{2r}\right)-\frac{h}{2r}\sqrt{1-\frac{h^2}{4r^2}}\right] \quad (A.3)$$

On the other hand, suppose that

$$w=\arcsin\left(\frac{h}{2r}\right),\text{ then }\tan w=\sqrt{1-\frac{h^2}{4r^2}},\quad\text{and}$$

$$w = \arctan w = \left(\frac{\frac{h}{2r}}{\sqrt{1-\frac{h^2}{4r^2}}} \right) \tag{A.4}$$

when (A.4) is substituted in (A.3)

$$\rho = 2r^2 \left[\frac{\pi}{2} - \arctan\left(\frac{\frac{h}{2r}}{\sqrt{1-\left(\frac{h}{2r}\right)^2}} \right) - \frac{h}{2r}\sqrt{1-\left(\frac{h}{2r}\right)^2} \right] \tag{A.5}$$

Suppose that,

$$\arctan y_1 = \frac{\pi}{2} \text{ then } y_1 = \infty$$

$$\arctan y_1 - \arctan y_2 = \arctan\left(\frac{y_1 - y_2}{1 + y_1 y_2} \right) = \arctan\left(\frac{1 - \frac{y_1}{y_2}}{y_2 + \frac{1}{y_1}} \right) \tag{A.6}$$

When

$$y_1 \Rightarrow \infty \arctan y_1 \Rightarrow \frac{\pi}{2},$$

so Eq. (A.6) becomes

$$\frac{\pi}{2} - \arctan y_2 = \arctan\left(\frac{\pi}{y_2} \right), \tag{A.7}$$

In Eq. (A.7),

$$y_2 = \frac{\frac{h}{2r}}{\sqrt{1-\left(\frac{h}{2r}\right)^2}}$$

so, the Eq. (A. 7) becomes

$$\frac{\pi}{2} - \arctan\left(\frac{\frac{h}{2r}}{\sqrt{1-\left(\frac{h}{2r}\right)^2}}\right) = \arctan\left(\frac{\sqrt{1-\left(\frac{h}{2r}\right)^2}}{\frac{h}{2r}}\right) \tag{A.8}$$

When (A.8) is substituted in (A.5)

$$\rho = 2r^2\left[\arctan\left(\frac{\sqrt{1-\left(\frac{h}{2r}\right)^2}}{\frac{h}{2r}}\right) - \frac{h}{2r}\sqrt{1-\left(\frac{h}{2r}\right)^2}\right] \tag{A.9}$$

On the other hand, since each cylinder is unit volume and height, we can state

$$V = \pi r^2 \cdot 1 = 1$$

then

$$2\pi r^2 = 2 \Rightarrow 2r^2 = \frac{2}{\pi}$$

Assume $z = (h/2r)$ and $2r^2 = (2/\pi)$, then (A.9) becomes

$$\rho(z) = \frac{2}{\pi}\left[\arctan\left(\frac{\sqrt{1-z^2}}{z}\right) - z\sqrt{1-z^2}\right]$$

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