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Derivations of Nest Algebras

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Introduction

The results presented below are generalisations of some well known results about von Neumann algebras. The first result, Theorem 2.1, yields a sufficient condition for automatic continuity of any derivation of an algebra of operators into a dual normal module.

The theorem is followed by a corollary which shows that reflexive algebras with commutative subspace lattices have only continuous derivations into dual normal modules.

The second result, Theorem 3.6, is quite independent of the former. In Section 3 we prove that, for any nest algebra $\mathscr{A}$ and any ultraweakly closed algebra $\mathscr{B}$ of operators, the first cohomology group $H^1(\mathscr{A},\mathscr{B})$ always vanishes. We ought perhaps to mention, that nest algebras are reflexive and do have commutative lattices of invariant subspaces.

1. Notation

We will always let H denote a complex Hilbert space, $B(H)$ stands for the algebra of all bounded operators on H, and the letters $\mathscr{A}$ and $\mathscr{B}$ refer to algebras of operators (bounded) on H.

Let $\mathscr{A}$ be an ultraweakly closed algebra of operators on a Hilbert space H. The Banach space M is said to be a dual normal Banach $\mathscr{A}$ module if M is a dual space, there exist continuous left and right multiplications from $\mathscr{A}\times M$ and $M\times\mathscr{A}$ into M, and for any m in M the maps $\mathscr{A}\ni a\to am$ and $\mathscr{A}\ni a\to ma$ are ultraweak to weak-star continuous.

A linear map δ of $\mathscr{A}$ into M is said to be a derivation if $\delta(ab)=a\delta(b)+\delta(a)b$ [12].

Let $\mathscr{A}$ be an algebra of operators on a Hilbertspace H; then $\mathrm{Lat}(\mathscr{A})$ denotes the lattice of closed invariant subspaces for $\mathscr{A}$. If L is a set of closed subspaces of H, then $\mathrm{Alg}(L)$ denotes the algebra of all operators which leave any space in L invariant. An algebra $\mathscr{A}$ is said to be reflexive if $\mathscr{A}=\mathrm{Alg}(\mathrm{Lat}(\mathscr{A}))$. A lattice L is said to be commutative if the corresponding orthogonal projections all commute.

We note, that a nest algebra is a reflexive algebra with totally ordered subspace lattice [8].

Finally we remark, that if x is an invertible element in an algebra $\mathscr{A}$ then $\mathrm{Ad}(x)$ denotes the isomorphism $\mathscr{A}\ni a\mapsto xax^{-1}$, and if x is arbitrary in $\mathscr{A}$ then $\mathrm{ad}(x)$ denotes the derivation $\mathscr{A}\ni a\mapsto[x,a]=xa-ax$.

2. Continuity of Derivations

The following section is very much inspired by Ringroses paper [11].

The details in our proof of automatic continuity of certain derivations are not identical, but the continuity ideal, the application of the uniform boundedness principle, and the $2^{-n}, 2^n$ argument are found in both papers.

Our result says nothing if an algebra only possesses a finite dimensional subspace of self-adjoint operators. Ringrose also had this kind of finite dimensional problem, but in the self-adjoint algebra case, the final step is elementary.

2.1. Theorem. *Let $\mathscr{A}$ be an ultraweakly closed algebra of operators on a Hilbert space H, M a dual normal Banach $\mathscr{A}$-module, and δ a derivation of $\mathscr{A}$ into M. Then there exists a central projection p in $\mathscr{A}\cap\mathscr{A}^*$ such that $(\mathscr{A}\cap\mathscr{A}^*)(I-p)$ is finite dimensional and the map $a\to\delta(ap)$ is bounded.*

Proof. Let $\mathscr{B}$ denote the weakly closed self-adjoint algebra $\mathscr{A}\cap\mathscr{A}^*$, and let P be the set of projections in $\mathscr{B}$ which have the property that for p in P the map $a\to\delta(ap)$ is bounded. The first part of the proof shows that P is both directed upwards and inductively ordered, so P has a last (largest) element.

Let $(e_\alpha)_{\alpha\in A}$ be a totally ordered set of projections from P, and let $e=\bigvee_{\alpha\in A} e_\alpha$.

The restriction of δ to $\mathscr{B}$ is bounded, say by a constant k, therefore, the derivation property shows that

$$\forall a\in\mathscr{A}\,\forall\alpha\in A: \|\delta(ae_\alpha)\| \leqq \|\delta(a)\| + k\|a\|. \tag{1}$$

It is now clear that the orbits $\{\delta(ae_\alpha)|\alpha\in A\}$ are bounded for any a in $\mathscr{A}$, so the Banach-Steinhaus Theorem yields that the set of mappings $a\to\delta(ae_\alpha)$ is bounded.

Using the derivation property and the ultraweak to weak-star continuity of the restriction of δ to $\mathscr{B}$, we see that e belongs to P.

$$\begin{aligned}\forall a\in\mathscr{A}: \delta(ae) &= \delta(a)e + a\delta(e) = \lim(\delta(a)e_\alpha + a\delta(e_\alpha))\\ &= \lim\delta(ae_\alpha).\end{aligned}$$

Now let e and f be projections in P and let g_n be the function $g_n(t)=t^{-1}$ for $n^{-1}\leqq t\leqq 2$ and zero elsewhere. Then the sequence of projections $h_n=(e+f)[g_n(e+f)]$ is increasing, with least upper bound equal to $e\vee f$. The previous discussion shows that $e\vee f$ belongs to P, and, by Zorn's lemma, P must have a last element, say p. It is rather obvious that p must be central in $\mathscr{B}$, but for the sake of completeness let us assume that u is a unitary in $\mathscr{B}$, and consider the map $a\to\delta(aupu^*)$. Here we find $\delta(aupu^*)=\delta(aup)u^*+aup\delta(u^*)$, so upu^* belongs to P, and p must be central in $\mathscr{B}$.

Let us now assume that $\mathscr{B}(I-p)$ is not finite dimensional, then there exists a sequence $(q_i)_{i\in\mathbb{N}}$ of nontrivial pairwise orthogonal projections in $\mathscr{B}$ with sum $(I-p)$. For each i in $\mathbb{N}$ the map $a\to\delta(aq_i)$ is not bounded, therefore, we find, as usual, a sequence $(a_i)_{i\in\mathbb{N}}$ such that $\|a_i\|<2^{-i}$ and $\|\delta(a_iq_i)\|>2^i$.

Let $a=\sum_{i=1}^{\infty} a_iq_i$; then

$$\delta(a_iq_i)=\delta(aq_i)=\delta(a)q_i+a\delta(q_i)$$

and

$$2^i<\|\delta(a_iq_i)\|\leqq\|\delta(a)\|+k\|a\|\,.$$

We have then obtained a contradiction, and $\mathscr{B}(1-p)$ must be finite dimensional.

2.2. Corollary. *If $\mathscr{A}\cap\mathscr{A}^*$ contains no finite dimensional central summand, then δ is continuous.*

2.3. Corollary. *If $\mathscr{A}$ is reflexive and* Lat($\mathscr{A}$) *is commutative, then δ is continuous.*

Proof. If Lat($\mathscr{A}$) is commutative and $\mathscr{A}$ is reflexive, then the set $\mathscr{E}$ of orthogonal projections onto the members of Lat($\mathscr{A}$) is contained in $\mathscr{A}$.

Suppose δ is not continuous; then $\mathscr{A}$ contains a minimal projection q which has the property that the map $a\to\delta(aq)$ is not continuous.

Since $\mathscr{A}=\mathrm{Alg}(\mathrm{Lat}(\mathscr{A}))$, we find that $\mathscr{A}\cap\mathscr{A}^*=\mathscr{E}'$, the commutant of the set $\mathscr{E}$, and, by ([5], Proposition 1, p. 16),

$$(\mathscr{E}'')_q=((\mathscr{E}')_q)'=B(qH)\,.$$

The commutativity of $\mathscr{E}''$ implies that qH must be one dimensional, and there exist finitely many pairwise orthogonal one dimensional projections $q_1\dots q_n$ in $\mathscr{A}$ such that $(I-p)=q_1+\dots+q_n$.

An easy check in the proof of Theorem 2.1 shows that there is a central projection r in $\mathscr{A}\cap\mathscr{A}^*$ and a set $s_1\dots s_m$ of pairwise orthogonal one dimensional projections such that $(I-r)=s_1+\dots+s_m$ and the map $a\to\delta(ra)$ is bounded. For any $(i,j)\in\{1,\dots,m\}\times\{1,\dots,n\}$ the map $\mathscr{A}\ni a\to\delta(s_iaq_j)$ is continuous, because $s_i\mathscr{A}q_j$ is an at most one dimensional subspace of $\mathscr{A}$. The following equality then shows that δ is continuous, and the corollary follows.

$$\delta(x)=\delta(xp)+\delta(rx(1-p))+\sum_{i=1}^{m}\sum_{j=1}^{n}\delta(s_ixq_i)\,.$$

3. Automorphisms and Derivations on Nest Algebras

Nest algebras were introduced by Ringrose in [9]. A reflexive algebra of operators on a Hilbert space is said to be a nest algebra if the lattice of invariant subspaces is totally ordered. In the following article [10] Ringrose showed that certain automorphisms of nest algebras are inner. We use this result together with a general result, Theorem 3.2 below, to show that any derivation and any automorphism on a nest algebra is inner.

The general content of this section is probably best understood if we introduce the following concept.

3.1. Definition. Let $\mathscr{A}$ be an ultraweakly closed unital algebra of operators on a Hilbert space H, and let r and s be positive real numbers.

$\mathscr{A}$ is said to have the automorphism implementation property (r, s) [AIP(r, s) for short], if any automorphism α of $\mathscr{A}$ with $\|\alpha - id\| \leqq r$ is implemented by an operator x in $\mathscr{A}$ such that $\|I - x\| \leqq s\|\alpha - \mathrm{id}\|$.

The procedure is now to show that if an algebra $\mathscr{A}$ has the AIP(r, s) for some pair (r, s), then any derivation on $\mathscr{A}$ is inner. Next, we turn to the case where the algebra $\mathscr{A}$ is reflexive, with commutative lattice. We show that $H^1(\mathscr{A}, \mathscr{A}) = H^1(\mathscr{A}, \mathscr{B})$ for any ultraweakly closed algebra $\mathscr{B}$ on H containing $\mathscr{A}$.

This is followed by the result that nest algebras have the AIP. Therefore, we finally obtain $H^1(\mathscr{A}, \mathscr{A}) = 0$ in these cases.

3.2. Theorem. *Let $\mathscr{A}$ be a weakly closed unital algebra of operators on a Hilbert space H. Suppose $\mathscr{A}$ has the* AIP(r, s) *for some pair (r, s) in $\mathbb{R}^+ \times \mathbb{R}^+$; then $H^1(\mathscr{A}, \mathscr{A}) = 0$.*

Proof. Let δ be a bounded derivation of $\mathscr{A}$ into $\mathscr{A}$; then $\alpha_t = \exp(t\delta)$ is a uniformly continuous automorphism group on $\mathscr{A}$ ([5], III, 9,4). The uniform continuity implies that there is a positive real t_0 such that $\|\mathrm{id} - \alpha_t\| \leqq r$ whenever $0 \leqq |t| < t_0$.

If $|t| < t_0$, then choose x_t in $\mathscr{A}$ such that $\alpha_t = \mathrm{Ad}(x_t)$ and $\|I - x_t\| \leqq s\|\mathrm{id} - \alpha_t\|$. The last inequality yields that the set of operators $\{(I - x_t)t^{-1} | 0 < t < t_0\}$ forms a bounded set. If we let t decrease to 0 and take an ultraweakly convergent subnet $((I - x_{t_\gamma})t_\gamma^{-1})_{\gamma \in D}$ with limit y, then, for any a in $\mathscr{A}$,

$$\begin{aligned} [-y, a] &= -\lim_{\gamma \in D} ([(I - x_{t_\gamma}), a] t_\gamma^{-1}) \\ &= \lim_{\gamma \in D} ([x_{t_\gamma}, a] t_\gamma^{-1}) = \lim_{\gamma \in D} (t_\gamma^{-1}(x_{t_\gamma} a x_{t_\gamma}^{-1} - a) x_{t_\gamma}) \\ &= \lim_{\gamma \in D} (t_\gamma^{-1}(\alpha_{t_\gamma}(a) - a) x_{t_\gamma}) = \delta(a). \end{aligned}$$

The last equality is standard and found, for instance, in [13], Theorems 13.35, 13.36.

3.3 Corollary. *Any uniformly continuous one parameter automorphism group on $\mathscr{A}$ has the form* Ad(exp(ta)).

Proof. Immediate.

3.4. Proposition. *Let $\mathscr{A}$ be a reflexive algebra on a Hilbert space H, and let $\mathscr{B}$ be any ultraweakly closed algebra on H which contains $\mathscr{A}$. If* Lat$(\mathscr{A})$ *is commutative, then $H^1(\mathscr{A}, \mathscr{A}) = H^1(\mathscr{A}, \mathscr{B})$.*

Proof. Let $\delta \in Z^1(\mathscr{A}, \mathscr{B})$; then; since $\mathscr{A} \cap \mathscr{A}^*$ is a type I von Neumann algebra, we can use the averaging technique ([12]) and find an x in $\mathscr{B}$ such that the derivation $\delta_1 = \delta - adx$ vanishes on $\mathscr{A} \cap \mathscr{A}^*$. On the other hand, all the orthogonal projections corresponding to the members of Lat$(\mathscr{A})$ are included in $\mathscr{A} \cap \mathscr{A}^*$, since Lat$(\mathscr{A})$ is commutative.

Therefore, for any a in $\mathscr{A}$, and any p in $\mathrm{Lat}(\mathscr{A})$,

$$\delta_1(a)p=\delta_1(ap)=\delta_1(pap)=p\delta_1(ap)$$

and we find, that $\delta_1(a)$ belongs to $\mathscr{A}$.

Since $B^1(\mathscr{A},\mathscr{A})$ is contained in $B^1(\mathscr{A},\mathscr{B})$ and $Z^1(\mathscr{A},\mathscr{A})$ is contained in $Z^1(\mathscr{A},\mathscr{B})$, there exists a canonical linear map κ of $H^1(\mathscr{A},\mathscr{A})$ into $H^1(\mathscr{A},\mathscr{B})$. The remarks above show that κ is surjective.

Suppose now that $\delta\in Z^1(\mathscr{A},\mathscr{A})$ and $\delta=\mathrm{ad}\,b$ for some b in $\mathscr{B}$, then for any projection p in $\mathrm{Lat}(\mathscr{A})$, p and $\delta(p)$ belong to $\mathscr{A}$; but this implies that $(1-p)bp=(1-p)[b,p]p=0$. The operator b then belongs to $\mathscr{A}$, and it is clear that κ is injective and hence an isomorphism.

3.5. Remark. The averaging techniques can be used to show that if $H^n(\mathscr{A},\mathscr{A})=0$ for some natural number n, then also $H^n(\mathscr{A},\mathscr{B})=0$. The only extra tool one needs is Lemma 4.1 of [12].

The rest of the paper studies AIP for nest algebras. The proof is, as mentioned above, based upon a result of Ringrose [10] which shows that certain automorphisms of nest algebras are always inner.

3.6. Lemma. *The commutant of a nest algebra is trivial.*

Proof. Let $\mathscr{A}$ be a nest algebra on a Hilbert space H and suppose p is a projection onto an invariant subspace for $\mathscr{A}$. The set of operators $pB(H)(I-p)$ is contained in $\mathscr{A}$ since every operator in the set will leave invariant any subspace that is either smaller or bigger than pH.

Assume x is in the commutant of $\mathscr{A}$, ξ is in $(I-p)H$ and η belongs to pH. If $\xi\otimes\eta$ denotes the operator $\gamma\to(\gamma|\xi)\eta$, then

$$\xi\otimes\eta\in pB(H)(1-p) \quad\text{and}\quad (x^*\xi)\otimes\eta=\xi\otimes(x\eta).$$

Since this equation holds for any such pair (ξ,η), it is clear that x must be a multiple of the identity.

3.7. Lemma. *Let $\mathscr{A}$ be a nest algebra on a Hilbert space H. Then, for any operator x in $B(H)$,*

$$\inf\{\|x-\lambda I\|\,|\,\lambda\in\mathbb{C}\}\leqq 3\sup\{\|xa-ax\|\,|\,a\in\mathscr{A},\ \|a\|\leqq 1\}.$$

Proof. If $\mathrm{Lat}(\mathscr{A})$ is trivial, $\mathscr{A}=B(H)$, and the Lemma is a consequence of Stampfli's result [15]. Suppose p is a nontrivial element in $\mathrm{Lat}(\mathscr{A})$; then we find, as above, that the set of operators $pB(H)(I-p)$ is contained in $\mathscr{A}$.

Let $\mathscr{E}$ be the von Neumann algebra generated by the projections in $\mathrm{Lat}(\mathscr{A})$. Then $\mathscr{E}$ is a commutative and $\mathscr{E}'$, the commutant, a type I von Neumann algebra.

For any x in $B(H)$ the ultraweakly closed convex hull of the set $\{uxu^*|u$ unitary in $\mathscr{E}'\}$ has a point, say y, in common with $(\mathscr{E}')'=\mathscr{E}$ ([3] or [12]).

This y also satisfies $\|\mathrm{ad}(y)|\mathscr{A}\|\leqq\|\mathrm{ad}(x)|\mathscr{A}\|$ $(\mathrm{ad}(y)(a)=[y,a])$, because for any unitary u in $\mathscr{E}'$, $\|[uxu^*,a]\|=\|[x,u^*au]\|$ and u^*au belongs to $\mathscr{A}$. Define $z=yp$ and $v=y(I-p)$, then z and v are normal operators in the abelian von Neumann algebra $\mathscr{E}$. Let $k=\|\mathrm{ad}(x)|\mathscr{A}\|$.

For any pair of vectors ξ *in* $(I-p)H$ and η in pH we find that

$$\mathrm{ad}(y)(\xi\otimes\eta)=\xi\otimes(z\eta)-(v^*\xi)\otimes\eta,$$

and

$$\|\xi\otimes(z\eta)-(v^*\xi)\otimes\eta\|\leqq k\|\xi\|\,\|\eta\|.$$

For any pair of unit vectors ξ,η in $(I-p)H$ and pH, respectively, we then obtain

$$|([\xi\otimes(z\eta)-(v^*\xi)\otimes\eta]\xi|\eta)|\leqq k$$

or

$$|(z\eta|\eta)-(v\xi|\xi)|\leqq k. \tag{1}$$

Let w_z and w_v denote the numerical ranges of the operators z and v on the spaces pH and $(I-p)H$. The spectrum of y is then contained in the union of the sets w_z and w_v, and the relation (1) shows that the diameter of the spectrum of y can not exceed $2k$. Since y is normal, we get that $\inf\|y-\lambda I\|\leqq 2k$ or $\inf\|x-\lambda I\|\leqq 3k=3\|\mathrm{ad}(x)|\mathscr{A}\|$.

3.8. Remark. The lemma above can be extended a bit to show that, for any nontrivial projection p in $B(H)$, the algebra $\mathscr{B}=\mathbb{C}p+\mathbb{C}(I-p)+pB(H)(I-p)$ has trivial commutant, and, moreover, for any selfadjoint z in $B(H)$,

$$\inf\{\|z-tI\|\,|\,t\in\mathbb{R}\}\leqq 6\|\mathrm{ad}(z)|\mathscr{B}\|.$$

3.9. Theorem. *Let $\mathscr{A}$ be a nest algebra on a Hilbert space H, and let Φ be a homomorphism of $\mathscr{A}$ into $B(H)$. Suppose $\|\Phi-\mathrm{id}\|\leqq 1/24$; then there exists an invertible operator x on H such that*

$$\|I-x\|\leqq 26\|\Phi-\mathrm{id}\|$$

and $\Phi=\mathrm{Ad}(x)|\mathscr{A}$.

Proof. We use the terminology from the lemma above. Since $\mathscr{E}'$ is a type I von Neumann algebra, the averaging technique applies. G_0 is found as an amenable subgroup of the unitaries in $\mathscr{E}'$, such that G_0 generates $\mathscr{E}'$ as a von Neumann algebra. The ultraweakly closed convex hull K of the set $\{\Phi(u)u^*|u\in G_0\}$ contains a point y with the following properties

$$\forall u\in G_0:\Phi(u)yu^*=y,\quad\text{and}\quad \|I-y\|\leqq\|\Phi-\mathrm{id}\|.$$

If $\|\Phi-\mathrm{id}\|<1/24$, then y is invertible and $\Psi=\mathrm{Ad}(y^{-1})\circ\Phi$ is an automorphism on $\mathscr{A}$ which leaves $\mathscr{E}'$ elementwise fixed and $\|\Psi-\mathrm{id}\|<4\|\Phi-\mathrm{id}\|$.
In fact take a in $\mathscr{A}$ and p in $\mathrm{Lat}(\mathscr{A})$; then $\Psi(a)p=\Psi(a)\Psi(p)=\Psi(ap)=p\Psi(ap)$, and $\Psi(a)$ belongs to $\mathscr{A}$.

$$\|I-y\|\leqq t=\|\Phi-\mathrm{id}\|\Rightarrow\|y^{-1}\|\leqq(1-t)^{-1},\ \|y\|\leqq 1+t.$$
$$\|I-y^{-1}\|=\|y^{-1}(1-y)\|\leqq t(1-t)^{-1}.$$
$$\|\Psi(a)-a\|\leqq\|y^{-1}(\Phi(a)-a)y)\|+\|y^{-1}ay-a\|\leqq(3t+t^2)(1-t)^{-1}\|a\|.$$

Since $t<1/24$, we obtain $\|\Psi-\mathrm{id}\|<4\|\Phi-\mathrm{id}\|$.

Since Ψ is an automorphism on $\mathscr{A}$ which leaves $\mathscr{E}'$ elementwise fixed, we can, by [12], Theorem 3.1, find z in $\mathscr{A}$ such that $\Psi = \mathrm{Ad}(z)|\mathscr{A}$ and $\|z\| = 1$. As before, we find, considering the maps as maps in $L(\mathscr{A}, \mathscr{A})$,

$$\|\mathrm{ad}(z)\| \leqq \|z\| \; \|\mathrm{Ad}(z) - \mathrm{id}\| \leqq \|\Psi - \mathrm{id}\| < 4\|\Phi - \mathrm{id}\| \leqq 4t\,.$$

Therefore, $d(z, \mathbb{C}) < 12t < \frac{1}{2}$.

Choose $\lambda \in \mathbb{C}$ such that $\|z - \lambda I\| < 12t$; hence,

$$|\lambda| \geqq 1 - 12t > \tfrac{1}{2} \quad \text{and} \quad \|\lambda^{-1} z - I\| < 24t \leqq 1\,.$$

Finally, we can conclude that, with $x = \lambda^{-1} yz$, $\mathrm{Ad}(x)|\mathscr{A} = \Phi$ and $\|x - I\| \leqq \|y(I - \lambda^{-1} z)\| + \|y - I\| \leqq 26\|\Phi - \mathrm{id}\|$.

3.10. Corollary. *$\mathscr{A}$ has the* AIP(1/24, 26).

Proof. Immediate.

3.11. Corollary. *Suppose $\mathscr{B}$ is an ultraweakly closed algebra of operators on H containing $\mathscr{A}$, then $H^1(\mathscr{A}, \mathscr{B}) = 0$.*

Proof. By Proposition 3.4, $H^1(\mathscr{A}, \mathscr{A}) = H^1(\mathscr{A}, \mathscr{B})$. Corollary 3.10, together with Theorem 3.2, yields that $H^1(\mathscr{A}, \mathscr{A}) = 0$.

3.12. Corollary. *Any uniformly continuous one parameter group of automorphisms on $\mathscr{A}$ has the form* Ad(exp(ta)) *for some a in $\mathscr{A}$.*

Proof. See Corollary 3.3.

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