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Derivation of Random Phase Approximation on the Basis of the Generalized Schwinger Representation of the Fermion-Pairs

Yutaka MIZOBUCHI, Seiya NISHIYAMA*) and Masatoshi YAMAMURA
Department of Physics, Kyoto University, Kyoto 606

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A derivation of the random phase approximation is given, which is based on the generalized Schwinger representation of the fermion-pairs. The method is made of two stages: First, the truncation of the boson space is done, and secondly comes the separation of boson operators into static and fluctuating parts. The pairing vibration in the super phase and the so-called quadrupole oscillation are treated by the method, which leads to the same results as those of RPA. Then the relations of these bosons to ones appearing in RPA are clarified.

§ 1. Introduction

One of the standard approaches to the theory of nuclear collective motion is the time dependent Hartree-Bogoliubov (TDHB) theory. Recently this theory seems to have been revived with the purpose of describing large amplitude collective motion.[1] According to the TDHB theory, motion of the system under consideration can be described in terms of the time-dependence of the generalized density matrix. Therefore, if we parametrize the density matrix with the help of $A_{\alpha i}$ and $B_{\alpha i}$, the coefficients of the generalized Bogoliubov transformation, the investigation of the time-dependence of $A_{\alpha i}$ and $B_{\alpha i}$ becomes important. If $A_{\alpha i}$ and $B_{\alpha i}$ can be regarded as boson operators, we can take into account the quantum fluctuations. Then we can obtain the quantized TDHB theory.

Along this line, the present authors (S. N. and M. Y.) have recently proposed a quantized TDHB theory (hereafter, referred to as (I)).[2] According to (I), the idea mentioned above can be completely established. An interesting point is that theory (I) is a natural generalization of the Schwinger representation of the quasi-spin[3] to the fermion-pair algebra. In the Schwinger representation, the quasi-spin operators are expressed in terms of the bilinear forms with respect to two kinds of bosons. In our case, the quantized $A_{\alpha i}$ and $B_{\alpha i}$ are regarded as natural extensions of the Schwinger bosons, and the fermion-pair operators can be completely expressed in terms of the bilinear forms of the bosons.

One of the simplest applications of the TDHB theory is the well-known random phase approximation (RPA). We know that, in RPA, boson operators appear as the result of the quantization. In many cases of analysing the an-

*) Present address: Department of Physics, Kochi University, Kochi 780.

harmonic effects, the conventional boson expansion theory based on the RPA bosons has been adopted. For the sake of the convergence problem, the results are reliable only for the case of small amplitudes.[4)] On the other hand, theory (I) does not contain the convergence problem, because the fermion-pairs can be expressed, as was already mentioned, in terms of the bilinear forms of the bosons. Therefore, we can expect that theory (I) is powerful even in the case of the large amplitudes. However, if in our framework we could not reproduce the results based on the conventional boson expansion theory for the case of the small amplitudes, theory (I) would be powerless for the practical applications in spite of the above-mentioned expectation. Therefore, it becomes an important problem to derive RPA in the framework of (I), that is, to connect our bosons with the ones appearing in RPA.

The main aim of the present paper is to show how the derivation of the conventional RPA is possible. For the practical purpose, we treat the case of sufficiently weak quadrupole force compared with pairing interaction. In the conventional approach, the system fluctuates around certain static values of the density matrix which can be determined by the Bogoliubov transformation and the fluctuations can be expressed in terms of the quasi-particle pairs. We adopt a similar picture. Our system fluctuates around certain static values with respect to the bosons introduced in (I). Then, the fluctuations can be taken into account in terms of the differences between the bosons and the static values. We can obtain the bosons used in RPA by making certain linear combinations of the fluctuations. This is our basic idea.

In the next section, we recapitulate the main results of (I) under the spherical tensor representation. Section 3 is devoted to giving the truncated forms of the generalized Schwinger representation which are suitable for our system already mentioned. In § 4, the separation of the boson operators into the static and the fluctuating parts is done. We derive RPA in §§ 5 and 6 for the cases of the pairing vibration in the superconducting phase and the so-called quadrupole oscillation, respectively. Finally some future problems are mentioned briefly as the concluding remarks. In the Appendix, we give a detailed interpretation of the kinematical constraints which govern our bosons.

§ 2. Spherical tensor representation

As mentioned in § 1, the main purpose of the present paper is to demonstrate how the conventional RPA can be constructed in the framework of the generalized Schwinger representation of the fermion-pair algebra. In this paper, we treat the cases of the pairing vibration in the superconducting phase and the so-called quadrupole oscillation in spherical nuclei. The first task for this purpose is to recapitulate the main results of (I) under the spherical tensor representation.

The essential idea of (I) is to construct the same algebra as that of the fermion-pairs ($so(2n)$) with the help of two kinds of bosons $\boldsymbol{A}^+_{\alpha i}$ and $\boldsymbol{B}^+_{\alpha i}$, which satisfy the following commutation relations:

$$\left.\begin{array}{l}[\boldsymbol{A}_{\alpha i}, \boldsymbol{A}^+_{\beta j}]_- = [\boldsymbol{B}_{\alpha i}, \boldsymbol{B}^+_{\beta j}]_- = \delta_{\alpha\beta}\delta_{ij}\,, \\ [\text{the other combinations}]_- = 0\,. \end{array}\right\} \tag{2·1}$$

Here the Greek subscript α denotes the set of the quantum numbers specifying the single-particle state $(n_a l_a j_a m_a)$. The Latin subscript i is the set of quantum numbers which can be obtained by an arbitrary unitary transformation from the original one. We assume that, among all levels under consideration, a given combination of $a(=(n_a l_a j_a))$-value occurs only once. It was mentioned in (I) that these bosons appear as the result of quantization of the TDHB theory. According to (I), the following set of the boson-pairs $\boldsymbol{E}(\boldsymbol{E}^\alpha{}_\beta, \boldsymbol{E}^{\alpha\beta}, \boldsymbol{E}_{\alpha\beta})$ obeys the same algebra as that of the fermion-pairs $\widehat{E}(\widehat{E}^\alpha{}_\beta, \widehat{E}^{\alpha\beta}, \widehat{E}_{\alpha\beta})$:

$$\left.\begin{array}{l}\boldsymbol{E}^\alpha{}_\beta = \sum_i (\boldsymbol{B}^+_{\alpha i}\boldsymbol{B}_{\beta i} - s_{\bar\beta}\boldsymbol{A}^+_{\bar\beta i} s_{\bar\alpha}\boldsymbol{A}_{\bar\alpha i})\,, \\ \boldsymbol{E}^{\alpha\beta} = \sum_i (\boldsymbol{B}^+_{\alpha i} s_{\bar\beta}\boldsymbol{A}_{\bar\beta i} - \boldsymbol{B}^+_{\beta i} s_{\bar\alpha}\boldsymbol{A}_{\bar\alpha i})\,, \\ \boldsymbol{E}_{\alpha\beta} = \sum_i (s_{\bar\alpha}\boldsymbol{A}^+_{\bar\alpha i}\boldsymbol{B}_{\beta i} - s_{\bar\beta}\boldsymbol{A}^+_{\bar\beta i}\boldsymbol{B}_{\alpha i})\,, \end{array}\right\} \tag{2·2 *)}$$

$$\left.\begin{array}{l}\widehat{E}^\alpha{}_\beta = \tfrac{1}{2}[c_\alpha{}^+, c_\beta]_-\,, \\ \widehat{E}^{\alpha\beta} = \tfrac{1}{2}[c_\alpha{}^+, c_\beta{}^+]_-\,, \\ \widehat{E}_{\alpha\beta} = \tfrac{1}{2}[c_\alpha, c_\beta]_-\,. \end{array}\right\} \tag{2·3}$$

For discussing the pairing and the quadrupole vibrations in spherical nuclei, it is convenient to adopt the spherical tensor representation of $\boldsymbol{E}$. For this aim, let $\{i\}$ be $\{\alpha\}$. Then, the following boson operators can be introduced:

$$\left.\begin{array}{l}\boldsymbol{A}_1{}^+ \equiv \boldsymbol{A}^+_{J_1M_1}(a_1b_1) = \sum\limits_{m_{a_1}m_{b_1}} (j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\,\boldsymbol{A}^+_{\alpha_1\beta_1}\,, \\ \boldsymbol{B}_1{}^+ \equiv \boldsymbol{B}^+_{J_1M_1}(a_1b_1) = \sum\limits_{m_{a_1}m_{b_1}} (j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\,\boldsymbol{B}^+_{\alpha_1\beta_1}\,. \end{array}\right\} \tag{2·4}$$

The commutation relations are given by

$$\left.\begin{array}{l}[\boldsymbol{A}_1, \boldsymbol{A}_2{}^+]_- = [\boldsymbol{B}_1, \boldsymbol{B}_2{}^+]_- = \delta_{12}(\equiv\delta_{a_1a_2}\delta_{b_1b_2}\delta_{J_1J_2}\delta_{M_1M_2})\,, \\ [\text{the other combinations}]_- = 0\,. \end{array}\right\} \tag{2·5}$$

The spherical tensor representation of $\boldsymbol{E}$ is the following:

$$\left.\begin{array}{lll}\mathscr{B}_1{}^+ \equiv & \mathscr{B}^+_{J_1M_1}(a_1b_1) & = \sum\limits_{m_{a_1}m_{b_1}} (j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\,s_{\bar\beta_1}\boldsymbol{E}^{\alpha_1}{}_{\bar\beta_1}\,, \\ s_{\bar 1}\mathscr{B}_{\bar 1} \equiv & (-)^{J_1+M_1}\mathscr{B}_{J_1\bar M_1}(a_1b_1) & = \sum\limits_{m_{a_1}m_{b_1}} (j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\,s_{\bar\alpha_1}\boldsymbol{E}^{\beta_1}{}_{\bar\alpha_1}\,, \\ \mathscr{A}_1{}^+ \equiv & \mathscr{A}^+_{J_1M_1}(a_1b_1) & = \sum\limits_{m_{a_1}m_{b_1}} (j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\,\boldsymbol{E}^{\alpha_1\beta_1}\,, \end{array}\right\} \tag{2·6}$$

*) The quantum number $\bar\alpha$ is obtained from α by changing the sign of m_a. The phase factor $s_{\bar\alpha}(\equiv(-)^{j_a+m_a})$ is necessary for the time reversal property, which was omitted in (I).

$$s_{\bar{1}}\mathcal{A}_{\bar{1}} \equiv (-)^{J_1+M_1}\mathcal{A}_{J_1\bar{M}_1}(a_1b_1) = \sum_{m_{a_1}m_{b_1}} (j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\, s_{\bar{\alpha}_1}s_{\bar{\beta}_1}\boldsymbol{E}_{\bar{\beta}_1\bar{\alpha}_1}\,.$$

Through Eqs. (2·2), $\mathcal{B}_1{}^+$, $s_{\bar{1}}\mathcal{B}_{\bar{1}}$, $\mathcal{A}_1{}^+$ and $s_{\bar{1}}\mathcal{A}_{\bar{1}}$ are expressed in terms of $\boldsymbol{B}_1{}^+$, $\boldsymbol{B}_1$, $\boldsymbol{A}_1{}^+$ and $\boldsymbol{A}_1$ as follows:

$$\left.\begin{aligned}
\mathcal{B}_1{}^+ &= \sum_{23} (\hat{p}_1\hat{p}_2 Y(123)\, \boldsymbol{B}_2{}^+ s_{\bar{3}}\boldsymbol{B}_{\bar{3}} - \hat{p}_2 Y(123)\, \boldsymbol{A}_2{}^+ s_{\bar{3}}\boldsymbol{A}_{\bar{3}})\,,\\
s_{\bar{1}}\mathcal{B}_{\bar{1}} &= \sum_{23} (\hat{p}_2 Y(123)\, \boldsymbol{B}_2{}^+ s_{\bar{3}}\boldsymbol{B}_{\bar{3}} - \hat{p}_1\hat{p}_2 Y(123)\, \boldsymbol{A}_2{}^+ s_{\bar{3}}\boldsymbol{A}_{\bar{3}})\,,\\
\mathcal{A}_1{}^+ &= \sum_{23} (1+\hat{p}_1)\, \hat{p}_2 Y(123)\, \boldsymbol{B}_2{}^+ s_{\bar{3}}\boldsymbol{A}_{\bar{3}}\,,\\
s_{\bar{1}}\mathcal{A}_{\bar{1}} &= \sum_{23} (1+\hat{p}_1)\, \hat{p}_2 Y(123)\, \boldsymbol{A}_2{}^+ s_{\bar{3}}\boldsymbol{B}_{\bar{3}}\,.
\end{aligned}\right\} \quad (2\cdot 7\text{a})^{*)}$$

We use also the following forms on several occasions:

$$\left.\begin{aligned}
&\mathcal{B}_1{}^{(\pm)} \equiv \frac{1}{2}(\mathcal{B}_1{}^+ \pm s_{\bar{1}}\mathcal{B}_{\bar{1}}), \qquad \mathcal{A}_1{}^{(\pm)} \equiv \frac{1}{2}(\mathcal{A}_1{}^+ \pm s_{\bar{1}}\mathcal{A}_{\bar{1}}),\\
&\mathcal{B}_1{}^{(\pm)} = \pm\sum_{23} \frac{1\pm\hat{p}_1}{2}\, \hat{p}_2 Y(123)\, (\boldsymbol{B}_2{}^+ s_{\bar{3}}\boldsymbol{B}_{\bar{3}} \mp \boldsymbol{A}_2{}^+ s_{\bar{3}}\boldsymbol{A}_{\bar{3}}),\\
&\mathcal{A}_1{}^{(\pm)} = \sum_{23} \frac{1+\hat{p}_1}{2}\, \hat{p}_2 Y(123)\, (\boldsymbol{B}_2{}^+ s_{\bar{3}}\boldsymbol{A}_{\bar{3}} \pm \boldsymbol{A}_2{}^+ s_{\bar{3}}\boldsymbol{B}_{\bar{3}})\,.
\end{aligned}\right\} \quad (2\cdot 7\text{b})$$

As was stressed in (I), it is necessary to form a connection between the fermion- and boson-space. In (I), for this aim we introduced certain relations which correspond to $R=R^2$ in the Hartree-Bogoliubov theory (R: the generalized density matrix). However, those relations are unsuitable for the practical purpose. As an alternative, we introduce the following set of the operators:

$$\left.\begin{aligned}
\boldsymbol{F}^{\alpha}{}_{\beta} &= \sum_i (\boldsymbol{B}^+_{\alpha i}\boldsymbol{B}_{\beta i} + s_{\bar{\beta}}\boldsymbol{A}^+_{\bar{\beta} i}s_{\bar{\alpha}}\boldsymbol{A}_{\bar{\alpha} i}) - \tfrac{1}{2}\delta_{\alpha\beta}\,,\\
\boldsymbol{F}^{\alpha\beta} &= \sum_i (\boldsymbol{B}^+_{\alpha i}s_{\bar{\beta}}\boldsymbol{A}_{\bar{\beta} i} + \boldsymbol{B}^+_{\beta i}s_{\bar{\alpha}}\boldsymbol{A}_{\bar{\alpha} i}),\\
\boldsymbol{F}_{\alpha\beta} &= \sum_i (s_{\bar{\alpha}}\boldsymbol{A}^+_{\bar{\alpha} i}\boldsymbol{B}_{\beta i} + s_{\bar{\beta}}\boldsymbol{A}^+_{\bar{\beta} i}\boldsymbol{B}_{\alpha i})\,.
\end{aligned}\right\} \quad (2\cdot 8)$$

We require that all the matrix elements of $\boldsymbol{F}$ should be vanished in the sub-space which corresponds to the fermion space. We call this sub-space the physical space ($|\text{phy}\rangle$). The detailed interpretation is given in the Appendix. The spherical tensor representation of $\boldsymbol{F}$ is the following:

$$\left.\begin{aligned}
\Phi_1{}^{(\pm)} &= \pm\sum_{23} \frac{1\pm\hat{p}_1}{2}\, \hat{p}_2 Y(123) \left(\boldsymbol{B}_2{}^+ s_{\bar{3}}\boldsymbol{B}_{\bar{3}} \pm \boldsymbol{A}_2{}^+ s_{\bar{3}}\boldsymbol{A}_{\bar{3}} - \frac{2}{\Omega} s_{\bar{3}}\delta_{2\bar{3}}\right),\\
\Psi_1{}^{(\pm)} &= -\sum_{23} \frac{1-\hat{p}_1}{2}\, \hat{p}_2 Y(123)\, (\boldsymbol{B}_2{}^+ s_{\bar{3}}\boldsymbol{A}_{\bar{3}} \pm \boldsymbol{A}_2{}^+ s_{\bar{3}}\boldsymbol{B}_{\bar{3}}),
\end{aligned}\right\} \quad (2\cdot 9)$$

*) $Y(123)$ is given by

$$Y(123) = \delta_{b_1b_2}\delta_{a_1a_3}\delta_{a_2b_3}\sqrt{(2J_3+1)(2J_2+1)}\, W(j_{a_1}j_{b_1}J_3J_2; J_1j_{a_2})\, (J_2M_2J_3M_3|J_1M_1).$$

The symbol $\hat{p}_1$ denotes the permutation operator, the action of which on arbitrary function $f(1)$ is defined by

$$\hat{p}_1 f(1) \equiv \hat{p}_1 f(a_1b_1J_1M_1) = -(-)^{j_{a_1}+j_{b_1}+J_1} f(b_1a_1J_1M_1).$$

where Ω denotes total number of the single-particle states $(\Omega\equiv\sum_a(2j_a+1))$. The definitions of $\Phi_1^{(\pm)}$ and $\Psi_1^{(\pm)}$ are the following:

$$\left.\begin{aligned}
&\Phi_1^{(\pm)}\equiv\tfrac{1}{2}(\Phi_1^+\pm s_{\bar 1}\Phi_{\bar 1}), \qquad \Psi_1^{(\pm)}\equiv\tfrac{1}{2}(\Psi_1^+\pm s_{\bar 1}\Psi_{\bar 1}),\\
&\Phi_1^+\equiv \quad \Phi_{J_1M_1}(a_1b_1) = \sum_{m_{a_1}m_{b_1}}(j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\, s_{\bar\beta_1}\boldsymbol{F}^{\alpha_1}{}_{\bar\beta_1},\\
&s_{\bar 1}\Phi_{\bar 1}\equiv(-)^{J_1+M_1}\Phi_{J_1\bar M_1}(a_1b_1) = \sum_{m_{a_1}m_{b_1}}(j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\, s_{\bar\alpha_1}\boldsymbol{F}^{\beta_1}{}_{\bar\alpha_1},\\
&\Psi_1^+\equiv \quad \Psi_{J_1M_1}(a_1b_1) = \sum_{m_{a_1}m_{b_1}}(j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\, \boldsymbol{F}^{\alpha_1\beta_1},\\
&s_{\bar 1}\Psi_{\bar 1}\equiv(-)^{J_1+M_1}\Psi_{J_1\bar M_1}(a_1b_1) = \sum_{m_{a_1}m_{b_1}}(j_{a_1}m_{a_1}j_{b_1}m_{b_1}|J_1M_1)\, s_{\bar\alpha_1}s_{\bar\beta_1}\boldsymbol{F}_{\bar\beta_1\bar\alpha_1}.
\end{aligned}\right\} \quad (2\cdot10)$$

Hereafter, we call the following relations kinematical constraints:

$$(\text{phy}|\Phi_1^{(\pm)}|\text{phy})=(\text{phy}|\Psi_1^{(\pm)}|\text{phy})=0. \qquad (2\cdot11)$$

These play an important role for the selection of the physical states.

§ 3. Truncation of the boson space

It is clear from Eqs. (2·7) that the operators $\mathscr{B}_1^+$ and $\mathscr{A}_1^+$ consist of many kinds of multipole bosons. Therefore, for actual application it may be necessary to approximate Eqs. (2·7) in relation with the dynamical property of the system under consideration. We are interested in the system of sufficiently weak quadrupole interaction compared with pairing force. First, let us consider how Eqs. (2·7) can be truncated for the above-mentioned system. For this end, we investigate the structure of the operators $\mathscr{B}_{00}^+(aa)$ and $\mathscr{A}_{00}^+(aa)$ which characterize the pairing interaction. If we pick up only the terms related to monopole bosons $\boldsymbol{A}_{00}^+(aa)$ and $\boldsymbol{B}_{00}^+(aa)$, the relations (2·7a) for $J_1=0$ are reduced to the following:

$$\left.\begin{aligned}
\mathscr{B}_{00}^+(aa)&=\frac{1}{\sqrt{\Omega_a}}(\boldsymbol{B}_{00}^+(aa)\boldsymbol{B}_{00}(aa)-\boldsymbol{A}_{00}^+(aa)\boldsymbol{A}_{00}(aa)),\\
\mathscr{A}_{00}^+(aa)&=\frac{2}{\sqrt{\Omega_a}}\boldsymbol{B}_{00}^+(aa)\boldsymbol{A}_{00}(aa),
\end{aligned}\right\} \qquad (3\cdot1)$$

where Ω_a denotes $(2j_a+1)$. We can see that these are nothing else but the extension of the original Schwinger representation of the quasi-spin to the many-j-level model, if we make the above expressions (3·1) correspond to the quasi-spin operators $\mathscr{S}_z(a)$ and $\mathscr{S}_\pm(a)$:

$$\left.\begin{aligned}
\mathscr{S}_z(a)&=\frac{1}{2}\sqrt{\Omega_a}\mathscr{B}_{00}^+(aa),\\
\mathscr{S}_+(a)&=(\mathscr{S}_-(a))^+=\frac{1}{2}\sqrt{\Omega_a}\mathscr{A}_{00}^+(aa).
\end{aligned}\right\} \qquad (3\cdot2)$$

A kinematical constraint is reduced to the following:

$$\Phi_{00}^{(+)}(aa)=\frac{2}{\sqrt{\Omega_a}}\left[\frac{1}{2}(\boldsymbol{B}_{00}^{+}(aa)\boldsymbol{B}_{00}(aa)+\boldsymbol{A}_{00}^{+}(aa)\boldsymbol{A}_{00}(aa))-\frac{\Omega_a}{4}\right]\ (=0). \qquad (3\cdot3)$$

The others are reduced to trivial relations. Equation (3·3) tells us that the magnitude of the quasi-spin $S(a)$ is equal to $\Omega_a/4$, i.e., the seniority $v_a=0$.*)

From the above argument, we can conclude that the states with $v_a=0$ can be exactly described within the framework of Eqs. (3·1) and (3·3) in the case of the pairing interaction. This fact gives us the following important suggestion: The monopole bosons $\boldsymbol{A}_{00}^{+}(aa)$ and $\boldsymbol{B}_{00}^{+}(aa)$ should lead the other multipole ones in the description of the low-lying states induced mainly by the strong pairing interaction.

Next, keeping in mind the above suggestion, we consider the case where the quadrupole force is weakly added. In this case, we pay attention to $\mathscr{B}_{2M}^{(+)}(ab)$ which characterizes the quadrupole force. Following the above suggestion, we pick up only the terms connected with the monopole bosons from $\mathscr{B}_{2M}^{(+)}(ab)$ given in Eq. (2·7b). Then, we have

$$\begin{aligned}\mathscr{B}_{2M}^{(+)}(ab)=&\frac{1}{2}\frac{1}{\sqrt{\Omega_b}}(\boldsymbol{B}_{00}(bb)\boldsymbol{B}_{2M}^{+}(ab)+\boldsymbol{B}_{00}^{+}(bb)(-)^M\boldsymbol{B}_{2\bar{M}}(ab)\\&-\boldsymbol{A}_{00}(bb)\boldsymbol{A}_{2M}^{+}(ab)-\boldsymbol{A}_{00}^{+}(bb)(-)^M\boldsymbol{A}_{2\bar{M}}(ab))\\&-\frac{1}{2}\frac{(-)^{j_a+j_b}}{\sqrt{\Omega_a}}(\boldsymbol{B}_{00}(aa)\boldsymbol{B}_{2M}^{+}(ba)+\boldsymbol{B}_{00}^{+}(aa)(-)^M\boldsymbol{B}_{2\bar{M}}(ba)\\&-\boldsymbol{A}_{00}(aa)\boldsymbol{A}_{2M}^{+}(ba)-\boldsymbol{A}_{00}^{+}(aa)(-)^M\boldsymbol{A}_{2\bar{M}}(ba)).\end{aligned} \qquad (3\cdot4)$$

This relation shows that in the case of the system we are interested in, it is enough to take into account only the quadrupole bosons $\boldsymbol{A}_{2M}^{+}(ab)$ and $\boldsymbol{B}_{2M}^{+}(ab)$ in addition to the monopole ones. On the other hand, the appearance of the quadrupole bosons disturbes the pairing correlations. Then, in order to take into account this disturbance, we pick up the terms related with the quadrupole bosons from (2·7a) and include them to Eqs. (3·1):

$$\left.\begin{aligned}\mathscr{B}_{00}^{+}(aa)=&\frac{1}{\sqrt{\Omega_a}}[(\boldsymbol{B}_{00}^{+}(aa)\boldsymbol{B}_{00}(aa)-\boldsymbol{A}_{00}^{+}(aa)\boldsymbol{A}_{00}(aa))\\&+\sum_{b,M}(\boldsymbol{B}_{2M}^{+}(ab)\boldsymbol{B}_{2M}(ab)-\boldsymbol{A}_{2M}^{+}(ab)\boldsymbol{A}_{2M}(ab))],\\\mathscr{A}_{00}^{+}(aa)=&\frac{2}{\sqrt{\Omega_a}}(\boldsymbol{B}_{00}^{+}(aa)\boldsymbol{A}_{00}(aa)+\sum_{b,M}\boldsymbol{B}_{2M}^{+}(ab)\boldsymbol{A}_{2M}(ab)).\end{aligned}\right\} \qquad (3\cdot5)$$

We can express the Hamiltonian with the pairing-plus-quadrupole interactions in terms of the monopole and the quadrupole bosons with the help of Eqs. (3·4)

*) Square of quasi-spin $\mathscr{S}(a)^2$ can be expressed as

$$\mathscr{S}(a)^2=\mathscr{S}(a)(\mathscr{S}(a)+1),\quad \mathscr{S}(a)=\tfrac{1}{2}(\boldsymbol{B}_{00}^{+}(aa)\boldsymbol{B}_{00}(aa)+\boldsymbol{A}_{00}^{+}(aa)\boldsymbol{A}_{00}(aa)).$$

and (3·5).

Following the above-mentioned truncation, we have the following relations as the kinematical constraints:

$$\left.\begin{aligned}
\Phi_{00}^{(+)}(aa) &= \frac{1}{\sqrt{\Omega_a}}\mathcal{F}_a \\
&= \frac{2}{\sqrt{\Omega_a}}\Big[\frac{1}{2}(\boldsymbol{B}_{00}^+(aa)\boldsymbol{B}_{00}(aa)+\boldsymbol{A}_{00}^+(aa)\boldsymbol{A}_{00}(aa)) \\
&\quad +\frac{1}{2}\sum_{b,M}(\boldsymbol{B}_{2M}^+(ab)\boldsymbol{B}_{2M}(ab)+\boldsymbol{A}_{2M}^+(ab)\boldsymbol{A}_{2M}(ab))-\frac{\Omega_a}{4}\Big]\ (=0), \\
\Phi_{2M}^{(\pm)}(ab) &= \frac{1}{2}\frac{1}{\sqrt{\Omega_b}}(\boldsymbol{B}_{00}(bb)\boldsymbol{B}_{2M}^+(ab)\pm\boldsymbol{B}_{00}^+(bb)(-)^M\boldsymbol{B}_{2\bar{M}}(ab) \\
&\quad \pm\boldsymbol{A}_{00}(bb)\boldsymbol{A}_{2M}^+(ab)+\boldsymbol{A}_{00}^+(bb)(-)^M\boldsymbol{A}_{2\bar{M}}(ab)) \\
&\quad \mp\frac{1}{2}\frac{(-)^{j_a+j_b}}{\sqrt{\Omega_a}}(\boldsymbol{B}_{00}(aa)\boldsymbol{B}_{2M}^+(ba)\pm\boldsymbol{B}_{00}^+(aa)(-)^M\boldsymbol{B}_{2\bar{M}}(ba) \\
&\quad \pm\boldsymbol{A}_{00}(aa)\boldsymbol{A}_{2M}^+(ba)+\boldsymbol{A}_{00}^+(aa)(-)^M\boldsymbol{A}_{2\bar{M}}(ba))\ \ (=0), \\
\Psi_{2M}^{(\pm)}(ab) &= \frac{1}{2}\frac{1}{\sqrt{\Omega_b}}(\boldsymbol{A}_{00}(bb)\boldsymbol{B}_{2M}^+(ab)\pm\boldsymbol{A}_{00}^+(bb)(-)^M\boldsymbol{B}_{2\bar{M}}(ab) \\
&\quad \mp\boldsymbol{B}_{00}(bb)\boldsymbol{A}_{2M}^+(ab)-\boldsymbol{B}_{00}^+(bb)(-)^M\boldsymbol{A}_{2\bar{M}}(ab)) \\
&\quad +\frac{1}{2}\frac{(-)^{j_a+j_b}}{\sqrt{\Omega_a}}(\boldsymbol{A}_{00}(aa)\boldsymbol{B}_{2M}^+(ba)\pm\boldsymbol{A}_{00}^+(aa)(-)^M\boldsymbol{B}_{2\bar{M}}(ba) \\
&\quad \mp\boldsymbol{B}_{00}(aa)\boldsymbol{A}_{2M}^+(ba)-\boldsymbol{B}_{00}^+(aa)(-)^M\boldsymbol{A}_{2\bar{M}}(ba))\ \ (=0).
\end{aligned}\right\}\tag{3·6}$$

The other types are reduced to trivial relations. It should be noted that the appearence of the quadrupole bosons forces us to set up kinematical constraints with $J_1=2$.

§ 4. Separation of the boson operators into static and fluctuating parts

Under the truncation of the boson space mentioned in the previous section, we consider an approximate diagonalization of the Hamiltonian trunscribed from the original one in the fermion space. Of course, our Hamiltonian consists of the single-particle energies and the pairing and the quadrupole interactions:

$$\left.\begin{aligned}
\mathcal{H} &= \mathcal{H}_0+\mathcal{H}_p+\mathcal{H}_q, \\
\mathcal{H}_0 &= \sum_a \epsilon_a^{(0)}\mathcal{N}_a, \\
\mathcal{H}_p &= -\frac{G}{4}\mathcal{P}^+\mathcal{P}, \\
\mathcal{H}_q &= -\frac{\chi}{2}\sum_M Q_M{}^+Q_M.
\end{aligned}\right\}\tag{4·1}$$

The quantities $\epsilon_a^{(0)}$, G and χ denote the single-particle energy of the state a and the strengths of the pairing and the quadrupole forces, respectively. We assume that G is much larger than χ. $\mathcal{N}_a$, $\mathcal{P}^+$ and $Q_M{}^+$ are given by

$$\left.\begin{aligned}
\mathcal{N}_a &= \sqrt{\Omega_a}\,\mathcal{B}_{00}^+(aa) + \frac{1}{2}\Omega_a\,,\\
\mathcal{P}^+ &= \sum_a \sqrt{\Omega_a}\,\mathcal{A}_{00}^+(aa)\,,\\
Q_M{}^+ &= \sum_{ab} q_{ab}\,\mathcal{B}_{2M}^{(+)}(ab)\,. \quad (q_{ab} = -(-)^{j_a+j_b}q_{ba})
\end{aligned}\right\} \tag{4·2}$$

Here the explicit forms of $\mathcal{B}_{00}^+(aa)$, $\mathcal{A}_{00}^+(aa)$ and $\mathcal{B}_{2M}^{(+)}(ab)$ are given in Eqs. (3·4) and (3·5).

As for an approximate diagonalization of the Hamiltonian (4·1), we adopt the following picture: Our system fluctuates around certain static values α_a and β_a for $(\boldsymbol{A}_{00}^+(aa), \boldsymbol{A}_{00}(aa))$ and $(\boldsymbol{B}_{00}^+(aa), \boldsymbol{B}_{00}(aa))$, respectively, and 0 for $(\boldsymbol{A}_{2M}^+(ab), \boldsymbol{A}_{2M}(ab))$ and $(\boldsymbol{B}_{2M}^+(ab), \boldsymbol{B}_{2M}(ab))$. Then, our bosons can be transformed into the following forms:

$$\begin{pmatrix}\boldsymbol{A}_{00}^+(aa)\\ \boldsymbol{A}_{00}(aa)\end{pmatrix} = \alpha_a + \begin{pmatrix}\widetilde{\boldsymbol{A}}_{00}^+(aa)\\ \widetilde{\boldsymbol{A}}_{00}(aa)\end{pmatrix}, \quad \begin{pmatrix}\boldsymbol{B}_{00}^+(aa)\\ \boldsymbol{B}_{00}(aa)\end{pmatrix} = \beta_a + \begin{pmatrix}\widetilde{\boldsymbol{B}}_{00}^+(aa)\\ \widetilde{\boldsymbol{B}}_{00}(aa)\end{pmatrix}, \tag{4·3a}$$

$$\begin{pmatrix}\boldsymbol{A}_{2M}^+(ab)\\ \boldsymbol{A}_{2M}(ab)\end{pmatrix} = \begin{pmatrix}\widetilde{\boldsymbol{A}}_{2M}^+(ab)\\ \widetilde{\boldsymbol{A}}_{2M}(ab)\end{pmatrix}, \quad \begin{pmatrix}\boldsymbol{B}_{2M}^+(ab)\\ \boldsymbol{B}_{2M}(ab)\end{pmatrix} = \begin{pmatrix}\widetilde{\boldsymbol{B}}_{2M}^+(ab)\\ \widetilde{\boldsymbol{B}}_{2M}(ab)\end{pmatrix}. \tag{4·3b}$$

Here the operators with the symbol $\sim$ denote the fluctuating parts and they are also bosons. The vacuum $|\phi_0)$ with respect to the bosons with the symbol $\sim$ is the coherent state for $\boldsymbol{A}_{00}(aa)$ and $\boldsymbol{B}_{00}(aa)$ with the eigenvalues α_a and β_a:

$$|\phi_0) = \exp\{-\tfrac{1}{2}\sum_a(\alpha_a{}^2+\beta_a{}^2)\}\cdot\exp\{\sum_a(\alpha_a\boldsymbol{A}_{00}^+(aa)+\beta_a\boldsymbol{B}_{00}^+(aa))\}|0), \tag{4·4}$$

where $|0)$ denotes the vacuum for the original bosons.

Our basic idea is to determine α_a and β_a by minimizing the expectation value of $\mathcal{H}$ with respect to $|\phi_0)$. Our Hamiltonian (4·1) commutes with $\sum_a\mathcal{N}_a$ and $\mathcal{F}_a$. However, $|\phi_0)$ is not the eigenstate for these operators. Therefore, we adopt the method of the Lagrange multiplier with the auxiliary conditions:

$$\left.\begin{aligned}
(\phi_0|\sum_a\mathcal{N}_a|\phi_0) &= \sum_a(\beta_a{}^2-\alpha_a{}^2+\tfrac{1}{2}\Omega_a) = N\,,\\
(\phi_0|\mathcal{F}_a|\phi_0) &= (\beta_a{}^2+\alpha_a{}^2-\tfrac{1}{2}\Omega_a) = 0\,,
\end{aligned}\right\} \tag{4·5}$$

where N denotes total number of nucleons. The Hamiltonian with the Lagrange miltiplier terms can be written as

$$\mathcal{H}' = \mathcal{H} - \lambda\sum_a\mathcal{N}_a + \sum_a\mu_a\mathcal{F}_a\,, \tag{4·6}$$

where λ and μ_a are the Lagrange multipliers. Performing the transformation (4·3), $\mathcal{H}'$ is decomposed as follows:

$$\mathcal{H}' = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} + \mathcal{H}_p{}^{(2)} + \mathcal{H}_q{}^{(2)} + \mathcal{H}^{(3)} + \mathcal{H}^{(4)}. \tag{4·7}$$

Here each component is given as

$$\mathcal{H}^{(0)} = \sum_a \epsilon_a\left(\beta_a{}^2 - \alpha_a{}^2 + \frac{\Omega_a}{2}\right) + \sum_a \mu_a\left(\beta_a{}^2 + \alpha_a{}^2 - \frac{\Omega_a}{2}\right) - \Delta \sum_\alpha \alpha_a\beta_a\,, \tag{4·8}$$

$$\mathcal{H}^{(1)} = \sum_a [(\mu_a - \epsilon_a)\alpha_a - \Delta\beta_a](\tilde{\boldsymbol{A}}_{00}^{+}(aa) + \tilde{\boldsymbol{A}}_{00}(aa))$$
$$+ \sum_a [(\mu_a + \epsilon_a)\beta_a - \Delta\alpha_a](\tilde{\boldsymbol{B}}_{00}^{+}(aa) + \tilde{\boldsymbol{B}}_{00}(aa)), \tag{4·9}$$

$$\mathcal{H}_p{}^{(2)} \equiv \mathcal{H}_p{}^{(c)} + \mathcal{H}_p{}^{(i)}\,, \tag{4·10a}$$

$$\mathcal{H}_p{}^{(c)} = \sum_a (\mu_a + \epsilon_a)\tilde{\boldsymbol{B}}_{00}^{+}(aa)\tilde{\boldsymbol{B}}_{00}(aa) + \sum_a (\mu_a - \epsilon_a)\tilde{\boldsymbol{A}}_{00}^{+}(aa)\tilde{\boldsymbol{A}}_{00}(aa)$$
$$- \Delta \sum_a (\tilde{\boldsymbol{B}}_{00}^{+}(aa)\tilde{\boldsymbol{A}}_{00}(aa) + \tilde{\boldsymbol{A}}_{00}^{+}(aa)\tilde{\boldsymbol{B}}_{00}(aa)), \tag{4·10b}$$

$$\mathcal{H}_p{}^{(i)} = -G \sum_{ab} (\alpha_a\tilde{\boldsymbol{B}}_{00}^{+}(aa) + \beta_a\tilde{\boldsymbol{A}}_{00}(aa))(\alpha_b\tilde{\boldsymbol{B}}_{00}(bb) + \beta_b\tilde{\boldsymbol{A}}_{00}^{+}(bb)), \tag{4·10c}$$

$$\mathcal{H}_q{}^{(2)} \equiv \mathcal{H}_q{}^{(c)} + \mathcal{H}_q{}^{(i)}, \tag{4·11a}$$

$$\mathcal{H}_q{}^{(c)} = \sum_{ab} (\mu_a + \epsilon_a) \sum_M \tilde{\boldsymbol{B}}_{2M}^{+}(ab)\tilde{\boldsymbol{B}}_{2M}(ab) + \sum_{ab} (\mu_a - \epsilon_a) \sum_M \tilde{\boldsymbol{A}}_{2M}^{+}(ab)\tilde{\boldsymbol{A}}_{2M}(ab)$$
$$- \Delta \sum_{ab}\sum_M (\tilde{\boldsymbol{B}}_{2M}^{+}(ab)\tilde{\boldsymbol{A}}_{2M}(ab) + \tilde{\boldsymbol{A}}_{2M}^{+}(ab)\tilde{\boldsymbol{B}}_{2M}(ab)), \tag{4·11b}$$

$$\mathcal{H}_q{}^{(i)} = -\frac{\chi}{2}\sum_{abcd} q_{ab}q_{cd}\frac{1}{\sqrt{\Omega_b\Omega_d}}$$
$$\cdot \sum_M [\beta_b(\tilde{\boldsymbol{B}}_{2M}^{+}(ab) + (-)^M\tilde{\boldsymbol{B}}_{2\bar{M}}(ab)) - \alpha_b(\tilde{\boldsymbol{A}}_{2M}^{+}(ab) + (-)^M\tilde{\boldsymbol{A}}_{2\bar{M}}(ab))]$$
$$\cdot [\beta_d(\tilde{\boldsymbol{B}}_{2M}(cd) + (-)^M\tilde{\boldsymbol{B}}_{2\bar{M}}^{+}(cd)) - \alpha_d(\tilde{\boldsymbol{A}}_{2M}(cd) + (-)^M\tilde{\boldsymbol{A}}_{2\bar{M}}^{+}(cd))], \tag{4·11c}$$

$$\mathcal{H}^{(3)} = \text{third order terms for the fluctuations}, \tag{4·12}$$

$$\mathcal{H}^{(4)} = \text{fourth order terms for the fluctuations}, \tag{4·13}$$

where ϵ_a and Δ are defined as

$$\epsilon_a \equiv \epsilon_a{}^{(0)} - \lambda\,, \quad \Delta \equiv G\sum_a \alpha_a\beta_a\,. \tag{4·14}$$

Assuming the fluctuations as small, hereafter we neglect $\mathcal{H}^{(3)}$ and $\mathcal{H}^{(4)}$.

Concerning the kinematical constraints, it is sufficient in our approximation to take into account the terms up to first order with respect to the fluctuations in relation with the approximation of the Hamiltonian. Relations (3·6) are reduced to the following, after performing the transformation (4·3):

$$\Phi_{00}^{(+)}(aa) = \frac{1}{\sqrt{\Omega_a}}\left(\beta_a{}^2 + \alpha_a{}^2 - \frac{\Omega_a}{2}\right)$$
$$+ \frac{1}{\sqrt{\Omega_a}}[\beta_a(\tilde{\boldsymbol{B}}_{00}^{+}(aa) + \tilde{\boldsymbol{B}}_{00}(aa)) + \alpha_a(\tilde{\boldsymbol{A}}_{00}^{+}(aa) + \tilde{\boldsymbol{A}}_{00}(aa))] \ (=0), \tag{4·15a}$$

$$\Phi_{2M}^{(\pm)}(ab)=\frac{1}{2}\frac{1}{\sqrt{\Omega_b}}[\beta_b(\tilde{\boldsymbol{B}}_{2M}^{+}(ab)\pm(-)^M\tilde{\boldsymbol{B}}_{2\bar{M}}(ab))\pm\alpha_b(\tilde{\boldsymbol{A}}_{2M}^{+}(ab)\pm(-)^M\tilde{\boldsymbol{A}}_{2\bar{M}}(ab))]$$

$$\mp\frac{1}{2}\frac{(-)^{j_a+j_b}}{\sqrt{\Omega_a}}[\beta_a(\tilde{\boldsymbol{B}}_{2M}^{+}(ba)\pm(-)^M\tilde{\boldsymbol{B}}_{2\bar{M}}(ba))\pm\alpha_a(\tilde{\boldsymbol{A}}_{2M}^{+}(ba)\pm(-)^M\tilde{\boldsymbol{A}}_{2\bar{M}}(ba))]$$

$$(=0), \tag{4·15b}$$

$$\Psi_{2M}^{(\pm)}(ab)=\frac{1}{2}\frac{1}{\sqrt{\Omega_b}}[\alpha_b(\tilde{\boldsymbol{B}}_{2M}^{+}(ab)\pm(-)^M\tilde{\boldsymbol{B}}_{2\bar{M}}(ab))\mp\beta_b(\tilde{\boldsymbol{A}}_{2M}^{+}(ab)\pm(-)^M\tilde{\boldsymbol{A}}_{2\bar{M}}(ab))]$$

$$+\frac{1}{2}\frac{(-)^{j_a+j_b}}{\sqrt{\Omega_a}}[\alpha_a(\tilde{\boldsymbol{B}}_{2M}^{+}(ba)\pm(-)^M\tilde{\boldsymbol{B}}_{2\bar{M}}(ba))\mp\beta_a(\tilde{\boldsymbol{A}}_{2M}^{+}(ba)\pm(-)^M\tilde{\boldsymbol{A}}_{2\bar{M}}(ba))]$$

$$(=0). \tag{4·15c}$$

The others are reduced to trivial relations.

In order to determine α_a and β_a, let us minimize $\mathcal{H}^{(0)}$ given in Eq. (4·8):

$$\left.\begin{aligned}\frac{1}{2}\frac{\partial\mathcal{H}^{(0)}}{\partial\alpha_a}&=(\mu_a-\epsilon_a)\alpha_a-\Delta\beta_a=0\,,\\ \frac{1}{2}\frac{\partial\mathcal{H}^{(0)}}{\partial\beta_a}&=-\Delta\alpha_a+(\mu_a+\epsilon_a)\beta_a=0\,.\end{aligned}\right\} \tag{4·16}$$

The solutions of Eq. (4·16) are given with the help of Eqs. (4·5) as follows:

$$\left.\begin{aligned}\mu_a{}^2&=\epsilon_a{}^2+\Delta^2\equiv E_a{}^2\,,\\ \alpha_a{}^2&=\frac{\Omega_a}{2}\cdot\frac{1}{2}\left(1+\frac{\epsilon_a}{E_a}\right)\equiv\frac{\Omega_a}{2}\cdot u_a{}^2\,,\\ \beta_a{}^2&=\frac{\Omega_a}{2}\cdot\frac{1}{2}\left(1-\frac{\epsilon_a}{E_a}\right)\equiv\frac{\Omega_a}{2}\cdot v_a{}^2\,.\end{aligned}\right\} \tag{4·17}$$

Δ and λ can be determined by Eqs. (4·14) and (4·5), the equations of which are given as

$$\left.\begin{aligned}\frac{1}{G}&=\frac{1}{4}\sum_a\frac{\Omega_a}{E_a}\,,\\ N&=\frac{1}{2}\sum_a\Omega_a\left(1-\frac{\epsilon_a{}^{(0)}-\lambda}{E_a}\right).\end{aligned}\right\} \tag{4·18}$$

These equations are completely the same as those given in the conventional BCS theory. We can see that $\mathcal{H}^{(1)}$ is vanished, if we substitute Eq. (4·16) into Eq. (4·9).

§ 5. Pairing vibration in the superconducting phase

We are now in a stage to diagonalize our Hamiltonian (4·7) with the use of the results of the previous section. This diagonalization leads us to RPA. $\mathcal{H}_p{}^{(2)}$ and $\mathcal{H}_q{}^{(2)}$ are independent of each other and, therefore, we can treat them separately. In this section, we consider mainly the case of $\mathcal{H}_p{}^{(2)}$.

First, let us diagonalize $\mathcal{H}_p{}^{(c)}$ given in Eq. (4·10b). This can be done by introducing the following linear combinations of $\tilde{\boldsymbol{B}}_{00}^{+}(aa)$ and $\tilde{\boldsymbol{A}}_{00}^{+}(aa)$:

$$\left.\begin{aligned}\boldsymbol{C}_{00}^{+}(aa)&\equiv u_a\tilde{\boldsymbol{B}}_{00}^{+}(aa)-v_a\tilde{\boldsymbol{A}}_{00}^{+}(aa),\\ \boldsymbol{D}_{00}^{+}(aa)&\equiv v_a\tilde{\boldsymbol{B}}_{00}^{+}(aa)+u_a\tilde{\boldsymbol{A}}_{00}^{+}(aa).\end{aligned}\right\} \tag{5·1}$$

The result becomes as follows:

$$\mathcal{H}_p{}^{(c)}=2\sum_a E_a\boldsymbol{C}_{00}^{+}(aa)\boldsymbol{C}_{00}(aa). \tag{5·2}$$

$\boldsymbol{C}_{00}^{+}(aa)$ and $\boldsymbol{D}_{00}^{+}(aa)$ are, of course, boson operators which are commutable. A characteristic of the result is the appearance of $\boldsymbol{D}_{00}^{+}(aa)$ boson with zero energy in contrast to $\boldsymbol{C}_{00}^{+}(aa)$ with energy $2E_a$.

Next, in addition to $\mathcal{H}_p{}^{(c)}$, we treat $\mathcal{H}_p{}^{(t)}$ given in Eq. (4·10c). With the use of $\boldsymbol{C}_{00}^{+}(aa)$ and $\boldsymbol{D}_{00}^{+}(aa)$, $\mathcal{H}_p{}^{(2)}$ can be rewritten in the following form:

$$\mathcal{H}_p{}^{(2)}=\mathcal{H}_p{}^{(v)}+\mathcal{H}_p{}^{(t)}, \tag{5·3a}$$

$$\begin{aligned}\mathcal{H}_p{}^{(v)}=&\,2\sum_a E_a\boldsymbol{C}_{00}^{+}(aa)\boldsymbol{C}_{00}(aa)\\ &-\frac{G}{2}\sum_{ab}\sqrt{\Omega_a\Omega_b}\,(u_a{}^2\boldsymbol{C}_{00}^{+}(aa)-v_a{}^2\boldsymbol{C}_{00}(aa))(u_b{}^2\boldsymbol{C}_{00}(bb)-v_b{}^2\boldsymbol{C}_{00}^{+}(bb)),\end{aligned} \tag{5·3b}$$

$$\begin{aligned}\mathcal{H}_p{}^{(t)}=&-\frac{G}{2}\sum_{ab}\sqrt{\Omega_a\Omega_b}\,u_av_a(u_b{}^2-v_b{}^2)\,(\boldsymbol{D}_{00}^{+}(aa)+\boldsymbol{D}_{00}(aa))\,(\boldsymbol{C}_{00}^{+}(bb)+\boldsymbol{C}_{00}(bb))\\ &-\frac{G}{2}[\sum_a u_av_a(\boldsymbol{D}_{00}^{+}(aa)+\boldsymbol{D}_{00}(aa))]^2.\end{aligned} \tag{5·3c}$$

The diagonalization of $\mathcal{H}_p{}^{(v)}$ leads us to the conventional RPA applied to the pairing vibration in the superconducting phase. It may be unnecessary to show the explicit diagonalization of $\mathcal{H}_p{}^{(v)}$, since this is already well known. It is interesting to see that $\boldsymbol{C}_{00}^{+}(aa)$ plays the same role as that of the quasi-particle pair coupled to $J=0$ under the boson approximation.

The final problem is how to understand $\mathcal{H}_p{}^{(t)}$ given in Eq. (5·3c). For this aim, we pay attention to the kinematical constraint (4·15a). In terms of $\boldsymbol{C}_{00}^{+}(aa)$ and $\boldsymbol{D}_{00}^{+}(aa)$, Eq. (4·15a) can be rewritten as

$$\boldsymbol{\Phi}_{00}^{(+)}(aa)=\frac{1}{\sqrt{2}}(\boldsymbol{D}_{00}^{+}(aa)+\boldsymbol{D}_{00}(aa))\quad(=0). \tag{5·4}$$

Under the requirement of $\boldsymbol{\Phi}_{00}^{(+)}(aa)$ being vanished, our physical state should be of the following form with respect to $\boldsymbol{D}_{00}^{+}(aa)$:

$$|\mathrm{phy})\sim\exp\{-\tfrac{1}{2}\sum_a\theta_{aa}^{(0)}(\boldsymbol{D}_{00}^{+}(aa))^2\}\,|\phi_0). \tag{5·5}$$

Here $\theta_{aa}^{(0)}$ is an arbitrary constant which obeys the condition, $|\theta_{aa}^{(0)}|<1$. We can prove that the expectation value of $\boldsymbol{\Phi}_{00}^{(+)}(aa)$ with respect to $|\mathrm{phy})$ is vanished.

The condition $|\theta_{aa}^{(0)}|<1$ means that $|\mathrm{phy})$ can be normalized to unity. Since $\mathcal{H}_p^{(t)}$ contains $\boldsymbol{D}_{00}^+(aa)$ and $\boldsymbol{D}_{00}(aa)$ in the form given in Eq. (5·4), $\mathcal{H}_p^{(t)}$ has no contribution to the excitations of the system. Therefore, for the practical application, it may be possible to omit $\mathcal{H}_p^{(t)}$, that is, we can leave $\boldsymbol{D}_{00}^+(aa)$ bosons out of consideration. Thus, we have been able to get completely the same result as that of the conventional RPA in the pairing vibration.

§ 6. Quadrupole vibration

As a final problem, we consider the diagonalization of $\mathcal{H}_q^{(2)}$ given in Eq. (4·11). This case also leads to the conventional RPA. The procedure is almost the same as that in the case of $\mathcal{H}_p^{(2)}$. Let us start with the diagonalization of $\mathcal{H}_q^{(c)}$. In this case, also by introducing the following independent bosons, the diagonalization can be proceed:

$$\left.\begin{aligned}\boldsymbol{C}_{2M}^+(ab)&=u_a\widetilde{\boldsymbol{B}}_{2M}^+(ab)-v_a\widetilde{\boldsymbol{A}}_{2M}^+(ab),\\ \boldsymbol{D}_{2M}^+(ab)&=v_a\widetilde{\boldsymbol{B}}_{2M}^+(ab)+u_a\widetilde{\boldsymbol{A}}_{2M}^+(ab).\end{aligned}\right\}\qquad(6\cdot1)$$

$\mathcal{H}_q^{(c)}$ can be rewritten as

$$\mathcal{H}_q^{(c)}=2\sum_{ab}E_a\sum_M\boldsymbol{C}_{2M}^+(ab)\boldsymbol{C}_{2M}(ab).\qquad(6\cdot2)$$

We can see that $\boldsymbol{D}_{2M}^+(ab)$ bosons have also zero energy. With the help of $\boldsymbol{C}_{2M}^+(ab)$ and $\boldsymbol{D}_{2M}^+(ab)$, $\mathcal{H}_q^{(2)}$ can be rewritten down as

$$\mathcal{H}_q^{(2)}=\mathcal{H}_q^{(v)}+\mathcal{H}_q^{(t)},\qquad(6\cdot3\mathrm{a})$$

$$\begin{aligned}\mathcal{H}_q^{(v)}=&\sum_{ab}(E_a+E_b)\sum_M\mathring{\boldsymbol{C}}_{2M}^+(ab)\mathring{\boldsymbol{C}}_{2M}(ab)\\ &-\frac{\chi}{4}\sum_{abcd}q_{ab}q_{cd}\xi_{ab}^{(+)}\xi_{cd}^{(+)}\sum_M(\mathring{\boldsymbol{C}}_{2M}^+(ab)+(-)^M\mathring{\boldsymbol{C}}_{2\bar{M}}(ab))\\ &\qquad\times(\mathring{\boldsymbol{C}}_{2M}(cd)+(-)^M\mathring{\boldsymbol{C}}_{2\bar{M}}^+(cd)),\end{aligned}\qquad(6\cdot3\mathrm{b})$$

$$\begin{aligned}\mathcal{H}_q^{(t)}=&\sum_{ab}(E_a-E_b)\sum_M(\mathring{\boldsymbol{C}}_{2M}^+(ab)\mathring{\boldsymbol{D}}_{2M}(ab)+\mathring{\boldsymbol{D}}_{2M}^+(ab)\mathring{\boldsymbol{C}}_{2M}(ab))\\ &+\sum_{ab}(E_a+E_b)\sum_M\mathring{\boldsymbol{D}}_{2M}^+(ab)\mathring{\boldsymbol{D}}_{2M}(ab)\\ &+\frac{\chi}{2}\sum_{abcd}q_{ab}q_{cd}\xi_{ab}^{(+)}\eta_{cd}^{(+)}\sum_M(\mathring{\boldsymbol{C}}_{2M}^+(ab)+(-)^M\mathring{\boldsymbol{C}}_{2\bar{M}}(ab))\\ &\qquad\times(\boldsymbol{D}_{2M}(cd)-(-)^{j_c+j_d}(-)^M\boldsymbol{D}_{2\bar{M}}^+(dc))\\ &-\frac{\chi}{4}\sum_{abcd}q_{ab}q_{cd}\eta_{ab}^{(+)}\eta_{cd}^{(+)}\sum_M(\boldsymbol{D}_{2M}^+(ab)-(-)^{j_a+j_b}(-)^M\boldsymbol{D}_{2\bar{M}}(ba))\\ &\qquad\times(\boldsymbol{D}_{2M}(cd)-(-)^{j_c+j_d}(-)^M\boldsymbol{D}_{2\bar{M}}^+(dc)),\end{aligned}\qquad(6\cdot3\mathrm{c})$$

where

$$\xi_{ab}^{(\pm)}=\pm\xi_{ba}^{(\pm)}\equiv u_av_a\pm v_au_b,\quad \eta_{ab}^{(\pm)}=\eta_{ba}^{(\pm)}\equiv u_au_b\mp v_av_b.\qquad(6\cdot4)$$

$\mathring{\boldsymbol{C}}_{2M}^{+}(ab)$ and $\mathring{\boldsymbol{D}}_{2M}^{+}(ab)$ are the antisymmetric and the symmetric bosons, respectively, defined by

$$\left.\begin{aligned}\mathring{\boldsymbol{C}}_{2M}^{+}(ab)&=-(-)^{j_a+j_b}\mathring{\boldsymbol{C}}_{2M}^{+}(ba)\equiv\tfrac{1}{2}(\boldsymbol{C}_{2M}^{+}(ab)-(-)^{j_a+j_b}\boldsymbol{C}_{2M}^{+}(ba)),\\ \mathring{\boldsymbol{D}}_{2M}^{+}(ab)&=(-)^{j_a+j_b}\mathring{\boldsymbol{D}}_{2M}^{+}(ba)\equiv\tfrac{1}{2}(\boldsymbol{C}_{2M}^{+}(ab)+(-)^{j_a+j_b}\boldsymbol{C}_{2M}^{+}(ba)),\end{aligned}\right\}\quad(6\cdot5\text{a})$$

$$\left.\begin{aligned}[\mathring{\boldsymbol{C}}_{2K}(ab),\mathring{\boldsymbol{C}}_{2\Lambda}^{+}(cd)]_-&=\tfrac{1}{2}(\delta_{ac}\delta_{bd}-(-)^{j_a+j_b}\delta_{ad}\delta_{bc})\delta_{K\Lambda},\\ [\mathring{\boldsymbol{D}}_{2K}(ab),\mathring{\boldsymbol{D}}_{2\Lambda}^{+}(cd)]_-&=\tfrac{1}{2}(\delta_{ac}\delta_{bd}+(-)^{j_a+j_b}\delta_{ad}\delta_{bc})\delta_{K\Lambda},\end{aligned}\right\}\quad(6\cdot5\text{b})$$

$$[\text{the other combinations}]_-=0\,.$$

The Hamiltonian $\mathcal{H}_q^{(v)}$ (6·3b) is just of the same form as that given in the boson approximation of the quasi-particle pairs. In this case, $\mathring{\boldsymbol{C}}_{2M}^{+}(ab)$ corresponds to the quasi-particle pair. The diagonalization may be unnecessary to show.

Finally, we mention how $\mathcal{H}_q^{(t)}$ given in Eq. (6·3c) can be undersood. We use the kinematical constraints (4·15b) and (4·15c). $\Phi_{2M}^{(\pm)}(ab)$ and $\Psi_{2M}^{(\pm)}(ab)$ can be rewritten in the following forms:

$$\left.\begin{aligned}\Phi_{2M}^{(\pm)}(ab)&=\frac{1}{\sqrt{2}}\xi_{ab}^{(\mp)}(\mathring{\boldsymbol{D}}_{2M}^{+}(ab)\pm(-)^M\mathring{\boldsymbol{D}}_{2\bar{M}}(ab))\\&\quad\pm\frac{1}{2\sqrt{2}}\eta_{ab}^{(\mp)}[(\boldsymbol{D}_{2M}^{+}(ab)-(-)^{j_a+j_b}(-)^M\boldsymbol{D}_{2\bar{M}}(ba))\\&\qquad\pm((-)^M\boldsymbol{D}_{2\bar{M}}(ab)-(-)^{j_a+j_b}\boldsymbol{D}_{2M}^{+}(ba))]\quad(=0),\\ \Psi_{2M}^{(\pm)}(ab)&=\frac{1}{\sqrt{2}}\eta_{ab}^{(\mp)}(\mathring{\boldsymbol{D}}_{2M}^{+}(ab)\pm(-)^M\mathring{\boldsymbol{D}}_{2\bar{M}}(ab))\\&\quad\mp\frac{1}{2\sqrt{2}}\xi_{ab}^{(\mp)}[(\boldsymbol{D}_{2M}^{+}(ab)-(-)^{j_a+j_b}(-)^M\boldsymbol{D}_{2\bar{M}}(ba))\\&\qquad\pm((-)^M\boldsymbol{D}_{2\bar{M}}(ab)-(-)^{j_a+j_b}\boldsymbol{D}_{2M}^{+}(ba))]\quad(=0).\end{aligned}\right\}\quad(6\cdot6)$$

From the condition of $\Phi_{2M}^{(\pm)}(ab)$ and $\Psi_{2M}^{(\pm)}(ab)$ being vanished in our physical space, we can see that the following important relations hold:

$$(\text{phy}|\mathring{\boldsymbol{D}}_{2M}^{+}(ab)|\text{phy})=(\text{phy}|\mathring{\boldsymbol{D}}_{2M}(ab)|\text{phy})=0\,,\quad(6\cdot7\text{a})$$

$$(\text{phy}|\boldsymbol{D}_{2M}^{+}(ab)-(-)^{j_a+j_b}(-)^M\boldsymbol{D}_{2\bar{M}}(ba)|\text{phy})=0\,.\quad(6\cdot7\text{b})$$

Relation (6·7a) tells us that our physical state should not contain $\mathring{\boldsymbol{D}}_{2M}^{+}(ab)$ bosons. Further, relation (6·7b) means that our physical state is of the following form with respect to $\boldsymbol{D}_{2M}^{+}(ab)$:

$$|\text{phy})\sim\exp\{\tfrac{1}{2}\sum_{ab}(-)^{j_a+j_b}\theta_{ab}^{(2)}\cdot\sum_M\boldsymbol{D}_{2M}^{+}(ab)\cdot(-)^M\boldsymbol{D}_{2\bar{M}}^{+}(ba)\}|\phi_0).\quad(6\cdot8)$$

Here $\theta_{ab}^{(2)}$ is an arbitrary constant with the condition $|\theta_{ab}^{(2)}|<1$. Therefore, in the same meaning as that given in the case of the pairing vibration, it may be possible to omit $\mathcal{H}_q^{(t)}$, that is, we can leave $\mathring{\boldsymbol{D}}_{2M}^{+}(ab)$ and $\boldsymbol{D}_{2M}^{+}(ab)$ out of consideration. In this way, we have been able to reformulate the conventional RPA in the framework of our generalized Schwinger representation.

§ 7. Concluding remarks

We have been able to derive the conventional RPA on the basis of the generalized Schwinger representation of the fermion-pairs proposed by the present authors (S. N. and M. Y.). Our basic idea on the approximations has been devided into two stages: First is the truncation of the boson space. Under the sufficiently weak quadrupole interaction compared with the pairing force, the system can be treated in terms of the monopole and the quadrupole bosons. Second is the separation of the bosons into the static and the fluctuating parts. The formers are determined by minimizing the expectation value of the Hamiltonian with respect to the coherent state for our bosons. Of course, the kinematical constraints are essential for the selection of the physical state.

In relation to these two stages of the approximations, we obtain some problems to be solved in the near future. Recently, Iwasaki et al. reported interesting results on the analysis of the anharmonicity in the framework of the quasi-particle Tamm-Dancoff approximation.[5] According to their results for the transition probabilities, the quasi-particle pairs show boson-like behaviour with certain attenuation factors even in rather highly excited states. Empirically, they showed that the factor has a simple dependence on the number of the quasi-particles of the state under consideration. This is the first problem.

Concerning the first stage of approximations, the first problem is the coupling between the monopole and the quadrupole bosons. The role of our monopole bosons is essentially different from the one appearing in the $SU(6)$ model by Janssen et al.[6] Theirs is a kinematical boson which is associated with the quadrupole degrees of freedom. Ours are dynamical bosons which create the pairing vibration. Therefore, it can be expected to describe the coupling between the pairing and the quadrupole vibrations. It is also interesting to investigate the *s*-*d* boson model by Arima and Iachello[7] in our framework.

The second is the validity of the truncation of the boson space. If the quadrupole force is stronger, the truncated forms of $\mathcal{B}_{2M}^{(+)}(ab)$, $\mathcal{B}_{00}^{+}(aa)$ and $\mathcal{A}_{00}^{+}(aa)$ shown in Eqs. (3·4) and (3·5) are no longer reliable. Therefore, it becomes important to consider which types of the multipole bosons are necessary in addition to the monopole and the quadrupole ones. It may be an interesting problem in relation to the rotational motion.

Finally, we stress that in our theory the fermion-pair operators can be completely expressed in terms of the microscopic quantities which exist in the original fermion system. Therefore, the Hamiltonian expressed with our bosons does not include any additional parameters in contrast to the $SU(6)$ model by Janssen et al.,[6] in which there are many parameters to be determined with the experimental data.

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Appendix

In the Appendix, we give a delaited interpretation of the kinematical constraints used in this paper. We define the following set of the operators $\widehat{F}(\widehat{F}^{\alpha}{}_{\beta}, \widehat{F}^{\alpha\beta}, \widehat{F}_{\alpha\beta})$ in our fermion space:

$$\left.\begin{aligned}&\widehat{F}^{\alpha}{}_{\beta}\equiv\tfrac{1}{2}[c_{\alpha}{}^{+}, c_{\beta}]_{+}-\tfrac{1}{2}\delta_{\alpha\beta}\,, \ (=0)\\ &\widehat{F}^{\alpha\beta}\equiv\tfrac{1}{2}[c_{\alpha}{}^{+}, c_{\beta}{}^{+}]_{+}\,, \ (=0)\\ &\widehat{F}_{\alpha\beta}\equiv\tfrac{1}{2}[c_{\alpha}, c_{\beta}]_{+}\,. \ (=0)\end{aligned}\right\} \tag{A·1}$$

They satisfy the relations

$$(\widehat{F}^{\alpha}{}_{\beta})^{+}=\widehat{F}^{\beta}{}_{\alpha}\,, \qquad (\widehat{F}^{\alpha\beta})^{+}=\widehat{F}_{\alpha\beta}\,, \qquad \widehat{F}^{\alpha\beta}=\widehat{F}^{\beta\alpha}. \tag{A·2}$$

Clearly, $\widehat{F}$ are nothing else than the anti-commutators. Therefore, they are identically vanished. Commutation relations of $\widehat{F}$ with $\widehat{E}$ are given formally as

$$\left.\begin{aligned}&[\widehat{F}^{\alpha}{}_{\beta}, \widehat{E}^{\gamma}{}_{\delta}]_{-}=\delta_{\beta\gamma}\widehat{F}^{\alpha}{}_{\delta}-\delta_{\alpha\delta}\widehat{F}^{\gamma}{}_{\beta}, \qquad [\widehat{F}^{\alpha\beta}, \widehat{E}^{\gamma}{}_{\delta}]_{-}=-\delta_{\alpha\delta}\widehat{F}^{\gamma\beta}-\delta_{\beta\delta}\widehat{F}^{\alpha\gamma}\,,\\ &[\widehat{F}_{\alpha\beta}, \widehat{E}^{\gamma}{}_{\delta}]_{-}=\delta_{\alpha\gamma}\widehat{F}_{\delta\beta}+\delta_{\beta\gamma}\widehat{F}_{\alpha\delta}, \qquad [\widehat{F}^{\alpha}{}_{\beta}, \widehat{E}^{\gamma\delta}]_{-}=\delta_{\beta\gamma}\widehat{F}^{\alpha\delta}-\delta_{\beta\delta}\widehat{F}^{\alpha\gamma}\,,\\ &[\widehat{F}^{\alpha\beta}, \widehat{E}^{\gamma\delta}]_{-}=0, \qquad [\widehat{F}_{\alpha\beta}, \widehat{E}^{\gamma\delta}]_{-}=\delta_{\alpha\gamma}\widehat{F}^{\delta}{}_{\beta}+\delta_{\beta\gamma}\widehat{F}^{\delta}{}_{\alpha}-\delta_{\alpha\delta}\widehat{F}^{\gamma}{}_{\beta}-\delta_{\beta\delta}\widehat{F}^{\gamma}{}_{\alpha}\,,\\ &[\widehat{F}^{\alpha}{}_{\beta}, \widehat{E}_{\gamma\delta}]_{-}=-\delta_{\alpha\gamma}\widehat{F}_{\beta\delta}+\delta_{\alpha\delta}\widehat{F}_{\beta\gamma}\,,\\ &[\widehat{F}^{\alpha\beta}, \widehat{E}_{\gamma\delta}]_{-}=\delta_{\alpha\gamma}\widehat{F}^{\beta}{}_{\delta}+\delta_{\beta\gamma}\widehat{F}^{\alpha}{}_{\delta}-\delta_{\alpha\delta}\widehat{F}^{\beta}{}_{\gamma}-\delta_{\beta\delta}\widehat{F}^{\alpha}{}_{\gamma}, \qquad [\widehat{F}_{\alpha\beta}, \widehat{E}_{\gamma\delta}]_{-}=0\,.\end{aligned}\right\} \tag{A·3}$$

Next, we introduce the following set of the operators $\boldsymbol{F}(\boldsymbol{F}^{\alpha}{}_{\beta}, \boldsymbol{F}^{\alpha\beta}, \boldsymbol{F}_{\alpha\beta})$ in our boson space:

$$\left.\begin{aligned}&\boldsymbol{F}^{\alpha}{}_{\beta}=x[\sum_{i}(\boldsymbol{B}^{+}_{\alpha i}\boldsymbol{B}_{\beta i}+\boldsymbol{A}^{+}_{\beta i}\boldsymbol{A}_{\alpha i})-z\delta_{\alpha\beta}],\\ &\boldsymbol{F}^{\alpha\beta}=y\sum_{i}(\boldsymbol{B}^{+}_{\alpha i}\boldsymbol{A}_{\beta i}+\boldsymbol{B}^{+}_{\beta i}\boldsymbol{A}_{\alpha i}),\\ &\boldsymbol{F}_{\alpha\beta}=y\sum_{i}(\boldsymbol{A}^{+}_{\alpha i}\boldsymbol{B}_{\beta i}+\boldsymbol{A}^{+}_{\beta i}\boldsymbol{B}_{\alpha i}),\end{aligned}\right\} \tag{A·4}$$

where x, y and z are certain constants to be determined later. The set $\boldsymbol{F}$ satisfies the same relations as those given in Eqs. (A·2). Further, we can see that the commutation relations of $\boldsymbol{F}$ with $\boldsymbol{E}$ are of the same forms as those in Eqs. (A·3). Then, we require that the set $\boldsymbol{F}$ plays the same role as that of $\hat{F}$ in the fermion space. This requirement permits us to set up the following relations:

$$(\mathrm{phy}|\boldsymbol{F}^{\alpha}{}_{\beta}|\mathrm{phy}) = (\mathrm{phy}|\boldsymbol{F}^{\alpha\beta}|\mathrm{phy}) = (\mathrm{phy}|\boldsymbol{F}_{\alpha\beta}|\mathrm{phy}) = 0\,. \qquad (\mathrm{A}\cdot 5)$$

Now, let us determine the constants x, y and z. For the sake of Eqs. (A·5), the constants x and y may be possible to set up, without loss of generality, as follows:

$$x = y = 1\,. \qquad (\mathrm{A}\cdot 6)$$

The problem is the determination of the constant z. For this aim, we introduce the operator $\boldsymbol{N}$ which corresponds to the total number of Fermions $\hat{N}$:

$$\hat{N} = \sum_{\alpha} c_{\alpha}{}^{+} c_{\alpha} \Leftrightarrow \boldsymbol{N} = \sum_{\alpha i} (\boldsymbol{B}^{+}_{\alpha i}\boldsymbol{B}_{\alpha i} - \boldsymbol{A}^{+}_{\alpha i}\boldsymbol{A}_{\alpha i}) + \frac{\Omega}{2}\,. \qquad (\mathrm{A}\cdot 7)$$

Since the zero fermion state $|0\rangle_F$ satisfies $\hat{E}_{\alpha\beta}|0\rangle_F = 0$, the boson state $|0)_B$ corresponding to $|0\rangle_F$ has the property $\boldsymbol{E}_{\alpha\beta}|0)_B = 0$. This means that $|0)_B$ does not contain any $\boldsymbol{B}^{+}$ bosons. Then, we have

$$\boldsymbol{N}|0)_B = (-\sum_{\alpha i} \boldsymbol{A}^{+}_{\alpha i}\boldsymbol{A}_{\alpha i} + \tfrac{1}{2}\Omega)\,|0)_B = 0\,. \qquad (\mathrm{A}\cdot 8)$$

This relation shows that $|0)_B$ consists of $(\Omega/2)$-$\boldsymbol{A}^{+}$ bosons. Operation of $\sum_{\alpha}\boldsymbol{F}^{\alpha}{}_{\alpha}$ to $|0)_B$ leads us to

$$\begin{aligned}\sum_{\alpha} \boldsymbol{F}^{\alpha}{}_{\alpha}|0)_B &= \sum_{\alpha}(\sum_{i} \boldsymbol{A}^{+}_{\alpha i}\boldsymbol{A}_{\alpha i} - z)\,|0)_B \\ &= (\sum_{\alpha i} \boldsymbol{A}^{+}_{\alpha i}\boldsymbol{A}_{\alpha i} - z\Omega)\,|0)_B = 0\,. \qquad (\mathrm{A}\cdot 9)\end{aligned}$$

From Eqs. (A·8) and (A·9), we get

$$z = \tfrac{1}{2}\,. \qquad (\mathrm{A}\cdot 10)$$

In this way, we can get Eqs. (2·8) under the consideration of the time reversal property. The condition that $\boldsymbol{F}$ should be vanished in the physical space has the following correpondence with the Hartree-Bogoliubov theory: If we replace $\boldsymbol{A}^{+}_{\alpha i}$ and $\boldsymbol{B}^{+}_{\alpha i}$ with c-number the condition is reduced to the orthonormality of the coefficients of the generalized Bogoliubov transformation.

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