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# **Deriving a Tree Growth Model from Any Existing Stand Growth Model**

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# Deriving a Tree-Level Growth Model from Any Existing Stand-Level Growth Model

## Abstract

In this study, a new method was developed to derive a tree survival and diameter growth model from any existing stand-level model, without the need for individual-tree growth data. Predictions from the derived tree model are constrained to match number of trees and basal area per ha as outputted by the stand model. The tree models derived from three different stand models were evaluated against a tree model, in both unadjusted and disaggregated forms.

For the same stand-level model, the derived tree model outperformed its counterpart, the disaggregated tree model. Furthermore, except for one stand model with poor performance, the tree models derived from the remaining two stand models delivered comparable results to those obtained from the unadjusted tree model. The tree model derived from one stand model even performed slightly better than the unadjusted tree model. This is significant because the coefficients of the unadjusted and disaggregated tree models had to be estimated from tree-level growth data, whereas the derived tree model required no tree growth data at all. The methodology presented in this study should be applicable when there is no ingrowth or recruitment.

**Keywords:** disaggregation; individual-tree model; least squares; seemingly unrelated regression.

## 22 **1. Introduction**

23 Growth and yield models have been extensively used by forest managers in order to make  
24 informed decisions on managing forest resources. These models can produce outputs that ranged  
25 from high-resolution (individual-tree simulation models), to medium-resolution (size-class  
26 models), to low-resolution (whole-stand models) (Burkhart and Tomé 2012).

27 Whole-stand models are relatively simple models that provide information for the entire  
28 stand. The predicted stand attributes can be stand survival (Zhang et al. 1993, Diéguez-Aranda  
29 et al. 2005, Tewari et al. 2014, Stankova 2016), basal area per unit area (Cao and Durand 1991,  
30 Barrio Anta et al. 2006, Naing 2020), or both (Somers and Farrar 1991, Erikäinen 2002, Garcia  
31 2011, Dean et al. 2013).

32 Size-class models deal with diameter classes. These models can be stand table-projection  
33 models that projects the number of trees in each diameter class into the future (Clutter and Jones  
34 1980, Nepal and Somers 1992, Cao and Baldwin 1999, Allen et al. 2011), or diameter-  
35 distribution models that use a probability density function (pdf) to model the frequency of tree  
36 diameters (Smalley and Bailey 1974, Matney and Sullivan 1982, Jiang and Brooks 2009,  
37 Carretero and Alvarez 2013).

38 Individual-tree models deliver detailed information for each tree. This information can  
39 be tree survival (Guan and Gerner 1991a, 1991b, Monserud and Sterba 1999, Kjell and Lennart  
40 2005, Cao 2006, 2017), tree diameter growth (Andreassena and Tomter 2003, Sánchez-González  
41 et al. 2006, Subedi and Sharma 2011, Bohora and Cao 2014), or both tree survival and diameter  
42 growth (Cao 1994, 2000, Palahía et al. 2003, Coble et al. 2012, Sun et al. 2019).

43 Because outputs from models of different resolutions might be inconsistent with one  
44 another, linking models having different levels of resolution have recently received a lot of

45 attention. Bridges have been established to connect a whole-stand model to a diameter-  
46 distribution model (Matney and Sullivan 1982; Baldwin and Feduccia 1987), to a stand table  
47 projection model (Clutter and Jones 1980, Nepal and Somers 1992, Cao and Baldwin 1999, Cao  
48 2007, Allen et al. 2011), or to an individual-tree model (Yue et al. 2008, Zhang et al. 2010,  
49 Hevia et al. 2015, Cao 2014, 2017). The latter is called the disaggregation approach (Ritchie and  
50 Hann 1997), in which information obtained from the tree model is used to disaggregate stand  
51 growth (predicted by a whole-stand model) among trees in the tree list. Recently, Cao (2019)  
52 showed how one can derive a tree survival model from any existing stand survival model; the  
53 level of accuracy and precision depended on the stand model performance and on whether or not  
54 tree-level survival data were available.

55 The objective of this study was to develop a method to derive a tree-level growth model  
56 for tree survival and diameter growth predictions from stand survival and basal area values  
57 predicted from any existing stand-level model.

58

## 59 **2. Data**

60 Data used in this study were from the Southwide Seed Source Study, which included 15  
61 loblolly pine (*Pinus taeda* L.) seed sources planted at 13 locations across 10 southern states  
62 (Wells and Wakeley 1966). A total of 200 plots were randomly selected from this data set; each  
63 0.0164 ha plot consisted of 49 trees, planted at a 1.8 m × 1.8 m spacing. Only one 5-year growth  
64 period was randomly selected for each plot to avoid correlation problems caused by repeated  
65 measurements. Measurements for growth periods 10-15, 15-20, and 20-25 years were randomly  
66 divided into two groups of 100 plots each. The distribution of number of plots for each growth

67 period is presented in Table 1. Table 2 shows the means and standard deviations of stand-level  
68 and tree-level attributes.

69 The two-fold evaluation scheme was applied in this study. Parameters of both stand and  
70 tree models were estimated from the fit data (group 1), and then used to predict for the validation  
71 data (group 2). The same procedure was repeated with group 2 being the fit data and group 1 the  
72 validation data. Predictions from both groups were finally pooled to compute evaluation  
73 statistics for the different methods.

74

### 75 **3. Methods**

76

#### 77 **3.1 Tree-Level Prediction**

##### 78 **3.1.1 Method 1: Deriving a Tree Model**

79 In this method, an individual-tree model was derived from an existing stand-level model,  
80 assuming that no tree survival and growth data was available. For this purpose, Cao (2019)  
81 employed the cumulative distribution function (CDF) of the negative exponential distribution to  
82 replace the often-used logistic function to model tree survival probability.

$$83 \quad p_{ij} = 1 - \exp(\alpha_1 d_{1ij}) , \quad (1)$$

84 where  $p_{ij}$  is the survival probability of tree  $j$  in plot  $i$  having diameter  $d_{1ij}$  (in cm) at time 1  
85 (beginning of the growth period); and  $\alpha_1$  is a coefficient to be determined so that the number of  
86 surviving trees sum up to the stand-level output.

87 In this study, preliminary analysis showed that better results were obtained when a  
88 location parameter ( $a$ ) was added to the CDF function as follows:

$$89 \quad p_{ij} = 1 - \exp[\alpha_1(d_{1ij} - a)] , \quad (2)$$

90 where  $a = 0.95 D_{min_{1i}}$ ; and  $D_{min_{1i}}$  = minimum diameter (cm) in plot  $i$ . The coefficient  
 91 0.95 above ensures that  $a$  is less than  $D_{min_{1i}}$ . A sensitivity analysis evaluating different  
 92 coefficient values from 0.90 to 0.99 revealed that 0.95 produced best results.

93 Future tree diameter ( $\hat{d}_{2ij}$ ) was predicted from current diameter ( $d_{1ij}$ ) by use of the  
 94 following simple function:

$$95 \quad \hat{d}_{2ij} = d_{1ij} \exp(\alpha_2 d_{1ij}), \quad (3)$$

96 By use of SAS proc MODEL (SAS Institute Inc. 2004), the parameters  $\alpha_1$  and  $\alpha_2$  in  
 97 equations (2) and (3) were solved such that

$$98 \quad \hat{N}_{2i} = \sum_{j=1}^{n_{1i}} p_{ij}/s, \text{ and} \quad (4)$$

$$99 \quad \hat{B}_{2i} = \sum_{j=1}^{n_{1i}} K p_{ij} d_{1ij}^2/s, \quad (5)$$

100 where  $\hat{N}_{2i}$  and  $\hat{B}_{2i}$  are stand survival (number of trees per ha) and basal area (m<sup>2</sup>/ha) of  
 101 plot  $i$  at time 2, respectively, predicted from any existing stand growth model;

102  $K = \pi/40000$ ; and  $s$  = plot size in ha.

103

### 104 **3.1.2 Method 2: Unadjusted Tree Model**

105 The tree model form used in this study consisted of the survival function by Cao  
 106 (2014) and the tree diameter growth function by Cao (2021):

$$107 \quad p_{ij} = [1 + \exp \{1 + \exp (b_0 + b_1 RS_{1i} + b_2 H_{1i} + b_3 d_{1ij}/Q_{1i})\}]^{-1}, \quad (6)$$

$$108 \quad \hat{d}_{2ij} = d_{1ij} \{1 + \exp [b_4 + b_5 N_{1i}/A_{1i} + b_6/A_{1i} + b_7 Q_{1i} + b_8 (d_{1ij}^2 - Q_{1i}^2)]\}, \quad (7)$$

109 where  $\hat{d}_{2ij}$  is predicted diameter at time 2 (end of the growth period);  $A_{1i}$ ,  $H_{1i}$ ,  $N_{1i}$ , and  $Q_{1i}$   
 110 are, respectively, age (years), dominant height (m), number of trees per ha, and quadratic mean

111 diameter (cm) for plot  $i$  at time 1;  $RS_{1i} = \frac{\sqrt{10000/N_{1i}}}{H_{1i}}$  = relative spacing; and the  $b$ 's are regression  
 112 coefficients.

113

### 114 3.1.3 Method 3: Disaggregating a Tree Model

115 Cao (2010) suggested the following method to adjust the predicted tree survival  
 116 probability ( $p_{ij}$ ) and diameter at the end of the growth period ( $\hat{d}_{2ij}$ ) such that the resulting  
 117 aggregated values match predicted number of trees per ha ( $\hat{N}_{2i}$ ) and basal area per ha ( $\hat{B}_{2i}$ ) from  
 118 any existing stand model, respectively:

$$119 \quad p_{ij}^* = p_{ij}^{\beta_{1i}}, \text{ such that } \sum_j p_{ij}^* = s_i \hat{N}_{2i}, \quad (8)$$

$$120 \quad d_{2ij}^{*2} = d_{1ij}^2 + \beta_{2i}(\hat{d}_{2ij}^2 - d_{1ij}^2), \text{ where } \beta_{2i} = \frac{(s_i \hat{B}_{2i}/K) - \sum_j (p_{ij}^* d_{1ij}^2)}{\sum_j [p_{ij}^* (\hat{d}_{2ij}^2 - d_{1ij}^2)]}, \quad (9)$$

121

## 122 3.2 Stand-Level Models

### 123 3.2.1 Model a: Cao (2021)

124 The growth model by Cao (2021) has components to predict stand survival ( $N$ , number of  
 125 trees per ha) and quadratic mean diameter ( $Q$ , cm) as follows:

$$126 \quad \hat{N}_{2i} = N_{1i} / [1 + \exp\{a_0 + a_1 RS_{1i} + a_2 H_{1i} + a_3 N_{1i}/A_{1i} + a_4/A_{1i}\}], \quad (10)$$

$$127 \quad \hat{Q}_{2i} = Q_{1i} \{1 + \exp[a_5 + a_6 N_{1i}/A_{1i} + a_7/A_{1i} + a_8 Q_{1i}]\}, \quad (11)$$

$$128 \quad \text{and} \quad \hat{B}_{2i} = K \hat{N}_{2i} \hat{Q}_{2i}^2, \quad (12)$$

129 where  $\hat{N}_{2i}$ ,  $\hat{B}_{2i}$ , and  $\hat{Q}_{2i}$  are, respectively, predicted number of trees and basal area ( $m^2$ ) per ha  
 130 and quadratic mean diameter (cm) for plot  $i$  at time 2; and the  $a$ 's are regression coefficients.

131



### 132 3.2.2 Model b: Clutter and Jones (1980)

133 Clutter and Jones (1980) predicted stand survival and basal area as follows:

$$134 \hat{N}_{2i} = 1000 \left\{ \left( \frac{N_{1i}}{1000} \right)^{a_1} + a_2 \left[ \left( \frac{A_{2i}}{10} \right)^{a_3} - \left( \frac{A_{1i}}{10} \right)^{a_3} \right] \right\}^{1/a_1}, \quad (13)$$

135 and

$$\hat{B}_{2i} = \exp \left\{ \left( \frac{A_{1i}}{A_{2i}} \right)^{a_4} \ln (B_{1i}) + a_5 \left[ 1 - \left( \frac{A_{1i}}{A_{2i}} \right)^{a_4} \right] \right\}. \quad (14)$$

136 Note that models *a* and *c* (below) predict future stand attributes for defined time intervals (5  
137 years in this case) and therefore do not need the future projection age ( $A_{2i}$ ). On the other hand,  
138 model *b* can be used for any projection length and consequently requires  $A_{2i}$ .

139

### 140 3.2.3 Model c: New model

141 A new stand-level growth model was developed in this study to predict stand survival and  
142 quadratic mean diameter as follows:

$$143 \hat{N}_{2i} = N_{1i} - \exp \left[ 1 + \exp \{ a_0 + a_1 R S_{1i} + a_2 H_{1i} + a_3 N_{1i} / A_{1i} + a_4 A_{1i} \} \right], \quad (15)$$

$$144 \hat{Q}_{2i} = Q_{1i} + \exp \{ a_5 + a_6 N_{1i} / A_{1i} + a_7 A_{1i} \}. \quad (16)$$

145 and

$$\hat{B}_{2i} = K \hat{N}_{2i} \hat{Q}_{2i}^2, \quad (17)$$

146 The Seemingly Unrelated Regressions (SUR) method (SAS proc MODEL, SAS Institute  
147 Inc., 2004) was used to estimate parameters of the systems of equations listed in the three stand  
148 models.

149

### 150 3.3 Evaluation

151 After the coefficients were obtained from one group, they were used to predict for the  
152 other group. Predicted values from both groups were then pooled for the computation of  
153 evaluation statistics.

154

155 **3.3.1 Stand-level prediction**

156 The following statistics were computed for evaluation of the three stand models:

157 **Mean difference:** 
$$MD = \frac{1}{m} \sum_i (y_{2i} - \hat{y}_{2i}), \quad (18a)$$

158 **Mean absolute difference:** 
$$MAD = \frac{1}{m} \sum_i |y_{2i} - \hat{y}_{2i}|, \quad (18b)$$

159 **Fit index:** 
$$FI = 1 - \frac{\sum_i (y_{2i} - \hat{y}_{2i})^2}{\sum_i (y_{2i} - \bar{y}_2)^2}, \quad (18c)$$

160 where  $m$  = number of plots;  $y_{2i}$  and  $\hat{y}_{2i}$  are, respectively, observed and predicted values of  $N$ ,  $Q$ ,  
161 or  $B$  of plot  $i$  at the end of the growth period; and  $\bar{y}_2$  = average of  $y_{2i}$ .

162

163 **3.2.2 Tree-level prediction**164 The seven methods (1a, 1b, 1c, 2, 3a, 3b, and 3c) were evaluated for tree-level prediction.  
165 Method 2 is independent of the stand models used. The remaining methods are combinations of  
166 tree-level and stand-level prediction methods. For example, method 3b refers to the tree model  
167 disaggregated from the Clutter and Jones (1980) model.168 Evaluation statistics for tree diameter predictions were similar to those presented in  
169 equations (18a–18c). Tree-level survival predictions were evaluated from:

170 **Mean difference:** 
$$MD = \frac{\sum_i \sum_j (y_{ij} - p_{ij})}{\sum_i n_{1i}}, \quad (19a)$$

171 where  $y_{ij} = 1$  if tree  $j$  in plot  $i$  was alive and 0 if it was dead;  $\sum_i$  denotes the sum for  $i$  from 1 to  $m$ ;  
172  $\sum_j$  denotes the sum for  $j$  from 1 to  $n_{1i}$ ; and  $n_{1i}$  = number of trees in plot  $i$  at the beginning of the  
173 growth period.

174 **Mean absolute difference:** 
$$MAD = \frac{\sum_i \sum_j |y_{ij} - p_{ij}|}{\sum_i n_{1i}}, \quad (19b)$$

175 *AUC*: area under the ROC (Receiver Operating Characteristic) curve. The range for  
176 *AUC* is between 0.5 (poorest fit) and 1 (perfect fit).

177 Poudel and Cao's (2013) relative rank system was used to describe the relative position  
178 of each method for stand- and tree-level prediction. The best and worst methods received  
179 relative ranks of 1 and  $m$ , respectively, in this ranking system for  $m$  methods. The remaining  
180 methods were ranked as real numbers between 1 and  $m$ . Because the magnitude as well as the  
181 order of each evaluation statistic were taken into consideration, this ranking system should  
182 provide more information than the traditional ordinal ranks.

183

#### 184 **4. Results and Discussion**

185 Table 3 shows parameter estimates by group for each of the three stand-level models.  
186 Parameter estimates of the individual-tree model for each group were also presented (Table 4).  
187 All parameter estimates were significant at the 5% level. Evaluation statistics are shown for  
188 predicting attributes at the stand level for the stand models (Table 5). After a relative rank was  
189 computed separately for each statistic of each method, an overall rank was calculated based on  
190 the sum of all ranks for each method. Based on the overall ranks, the new stand model ( $c$ ) was  
191 first, achieving the best statistics in all categories but two (MD for  $N$  and  $B$ ). Model  $a$  (Cao  
192 2021) was second with a rank of 1.80, and model  $b$  (Clutter and Jones 1980) was a distant third  
193 (Table 5).

194 Table 6 presents evaluation statistics for predicting tree diameter and survival probability,  
195 for each of the seven methods. Method 1c had the best overall rank (1.00), followed closely by  
196 method 2 (1.68) and method 1a (1.77). The bottom methods include method 1b (4.89) and  
197 method 3b (7.00), both associated with stand model  $b$  (Clutter and Jones 1980).

198

199 **4.1 Method 2 versus method 3**

200 Method 2 is the unadjusted tree model, whereas method 3 is the disaggregated tree  
201 model. The success of disaggregation depends largely on how well the stand attributes are  
202 predicted. Using observed stand attributes (to simulate a perfect stand model) for adjustment  
203 resulted in improvement of tree-level predictions (Cao 2010). On the other hand, disaggregation  
204 from a poor stand-level model might hurt rather than help the performance of the tree model  
205 (Cao 2017). The tree model (method 3*b*) that was disaggregated from the worst stand model in  
206 this study (Clutter and Jones 1980, Table 5) also ranked last among the seven tree models (Table  
207 6).

208 The disaggregated tree models over-predicted tree survival, which is a direct result of  
209 over-prediction by the stand survival models (negative MD for both tree and stand levels). The  
210 fact that method 2 was better than method 3 in terms of MD for tree survival (Table 5) is  
211 consistent with findings from Cao (2017). He stated that tree survival MDs was better for the  
212 disaggregated tree models if the FI from the stand survival model exceeded 0.93, which was not  
213 the case for any of the three stand survival models tested in this study. Cao (2017) also found  
214 through simulation that the disaggregated tree models produced better tree survival MADs and  
215 AUCs if the FI from the stand survival model exceeded 0.81. From Table 5, this was true for  
216 models *a* and *c* (FI = 0.85 and 0.86, respectively), whereas the reverse was true for model *b* (FI =  
217 0.74).

218 Similar to the stand survival component, the three stand-level models over-predicted  
219 basal area per ha (negative MD, Table 5), leading to over-prediction of tree diameters by the  
220 disaggregated tree models (Table 6). On the other hand, except for method 3*b*, the disaggregated

221 tree models (methods 3a and 3c) outperformed the unadjusted tree model (method 2) in terms of  
222 MAD and FI.

223

#### 224 **4.2 Method 1 versus method 3**

225 The derived tree survival function (Equation 1) was revised from the one by Cao (2019)  
226 by adding a location parameter ( $a = 0.95 Dmin_{1i}$ ). This simple modification improved the AUC  
227 range from 0.70 – 0.72 (Cao 2019) to 0.76 – 0.81 in this study (Table 6). The coefficients ( $\alpha_1$  in  
228 Equation 1 for tree survival and  $\alpha_2$  in Equation 2 for tree diameter) were solved such that the  
229 tree-level predictions summed up to outputs obtained from the stand models.

230 Methods 1 and 3 are similar in that their individual tree predictions summed up to the  
231 predictions from the stand-level models. In this respect, the derived tree models (method 1) can  
232 be considered a form of disaggregated tree models. However, the main difference between the  
233 two methods is that method 3 requires individual tree growth and survival data whereas method  
234 1 does not.

235 For the same stand-level model, the derived tree models (method 1) always fared better  
236 than the disaggregated models (method 3): overall rank of 1.77 vs. 2.83 for Cao (2021), 4.89 vs.  
237 7.00 for Clutter and Jones (1980), and 1.00 vs. 2.02 for the new stand model. Similar to the  
238 disaggregated models, the performance of the derived tree models depended on the quality of the  
239 corresponding stand models.

240

#### 241 **4.3 Method 1 versus method 2**

242 With the exception of method 1b (derived from Clutter and Jones 1980), the derived tree  
243 model compared favorably with the unadjusted tree model (method 2). The overall rank of

244 method 2 (1.68) was sandwiched between methods 1a (1.77) and 1c (1.00). It is amazing that  
245 the derived tree models did that well, considering that they required no tree-level growth data,  
246 and were completely based on existing stand models. In fact, method 1c, which was derived  
247 from the best-ranked stand model (c), even outperformed method 2. Figure 1 shows 5-year  
248 survival probabilities and future diameters, derived from method 1c, for current diameters of  
249 trees in three different plots.

250

## 251 **5. Summary and Conclusions**

252 In this study, a new method was developed to derive a tree survival and diameter growth  
253 model from any existing stand-level model, without the need for individual-tree growth data.  
254 Predictions from the derived tree model are constrained to match number of trees and basal area  
255 per ha as outputted by the stand model. The tree models derived from three different stand  
256 models were evaluated against a tree model, in both unadjusted and disaggregated forms.

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258 disaggregated tree model. Furthermore, except for one stand model with poor performance, the  
259 tree models derived from the remaining two stand models delivered comparable results to those  
260 obtained from the unadjusted tree model. The tree model derived from one stand model even  
261 performed slightly better than the unadjusted tree model. This is significant because the  
262 coefficients of the unadjusted and disaggregated tree models had to be estimated from tree-level  
263 growth data, whereas the derived tree model required no tree growth data at all. The  
264 methodology presented in this study should be applicable when there is no ingrowth or  
265 recruitment.

266

267 **References**

- 268 Allen II, M.G., Coble, D.W., Cao, Q.V., Yeiser, J., and Hung, I. 2011. A modified stand table  
269 projection growth model for unmanaged loblolly and slash pine plantations in east Texas.  
270 So. J. Appl. For. 35:115-122.
- 271 Andreassena, K., and Tomter, S. M. 2003. Basal area growth models for individual trees of  
272 Norway spruce, Scots pine, birch and other broadleaves in Norway. For. Ecol. Mgt. 180:11-  
273 24.
- 274 Baldwin, V.C., Jr., and Feduccia D.P. 1087. Loblolly pine growth and yield prediction for  
275 managed West Gulf plantations. USDA For. Ser. Res. Pap. SO-236. 27 p.
- 276 Barrio Anta, M., Castedo Dorado, F., Diéguez-Aranda, U., Álvarez González, J.G, Parresol,  
277 B.R., and Rodríguez Soalleiro, R. 2006. Development of a basal area growth system for  
278 maritime pine in northwestern Spain using the generalized algebraic difference approach.  
279 Can. J. For. Res. 36:1461-1474.
- 280 Bohora, S.B., and Cao, Q.V., 2014. Prediction of tree diameter growth using quantile regression  
281 and mixed-effects models. For. Ecol. Mgt. 319:62-66.
- 282 Burkhart, H.E., and Tomé, M. 2012. Modeling forest trees and stands. Springer Science and  
283 Business Media, Dordrecht, The Netherlands, 458 p.
- 284 Cao, Q. V. 1994. A tree survival equation and diameter growth model for loblolly pine based on  
285 the self-thinning rule. J. Appl. Ecol. 31:693-698.
- 286 Cao, Q. V. 2000. Prediction of annual diameter growth and survival for individual trees from  
287 periodic measurements. For. Sci. 46:127-131.
- 288 Cao, Q.V. 2006. Predictions of individual-tree and whole-stand attributes for loblolly pine  
289 plantations. For. Ecol. Mgt. 236:342-347.
- 290 Cao, Q. V. 2007. Incorporating whole-stand and individual-tree models in a stand-table  
291 projection system. For. Sci. 53:45-49.

- 292 Cao, Q.V. 2010. Adjustments of individual-tree survival and diameter growth equations to match  
293 whole-stand attributes. P. 369-373 in Proc. of the South. Silv. Res. Conf. USDA For. Serv.  
294 Gen. Tech. Rep. SRS-121.
- 295 Cao, Q.V. 2014. Linking individual-tree and whole-stand models for forest growth and yield  
296 prediction. For. Ecosys. 1:18.
- 297 Cao, Q. V. 2017. Evaluation of methods for modeling individual tree survival. For. Sci. 63:356-  
298 361.
- 299 Cao, Q. V. 2019. A method to derive a tree survival model from any existing stand survival  
300 model. Can. J. For. Res. 49:1598-1603.
- 301 Cao, Q. V. 2021. A unified system for tree- and stand-level predictions. For. Ecol. Mgt.  
302 481:118713.
- 303 Cao, Q.V., and Baldwin, V.C. 1999. A new algorithm for stand table projection models. For. Sci.  
304 45:506-511.
- 305 Cao, Q.V., and Durand, K.M. 1991. A growth and yield model for improved eastern cottonwood  
306 plantations in the lower Mississippi Delta. South. J. Appl. For. 15:213-216.
- 307 Carretero, A.C., and Alvarez, E.T. 2013. Modelling diameter distributions of *Quercus suber* L.  
308 stands in “Los Alcornocales” Natural Park (Cádiz-Málaga, Spain) by using the two-  
309 parameter Weibull functions. For. Syst. 22:15-24.
- 310 Clutter, J.L., and Jones, E.P.:1980. Predicting of growth after thinning in old-field slash pine  
311 plantations. USDA Forest Serv. Res. Pap. SE-217, 14 p.
- 312 Coble, D.W., Cao, Q.V., and Jordan, L., 2012. An annual tree survival and diameter growth  
313 model for loblolly and slash pine plantations in east Texas. South. J. Appl. For. 36:79-84.
- 314 Dean, T.J., Jerez, M., and Cao, Q.V., 2013. A simple stand growth model based on canopy  
315 dynamics and biomechanics. For. Sci. 59:335-344.



- 316 Diéguez-Aranda, U., Castedo Dorado, F., Álvarez González, J.G., and Rodríguez Soalleiro, R.  
317 2005. Modelling mortality of Scots pine (*Pinus sylvestris* L.) plantations in the northwest of  
318 Spain. *Eur. J. For. Res.* 124:143–153.
- 319 Erikäinen K. 2002. A site dependent simultaneous growth projection model for *Pinus kesiya*  
320 plantations in Zambia and Zimbabwe. *For. Sci.* 48:518-529.
- 321 Garcia, O. 2011. A parsimonious dynamic stand model for interior spruce in British Columbia.  
322 *For. Sci.* 57:265-280.
- 323 Guan, B.T., and Gertner, G. 1991a. Using a parallel distributed processing system to model  
324 individual tree mortality. *For. Sci.* 37:871-885.
- 325 Guan, B.T., and Gertner, G., 1991b. Modeling red pine tree survival with an artificial neural  
326 network. *For. Sci.* 37:1429-1440.
- 327 Hevia, A., Cao, Q. V., Álvarez-González, J. G., Ruiz-González, A. D., and von Gadow, K. 2015.  
328 Compatibility of whole-stand and individual-tree models using composite estimators and  
329 disaggregation. *For. Ecol. Mgt.* 348:46-56.
- 330 Jiang, L.C., and Brooks, J.R., 2009. Predicting diameter distributions for young longleaf pine  
331 plantations in southwest Georgia. *South. J. Appl. For.* 33:25-28.
- 332 Kjell, K., and Lennart, N. 2005. Modelling survival probability of individual trees in Norway  
333 spruce stands under different thinning regimes. *Can. J. For. Res.* 35:113-121.
- 334 Matney, T.G., and Sullivan, A.D. 1982. Compatible stand and stock tables for thinned and  
335 unthinned loblolly pine stands. *For. Sci.* 28:161-171.
- 336 Monserud, R.A., and Sterba, H. 1999. Modeling individual tree mortality for Austrian forest  
337 species. *For. Ecol. Mgt.* 113:109-123.
- 338 Naing, Y.M. 2020. Growth and yield models for thinned teak stands in Taungoo District, Bago  
339 region of Myanmar. *Int. J. Sci. Res. Publications.* 10:546-557.

- 340 Nepal, S.K., and Somers, G.L. 1992. A generalized approach to stand table projection. For. Sci.  
341 38:120-133.
- 342 Palahía, M., Pukkala, T., Miinac, J., and Montero, G. 2003. Individual-tree growth and  
343 mortality models for Scots pine (*Pinus sylvestris* L.) in north-east Spain. Ann. For. Sci. 60:1-  
344 10.
- 345 Poudel, K., and Cao Q. V. 2013. Evaluation of methods to predict Weibull parameters for  
346 characterizing diameter distributions. For. Sci. 59:243-252.
- 347 Ritchie, M.W., and Hann, D.W. 1997. Implications of disaggregation in forest growth and yield  
348 modeling. For. Sci. 43:223-233.
- 349 Sánchez-González, M., delRío, M., Cañellas, I., and Montero, G. 2006. Distance independent  
350 tree diameter growth model for cork oak stands. For. Ecol. Mgt. 225:262-270.
- 351 SAS Institute Inc. 2004. SAS/ETS 9.1 User's Guide. SAS Institute Inc., Cary, NC, 2416 p.
- 352 Smalley, G.W., and Bailey, R.L. 1974. Yield tables and stand structure for loblolly pine  
353 plantations in Tennessee, Alabama, and Georgia highlands. USDA For. Serv. Res. Paper  
354 SO-96, 96 p.
- 355 Somers, G.L., and Farrar R.M., 1991. Biomathematical growth equations for natural longleaf  
356 pine stand. For. Sci. 37:227-244.
- 357 Stankova, T.V. 2016. A dynamic whole-stand growth model, derived from allometric  
358 relationships. Silva Fennica vol. 50:1-21.
- 359 Subedi, N., and Sharma, M. 2011. Individual-tree diameter growth models for black spruce and  
360 jack pine plantations in northern Ontario. For. Ecol. Mgt. 261:2140-2148.
- 361 Sun, S., Cao, Q.V., and Cao, T. 2019. Evaluation of distance-independent competition indices in  
362 predicting tree survival and diameter growth. Can. J. For. Res. 49:440-446.
- 363 Tewari, V.P., Álvarez-González, J.G., and García, O. 2014. Developing a dynamic growth  
364 model for teak plantations in India. For. Ecosyst. 1:9.

- 365 Wells, O.O., and Wakeley, P. C. 1966. Geographic variation in survival, growth, and fusiform  
366 rust infection of planted loblolly pine. For. Sci. Monograph 11. 40 p.
- 367 Yue, C., Kohnle, U., and Hein, S., 2008. Combining tree-and stand-level models: A new  
368 approach to growth prediction. For. Sci. 54:553-566.
- 369 Zhang, X., Lei, Y., and Cao, Q. V., 2010. Compatibility of stand basal area predictions based on  
370 forecast combination. For. Sci. 56:552-557.
- 371 Zhang, L.; Moore, J.A., and Newberry, J.D. 1993. A whole-stand growth and yield model for  
372 interior Douglas-fir. West. J. Appl. For. 8:120-125.
- 373

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Table 1. Distribution of 200 plots, by starting age and group.

Starting age	Ending age	Group 1	Group 2
		--- Number of plots ---	
10	15	33	33
15	20	33	33
20	25	34	34
Total		100	100

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Table 2. Means (and standard deviations) of stand and tree attributes, by group and age at the beginning of the growth period.

Group	Age	Dominant height (m)	Number of trees/ha	Basal area (m <sup>2</sup> /ha)	Tree diameter (cm)
1	10	9.3 (1.1)	2063 (646)	22.9 (5.7)	11.6 (2.7)
	15	13.2 (1.9)	1713 (714)	31.7 (9.2)	14.8 (3.9)
	20	16.3 (2.1)	1256 (370)	33.5 (8.2)	17.9 (4.4)
2	10	9.2 (1.5)	2065 (608)	22.4 (7.2)	11.4 (2.7)
	15	13.3 (1.7)	1631 (463)	30.9 (5.9)	15.1 (3.8)
	20	16.8 (1.7)	1337 (326)	34.5 (7.0)	17.7 (4.2)

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Table 3. Parameter estimates (and standard errors), by stand model and group.

Parameter estimate	Model <i>a</i> (Eq. 10 – 11) Cao (2021)		Model <i>b</i> (Eq. 13 – 14) Clutter and Jones (1980)		Model <i>c</i> (Eq. 15 – 16) New model	
	Group 1	Group 2	Group 1	Group 2	Group 1	Group 2
	$a_0$	21.9167 (2.2756)	18.4259 (2.3563)			6.1444 (0.4326)
$a_1$	-58.8599 (6.5739)	-53.0888 (6.9893)	-0.9715 (0.3473)	-0.5457 (0.3698)	-9.7324 (0.7245)	-9.6260 (0.9511)
$a_2$	-0.8944 (0.0959)	-0.7785 (0.0951)	0.0650 (0.0228)	0.0902 (0.0365)	-0.1475 (0.0115)	-0.1380 (0.0130)
$a_3$	-0.0254 (0.0044)	-0.0262 (0.0044)	2.0236 (0.4175)	1.2577 (0.4394)	-0.0032 (0.0005)	-0.0037 (0.0006)
$a_4$	37.8062 (12.0869)	51.5813 (11.9887)	1.3479 (0.2361)	0.8344 (0.1386)	-0.0283 (0.0088)	-0.0472 (0.0099)
$a_5$	-1.7207 (0.2855)	-1.3051 (0.2815)	3.8760 (0.0818)	4.2214 (0.1267)	2.8552 (0.1358)	2.8173 (0.1454)
$a_6$	-0.0028 (0.0005)	-0.0033 (0.0004)			-0.0032 (0.0003)	-0.0033 (0.0004)
$a_7$	17.8976 (1.6905)	16.2977 (1.8217)			-0.0970 (0.0074)	-0.0918 (0.0076)
$a_8$	-0.0602 (0.0114)	-0.0744 (0.0111)				

Table 4. Parameter estimates of the individual-tree model (Eq. 6 – 7), by group.

Parameter estimate	Group 1	Group 2
$b_0$	10.2765 (0.9423)	12.1540 (0.7658)
$b_1$	-0.2256 (0.0311)	-0.2773 (0.0261)
$b_2$	-21.9825 (2.5434)	-25.5995 (2.0180)
$b_3$	-5.0864 (0.3482)	-5.9253 (0.3098)
$b_4$	-2.6091 (0.1325)	-1.3919 (0.1373)
$b_5$	-0.0029 (0.0002)	-0.0038 (0.0002)
$b_6$	23.7870 (0.8111)	19.0524 (0.7664)
$b_7$	-0.0455 (0.0056)	-0.0954 (0.0056)
$b_8$	0.0011 (0.0001)	0.0013 (0.0001)

Table 5. Evaluation statistics for stand-level prediction, by variable and model.

Variable <sup>1/</sup>	Evaluation Statistic <sup>2/</sup>	Cao (2021)	Clutter and Jones (1980)	New model
<i>N</i>	<i>MD</i>	<u>-13.115</u>	<b>-8.080</b>	-10.239
	<i>MAD</i>	148.649	<u>195.740</u>	<b>145.034</b>
	<i>FI</i>	0.8545	<u>0.7439</u>	0.8581
<i>B</i>	<i>MD</i>	<u>-0.3354</u>	<b>-0.1994</b>	-0.2502
	<i>MAD</i>	3.2008	<u>3.7804</u>	<b>3.0246</b>
	<i>FI</i>	0.7674	<u>0.6222</u>	<b>0.7854</b>
<i>Q</i>	<i>MD</i>	0.0422	<u>-0.0625</u>	<b>0.0119</b>
	<i>MAD</i>	0.5519	<u>0.6366</u>	<b>0.5435</b>
	<i>FI</i>	0.9577	<u>0.9467</u>	<b>0.9594</b>
Sum of the ranks		15.53	<u>23.00</u>	<b>10.61</b>
Overall rank		1.80	<u>3.00</u>	<b>1.00</b>

<sup>1/</sup> *N* = number of trees per ha; *B* = basal area (m<sup>2</sup>/ha); *Q* = quadratic mean diameter (cm).

<sup>2/</sup> *MD* = mean difference; *MAD* = mean absolute difference; *FI* = fit index.

For each evaluation statistic, a bold, italic number denotes the best statistic, and an underlined number denotes the worst.



Table 6. Evaluation statistics<sup>1/</sup> for tree-level prediction, by method and variable.

Method	Tree diameter			Tree survival			Sum of the ranks	Overall ranks
	<i>MD</i>	<i>MAD</i>	<i>FI</i>	<i>MD</i>	<i>MAD</i>	<i>AUC</i>		
1a	-0.0152	0.8576	0.9416	<u>-0.0078</u>	0.2030	0.8027	16.21	1.77
1b	<b><i>0.0007</i></b>	0.8935	<u>0.9376</u>	-0.0048	0.2174	0.7608	29.62	4.89
1c	-0.0095	<b><i>0.8537</i></b>	0.9420	-0.0061	<b><i>0.2020</i></b>	0.8078	<b><i>12.91</i></b>	<b><i>1.00</i></b>
2	0.0022	0.8695	0.9420	<b><i>0.0005</i></b>	0.2202	0.7929	15.83	1.68
3a	-0.0958	0.8608	0.9431	<u>-0.0078</u>	0.2090	0.7992	20.79	2.83
3b	<u>-0.1277</u>	<u>0.9009</u>	0.9384	-0.0048	<u>0.2284</u>	<u>0.7566</u>	<u>38.71</u>	<u>7.00</u>
3c	-0.1020	0.8553	<b><i>0.9438</i></b>	-0.0061	0.2094	<b><i>0.8093</i></b>	17.28	2.02

<sup>1/</sup> For each evaluation statistic, a bold, italic number denotes the best statistic, and an underlined number denotes the worst.

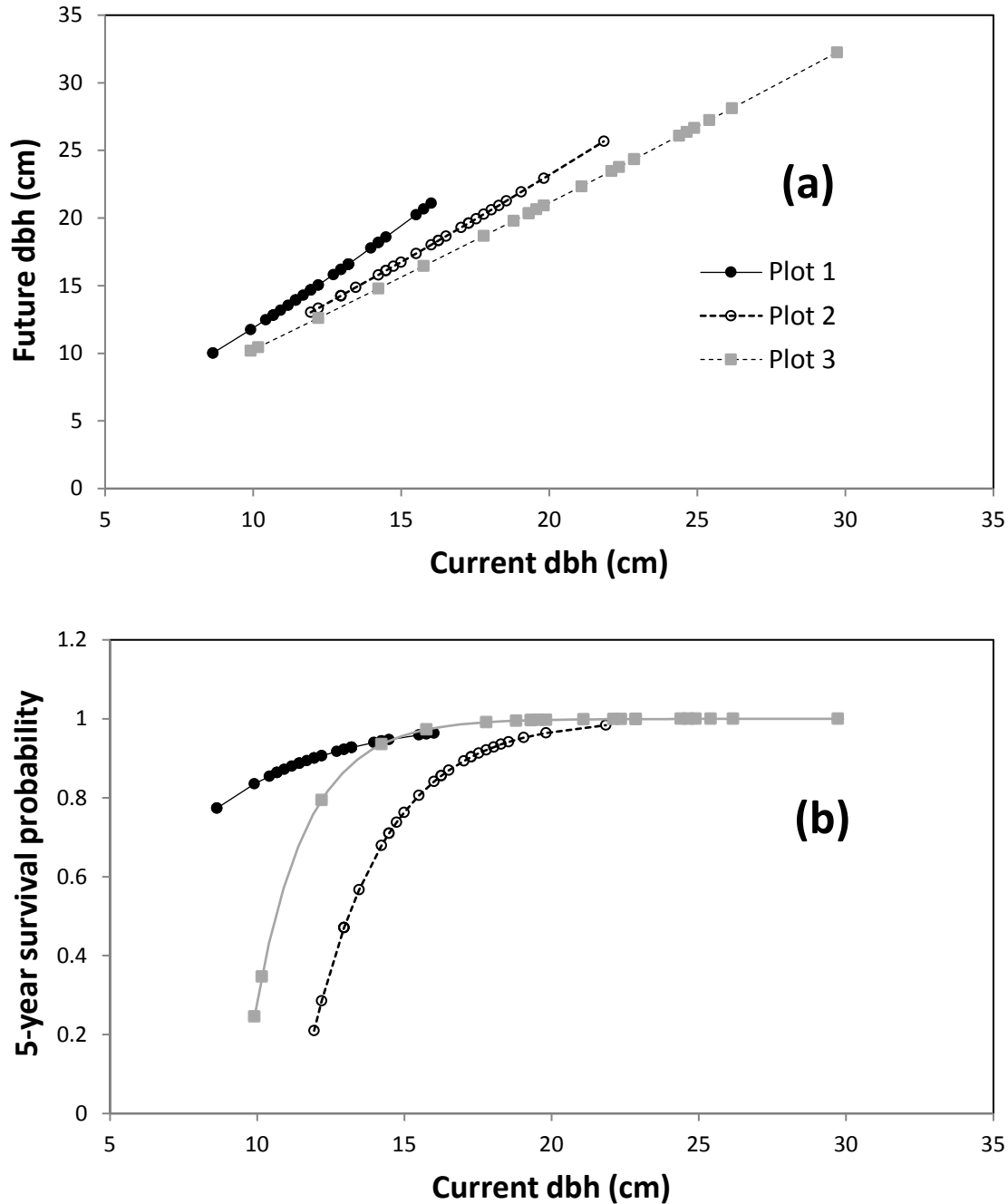
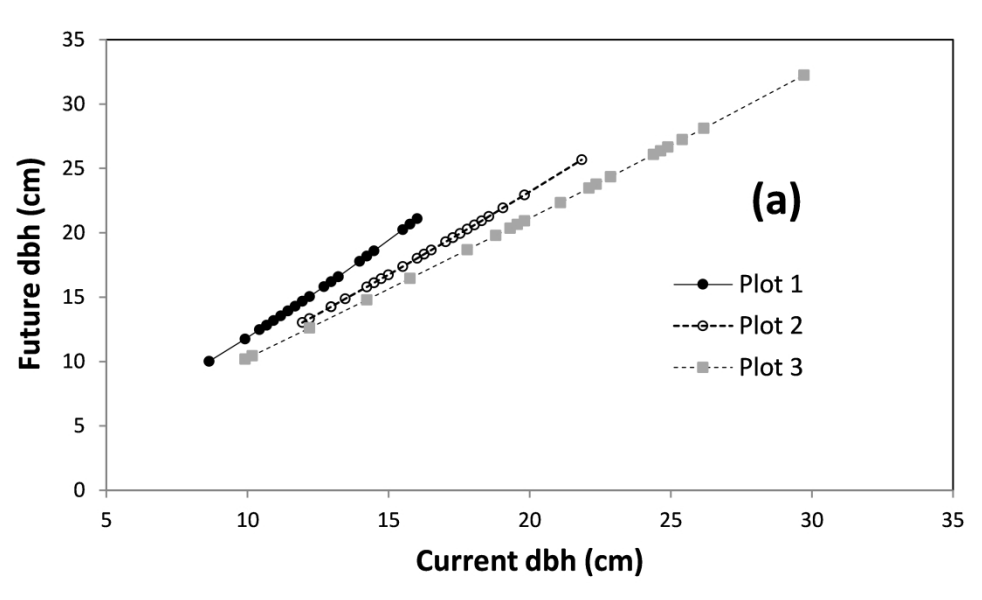
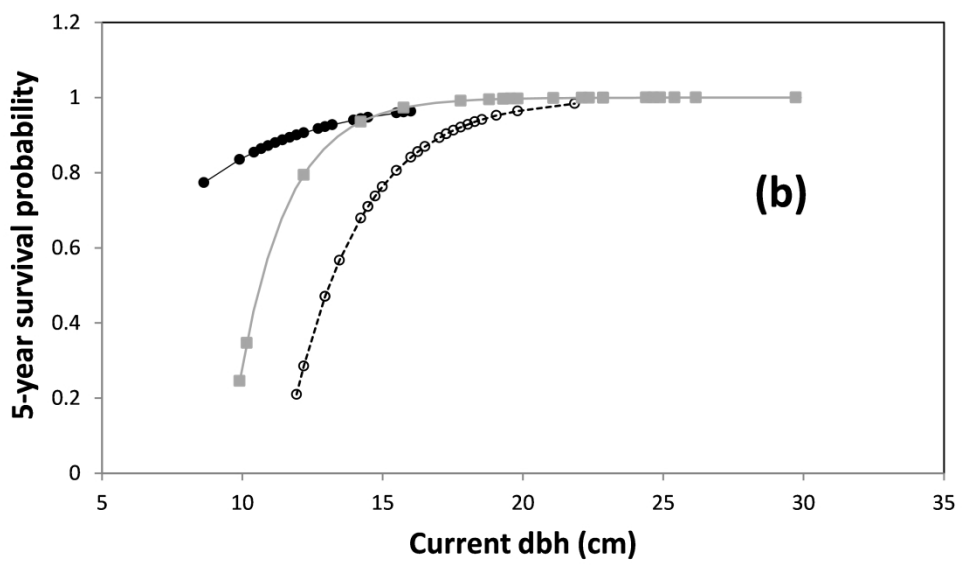


Figure 1. Curves for 5-year diameter growth (a) and survival probability (b), as derived from the new stand model, for trees in plot 1 ( $A_1 = 10$  yrs;  $H_1 = 9.2$  m;  $N_1 = 2074$  trees/ha;  $B_1 = 24.5$  m<sup>2</sup>/ha), plot 2 ( $A_1 = 15$  yrs;  $H_1 = 12.3$  m;  $N_1 = 1647$  trees/ha;  $B_1 = 33.9$  m<sup>2</sup>/ha), and plot 3 ( $A_1 = 20$  yrs;  $H_1 = 16.9$  m;  $N_1 = 1342$  trees/ha;  $B_1 = 45.5$  m<sup>2</sup>/ha).



389x231mm (236 x 236 DPI)



389x231mm (236 x 236 DPI)