Description of a program for solving problems of nonlinear programming

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The paper describes a method of solving the problem of nonlinear programming. By parametrizing the constraint conditions the problem is transformed into an iterative sequence of partial solutions. Simple examples are given for illustration.

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1. Introduction

We shall be dealing with the nonlinear programming problem in the following form (1)

$$g_k(\mathbf{x}) \leq 0 \qquad k = 1, 2, \ldots, m \tag{1a}$$

$$f(\mathbf{x}) \rightarrow \min . \tag{1b}$$

where $g(\mathbf{x})$ and $f(\mathbf{x})$ are non-linear functions of the independent variable $\mathbf{x} = (x_1, x_2, \ldots, x_t, \ldots, x_n)$.

2. Methods of nonlinear problem solution

A number of methods exist for a solution of equation (1). We shall consider one standard approach by which the problem is reduced to a sequence of problems of step by step reaching the feasible domain. It introduces an additional constraint

$$g_{fj}(\mathbf{x}) = f(\mathbf{x}) - \Delta_j \tag{2}$$

and the barrier function F. (See **Fig. 1.**)

$$F_j(\mathbf{x}) = \sum_{k=1}^{m} e^{g_k} + e^{g_{fj}} \tag{3}$$

The process is divided into steps j. Point $\mathbf{x}_j$ (at the beginning it will be an arbitrarily chosen point $\mathbf{x}_0$; also $\Delta_{j=1}$ and $\bar{\Delta}$ are chosen) enters the searching block S (the program for unconstrained optimisation) that searches for the 'lowest depression' (which is the feasible domain) on the F surface. The point found having smallest value F is designated as $\mathbf{x}_{j,N}$. If this point is feasible, the new step j takes place (when Δ_j is reduced by $\bar{\Delta}$ in g_{fj}). In the opposite case, the end is reached and the extreme is the last found feasible point.

3. Parametrisation of constraint conditions

In the presented approach the solution will basically reduce, from the geometrical point of view, to finding the 'lowest depression' on the barrier surface F.

A guarantee of success of finding the global extreme ($\mathbf{x}_{eG}$) is only valid if functions $g_k(\mathbf{x})$ are convex for all k and the $f(\mathbf{x})$ function is convex (and thus also $F(x)$).

Conversely it may be expected only of a smaller or larger probability of reaching a local extreme, more or less distant from the global extreme.

We shall try to increase this probability by an iterative process in which we should solve a sequence of problems instead of one problem. It is intended that the character of problems will approach, in the first phases, a favourable convex character, and gradually reduce into the initial problem at the end. At the same time, during the process, the arbitrarily chosen starting point $\mathbf{x}_0$ should gradually migrate to a closer vicinity of $\mathbf{x}_{eG}$.

For the sake of simplicity, we shall limit ourselves to the assumption that the constraint functions are defined by polynomials (this assumption could eventually be extended). One function from equation (1) can then be written

$$g(\mathbf{x}) = \sum_{i=1}^{r} A_i(\mathbf{x}) \tag{4}$$

where $A_i(\mathbf{x})$ are individual terms of the polynomial.

Now, we shall transform the function $g(\mathbf{x})$ by a suitable parametrisation. We shall introduce two groups of parameters $\{l_i\}$ and $\{T\}$. By their choice we shall achieve the parametrised function g_p to form the required sequence of problems having properties 1 and 2.

1. By the election of $\{T\}$ (at the arbitrarily elected $\{l_i\}$), such a sequence of functions g_{PT} should be achieved, that the g_{PT} would change from the mostly deformed form to the initial $g(\mathbf{x})$ one.

   This can be achieved practically through function (4) being parametrised into the form:

$$g_{P,T}(\mathbf{x}; L_i) = \sum_{i=1}^{r} L_i A_i(\mathbf{x}) \tag{5}$$

   where

$$L_i = \frac{l_i - 1}{T_1} T + 1 \ ; \quad \{T\} = (T_1, T_2, \ldots, T_M) \tag{6a, b}$$

   The sequence T, according to (6b), is chosen arbitrarily with only one limitation: that it should have decreasing character in its magnitudes and that the last T_M should be zero.

2. Now it remains to determine parameters $\{l_i\}$, that is l_i; $i = 1, 2, \ldots r$. These parameters will be determined so that the parametrised function meets the three following requirements:

   (*a*) that it should be convex;

   (*b*) that it should, at the same time, be a mean surface of the original surface $g(\mathbf{x})$ (with the same position on average) in a region that will represent the vicinity of a particular

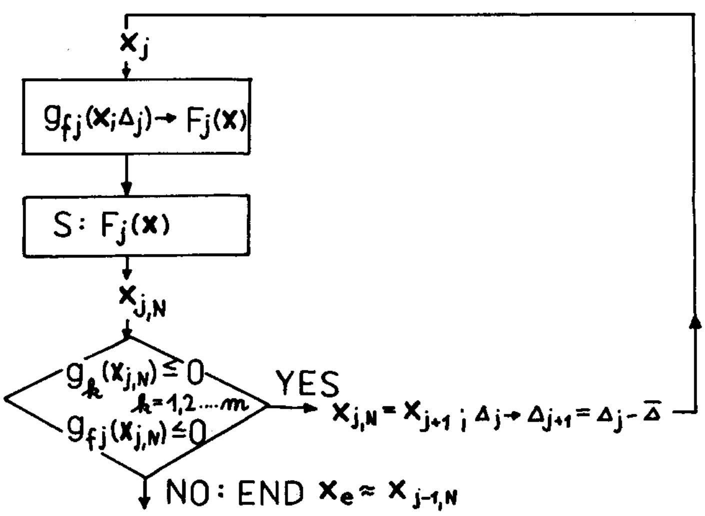

Fig. 1

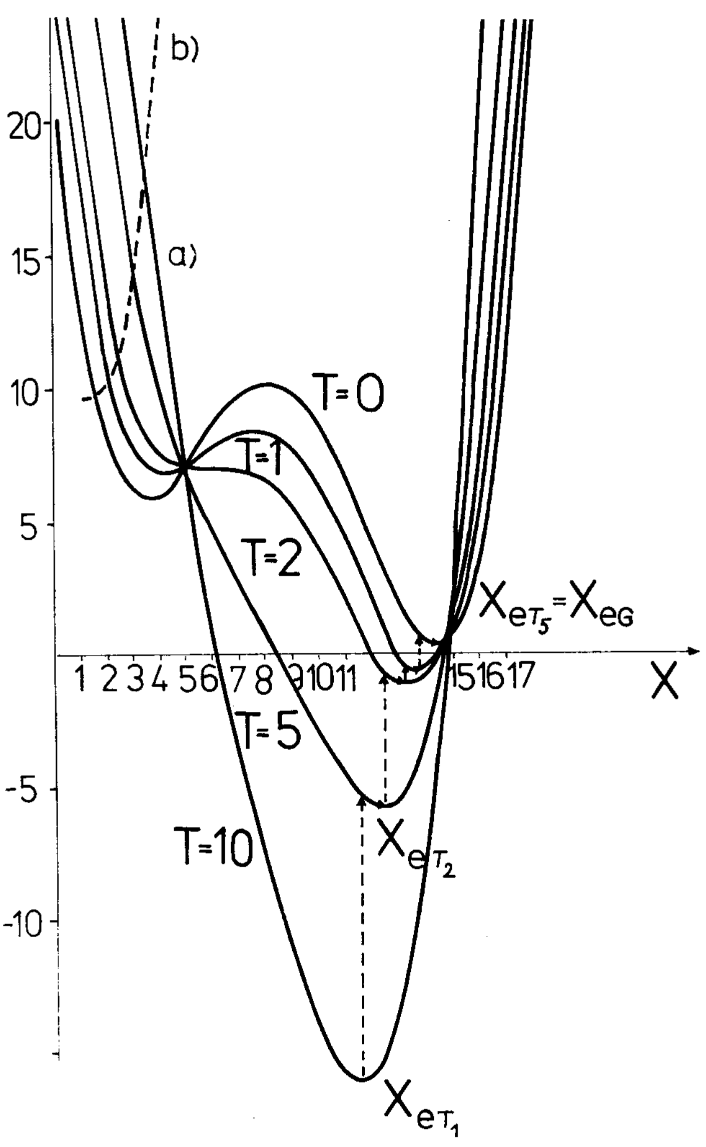

Fig. 2

point while the volume of this region will be of the dimension H_T;

(*c*) the magnitude of the dimension H_T will decrease, at the decreasing T, according to the chosen progression.

Now, instead of solving one optimisation problem with function (4) we shall solve the sequence of optimisation problems with functions $g_{P,T}(\mathbf{x}; L_i)$ for individual T. The extreme found in one iteration will be the initial starting point for the following iteration. The searching $\mathbf{x}_0 \rightarrow \mathbf{x}_{eG}$ will be substituted by the sequence of searching in assumed shorter distances $\mathbf{x}_0 = \mathbf{x}_{T_1} \rightarrow \mathbf{x}_{eT_1} = \mathbf{x}_{T_2} \rightarrow \mathbf{x}_{eT_2} = \mathbf{x}_{T_3} \rightarrow \ldots \rightarrow \mathbf{x}_{eG}$. This process is illustrated in **Fig. 2.** The practical realisation of the sequence of iterations will consist of the following steps:

1. We shall choose the starting point $\mathbf{x}_0 = \mathbf{x}_{T_1}$. Let us now take $T = T_1$ (the first from the T sequence) $\rightarrow L_i = l_i$. Then

$$g_{P\ T_1}(\mathbf{x}; L_i) = g_P(\mathbf{x}; l_i) = \sum_{i=1}^{r} l_i A_i(\mathbf{x}) \qquad (7)$$

Now, we shall determine parameters l_i in (7) so that we will satisfy both conditions, 2(*a*) as well as 2(*b*). At the same time, we shall require satisfaction of the 2(*b*) condition in the neighbourhood of the point $\mathbf{x}_{T_1}$. The magnitude of this area is chosen with a relatively large dimension H_{T_1}.

We solve the optimisation problem with functions of the $g_{P_1 T_1}$ type by any program and obtain the solution $\mathbf{x}_{eT_1} = \mathbf{x}_{T_2}$.

2. The point $\mathbf{x}_{T_2}$ serves as an initial point for the following iteration. We take the next quantity from the T sequence, i.e. T_2.

Parameters l_i in (7) (i.e. at $T = T_1$) will be determined so that, again, the condition 2(*a*) is satisfied. The satisfaction of 2(*b*) is required now in the region around the point $\mathbf{x}_{T_2}$. The magnitude H_T of its dimension will reduce to H_{T_2}. Utilising these l_i, $i = 1, 2, \ldots, r$ and T_2 we form

$$L_i = \frac{l_i - 1}{T_1} T_2 + 1 \ ,$$

and from these, we create $g_{P_1 T_2}(\mathbf{x};\ L_i)$ according to (5) and (6a).

By the solution of the optimisation problem with these functions we can obtain $\mathbf{x}_{eT_2} = \mathbf{x}_{T_3}$.

In the following, this process repeats itself for individual T as far as T_M.

During each step T, we check whether we have reached the feasible region or not (in this case we use the method according to Section 1 for the optimisation algorithm). We should reach this at $T = T_M$. If it is not achieved, the whole process of searching is repeated with the new $\mathbf{x}_0 = \mathbf{x}_{e,\,T_M}$.

The assumption about the possibility of an improvement in the probability of success in the iterative process is based upon the following geometrical consideration.

Since the function $g_{P,T}(\mathbf{x}, L_i)$, for first quantities of T, will be more convex (the requirement 2(*a*)), the probability of finding corresponding $\mathbf{x}_{eT} = \mathbf{x}_T$ (the global solution for the T stage) is considerable.

A sequence of these $\mathbf{x}_{eT}$ will form a path that can approach the point $\mathbf{x}_{eG}$. It will divide the path of the original problem $\mathbf{x}_0 \rightarrow \mathbf{x}_{eG}$ into shorter intervals. Since the probability of stopping (in the local minimum) decreases roughly with the distance of the points (from which we start and which we are searching) the total probability of success increases. During the decrease of T the convexity will decrease, though by a selection of a suitable progression for the sequence T we can achieve in this stage an increasing shortening of distances of points $\mathbf{x}_{eT} \rightarrow \mathbf{x}_{e(T+1)}$.

For an increase of the probability of a success it will also be essential to satisfy requirement 2(*b*).

During the first T_1 the same position on average of $g_{P,T_1}(\mathbf{x};\ L_i)$ with $g(\mathbf{x})$ will take place in the surrounding region of the point x_{T_1} with a large dimension H_{T_1}. The same position on average of g_{PT} and g will thus have more global character and will react less sensitively to the existence of the 'lowest trough' of the F surface. In most cases it can be supposed that in the particular broader neighbourhood of a locally lowest trough (i.e. the feasible domain) the values of the F function are, on average, mostly smaller than in any other position. (The character of the F surface may be undulating and non-convex.)

Under this assumption there is a considerable probability that, in a global scale, the resulting point $\mathbf{x}_{T_2}$ can approach the point $\mathbf{x}_{eG}$.

In step T_2 the same position, on an average of g and g_P, will be made around the point $\mathbf{x}_{T_2}$ (thus already closer to $\mathbf{x}_{eG}$ then was $\mathbf{x}_{T_1}$) in the region of decreased dimension H_{T_2}. This will cause the reaction to the lowest trough of the F surface (where there is the global extreme) to be more locally sensitive and the resulting point $\mathbf{x}_{T_3}$ could get closer to the lowest trough, etc.

An illustration that besides condition 2(*a*), it is necessary to satisfy also 2(*b*) is in Fig. 2. The curve b is the curve $g_{P,\,T_1}$, where the selection of l_i has been limited by condition 2(*a*) only; whereas the choice of parameters l_i for the curve a has been limited by both 2(*a*) and 2(*b*).

It can be assumed that, in a number of situations, the described procedure could increase the probability of success. Even from a clearly geometrical point of view (see basically the searching for the lowest trough of the generally undulated surface F) it would certainly be possible to mention special situations when even this approach would be less successful and would depend more on the selection of the starting point, $\mathbf{x}_0$.

4. Conditions for a convexity

After outlining the essential features of the program, we may return to consolidate condition 2(*a*), i.e. how to choose properly l_i; $i = 1, 2, \ldots, r$, to cause $g_P(\mathbf{x}; l_i)$ (in the sense of (7)) to get closer to a convex character.

From the geometrical point of view, the condition for a convexity of the g_P function in the particular region may be substituted by the condition that in any position of this region the function g_P should have an elliptical point (and neither hyperbolic nor parabolic).

For a two-dimensional problem the condition for an elliptical point in a general position $\mathbf{x}$ is, according to Vojtech (1946) as follows:

$$\frac{1}{R_1}\frac{1}{R_2} = \frac{g_{PX_1X_1}(\mathbf{x}; l_i)\, g_{PX_2X_2}(\mathbf{x}; l_i) - g^2_{PX_1X_2}(\mathbf{x}; l_i)}{(g^2_{PX_1}(\mathbf{x}; l_i) + g^2_{PX_2}(\mathbf{x}; l_i) + 1)^2} \geqslant 0 \qquad (8)$$

where R_1, R_2 are main curvatures of the g_P surface in the point $\mathbf{x}$; $g_{PX_1X_1}$ is its second partial derivation with respect x_1; $g_{PX_1X_2}$ is second mixed derivation.

Since the denominator of expression (8) is always positive, condition (8) can be written:

$$g_{PX_1X_1}(\mathbf{x}; l_i)\, g_{PX_2X_2}(\mathbf{x}; l_i) - g^2_{PX_1X_2}(\mathbf{x}; l_i) \geqslant 0 \qquad (9)$$

that, again, must be valid for any $\mathbf{x}$ from the region considered.

We shall distinguish three cases:

1. In the case that the g_P function is separable (i.e. it does not contain mixed members, but only members containing only one variable) the $g_{PX_1X_2} = 0$, and condition (9) is in the form

$$g_{PX_1X_1}(\mathbf{x}_T; l_i) \geqslant 0 \;;\quad g_{PX_2X_2}(\mathbf{x}_T; l_i) \geqslant 0 \qquad (10)$$

In order that condition (10) is a function consisting of only l_i parameters, point $\mathbf{x}_T$ was substituted for general $\mathbf{x}$. Here, $\mathbf{x}_T$ is the centre point of the region where it is necessary to satisfy the condition. The approximation originated by this substitution can be partly compensated by satisfying requirement (10) with a larger reserve.

2. In the case of a general function, it seems useful to choose l_i so that in the first stage we choose the parameters l_i which are multipliers of the mixed terms g_P in the form $l_i \ll 1$ ($l_i \geqslant 0$). Through this, these members will be suppressed in comparison with separable members and thus the problem is transformed into a problem of the first type (where remaining l_i are determined from condition (10)). This approach can be generalised in the form of the following constraining condition:

Let us designate:

$$g_P(\mathbf{x}; l_i) = g_P^{\mathrm{SEP}} + g_P^{\mathrm{MIX}}$$

where g_P^{SEP} is the part of g_P with separable members, g_P^{MIX} is the part of g_P with mixed members. Therefore, our condition can be expressed approximately:

$$g_P^{\mathrm{MIX}}(\mathbf{x}_T; l_i) + C - \epsilon_M \leqslant g_P^{\mathrm{SEP}}(\mathbf{x}_T; l_i) \qquad (11)$$

with additional condition

$$C - \epsilon_M \geqslant 0 \qquad (12)$$

where C is the chosen constant, ϵ_M is an auxiliary parameter which, to satisfy (11), we shall try to achieve as small as possible.

3. When $g_P(\mathbf{x}; l_i)$ consists of only mixed members, the most approximate way is taken and it is only necessary to satisfy condition (10) (eventually again with a larger reserve).

For a general case of n coordinates, condition (10) can be made general to the form:

$$g_{PX_tX_t}(\mathbf{x}_T; l_i) \geqslant 0 \quad t = 1, 2, \ldots, n \qquad (13)$$

It can be seen that in order to make $g_P(\mathbf{x}; l_i)$ approach a convex character, the selection of l_i must satisfy limitations (13) and eventually (13) and (11). This selection can then be made in different ways, thus it is a variant problem. Let us first elaborate the problem of achieving the same position on average of functions g_P and g.

5. Conditions for the same position on average of functions g_P and g

The aim is to find additional conditions for the selection of l_i so the $g_P(\mathbf{x}, l_i)$ may be on average in the same position as g. Let us introduce the following designation

$$A = \int_{V[H_T:\,\mathbf{x}_T]} [g(\mathbf{x}) - g_P(\mathbf{x}; l_i)]\, dV =$$

$$= \int_{X_{1(T)}-H_T/2}^{X_{1(T)}+H_T/2} \int_{X_{2(T)}-H_T/2}^{X_{2(T)}+H_T/2} \cdots\cdots$$

$$\int_{X_{n(T)}-H_T/2}^{X_{n(T)}+H_T/2} [\mathbf{g}(\mathbf{x}) - \mathbf{g}_P(\mathbf{x}; l_i)]\, dx_1\, dx_2 \ldots dx_n \qquad (14)$$

where $g(\mathbf{x})$ is again a non-parametrised function in the sense of (14), $g_P(\mathbf{x}, l_i)$ is a parametrised function in the sense of (7), A is the difference in volumes of bodies limited by the $g(\mathbf{x})$ and $g_P(\mathbf{x}, l_i)$ surfaces in the V region being of H_T dimension about the centre of point x_T.

Similarly

$$S_{X_t} = \int [g(\mathbf{x}) - g_P(\mathbf{x}; l_i)]\, x_t\, dV$$

is a difference of static moments of bodies limited by g and g_P to the coordination axis x_t. We shall define analogically the difference of moments of inertia, products of inertia, etc.

$$J_{X^2_t} = \int_V [g(\mathbf{x}) - g_P(\mathbf{x}; l_i)]\, x_t^2\, dV \;;$$

$$J_{X_tX_{t'}} = \int_V [g(\mathbf{x}) - g_P(\mathbf{x}; l_i)]\, x_t\, x_{t'}\, dV \ldots$$

Let us now require the volumes of bodies limited by $g(\mathbf{x})$ and $g_P(\mathbf{x}, l_i)$ to be equal and their static moments, with respect to individual coordinate axes, to be equal. The same would be valid for their moments of inertia, products of inertia and gradually moments of higher order. Then, obviously, both $\mathbf{g}(\mathbf{x})$ and $g_P(\mathbf{x}, l_i)$ will come mutually closer and closer through these characteristics, i.e. they will identify themselves closer and closer through the average.

Our requirements can thus be written as follows:

$$\begin{aligned} -\epsilon_A &\leqslant A \leqslant \epsilon_A & t &= 1, 2, \ldots, n \\ -\epsilon_{X_t} &\leqslant S_{X_t} \leqslant \epsilon_{X_t} & &\text{,,} \\ -\epsilon_{X^2_t} &\leqslant J_{X^2_t} \leqslant \epsilon_{X^2_t} & &\text{,,} \qquad (15) \\ -\epsilon_{X_tX_{t'}} &\leqslant J_{X_tX_{t'}} \leqslant \epsilon_{X_tX_{t'}} & &\text{,,} \\ &\vdots \end{aligned}$$

Conditions (15) cannot be written in the form of the equations, but in the form of inequalities, because the number of parameters l_i being determined will generally differ from the number of conditions (15). Best results in satisfying conditions (15) will be achieved by trying to get the smallest magnitudes of auxiliary parameters ϵ.

Alternative formulation of identification conditions (15) (i.e. achieving the same position on an average of functions g and g_P) can be by the least squares method. It can be understood as a special case of a more general approach (15),

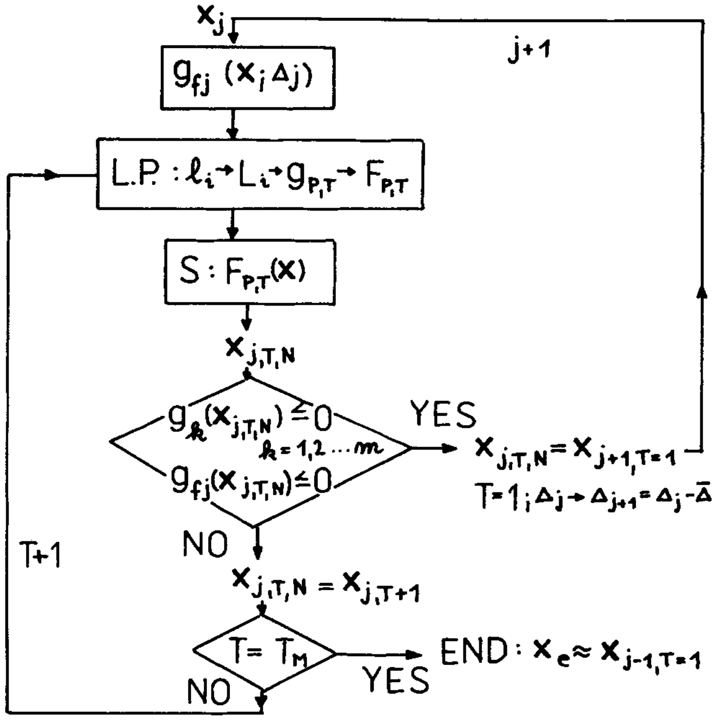

Fig. 3

but will thus be substantially more economical. It can be stated

$$L = \int_V [g(\mathbf{x}) - g_P(\mathbf{x}; l_i)]^2 \, dV \to \min \to \frac{\partial L}{\partial l_i} = 0$$

$$\text{for } i = 1, 2, \ldots, r \qquad (16)$$

thus

$$\int_V [g(\mathbf{x}) - g_P(\mathbf{x}; l_i)] \frac{\partial g_P(\mathbf{x}; l_i)}{\partial l_i} dV = 0 \ , \quad i = 1, 2, \ldots, r$$

In conditions (16) the number of l_i being looked for is equal to the number of conditions. In order to determine l_i, though, conditions (13) and (11) for convexity must be respected at the same time; for this reason conditions (16) are necessarily expressed as inequalities.

$$-\epsilon_i \leqslant \int_V [g(\mathbf{x}) - g_P(\mathbf{x}; l_i)] \frac{\partial g_P(\mathbf{x}; l_i)}{\partial l_i} dV \leqslant \epsilon_i$$

$$i = 1, 2, \ldots, r \qquad (17)$$

The selection of parameters l_i in such a way that g_P is approaching convexity and, at the same time is in the same position on average with g, is subjected, consequently, to constraints (11), (13), (17). A fair satisfaction of these conditions requires

$$f = C_M \epsilon_M + \sum_i C_i \epsilon_i \to \min \qquad (18)$$

where C_i are weights put on to compromise relative meeting of individual conditions. To conditions (11), (13), (17) stated above, additional conditions (12) and (19) are included.

$$l_i \geqslant 0, \epsilon_i \geqslant 0 \ ; \quad i = 1, 2, \ldots, r \qquad (19)$$

It can thus be seen that the problem of finding l_i is reduced to an auxiliary problem of linear programming.

6. A flowchart of the program

Now the rough scheme of the process can be introduced. If the method defined in Section 1 is to be used to solve an optimisation process itself in every stage T, then the scheme illustrated in Fig. 1 becomes the scheme shown in **Fig. 3**.

There will be a change in the procedure in that each step j (reaching the feasible region) will split into M steps of T. The change of stages of a working point from $\mathbf{x}_j$ to $\mathbf{x}_{j,T}$ corresponds. The process is shown in Section 3.

In the $L.P$ block, during the T stage, the de-formation of the barrier function $F(\mathbf{x}) \to F(\mathbf{x}; L_i)$ is performed by means of l_i parameters. These parameters are determined for each particular step T by an auxiliary problem of linear programming. Any integration necessary, in the sense of Section 5 on the assumption that conditions (4) hold, can be done automatically by programming.

In the block $S : F$ the lowest point $\mathbf{x}_N$ is being looked for on this surface $F_{P,T}$. The process could be modified in a variety of ways. For example a more approximate, but more economical version would be the solution of the auxiliary linear programming problem for finding l_i only in case T_1.

7. Illustration problems

Problem 1

For the sake of geometrical clearness in the process following Section 3 let us first consider a simple one-dimensional case. The limitation (4) is chosen in the following form:

$$g(\mathbf{x}) = ax_1^4 + bx_1^3 + cx_1^2 + dx_1 + e \qquad (20)$$

where

$$a = 0{,}00792, b = -0{,}245, c = 2{,}373 \ ,$$
$$d = -7{,}929, e = 14{,}401$$

Then (5) and (7) are in the form:

$$g_{P,T}(\mathbf{x}; L_i) = L_1 \, ax_1^4 + L_2 \, bx_1^3 + L_3 \, cx_1^2 + L_4 \, dx_1 + L_5 \, e \qquad (21)$$

$$g_{P,T_1}(\mathbf{x}; L_i) = g_P(\mathbf{x}; l_i) = l_1 \, ax_1^4 + l_2 \, bx_1^3 + l_3 \, cx_1^2 + l_4 \, dx_1 + l_5 \, e \qquad (22)$$

If a simplified cycle T is chosen (i.e. parametrisation at T_1 only), the condition for achieving the convexity in determining l_i according to (13) is in the form ($\mathbf{x}_0 = \mathbf{x}_{T_1} = 8{,}5$)

$$g_{PX_1X_1}(\mathbf{x}_{T_1}; l_i) = 12 \, l_1 \, a_1 \, x_{1(T_1)}^2 + 6 l_2 \, bx_{1(T_1)} + 2 l_3 \, c = = 6{,}86 \, l_1 - 12{,}49 \, l_2 + 4{,}74 \, l_3 \geqslant 0 \qquad (23)$$

Alternative selections l_1, l_2, l_3 exist for satisfying the condition (23). They are based on trying to enlarge positive members in (23) (through the choice of corresponding $l_i > 1$) and to reduce negative ones (through the choice of corresponding $l_i < 1$; at $l_i \geqslant 0$). From this point of view any variant can be taken from the above mentioned alternatives, according to (24).

l_1	l_2	l_3
1	0,85	1
1	1	1,4
	⋮	

(24)

The remaining l_4, l_5 can be determined from conditions of the same position on average of functions g_P and g (SPA of g and g_P) (15). Only the first two of them will be taken which will enable their interpretation as equations (two equations for two remaining unknown l_4, l_5) under the particular selection of (24).

$$A = 0 \ , \quad S_{X_1} = 0 \qquad (25)$$

The form of (25) (at $H_T = 8{,}5$) is as follows:

$$\int_{h_1=0}^{h_1=17} [g_P(\mathbf{x}; l_i) - g(\mathbf{x})] \, dx_1 = 0 \ ,$$

$$\int_{h_1=0}^{h_1=17} [g_P(\mathbf{x}; l_i) - g(\mathbf{x})] \, x_1 \, dx_1 = 0$$

If the first variant $l_1 = 1$, $l_2 = 0{,}85$, $l_3 = 1$ from (24) is chosen and substituted into (25); then from solving (25) it follows $l_4 = 2{,}204$, $l_5 = 3{,}504$.

A small number of parameters enabled us, in this case, to solve a problem of the convexity and (SPA of g and g_P) simply as the selection of l_i and as the subsequent solving of a system of linear equations. An approach through linear programming is obviously more general.

The procedure itself is illustrated graphically in Fig. 2. Individual $g_{P,T}(\mathbf{x}; L_i)$ are graphically displayed there according to (21) for individual $T = 10, 5, 2, 1, 0$. The sequence of points $\mathbf{x}_0 \to \mathbf{x}_{eT_1} \to \mathbf{x}_{eT_2} \to \ldots$ in the sense of Section 3 is also evident.

Curve b represents (22) where, as a contrary to curve a, only conditions for convexity are respected (l_1, l_2, l_3 according to the first variant (24)) and where conditions for (SPA of g and g_P) were not respected ($l_4 = l_5 = 1$).

Example 2

This is again a one-dimensional case to illustrate the influence

Table 1

No. var	l_1	l_2	l_3	l_4	l_5	h_1	h_2
1	1	0,85	1	1,269	0,475	0	6
2	1	0,85	1	1,907	− 2,229	0	11
3	1	0,85	1	3,167	−10,920	0	17
4	1	0,85	1	4,966	−28,520	0	23

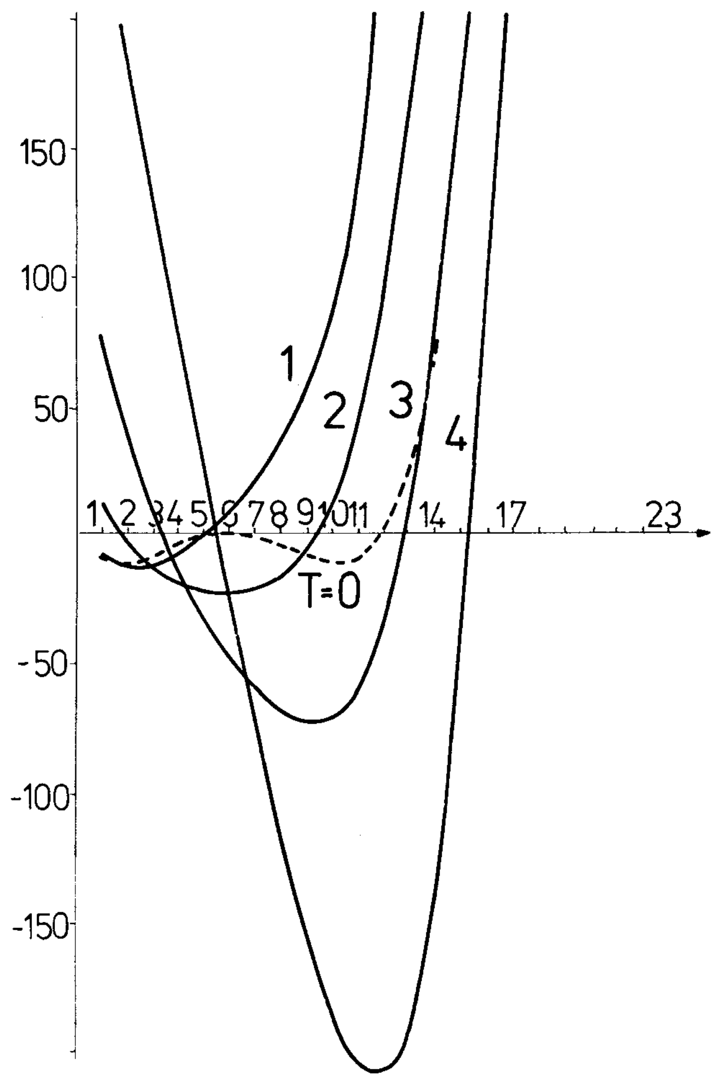

Fig. 4

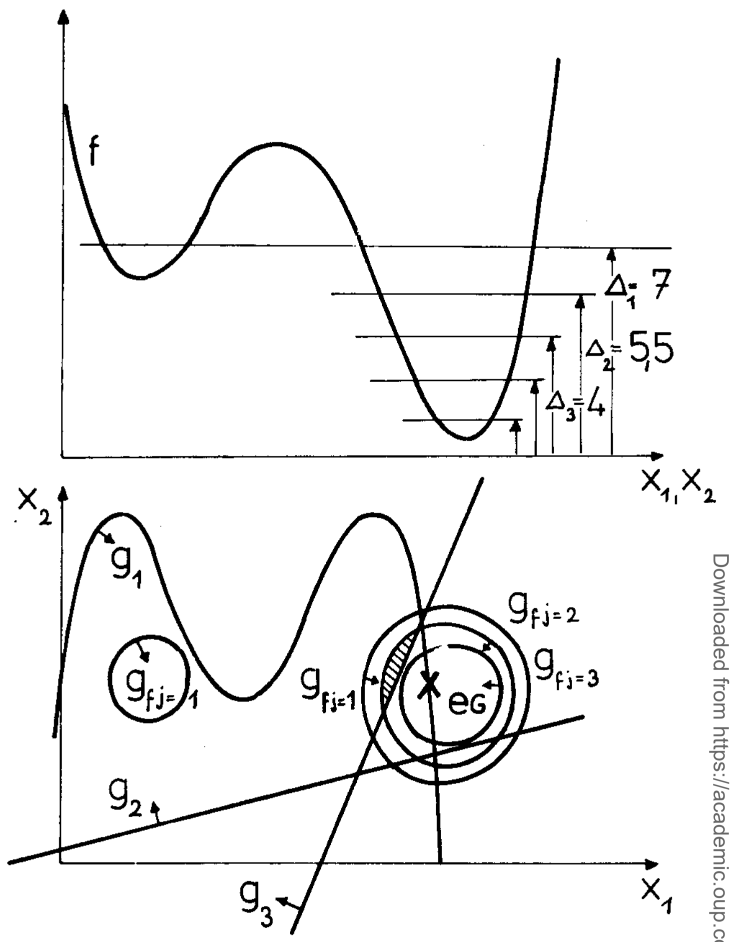

Fig. 5

of the magnitude of H_T (i.e. the integration interval) on the character of identification.

Expressions (20) to (23) will remain the same, only the following quantities have been changed:

$$a = 0{,}0370, \quad b = -0{,}889, \quad c = 6{,}674,$$
$$d = -16{,}02, \quad e = -11{,}002$$

We choose again a simplified interpretation in the choice of the first three parameters l_1, l_2, l_3 in order to satisfy condition (23) (now for new values of $a, b, c, \ldots$) and we compute the values of l_4, l_5 from (25). During this procedure (using (25)) H_T will alternatively be changed (h_1, h_2 are given, i.e. lower and upper limits of integration of expressions (25)).

Alternative choices l_1, l_2, l_3 in the sense of (24) are listed in **Table 1** together with alternative choices of H_T (during the solution of (25)).

In **Fig. 4**, curves (22) (i.e. stages of T_1 only) are shown. The number appearing at the curve in Fig. 4 corresponds to the number of the alternative in the first column of Table 1. The curve for $T = 0$, i.e. $g(\mathbf{x})$, is shown as the dashed line. It is obvious from Fig. 4 that for the larger H_T the (SPA of g_P and g) is of a more global character. At decreasing H_T the (SPA of g_P and g) has a more local character with increasing respect for details in the course of $g(\mathbf{x})$ in the particular region.

Example 3

This is a two-dimensional case that illustrates the case of separable functions.

In **Fig. 5** the graphical interpretation of the method according to Section 1 illustrates beforehand the position of $\mathbf{x}_{eG}$ that was looked for.

In stating the problem the parametrised form is given directly in the sense of (21):

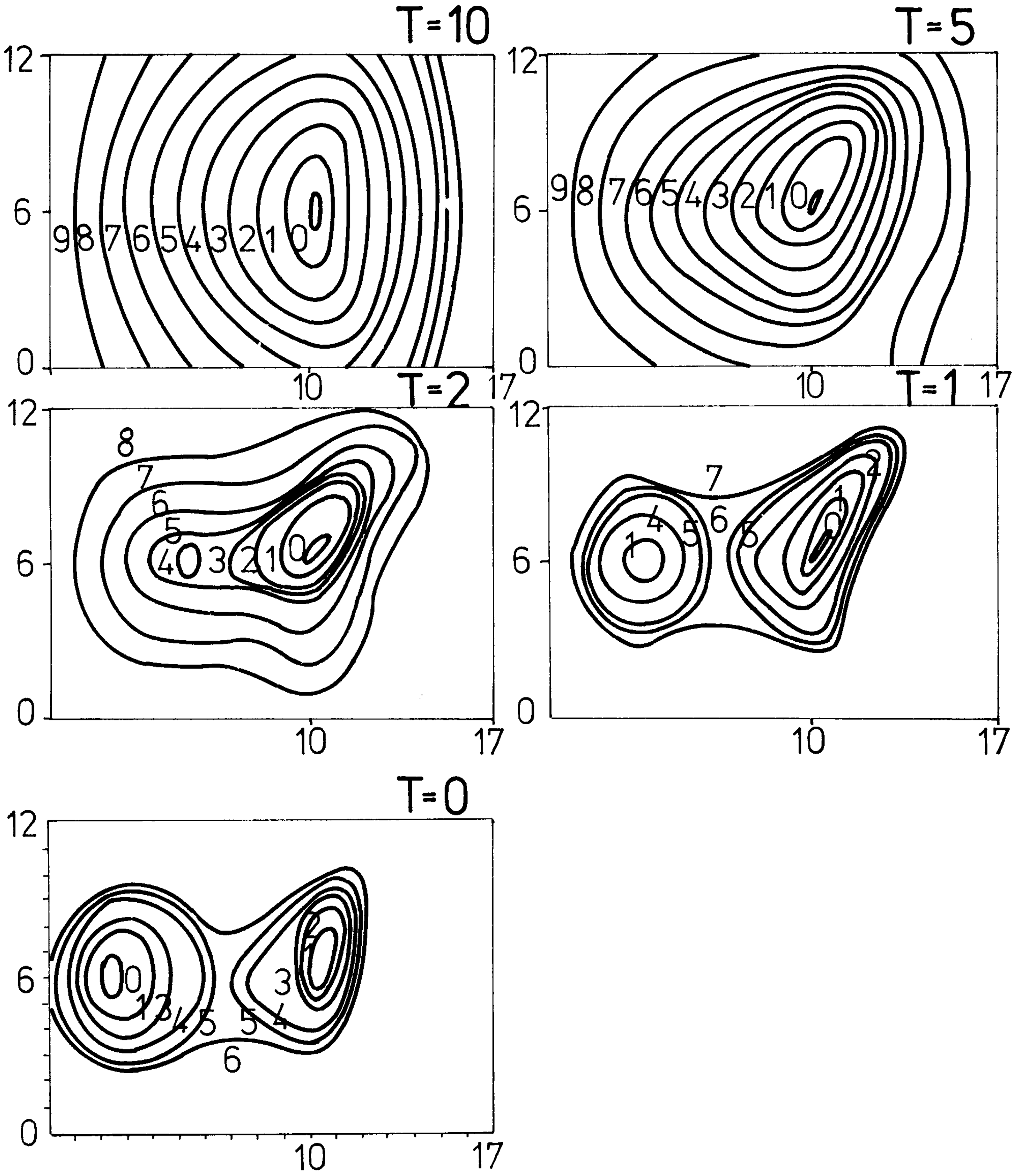

Fig. 6

$$\left.\begin{matrix} g_{fj,P,T}(\mathbf{x}; L_l) \\ g_{1,P,T}(\mathbf{x}; L_l) \end{matrix}\right\} = L_1\, ax_1^4 + L_2\, bx_1^3 + L_3\, cx_1^2 + L_4\, dx_1 + L_5\, e + fx_2^2 + gx_2 + h \qquad (26)$$

where

	a	b	c	d		e	f	g	$h = -\Delta_{j=1}$
g_{fj}	0,00792$_j$	—0,245	2,373	−7,929	g_{fj}	30,601	0,45	−5,4	−7,5
g_1	0,0370	−0,889	6,674	−16,020	g_1	−11,002	0	2	0

$$g_{2;P,T}(\mathbf{x}; L_i) = -2x_2 + 0{,}5x_1 + 1 + L_6 - 1$$
$$g_{3;P,T}(\mathbf{x}; L_i) = -2x_2 + 4{,}84x_1 - 41 + L_7 - 1$$

Functions g_2, g_3 are convex (planes), which means that there was no need to parametrise them. In order to achieve the effect of 'gradual coming back of the system to the original form' they were still augmented by the parameter $L_i - 1$ (with the choice $l_6 = l_7 = -25$).

Functions g_{fj} and g_1 are non-convex only in the part containing x_1 that was parametrised; the part containing x_2 is convex, which means that there was no need to parametrise it. At the same time, it represents a typical case of separable functions. In order to achieve their convexity, only condition (13) is thus used

$$g_{fj,P,x_1x_1}(\mathbf{x}_T; l_i) \geqslant 0 \ , \quad g_{1,P,x_1x_1}(\mathbf{x}_T; l_i) \geqslant 0 \qquad (27)$$

for the numerical satisfaction of condition (27) at $\mathbf{x}_T = (8{,}5; 6)$, to choose l_1, l_2, l_3 according to **Table 2** is satisfactory.

In order to achieve (SPA of g_P and g), the first two conditions from (15) are again used in the form of equations (because only l_4, l_5 remain to be determined). For this, the condition S_{x_2} is not necessary because the direction of x_2 is convex.

Table 2

	l_1	l_2	l_3	l_4	l_5
g_{fj}	1	0,85	1	2,205	2,180
g_1	1	0,85	1	3,167	−10,920

$$A = 0, S_{x_1} = 0 \qquad (28)$$

in the form

$$\int_0^{17}\int_0^{12} [g_{fj;P}(\mathbf{x}; l_i) - g_{fj}(\mathbf{x})]\, dx_1\, dx_2 = 0\,,$$
$$\int_0^{17}\int_0^{12} [g_{fj,P}(\mathbf{x}; l_i) - g_{fj}(\mathbf{x})]\, x_1\, dx_1\, dx_2 = 0$$

For the choice of l_1, l_2, l_3 according to Table 2, l_4, l_5 were obtained by solving (28). Analogically, l_4, l_5 for g_1 were obtained as well. The simplified algorithm was again used (the parametrisation only for T_1) that is illustrated in **Fig. 6** showing the F function according to (2) for $T = 10, 5, 2, 1, 0$. The above mentioned point has reached $\mathbf{x}_{eG}$ at $j = 1$ already, so further decreases of Δ_j in the sense of Section 2 were not necessary.

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Book review

Realization of Data Protection in Health Information Systems, edited by G. Griesser, 1977; 214 pages. (*North Holland Publishing Company*, $24·00)

This book represents the proceedings of the IFIP Working Group 4.2 conference held at Kiel in June 1976. In the editing, Professor Griesser of Kiel was assisted by Messrs J. Anderson (UK), F. Gremy (France), H. Peterson (Sweden), K. Sauter (Germany). The organisers of the conference adopted the procedure whereby the papers were circulated beforehand and the conference time was devoted to comprehensive discussions thereon. In consequence, the publication consists of the twenty papers (pages 1 to 138) followed by forty pages covering the detailed contributions made in the five discussion sessions. These latter cover data protection by hardware precautions, software techniques (two sessions), organisations means, (ORGWARE) and interdependencies. Workers in the medical field were introduced to the word ORGWARE, i.e. organisational methods. This covers topics such as authorisation of personnel, categorisation of terminals according to their location, and choice of correct hardware for the particular job to be done at that location.

The contributed papers cover data protection in group practices, medical research, social insurance, a cancer registry and, of course, a total hospital information system. A few extracts will give readers a flavour of the conference. 'The error rates in messages were lower than in the written record and the amount of data held in the computer record increased appreciably. However, there was an overhead of 30% or more in the time that the junior doctors required to provide accurate data patient information. There were many who felt this was far too high a price to pay for the resulting improved record even when allowing for its accuracy and its increased manipulative capability.' 'One hospital uses minicomputers to accept input data, validate it, record the data on cassettes, these latter then being taken by road to the DP Centre. However, central computing is proving expensive.' 'It has been claimed that the structure of files designed for research is much simpler than files for health care institutions. According to our experience, that does not seem correct: we deal in our centre with every kind of complexity, . .' 'That should imply that the data files need not be kept any longer. Indeed we used to destroy these files, but experience showed us that one must never believe a clinician who claims that his study is finished . . . and henceforth, we do keep the files.' '. . . the protection system of the files within our computer, the PDP 10, is rather sophisticated, implying three levels of protection: . . .' 'In general, the neighbourhood health centres serve small populations (average of 15,000). Since the target areas for the centres often have characteristics of a small town, sensitive information may spread quickly and cause patients embarrassment.'

In the discussion the distinction was made between 'safety', i.e. destruction, falsification, and theft of the media; and 'security', meaning theft of contents, unauthorised access and misuse. Speakers recognised that these problems were not particular to the field of medicine. Opinions were divided as to whether a medical information system/data base could safely share a computer with any other system. On the software side, experiments have been done with 'scrambling' and this has been shown to involve only a 0·1% increase in processing time when passing records to and fro through the scrambler program. One speaker pointed out that, with validation checks in use, the data in the computer is often more accurate and reliable than that in manual records.

The final chapter contains the important conclusions reached after three days of profitable discussions. The meeting recognised that the doctor/patient relationship is a personal relationship based on trust and this trust must be protected by any new system. Computer professionals must come under the same rules of confidentiality as other medical personnel. Computer professionals have a duty to provide systems which protect the doctors/patients from a loss or corruption of data. In the past computer people have let their profession down by letting down the very people they are trying to serve.

The conference has served a useful purpose not only reviewing the state of the art for those involved but also, through this record of the Proceedings, making this knowledge available to those responsible for originating, funding, or managing medical information systems. This book should be read by all those in these fields.

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