Derivation of Reaction Theorems for Scattered Fields

Johan Malmström

***Abstract*—Two novel formulations of the reaction are derived. The formulations decompose the electromagnetic fields in scattered components based on the location of the sources of the scattered fields. It is shown that some of the scattering components do not contribute to the reaction. The novel formulations of the reaction are derived by excluding these noncontributing components from the classical reaction formulations. The correctness of one of the formulations is verified with a numerical example. It is observed from one of the novel formulations that the first-order scattered fields do not contribute to the reaction. This result legitimizes the approximation to neglect multiple scattering, which is a common assumption when using reaction theorems. The novel formulations are also important for a conceptual understanding of the reaction.**

***Index Terms*—Antennas, electromagnetic scattering, electromagnetic theory, mutual coupling, mutual impedance.**

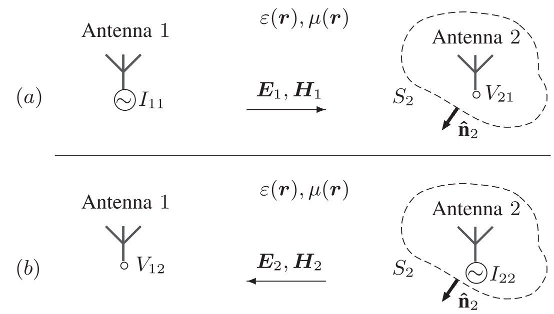

Fig. 1. Original reaction (1). (a) Antenna 1 transmits with a current I_{11}, while Antenna 2 is open circuit. (b) Antenna 2 transmits with a current I_{22}, while Antenna 1 is open circuit. The media is the same in (a) and (b).

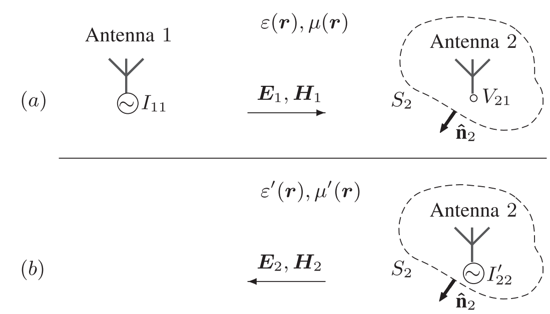

Fig. 2. Generalized reaction (2). (a) Antenna 1 transmits with a current I_{11} in the original environment with Antenna 2 open circuit. (b) Antenna 2 transmits with a current I'_{22} in an alternative environment. The media inside S_2 must be the same in the two environments but can differ outside S_2.

I. Introduction

THE mutual impedance between antennas, and the closely related mutual coupling, is of high importance when installing antennas for simultaneous transmission on platforms [1]. A high mutual coupling means that much of the transmitted energy leaks into the other antenna [2], something that can severely degrade system's performance.

The *reaction* is a measure of the interaction between two pairs of electric and magnetic sources, carried by their corresponding electromagnetic (EM) fields. The reaction, denoted by $\langle 2,1\rangle$, is defined as [3]

$$\langle 2,1\rangle = \oint_{S_2} (\boldsymbol{E}_2 \times \boldsymbol{H}_1 - \boldsymbol{E}_1 \times \boldsymbol{H}_2) \cdot \hat{\mathbf{n}}_2 \, dS \qquad (1)$$

where fields $\boldsymbol{E}_1, \boldsymbol{H}_1$ are generated by Antenna 1 and $\boldsymbol{E}_2, \boldsymbol{H}_2$ by Antenna 2, as shown in Fig. 1. Integration surface S_2, with normal $\hat{\mathbf{n}}_2$, must completely enclose one and only one of the antennas (in this case Antenna 2). To be able to handle inhomogeneous regions, such as a platform or other structure, the complex values $\varepsilon(\boldsymbol{r}), \mu(\boldsymbol{r})$ depend on position $\boldsymbol{r}$.

Reaction $\langle 2,1\rangle$ does not depend on the environment outside integration surface S_2, when generating $\boldsymbol{E}_2, \boldsymbol{H}_2$ [4], which is used in the generalized reaction [4]

$$\langle 2,1\rangle = \oint_{S_2} (\boldsymbol{E}'_2 \times \boldsymbol{H}_1 - \boldsymbol{E}_1 \times \boldsymbol{H}'_2) \cdot \hat{\mathbf{n}}_2 \, dS \qquad (2)$$

where unprimed variables are from the original environment and primed variables are from an alternative environment, as depicted in Fig. 2. The structure outside integration surface S_2, described by $\varepsilon'(\boldsymbol{r}), \mu'(\boldsymbol{r})$, can be changed between the original and alternative environments, but structure enclosed by S_2 must remain unchanged.

The *reaction theorem* relates the field quantities in (1) and (2) with circuit quantities, i.e., currents and voltages defined on the terminal of antennas, see Figs. 1 and 2. For example, it can relate reaction $\langle 2,1\rangle$ with mutual impedance Z_{21} between two sources or antennas

$$Z_{21} = -\frac{\langle 2,1\rangle}{I_{11} I_{22}}. \qquad (3)$$

Terminal current I_{21} in Antenna 2, when Antenna 1 transmits, is assumed to be zero, i.e., an open circuit. Terminal current I_{22} is replaced with I'_{22} in the generalized reaction theorem (2).

The reaction theorem has been used for calculating mutual impedance in several works recently [5]–[8]. It is observed that multiple scattering is neglected in these works, to make the reaction theorem applicable. It is hence interesting to investigate the effects of multiple scattering on the reaction, which is a goal of this paper.

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J. Malmström is with Saab Surveillance, Stockholm S-175 88, Sweden, and also with the Electromagnetic Engineering Laboratory, KTH Royal Institute of Technology, Stockholm 114 28, Sweden (e-mail: johan.malmstrom@ee.kth.se).

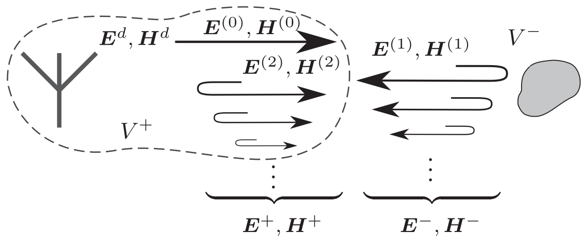

Fig. 3. Decomposition of fields $\boldsymbol{E}, \boldsymbol{H}$ generated from an antenna in V^+ in $\boldsymbol{E}^+, \boldsymbol{H}^+$ and $\boldsymbol{E}^-, \boldsymbol{H}^-$ based on source origin according to (5)–(8).

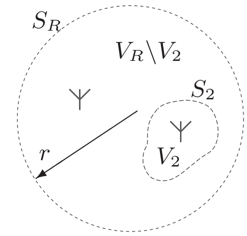

Fig. 4. Surfaces S_R and S_2, enclose volumes V_R and V_2, respectively. Volume $V_R \backslash V_2$ is between the two surfaces S_R and S_2.

II. Reaction for Scattered Fields

A. Field Decomposition

We consider an antenna generating fields $\boldsymbol{E}, \boldsymbol{H}$ that are scattered on some objects, as shown in Fig. 3. We use a fictitious surface to separate the transmitting antenna (if necessary, also including parts of the platform) and scatterers in two disjoint regions V^+ and V^-, as shown in Fig. 3.

The fields from the antenna will induce a current $\boldsymbol{J}$ on the platform and all scattering objects. We note that, from the method of moment approach, given the induced current $\boldsymbol{J}$ and material parameters $\varepsilon(\boldsymbol{r}), \mu(\boldsymbol{r})$, fields $\boldsymbol{E}, \boldsymbol{H}$ can be uniquely determined [9]. Clearly, $\boldsymbol{E}$, $\boldsymbol{H}$ are linearly dependent on $\boldsymbol{J}$. Current $\boldsymbol{J}$ is decomposed into two parts

$$\boldsymbol{J} = \boldsymbol{J}^+ + \boldsymbol{J}^- \tag{4}$$

based on the origin of the currents, so that

$$\boldsymbol{J}^+ = \boldsymbol{J}, \text{ in } V^+ \tag{5}$$

$$\boldsymbol{J}^- = 0, \text{ in } V^+ \tag{6}$$

$$\boldsymbol{J}^+ = 0, \text{ in } V^- \tag{7}$$

$$\boldsymbol{J}^- = \boldsymbol{J}, \text{ in } V^-. \tag{8}$$

Based on the superposition principle, we consider fields $\boldsymbol{E}^+$, $\boldsymbol{H}^+$ to be generated by sources $\boldsymbol{J}^+$ and fields $\boldsymbol{E}^-$, $\boldsymbol{H}^-$ by sources $\boldsymbol{J}^-$, so that

$$\boldsymbol{E} = \boldsymbol{E}^+ + \boldsymbol{E}^- \tag{9}$$

$$\boldsymbol{H} = \boldsymbol{H}^+ + \boldsymbol{H}^-. \tag{10}$$

Scattering components $\boldsymbol{E}^{(n)}$ and $\boldsymbol{H}^{(n)}$ are denoted with their scattering order n, i.e., the number of reflections they have been undergoing, as illustrated in Fig. 3. Summing scattered components with even scattering order produces fields $\boldsymbol{E}^+, \boldsymbol{H}^+$, whereas summing components with odd scattering order produces fields $\boldsymbol{E}^-, \boldsymbol{H}^-$, i.e.,

$$\boldsymbol{E}^+ = \sum_{n=0,2,4,\ldots}^{N} \boldsymbol{E}^{(n)} \tag{11}$$

$$\boldsymbol{E}^- = \sum_{n=1,3,5,\ldots}^{N} \boldsymbol{E}^{(n)} \tag{12}$$

and equivalently for magnetic fields $\boldsymbol{H}^+, \boldsymbol{H}^-$. Consequently, fields $\boldsymbol{E}^+$, $\boldsymbol{H}^+$ are source free in V^-, and fields $\boldsymbol{E}^-$, $\boldsymbol{H}^-$ are source free in V^+. The transmitting antenna is assumed to be in volume V^+. Hence, the direct components $\boldsymbol{E}^d$, $\boldsymbol{H}^d$

$$\boldsymbol{E}^d = \boldsymbol{E}^{(0)} \tag{13}$$

$$\boldsymbol{H}^d = \boldsymbol{H}^{(0)} \tag{14}$$

are included in $\boldsymbol{E}^+, \boldsymbol{H}^+$.

B. Decomposing the Reaction

We apply decomposition (9), (10) to $\boldsymbol{E}_1, \boldsymbol{H}_1$ and $\boldsymbol{E}_2, \boldsymbol{H}_2$ in (1) and let the surface that separates V^+ and V^- coincide with integration surface S_2 in Figs. 1 and 2. Reaction (1) separates into four terms

$$\langle 2,1\rangle = \oint_{S_2} \left(\boldsymbol{E}_2^+ \times \boldsymbol{H}_1^+ - \boldsymbol{E}_1^+ \times \boldsymbol{H}_2^+\right) \cdot \hat{\mathbf{n}}_2 \, dS \tag{15}$$

$$+ \oint_{S_2} \left(\boldsymbol{E}_2^+ \times \boldsymbol{H}_1^- - \boldsymbol{E}_1^- \times \boldsymbol{H}_2^+\right) \cdot \hat{\mathbf{n}}_2 \, dS \tag{16}$$

$$+ \oint_{S_2} \left(\boldsymbol{E}_2^- \times \boldsymbol{H}_1^+ - \boldsymbol{E}_1^+ \times \boldsymbol{H}_2^-\right) \cdot \hat{\mathbf{n}}_2 \, dS \tag{17}$$

$$+ \oint_{S_2} \left(\boldsymbol{E}_2^- \times \boldsymbol{H}_1^- - \boldsymbol{E}_1^- \times \boldsymbol{H}_2^-\right) \cdot \hat{\mathbf{n}}_2 \, dS. \tag{18}$$

C. Radiation Condition

First, we will focus on the second of these four integrals, i.e., (16). Integral (16) depends on fields $\boldsymbol{E}_2^+, \boldsymbol{H}_2^+$ from Antenna 2, and fields $\boldsymbol{E}_1^-, \boldsymbol{H}_1^-$ from Antenna 1 that are scattered on objects enclosed by S_2.

We let a sphere S_R enclose a volume V_R that includes both antennas and surface S_2, see Fig. 4, and calculate reaction (1) over surface S_R. When radius r of the sphere goes to infinity $r \to \infty$, the radiated fields decay as r^{-1} and the cross products as r^{-2}. Since the area of integration goes as r^2, the integral will not obviously converge. We have to convince ourselves that the reaction over the large sphere S_R with the decomposed fields is zero.

We assume that the medium far away from the antennas is homogeneous and isotropic and that the sources are localized. With these assumptions, the EM fields will propagate outwards from the antennas and, asymptotically (i.e., infinitely far away), form a locally plane wave.

For large but finite distances from the source, the wave will not be perfectly plane. We consider the Silver–Müller radiation condition, which gives information about the asymptotic behavior of the fields far from the sources. In [10], the Silver–Müller radiation condition is given as (with a omitted/normalized impedance $Z = 1$)

$$\lim_{r\to\infty} r\,(\boldsymbol{H}\times\hat{\mathbf{n}} - \boldsymbol{E}) = 0. \tag{19}$$

Using the little-o notation,[1] the radiation condition (19) for a large, but finite, r and a general impedance Z can be formulated as a plane wave including a correction term as

$$\boldsymbol{E} = Z(\boldsymbol{H}\times\hat{\mathbf{n}}) + o(r^{-1}) \tag{20}$$

in which it is understood that each field component goes as $o(r^{-1})$. Using (20), we can write the integrand in (16) as

$$\begin{aligned}
&\boldsymbol{E}_2^+\times\boldsymbol{H}_1^- - \boldsymbol{E}_1^-\times\boldsymbol{H}_2^+ \\
&= Z(\boldsymbol{H}_2^+\times\hat{\mathbf{n}}) + o(r^{-1}))\times\boldsymbol{H}_1^- \\
&\quad - (Z(\boldsymbol{H}_1^-\times\hat{\mathbf{n}}) + o(r^{-1}))\times\boldsymbol{H}_2^+ = \\
&= Z\big(\boldsymbol{H}_2^+(\boldsymbol{H}_1^-\cdot\hat{\mathbf{n}}) - \boldsymbol{H}_1^-(\boldsymbol{H}_2^+\cdot\hat{\mathbf{n}}) + (\boldsymbol{H}_1^- - \boldsymbol{H}_2^+)o(r^{-1})\big).
\end{aligned} \tag{21}$$

Far from the source, the dominant parts of $\boldsymbol{H}_1^-$ and $\boldsymbol{H}_2^+$ are orthogonal to the direction of propagation $\hat{\mathbf{n}}$. We know that $\boldsymbol{H}_1^-$, $\boldsymbol{H}_2^+$, and $(\boldsymbol{H}_1^- - \boldsymbol{H}_2^+)$ all decay as r^{-1} for large r. Radial components, $\boldsymbol{H}_1^-\cdot\hat{\mathbf{n}}$ and $\boldsymbol{H}_2^+\cdot\hat{\mathbf{n}}$, decay faster, so that $\boldsymbol{H}_1^-\cdot\hat{\mathbf{n}} = o(r^{-1})$ and $\boldsymbol{H}_2^+\cdot\hat{\mathbf{n}} = o(r^{-1})$. Hence, all terms in (21) are $o(r^{-2})$, and so is the whole integrand, i.e.,

$$\boldsymbol{E}_2^+\times\boldsymbol{H}_1^- - \boldsymbol{E}_1^-\times\boldsymbol{H}_2^+ = o(r^{-2}). \tag{22}$$

This relation implies that integral (16) over the large sphere S_R goes to zero when $r\to\infty$

$$\oint_{S_R}\big(\boldsymbol{E}_2^+\times\boldsymbol{H}_1^- - \boldsymbol{E}_1^-\times\boldsymbol{H}_2^+\big)\cdot\hat{\mathbf{n}}\,dS = 0. \tag{23}$$

Now, still with integral (16) in focus, we consider the region $V_R\backslash V_2$ between surface S_2 and sphere S_R (see Fig. 4)

$$\oint_{S_R - S_2}\big(\boldsymbol{E}_2^+\times\boldsymbol{H}_1^- - \boldsymbol{E}_1^-\times\boldsymbol{H}_2^+\big)\cdot\hat{\mathbf{n}}\,dS. \tag{24}$$

Recall that fields $\boldsymbol{E}_2^+, \boldsymbol{H}_2^+, \boldsymbol{E}_1^-, \boldsymbol{H}_1^-$ per definition (see Fig. 3) are source free, $\boldsymbol{J}_1^- = 0$ and $\boldsymbol{J}_2^+ = 0$, in volume $V_R\backslash V_2$ outside S_2. By considering the integral form of the Lorentz reciprocity theorem [12] in the source-free region $V_R\backslash V_2$, we immediately see that integral (24) evaluates to zero

$$\oint_{S_R - S_2}\big(\boldsymbol{E}_2^+\times\boldsymbol{H}_1^- - \boldsymbol{E}_1^-\times\boldsymbol{H}_2^+\big)\cdot\hat{\mathbf{n}}\,dS = 0. \tag{25}$$

We know from (23) that the surface integral over the large sphere S_R is zero. Combined with (25), it follows that the integral over S_2, i.e., integral (16), must be zero

$$\oint_{S_2}\big(\boldsymbol{E}_2^+\times\boldsymbol{H}_1^- - \boldsymbol{E}_1^-\times\boldsymbol{H}_2^+\big)\cdot\hat{\mathbf{n}}_2\,dS = 0. \tag{26}$$

Changing our focus to the third integral (17) in the decomposed reaction, we see that fields $\boldsymbol{E}_2^+, \boldsymbol{H}_1^-, \boldsymbol{E}_1^-, \boldsymbol{H}_2^+$ are all source free inside integration surface S_2. We can again consider the integral form of the Lorentz reciprocity theorem [12], to see that integral (17) evaluates to zero

$$\oint_{S_2}\big(\boldsymbol{E}_2^-\times\boldsymbol{H}_1^+ - \boldsymbol{E}_1^+\times\boldsymbol{H}_2^-\big)\cdot\hat{\mathbf{n}}_2\,dS = 0. \tag{27}$$

D. Resulting Reaction

Using (23) and (27), reaction $\langle 2,1\rangle$ can be formulated as

$$\begin{aligned}
\langle 2,1\rangle = &\oint_{S_2}\big(\boldsymbol{E}_2^+\times\boldsymbol{H}_1^+ - \boldsymbol{E}_1^+\times\boldsymbol{H}_2^+\big)\cdot\hat{\mathbf{n}}_2\,dS \\
&+ \oint_{S_2}\big(\boldsymbol{E}_2^-\times\boldsymbol{H}_1^- - \boldsymbol{E}_1^-\times\boldsymbol{H}_2^-\big)\cdot\hat{\mathbf{n}}_2\,dS.
\end{aligned} \tag{28}$$

Note that there is no interaction between fields $(\cdot)^+$ and $(\cdot)^-$ from the different antennas.

The generalized reaction (2) can also be separated into scattering components. It becomes especially simple if the alternative environment where Antenna 2 transmits, as depicted in Fig. 2(b), is chosen to be homogeneous (e.g., vacuum) outside integration surface S_2. With this choice, there will be no scattering from outside S_2, and hence, $\boldsymbol{E}_2^+ = \boldsymbol{E}_2^d$, $\boldsymbol{H}_2^+ = \boldsymbol{H}_2^d$, $\boldsymbol{E}_2^- = 0$, $\boldsymbol{H}_2^- = 0$, using the notation introduced in (13) and (14) and Fig. 3. Fields $\boldsymbol{E}_2^d$, $\boldsymbol{H}_2^d$, $\boldsymbol{E}_1^-$, $\boldsymbol{H}_1^-$ have their sources within S_2 and are source free outside S_2. The same derivations as in the previous sections can be repeated for the generalized reaction (2), resulting in the following expression:

$$\langle 2,1\rangle = \oint_{S_2}\big(\boldsymbol{E}_2^d\times\boldsymbol{H}_1^+ - \boldsymbol{E}_1^+\times\boldsymbol{H}_2^d\big)\cdot\hat{\mathbf{n}}\,dS \tag{29}$$

which is valid for an empty alternative environment. Note that components $\boldsymbol{E}_1^-, \boldsymbol{H}_1^-$, generated by Antenna 1 and scattered inside surface S_2, do not contribute to the reaction.

III. VERIFICATION

We verify the results for a pair of Hertzian dipoles between infinite ground planes, as depicted in Fig. 5. The configuration will give strong multiple scattering. The EM fields can be determined using the method of mirroring and the analytically known fields from a Hertzian dipole in free space [12].

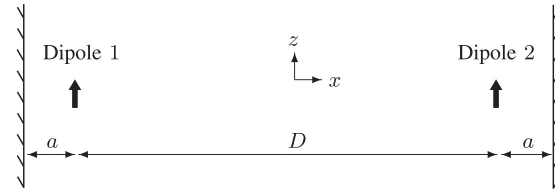

Fig. 5. Two high-scattering antennas, each consisting of a Hertzian dipole in front of an infinite ground plane, facing each other.

[1] For non-zero $g(r)$, the little-o notation $f(r) = o\big(g(r)\big)$ implies that

$$\lim_{r\to\infty}\frac{f(r)}{g(r)} = 0.$$

Additionally, the little-o notation is related to the big-O notation. If a function $f(r) = o(r)$, it implies that $f(r) = \mathcal{O}(r)$ [11].

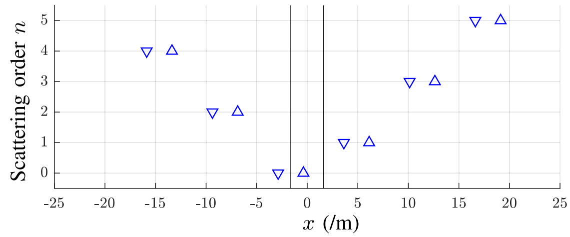

Fig. 6. Mirroring of Dipole 1 with positive (up-pointing triangle) and negative (down-pointing triangle) Hertzian dipoles between two ground planes at $x = \pm 2$ m. The vertical axis denotes scattering order n.

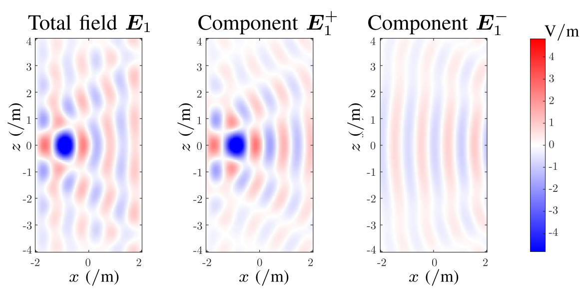

Fig. 7. z Component of electric field $\Re\boldsymbol{E}_1$ (left) from Dipole 1 decomposed in field $\Re\boldsymbol{E}_1^+$ (middle), and left-propagating field $\Re\boldsymbol{E}_1^-$ (right), illustrated on plane $x = [-2, 2], y = 0, z = [-4, 4]$.

Starting with Dipole 1, we determine the fields after mirroring in the left ground plane. The resulting two Hertzian dipoles in Fig. 6 with scattering order $n = 0$ produce fields $\boldsymbol{E}_1^{(0)}$, $\boldsymbol{H}_1^{(0)}$ in the region between the ground planes. Next, the two Hertzian dipoles with scattering order $n = 0$ are mirrored in the right ground plane. The resulting two Hertzian dipoles, with scattering order $n = 1$ in Fig. 6, produce left-propagating fields $\boldsymbol{E}_1^{(1)}$, $\boldsymbol{H}_1^{(1)}$ between the ground planes.

By continuing the alternating mirroring on the two ground planes, we get mirrored dipoles for higher scattering orders n, as illustrated in Fig. 6. The sources will be farther away for each mirroring and give weaker contribution to the fields in the region between the ground planes. Referring to Fig. 6, we can convince ourselves that summing the fields with scattering order $n = 0, n = [1, 2], n = [3, 4], \ldots$ produces null-fields on the left ground plane, whereas summing the fields with scattering order $n = [0, 1], n = [2, 3], \ldots$ produces null-fields on the right ground plane.

We let the integration surface S_2 enclose the half-space $x > 0$ and use S_2 is used to decompose $\boldsymbol{E}_1, \boldsymbol{H}_1$ generated by Dipole 1 in scattering components $\boldsymbol{E}_1^+$, $\boldsymbol{E}_1^-$, and $\boldsymbol{H}_1^+$, $\boldsymbol{H}_1^-$, according to (9), (10) and (11), (12).

The same procedure is repeated with Dipole 2, to decompose fields $\boldsymbol{E}_2, \boldsymbol{H}_2$ into $\boldsymbol{E}_2^+$, $\boldsymbol{E}_2^-$ and $\boldsymbol{H}_2^+$, $\boldsymbol{H}_2^-$.

Based on fields $\boldsymbol{E}_1$, $\boldsymbol{H}_1$, $\boldsymbol{E}_2$, $\boldsymbol{H}_2$, and their decomposed fields $\boldsymbol{E}_1^+$, $\boldsymbol{H}_1^+$, $\boldsymbol{E}_1^-$, $\boldsymbol{H}_1^-$, $\boldsymbol{E}_2^+$, $\boldsymbol{H}_2^+$, $\boldsymbol{E}_2^-$, $\boldsymbol{H}_2^-$, we evaluate two formulations of the reaction; the original reaction (1), and the reactions for scattered fields (28) derived in this letter. We use a dipole separation distance $D = 1.8$ m, with each dipole placed $a = 1.1$ m from the ground plane, and a wavelength $\lambda = 1$ m. Scattering components up to order $N = 12$ are included. Field

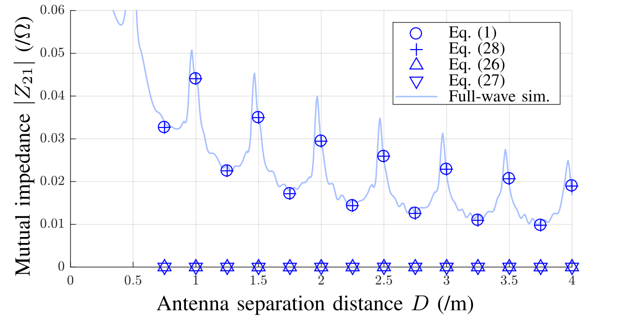

Fig. 8. Mutual impedance magnitude $|Z_{21}|$ calculated with the reaction as in (1), and (28). For illustration, integrals (26) and (27) are also plotted. We see that the agreement of (1) and (28) is excellent and that (26) and (27) are both very close to zero.

$\boldsymbol{E}_1$ from Dipole 1, decomposed into fields $\boldsymbol{E}_1^+$ and $\boldsymbol{E}_1^-$, is depicted in Fig. 7.

The results, in terms of mutual impedance using (3), are depicted in Fig. 8. We see that the agreement between the mutual impedance calculated with (1) and (28) is excellent. The results are also close to the mutual impedance calculated with a full wave method. As expected, the reaction from the scattering components (26) and (27) are very close to zero. The results from this strong scattering case indicate that the derived relation (28) for the reaction is correct.

IV. Discussion and Conclusion

Reaction theorems can be formulated in several ways, where formulations (1) and (2) are well known. In this letter, we derive two novel formulations of the reaction for scattered fields, see (28) and (29). The derived formulations of the reaction are expressed for fields decomposed in their scattered components, where the decomposition is based on the location of sources. It is shown that not all scattering components contribute to the reaction.

The resulting reaction between sources must not change depending on the chosen formulation of the reaction. The classical formulations of the reaction, (1) and (2), include combinations of scattering components that do not contribute to the reaction.

In (28), we see that scattered fields from sources on one side of the integration surface only interact with scattered fields from sources on the opposite side of the surface.

With formulation (29) of the reaction, we see that none of the odd scattering components ($\boldsymbol{H}_1^-$, $\boldsymbol{E}_1^-$) contribute to the reaction, if the alternative environment where Antenna 2 transmits is chosen to be homogeneous outside integration surface S_2.

If multiple scattering is neglected, as in [5]–[8], we see in the novel formulation (29) that the lowest scattering component that contributes to the approximation error is not of first but of second order.

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