Design and Application of Fuzzy PSS for Power Systems Subject to Random Abrupt Variations of the Load

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Abstract— In this paper, a design method and application of fuzzy power system stabilizers for electrical power systems subject to random abrupt variations of loads are considered. A new control design method that uses recently developed techniques based on linear matrix inequalities with damping and control input constraints for fuzzy logic control design is proposed. Random abrupt variations caused by fluctuations in the load patterns are described as Markovian jumps in the parameters of the electrical power system. To illustrate the effectiveness of the control design method, simulation results on a single-machine infinite-bus model are presented and compared to the results of a classical power system stabilizer.

I. INTRODUCTION

Power system stabilizers (PSS's) have long been regarded as an effective way to enhance the damping of electromechanical oscillations in power systems. The PSS's were developed to extend stability limits by modulating the generator excitation to provide additional damping to the oscillations of synchronous machine rotors [1]. Many methods have been used in the design of PSS, such as root locus and sensitivity analysis [1], pole placement [2], adaptive control [3] and robust control [4], [5]. The main problem with these methods is that the control law is based on a linearized machine model and the control parameters are tuned to some nominal operating conditions. In case of a large disturbance, the system conditions will change in a highly nonlinear manner and the controller parameters are no longer valid. In this case, the controller may even add a destabilizing effect to the disturbance by, for example, adding negative damping. To overcome this, the control design method must consider the nonlinear dynamics of the power system. In this context, some stabilizing control solutions for power systems have appeared [6], [7], [8], [9].

Fuzzy logic controllers (FLC) have appeared as an effective tool to stabilize power systems [10], [11], [12], [13]. In the context of PSS's, FLC has been shown to be an efficient tool to reduce electromechanical oscillations [10], [13]. It has been used in generating unit controls coordinated to the automatic voltage regulator (AVR). The FLC combine qualitative and quantitative knowledge about the system operating through some hierarchy. Basically, there exist two kinds of FLC: the Mamdani FLC [14] and the Takagi-Sugeno FLC [15]. They are both flexible in the control

solution, but they differ in the control design. The Mamdani FLC design is heuristic, based on human knowledge and the Takagi-Sugeno FLC design is more systematic, based on nonlinear and robust control design methods as well as on human knowledge.

In this paper, we propose a new FLC to reduce oscillations of electrical power systems subject to random abrupt variations of loads. The FLC uses knowledge about the nominal operating conditions and the probability of occurrence of the random abrupt variations caused by fluctuations in the local load patterns, which are described as Markovian jumps in the parameters of the electrical power system. The stochastic stabilization of the power system is accomplished by using a fuzzy-model-based control design which employs recently developed techniques based on linear matrix inequalities (LMI's) with damping and control input constraints. Simulation results on a single-machine infinite-bus (SMIB) power system illustrate the effectiveness of the proposed control design method, which are compared to the results of a classical PSS.

II. ELECTRICAL POWER SYSTEM MODELING

Consider a SMIB power system as shown in Figure 1 whose dynamics is obtained from the classical Park model of a synchronous machine [16] adopting balanced conditions and neglecting the stator winding resistance, voltages due to magnetic flux derivatives, damper windings, saturation effects and the frequency deviation in speed voltage terms. In addition, the transmission line resistance is neglected. Also consider that the generating unit control consists of a AVR coordinated to a PSS. Under these considerations, the SMIB power system dynamics is described as follows

$$\dot{\delta} = \omega_0 \, \omega \qquad (1)$$

$$\dot{\omega} = \frac{1}{2H} \left[P_m - E'_q I_q \right] \qquad (2)$$

$$\dot{E}'_q = \frac{1}{\tau'_{do}} \left[E_{fd} - E'_q + (x_d - x'_d) I_d \right] \qquad (3)$$

$$\dot{E}_{fd} = \frac{1}{T_e} \left[K_e (V_{ref} - |V_t| + V_s) - E_{fd} \right] \qquad (4)$$

where $|V_t| = \left(V_q^2 + V_d^2 \right)^{1/2}$, $V_q = E'_q + x'_d I_d$ and $V_d = -x'_d I_q$, with δ the power angle of the generator, [rad]; ω the rotor speed of the generator, [rad/s]; ω_0 the synchronous machine speed, [rad/s]; P_m the mechanical input power, [p.u.]; E'_q the transient EMF in the quadrature axis of the generator, [p.u.]; E_{fd} the equivalent EMF in the excitation

coil, [p.u.]; I_q the quadrature axis current, [p.u.]; I_d the direct axis current, [p.u.]; V_q the quadrature axis voltage, [p.u.]; V_d the direct axis voltage, [p.u.]; H the inertia constant, [p.u.]; τ'_{do} the direct axis transient open circuit time constant, [s]; T_e the AVR time constant, [s]; K_e the AVR gain; V_{ref} the AVR reference voltage, [p.u.]; V_t the generator terminal voltage, [p.u.]; V_s the PSS voltage, [p.u.]; V_∞ the infinite-bus voltage, [p.u.]; x'_d the direct axis transient reactance, [p.u.]; x_d the direct axis reactance, [p.u.] and x_e the external equivalent reactance, [p.u.]. In what follows, we obtain the currents I_q and I_d considering the effects of random abrupt variations in the load-bus of the SMIB power system.

Suppose that different loads coupled to the load-bus can be active or not during a period of operating of the SMIB power system. The dynamics of the load-bus affect the dynamics of the nominal operating conditions of this system. The frequency of occurrence of each load in the bus can be determined and then the operating conditions of the system can be divided according to which load it supplies. This modeling is useful, for example, in the operating of industrial power plant in a co-generation scheme, where the power excess can be sent to the global network. In this case, the control solution must guarantee efficiency in reducing electromechanical oscillations and stability of the power plant during the load variations. Figure 2 illustrates the load configuration in the load-bus adopted.

Now, consider a finite set $\mathbb{S} = \{1, 2, \ldots, N\}$ as a set of operating modes of the SMIB power system, with N the number of these modes. Also consider that the load variations are governed by a stochastic process $\{r\}$ which is a finite-mode Markovian jump process taking values in the set $\mathbb{S}$ with transition probability given by

$\Pr\{r(t + \Delta) = j | r(t) = i\} :=$

$$
\begin{cases}
\pi_{ij}\Delta + o(\Delta), & i \neq j \\
1 - \pi_i\Delta + o(\Delta), & i = j
\end{cases} \tag{5}
$$

where $\Delta > 0$, $\lim_{\Delta \to 0} o(\Delta)\Delta^{-1} = 0$, $\pi_{ij} \geq 0$ is the probability rate between modes i and j, for $i \neq j$; $i, j \in \mathbb{S}$ and $\forall i \in \mathbb{S}$, $\pi_i := -\pi_{ii} = \sum_{j=1, j \neq i}^{N} \pi_{ij}$. The matrix $\Pi := [\pi_{ij}]$ is called a transition rate matrix. We assume that the Markov process $\{r\}$ has initial distribution $\mu := (\mu_1, \mu_2, \ldots, \mu_N)$, with $\mu_i := \Pr\{r_0 = i\}$ and that $r_0 = r(0)$ is a non random initial condition. The initial distribution yields the probability of each operating mode to remain in its initial mode. Using this description, each transition between the modes i and j has a probability rate π_{ij}, which depends on the operating conditions of the SMIB power system during a period of time. We suppose that the load activation instants are known. Figure 3 illustrates the transition among modes of the SMIB power system according to the load variations.

Now consider that the SMIB power system supplies two different random loads: $Z_1 = jX_1$ and $Z_2 = jX_2$ with X_1 and X_2 the reactances [p.u.]. Thus, it is possible to
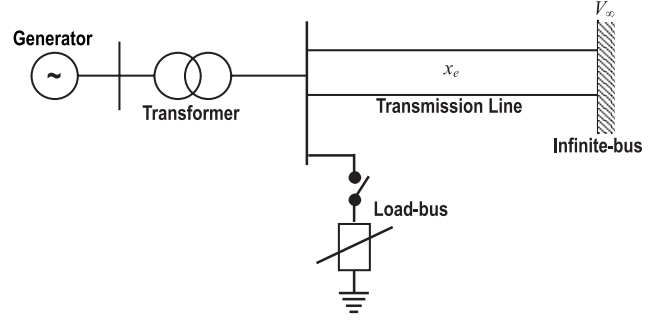

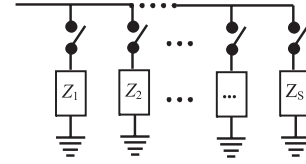Fig. 1. Unifilar diagram of the SMIB power system.

Fig. 2. Configuration of the load-bus.

determine the currents I_q and I_d as

$$
I_q = \frac{X_{eq}V_\infty \sin\delta}{x'_d(x_e + X_{eq}) + x_e X_{eq}}
$$

$$
I_d = \frac{X_{eq}V_\infty \cos\delta - (x_e + X_{eq})E'_q}{x'_d(x_e + X_{eq}) + x_e X_{eq}}
$$

where k_ℓ, $\ell = 1, 2, ..., 6$, are parameters which vary according to the equivalent load X_{eq} in the load bus of the SMIB power system. The operating conditions of the system according to the load variations are modeled as a Markov chain with three different operating modes ($N = 3$), which correspond to the possible combinations between loads Z_1 and Z_2 as following: mode 1 - only load Z_1, mode 2 - only load Z_2 and mode 3 - both loads Z_1 and Z_2. We consider that the system under nominal operating conditions is given by mode 3. However, during a period of time, the dynamics of the system can be changed following the process $\{r\}$ with the following initial distribution
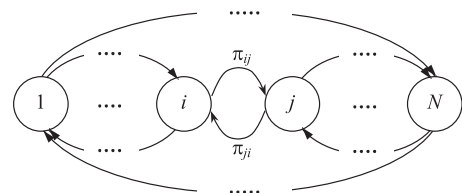
$$
\mu = (0.30, \ 0.20, \ 0.50) \tag{6}
$$

Fig. 3. Transition modes of the SMIB power system.

Mode	k_1	k_2	k_3	k_4	k_5	k_6	V_{ref}
1	0.86	1.82	1.10	0.50	0.27	0.27	1.42
2	0.76	2.06	0.97	0.44	0.24	0.24	1.39
3	0.61	2.44	0.79	0.36	0.20	0.20	1.35

and the transition rate matrix

$$\Pi = \begin{bmatrix} -3.60 & 1.40 & 2.20 \\ 2.10 & -2.80 & 0.70 \\ 1.32 & 0.28 & -1.60 \end{bmatrix} \quad (7)$$

The numerical values of physical parameters are: $\omega_0 = 377$, $H = 1.50$, $P_m = 1$, $\tau'_{do} = 6$, $T_e = 0.01$, $K_e = 100$, $V_\infty = 1.20$, $x'_d = 0.32$, $x_d = 1.60$, $x_e = 0.70$, $X_1 = 0.60$ and $X_2 = 0.40$. Table I shows the numerical values of parameters k_ℓ, $\ell = 1, 2, ..., 6$ and V_{ref} for each mode. In the control literature, the SMIB power system (1) - (7) can be recognized as a typical example of Markovian jump nonlinear systems [17], a class of hybrid systems, which has different operating modes governed by the Markovian jump process $\{r\}$ defined before. In what follows, we present a new approach to stabilize this class of systems using fuzzy-model-based control design method.

III. FUZZY-MODEL-BASED CONTROL

The FLC design proposed here considers a non local approach, which is conceptually simple and straightforward, following the Takagi-Sugeno (TS) fuzzy systems theory wherein linear feedback control techniques can be used. TS fuzzy systems are used to approximate nonlinear systems by a fuzzy blending of local linear subsystems. In fact, it is proved that the TS fuzzy systems are universal approximators [18]. The control design is based on the fuzzy system modeling via the so-called parallel distributed compensation (PDC) scheme, where a linear controller is designed for each local linear subsystem. The overall controller is again a fuzzy blending of all local linear controllers, which is nonlinear in general. Our goal is to represent the nonlinear dynamics of the SMIB power system (1) - (7) in each operating mode by a TS fuzzy system in order to obtain a FLC to reduce the electromechanical oscillations. Figure 4 shows the block diagram using the FLC proposed.

A. Fuzzy System Modeling

Adopting $x = [\delta, \ \omega, \ E'_q, \ E_{fd}]^T$ as the vector of state variables and $u = V_s$ as the input vector, an N-mode SMIB power system can be described in a generic state-space representation as

$$\dot{x} = f(x, u, r); \ x_0 = x(0); \ r_0 = r(0) \quad (8)$$

where $f(\cdot, \cdot, \cdot)$ is a vector of smooth nonlinear functions with respect to the first and the second arguments, $r \in \mathbb{S}$, r and $\mathbb{S}$ are as defined before, x_0 is the initial state which,
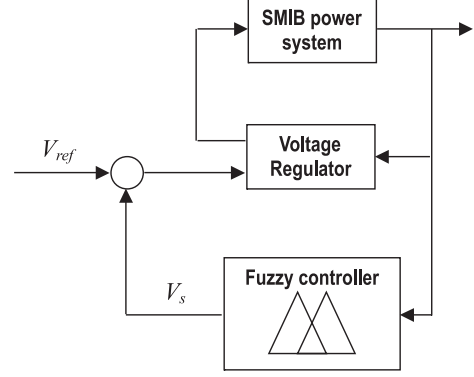

Fig. 4. Control system block diagram of the SMIB power system.

for simplicity, is a fixed non random constant vector and r_0 is the initial operating mode of the SMIB power system.

The fuzzy system proposed to approximate system (8) has two levels in its structure, one representing the Markovian jump process $\{r\}$ and the other representing the nonlinearities in the state variables x, which is given by

$$\dot{x} = \hat{f}(x, u, r)$$
$$= \sum_{i=1}^{N} \sum_{j=1}^{R} m_i(r) n_{ij}(x) \left(A_{ij} x + B_{ij} u \right) \quad (9)$$

where x and u are as defined before, A_{ij} and B_{ij}, $i \in \mathbb{S}$, $j = 1, 2, \ldots, R$ are matrices of appropriated dimensions representing the local linear approximations of the system nonlinearities with

$$m_i(r) = \frac{M_i(r)}{\sum_{\ell=1}^{N} M_\ell(r)} \quad (10)$$

and

$$n_{ij}(x) = \frac{\prod_{k=1}^{n} N_{ijk}(x_k)}{\sum_{j=1}^{R} \prod_{k=1}^{n} N_{ijk}(x_k)} \quad (11)$$

the mode indicator and normalized membership functions, respectively, $m_i(r) \in \{0, 1\}$ is the grade of membership of r in a crisp set M_i and $N_{ijk}(x_k) \in [0, 1]$ is the grade of membership of x_k in a fuzzy set N_{ijk}. TS fuzzy systems have been successfully used to approximate the nonlinearities of real systems [18], [19]. In the present approach, the approximation error in each mode i $e = f(x, u, i) - \hat{f}(x, u, i)$ can be reduced considering more local linear approximations and/or other type of membership functions to choose regions of the subspace that better represent the dynamics of the nonlinear system under random abrupt variations.

We consider the nominal operating conditions of system (1) - (7) for $\delta \cong \pi/6 \pm 20\%$ which assures the real and reactive powers as $P = 1.0 \pm 20\%$ p.u. and $Q = 0.5 \pm 20\%$ p.u., respectively, and two local linear approximations ($R = 2$) around the following linearization points $\bar{x}$: mode 1 - $\bar{x}_{R=1} = [2\pi/15 \ 0 \ 2.86 \ 1.49]^T$ and $\bar{x}_{R=2} = [\pi/5 \ 0 \ 1.98 \ 0.71]^T$; mode 2 - $\bar{x}_{R=1} = [2\pi/15 \ 0 \ 3.24 \ 1.27]^T$
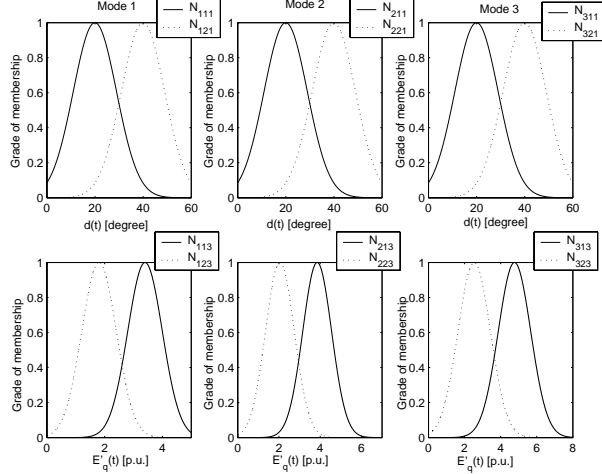
Fig. 5. Membership functions adopted.

and $\bar{x}_{R=2} = [\pi/5 \; 0 \; 2.24 \; 0.52]^T$ and mode 3 - $\bar{x}_{R=1} = [2\pi/15 \; 0 \; 4.01 \; 0.92]^T$ and $\bar{x}_{R=2} = [\pi/5 \; 0 \; 2.77 \; 0.20]^T$. In order to obtain matrices A_{ij} and B_{ij}, $i = 1, 2, 3$ and $j = 1, 2$, we use the Teixeira & Żak linearization formula [19].

The mode indicator membership functions $m_i(.)$, $i = 1, 2, 3$ are crisp functions which represent the operating modes, in this case $m_i(r) = 1$, if $r = i$ and $m_i(r) = 0$, otherwise. The normalized membership functions $n_{ij}(.)$, $j = 1, 2$ are obtained from standard membership functions available in the Matlab Fuzzy Control Toolbox, which represent the range of the state variables x_1 and x_3 in each transition mode as shown in Figure 5.

B. Fuzzy PSS Design

The fuzzy PSS proposed here shares the same structure of the fuzzy system (9) which is given by

$$u = -\sum_{i=1}^{N} \sum_{j=1}^{R} m_i(r) n_{ij}(x) F_{ij} x \qquad (12)$$

where $F_{ij} \in R^{m \times n}$, $i \in \mathbb{S}$, $j = 1, 2, \ldots, R$, are the state feedback gains to be designed, $m_i(.)$ and $n_{ij}(.)$ are as in (10) and (11), respectively. Substituting (12) in (9) and using the fact that $m_i(z)m_\ell(r) = 0$, $i \neq \ell$, i, $\ell \in \mathbf{S}$, it results
$\dot{x} =$

$$\sum_{i=1}^{N} m_i(r) \left[\sum_{j=1}^{R} \sum_{k=1}^{R} n_{ij}(x) n_{ik}(x) (A_{ij} - B_{ij} F_{ik}) \right] x. \qquad (13)$$

Defining $G_{ij} := A_{ij} - B_{ij} F_{ij}$ and $H_{ijk} := A_{ij} - B_{ij} F_{ik} + A_{ik} - B_{ik} F_{ij}$, $i \in \mathbb{S}$, $j, k = 1, 2, \ldots, R$ and using the fact that $n_{ij}(x) n_{ik}(x) = n_{ik}(x) n_{ij}(x)$, after some algebraic manipulations, system (13) can be written as
$\dot{x} =$

$$\sum_{i=1}^{N} m_i(r) \left[\sum_{j=1}^{R} n_{ij}^2(x) G_{ij} + \sum_{j<k}^{R} n_{ij}(x) n_{ik}(x) H_{ijk} \right] x. \qquad (14)$$

In (14), for instance for $R = 3$, $\sum_{j<k}^{3} a_{jk} = a_{12} + a_{13} + a_{23}$.

Using the concept of stochastic stabilizability and a coupled Lyapunov function of the type $V(x, i) = x^T P_i x$, $P_i = P_i^T$, $P_i > 0$, it is possible to derive convex problems using LMI's in order to obtain the state feedback gains F_{ij}, $i \in \mathbb{S}$ and $j, k = 1, 2, \ldots, R$. In our approach, we use the stabilizing fuzzy control design including the following performance indices: decay rate, in order to reduce the speed of response and control input constraints to guarantee the operating conditions of the SMIB power system (see the appendix for the preliminary results).

Design Problem: Given α_i and γ_i, $i \in \mathbb{S}$, find a set of positive definite matrices X_i and a set of matrices Y_{ij} of appropriated dimensions satisfying the following LMI's $\forall i \in \mathbb{S}$

$$\begin{bmatrix} T_{ij} & Z_i \\ Z_i^T & -W_i \end{bmatrix} < -2\alpha_i \begin{bmatrix} X_i & 0 \\ 0 & 0 \end{bmatrix};$$
$$j = 1, 2, \ldots, R \qquad (15)$$

$$\begin{bmatrix} U_{ijk} & Z_i \\ Z_i^T & -W_i \end{bmatrix} < -2\alpha_i \begin{bmatrix} X_i & 0 \\ 0 & 0 \end{bmatrix};$$
$$j < k; \; j, k = 1, 2, \ldots, R \qquad (16)$$

$$\begin{bmatrix} 1 & x_0^T \\ x_0 & X_i \end{bmatrix} \geq 0; \qquad (17)$$

$$\begin{bmatrix} X_i & Y_{ij}^T \\ Y_{ij} & \gamma_i I \end{bmatrix} \geq 0;$$
$$j = 1, 2, \ldots, R \qquad (18)$$

where

$$T_{ij} := X_i A_{ij}^T + A_{ij} X_i - Y_{ij}^T B_{ij}^T - B_{ij} Y_{ij} - \pi_i X_i$$
$$U_{ijk} := \begin{array}{l} X_i A_{ij}^T + A_{ij} X_i - Y_{ik}^T B_{ij}^T - B_{ij} Y_{ik} \\ + X_i A_{ik}^T + A_{ik} X_i - Y_{ij}^T B_{ik}^T - B_{ik} Y_{ij} - \pi_i X_i \end{array}$$
$$Z_i := \begin{bmatrix} \pi_{i1}^{1/2} X_i & \ldots & \pi_{ii-1}^{1/2} X_i & \pi_{ii+1}^{1/2} X_i & \ldots & \pi_{iN}^{1/2} X_i \end{bmatrix}$$
$$W_i := diag\{ X_1 \; \ldots \; X_{i-1} \; X_{i+1} \; \ldots \; X_N \}$$
$$Y_{ij} := F_{ij} X_i$$
$$X_i := P_i^{-1}.$$

IV. SIMULATION RESULTS

In order to test the efficiency of the fuzzy PSS proposed, we obtain the solution of (15-18) using the Matlab LMI toolbox for decay rates $\alpha_1 = \alpha_2 = 4$, $\alpha_3 = 0$, control input constraint $\gamma_i = 0.1$, $i = 1, 2, 3$ and the following initial conditions $x_0 = [\pi/6 \; -0.001 \; 3.3 \; 4.5]^T$ and $r_0 = 3$. We consider the nominal operating conditions of the SMIB power system in mode 3 as $\delta \cong \pi/6$ ($\cong 30°$) for $P \cong 1.0$ and $Q \cong 0.5$. Using μ and Π previously defined, we use the software provided in [20] to simulate transitions among operating modes which are given by the Markovian jump process $\{r\}$. Figure 6 shows load variations during a period of operation of the SMIB power system. Table II presents the control design results. We compare the results obtained using the fuzzy PSS with the results obtained using a classical PSS which was tuned according to [1]. Figures 7 and 8 show the main system responses.
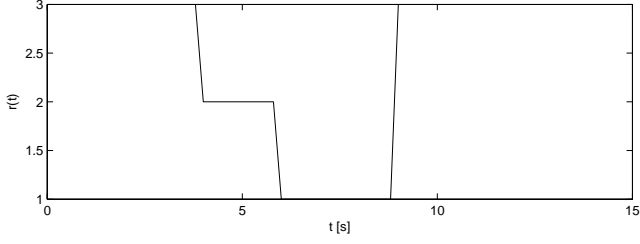
Fig. 6. Load variations during a period of operation of the SMIB power system.

TABLE II

CONTROL DESIGN RESULTS.

Mode	Feedback gains			
1	$F_{11} = [-0.10$	-0.47	0.49	$0.01]$
	$F_{12} = [-0.13$	-0.62	0.49	$0.01]$
2	$F_{21} = [-0.09$	-0.49	0.43	$0.01]$
	$F_{22} = [-0.12$	-0.60	0.43	$0.01]$
3	$F_{31} = [-0.08$	-0.61	0.35	$0.01]$
	$F_{32} = [-0.10$	-0.67	0.35	$0.01]$

Note that the error e given by the fuzzy approximation to the SMIB power system is small and can be further decreased considering more linearization points or other type of membership functions. The use of decay rate and control input constraints in the stabilizing control design problem reduces electromechanical oscillations in the state variables x and control input u. The advantage of using Markov jump systems to model the SMIB power system can be clearly seen, for instance, we could include in the SMIB power system a more refined description of the variation in the load-bus. Taking into account this information, we can provide less restrictive conditions for stability using controllers which provide better performance. Another important point concerns the stability of the SMIB power system. Using the conventional methods, the power system must be stable for all deviations of the load-bus, whereas in the stochastic stability framework, stability of all operating modes is not even required. In this case, stability in each mode is given in terms of matrices (A_{ij}, B_{ij}, Π), $\forall i \in \mathbb{S}$ and $j = 1, 2, \ldots, R$, i.e., the stability in each mode of the SMIB power system is verified whenever $\lambda_{max}[1/2\pi_i\mathbf{I}] > \lambda_{max}[A_{ij}]$, whereas in the conventional methods, stability in each mode is verified only if $\lambda_{max}[A_{ij}] < 0$.

V. DISCUSSION AND CONCLUSIONS

The fuzzy PSS proposed here is proved to be very effective in reducing electromechanical oscillations. The simulation results for both fuzzy and classical PSS show that the LMI-fuzzy-model-based control design achieves better regulation. Moreover, the fuzzy PSS is more flexible as it uses in the design knowledge of the system operation. The design method presented in this paper can be accomplished by a systematic procedure using convex programming with efficient interior-point algorithms. Further work includes
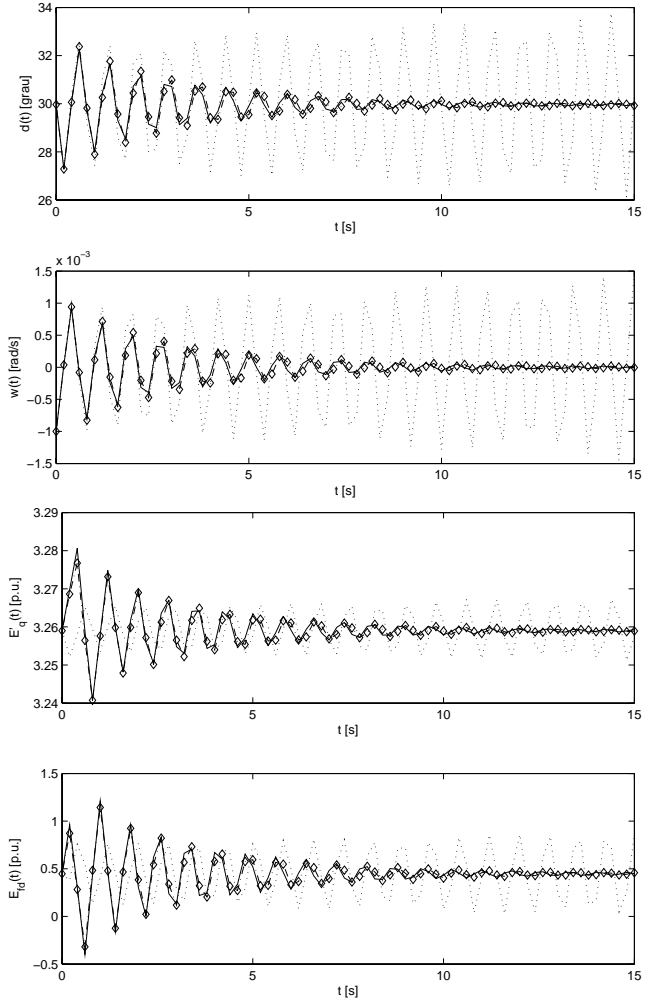

Fig. 7. State variables: '—' SMIB power system with fuzzy PSS, '···' SMIB power system with classical PSS and '-◇-' Fuzzy system model with fuzzy PSS.
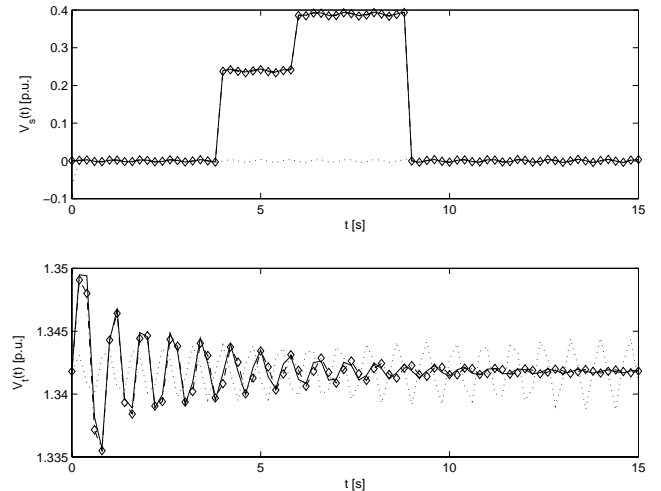

Fig. 8. Control input and terminal voltage: '—' SMIB power system with fuzzy PSS, '···' SMIB power system with classical PSS and '-◇-' Fuzzy system model with fuzzy PSS.

the development of a dynamic feedback FLC to consider incomplete information of the state of a system using the same fuzzy modeling presented.

Acknowledgments

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Appendix

In this appendix we present the preliminary results for the stabilizing fuzzy control design proposed here.

Definition 1: [17] The fuzzy system (9) is said to be stochastically stable if there exists a state feedback fuzzy control law (12) satisfying

$$\lim_{T \to \infty} E \left[\int_0^T x(t, x_0, r_0, u)^T x(t, x_0, r_0, u) \; dt | x_0, r_0 \right]$$
$$\leq x_0^T M x_0 \quad (19)$$

for some symmetric positive definite matrix M of appropriated dimensions.

Proposition 1: The fuzzy system (9) is stochastically stabilizable with state feedback fuzzy control law (12) if there exist a set of positive definite matrices X_i and a set of matrices Y_{ij} of appropriated dimensions, $i \in \mathbb{S}$ and $j = 1, 2, \ldots, R$, satisfying the following LMI's $\forall i \in \mathbb{S}$

$$X_i A_{ij}^T + A_{ij} X_i - Y_{ij}^T B_{ij}^T - B_{ij} Y_{ij}$$
$$-1/2 \pi_i X_i + \sum_{\ell=1; \; \ell \neq i}^N X_i (\pi_{i\ell} X_\ell^{-1}) X_i < 0;$$
$$j = 1, 2, \ldots, R \quad (20)$$
$$X_i A_{ij}^T + A_{ij} X_i - Y_{ik}^T B_{ij}^T - B_{ij} Y_{ik}$$
$$+ X_i A_{ik}^T + A_{ik} X_i - Y_{ij}^T B_{ik}^T - B_{ik} Y_{ij}$$
$$-1/2 \pi_i X_i + \sum_{\ell=1; \; \ell \neq i}^N X_i (\pi_{i\ell} X_\ell^{-1}) X_i < 0;$$
$$j < k; \; j, k = 1, 2, \ldots, R \quad (21)$$

where X_i and Y_{ij} are as defined before.

Proof: The proof is omitted due to lack of space. ■

Proposition 2: Assume that the decay rate $\alpha_i := \alpha_{r=i}$, $\alpha_i > 0$, $i \in \mathbb{S}$ is known. The condition

$$\mathcal{A}V(x, i) \leq -2\alpha_i V(x, i) \quad (22)$$

is enforced to all trajectories of the fuzzy system (9) with state feedback fuzzy control law (12), if there exist a set of positive definite matrices X_i and a set of matrices Y_{ij} of appropriated dimensions satisfying LMI's (15) and (16) $\forall \; i \in \mathbb{S}$.

Proof: The proof is omitted due to lack of space. ■

Proposition 3: Assume that the initial condition x_0 is known. The constraint

$$E[u^T u | x, r = i] \leq \mu_i^2 \quad (23)$$

is enforced to all trajectories of the fuzzy system (9) with state feedback fuzzy control law (12), if LMI's (17) and (18) hold $\forall i \in \mathbb{S}$.

Proof: The proof is omitted due to lack of space. ■