Design and Implementation of Forward and Inverse Gravity Modeling Tool

Amanuel Zewge*, **Elias Lewi and Dida Midekso**

Addis Ababa University, Department of Computer Science, P O Box 1888, Addis Ababa, Ethiopia

Abstract: The implementation of a tool to perform two-dimensional forward and inverse gravity data modeling that can be used to interpret the subsurface geologic structure is presented in this article. The approach subdivides the subsurface into regular shape prisms and reconstructs the geologic structures by assigning variable densities to the different prisms. To obtain the subsurface density distribution one will first use the forward modeling tool and generate a plausible model of the subsurface to use it later as the initial model in the inversion program. The inversion tool makes use of the compact gravity data inversion algorithm to iteratively model the subsurface. The advantage of this approach is that the desired geological characteristics are automatically incorporated into the model with a minimum subjective judgment on the part of the interpreter. The method was demonstrated by inversion of synthetic and real data. The synthetic data is generated from a two–dimensional model consisting of a regular array of identical blocks whose densities can be individually specified. While testing the application on real data that were collected around Filwoha (Addis Ababa), the resulting subsurface structural model produced gravity data that matched with the observed gravity data, within a predefined acceptable root mean square error.

Keywords: Gravity Anomaly; Forward Gravity Modeling; Inverse Gravity Modeling

1. Introduction

The physical properties of the earth's interior can be deal with different methods such as gravity, resistivity, electromagnetic and seismic reflection. Among these methods, the gravity technique is used to map the subsurface structure of the earth (density contrast) by measuring the gravitational field variation on the surface of the earth. The basis for this method is Newton's law of gravitation, which states that the gravitational attraction between two particles is directly proportional to the mass of the two particles and inversely proportional to the square of the distance between their centers of mass of the particles (Bouguer, 1977). The proportionality constant is the gravitational constant G. A gravity survey mainly involves the collection of data in the target area using a relative gravimeter, which can be referred to an absolute gravity value at a base station. The measurement of the gravitational field at a series of measurement points demands the determination of station spacing before the survey is carried out. The selection of the station spacing depends upon the problem at hand and it should be chosen optimally to prevent an aliasing effect (Willian, 1987, XU, 2005). The collected data are then reduced to remove all undesired temporal and spatial variations that are not related to the body of interest to produce what is called the complete Bouguer Anomaly. Depending on the need, this anomaly can be separated further into long and short wavelength signals that are related to deep and shallow seated causative bodies, respectively.

It is then possible to carry out qualitative and quantitative interpretation of the gravity data to determine the shape, size and location of the causative body. However, the determination of the size, shape and position of the subsurface structure is a challenging process for the problem is non-unique in nature. To overcome this problem, it is necessary to constrain the inversion process using a priori information from previous geophysical surveys, borehole data or any other geological information.

In the modeling process different geological models are constructed by fitting the model generated with the measured data. Among the plausible different models the one that best define the a priori information is chosen to represent the subsurface physical property. The means that, even if the density of the subsurface cannot be uniquely determined from gravity measurements, one can often develop a class of models which will give a 'closest fit 'to the anomaly based on a priori information. In this process one can try to find a best fit of measured and computed gravity curves using either the density parameter of the XY-coordinates or the geometry of the model (Nagy, 1966; Carmichael, 1977; Casten *et al.,* 2004). There are in general two approaches which enable either the shape or the density of the causative body to be fixed. The approach that we have chosen in this work is to subdivide the subsurface using prismatic bodies and fix the shape and size of the prisms to vary their density so that the horizontal variation of the density can be modeled.

The modeling can be done using either forward and inverse gravity modeling or both. In forward modeling, the interpreter starts from an initial model, Mo, computes the predicted data values and compares them with the observed ones manually. Then, by considering all available information and using personal judgment, one can apply corrections to M_o in order to minimize the misfits between the computed and observed data. The procedure is repeated until a satisfactory result is obtained. On the other hand, the inversion modeling automatically

*Corresponding author. E-mail: azewgee@gmail.com

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determines the position, the density contrast and the geometry of a causative body, provided that sufficient prior information about the source is incorporated by the method (Tarantola, 1987; Krzysztof, 2005).

A classical problem in gravity exploration is the computation of theoretical anomalies caused by idealized models of known shapes. Towards this end, many researchers have published different methods for carrying out such computation, and textbooks on potential theory, e.g. (Parker, 2005) have used numerical integration techniques for the computation of the fields due to models of arbitrary shape by dividing them into polygonal prisms or laminas. (Valeria *el al.,* 1994) tried to find depth and density values using gravity data. (Last *el al.,* 1993) estimated subsurface density distribution with recursive inverse solution techniques. (Krzysztof, 2005) used the Genetic algorithm

In this article the aim is to present a new modeling tool that has been developed based on the compact gravity inversion algorithm (Elias, 2007; Last *el al., 1993*), which makes use of fixed shapes to estimate the physical parameters of buried objects in terms of maximum compactness of the anomalous sources. The modeling tool that can be used in the forward and inverse modeling mode and it has the capacity to use the forward developed model as an initial model for the inverse modeling algorithm.

According to Willian (1987) XU (2005), gravity methods have played increasingly important roles in the search for new reserves of ore since the development of highly portable gravimeters that have a high degree of precision. Measurements of gravity data provide information about densities of rocks in the earth's subsurface. For example, rocks of differing densities may occur on the opposite sides of a fault. Hence, the developed gravity modeling tool can be applied to find the offset of the bedrock caused by faulting, which might control water of the ascending geothermal system. In general, with the help of gravimeter, the end product of this work will be useful to the researcher in acquiring knowledge about the subsurface geological structure or at least some of its elements, in terms of the anomalous mass distribution (density contrast) without destroying the environment.

2. Methodology and Procedures

Most geologic problems demand critical observation, which will lead to inquiry as to the cause of the observed facts. Hence, the examination of subsurface properties of the earth, using geophysical data is generally made using geophysical inversion methods**,** and demands the experience of the geophysicist as well as the available a priori information so that result will be unique in the loose sense. There are a number of subsurface inversion techniques and in each assumes that that a physical law holds. In gravitational- field inversion, for example, the law are described by the gravitational attraction. With this technique an algorithm based on a physical law enables us mathematically describe an observed physical process. With this assumption, each geophysical data set is inverted to invert the observed data for the subsurface characteristics which gave rise to the observations. To this end, we classified our procedures for gravity modeling and quantitative interpretation into two namely, Data Filtering and Model-Fitting

2.1. Data Filtering

Before we proceed to model-fitting, the gravity data must be corrected systematically for all factors that influence the magnitude of gravity at any particular location, other than those which represent subsurface densities. The filtered gravity value is then obtained after the elevation (Free-air) correction, correction for latitude, and the Bouguer correction have been done (Tarantola, 1987).

2.2. Model-fitting

Once we have removed the effect of all factors except the variation in the lateral density contrast of the subsurface from the measured gravity data, we can then model the subsurface structure that caused the gravity anomalies. This was done in three steps.

2.2.1. Parameterization

This refers to identifying an appropriate list of parameters that determine the model characteristics such as *geometry, density, depth and station interval.*

2.2.2. Selecting Modeling method

This determines the approach to be used in deciding the method for computing the theoretical data using the identified model parameters. To obtain the gravity effect of a given mass distribution using precise mathematical expression we used Regular Shaped Methods**.** It considers the earth (the geological volume to be simulated) is composed of many simple elements, for example equally sized rectangular prisms. Because the geometry of the model structure underlying the gravity profile remains constant, only the variable parameters are the densities of the blocks. Hence, the subsurface gravity is then the sum of the gravity effects of each cell. By changing the density of the selected cells, the interpreter changes the modeling structure which is assumed to be simple and good in realizing the expected model.

2.2.3. Modeling

This is the reconstruction of the model parameters iteratively based on certain criteria so as to obtain a good match with the observed data. It can be performed in two major ways, namely, forward and inverse gravity modeling.

2.2.3.1. The Forward gravity modeling

The forward gravity modeling procedure consists of a code that computes gravity fields from an assumed subsurface density distribution. To make it clear, let us denote the forward gravity modeling process as a transformation $f = T(x)$, where f is the model response, x is a vector containing the set of subsurface model parameters, and T is some transformation which we will with a forward model selected to simulate the particular physical process producing the recordings. Thus, a gravity simulation algorithm might produce a synthetic gravity field that is to be matched to a set of observed gravity readings. All such algorithms are designed to minimize some measure of the difference between the observed and the computed data (Krzysztof, 2005; XU, 2005).

In order to minimize the difference between the observed and the computed data, we start out with an initial guess of the model parameters and for all successive steps the optimization algorithm yields a set of adjusted or updated parameter estimates. These updated parameters are then "plugged" into the theoretical model, and the resulting new theoretical response should produce an improved match to the data. If this happens, the inversion is said to converge; if not, it is necessary to do these calculations iteratively. The above procedure must be applied many times in succession until a satisfactory degree of agreement between the theoretical and the recorded gravity responses has been achieved.

In this work, a finite region in the X-Z plane (Figure 1) is divided into *M* rectangular prisms. Each has constant density but density variations among different prisms are allowed.

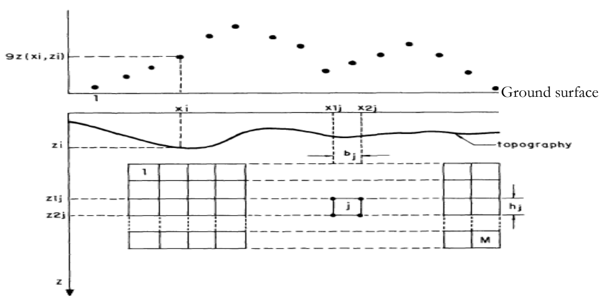

FIG. 1. Gravity field at N points (above) and physical domain discretized into *M* two-dimensional prisms (below).

Based on Figure 1, in order for forward gravity modeling to compute the vertical gravity anomaly at a point due to an arbitrary body of two-dimensional (rectangular prism) shape we used the algorithm which is developed by (Nagy, 1966); that is, let the horizontal and vertical distance from the center of the reference system to the center of the j^{th} rectangular prism be X_j and Z_j respectively. If the width and height of the rectangle are W and H, respectively, the vertical gravity anomaly at the i^{th} measurement point due to the entire j^{th} rectangular block can be obtained by

$$g_i = \sum_{j=1}^{m} a_{ij}\rho_j + e_i \text{ ; I = 1,2,…..N …………...….(1)}$$

where g_i = is the vertical gravity anomaly at the i^{th} measuring point due to the j^{th} rectangular prism; ρ_j = is the density of the j^{th} rectangular block; a_{ij} = is the geometric effect of the j^{th} rectangular block at the i^{th} station; e_i = is the random error term at a station i (the difference between the observed and estimated gravitational field at the i^{th} station).

The exact expression for a_{ij} is given as

$$a_{ij} = 2*G*\left[\left(X_i + \frac{W}{2}\right)\log\left(\frac{r_2 r_3}{r_1} r_4\right) + W\log\left(\frac{r_4}{r_3}\right) - \left(Y_j + \frac{H}{2}\right)(\theta_4 - \theta_2) + \left(Y_j - \frac{H}{2}\right)(\theta_3 - \theta_1)\right]$$

Where G = The universal gravitational constant; W = Prism width; H = Prism Height; X_i = The horizontal distance from the center of the reference point to the center of the rectangular prism; Y_j = The vertical distance from the center of the reference point to the center of the rectangular prism.

$$r_1^2 = \left(Y_j - \frac{H}{2}\right)^2 + \left(X_i - X_j + \frac{W}{2}\right)^2$$

$$r_2^2 = \left(Y_j + \frac{H}{2}\right)^2 + \left(X_i - X_j + \frac{W}{2}\right)^2$$

$$r_3^2 = \left(Y_j - \frac{H}{2}\right)^2 + \left(X_i - X_j - \frac{W}{2}\right)^2$$

$$r_4^2 = \left(Y_j + \frac{H}{2}\right)^2 + \left(X_i - X_j - \frac{W}{2}\right)^2$$

$$\theta_1 = \arctan\left(\frac{X_i - X_j + \frac{W}{2}}{Y_j - \frac{H}{2}}\right)$$

$$\theta_2 = \arctan\left(\frac{X_i - X_j + \frac{W}{2}}{Y_j + \frac{H}{2}}\right)$$

$$\theta_3 = \arctan\left(\frac{X_i - X_j - \frac{W}{2}}{Y_j - \frac{H}{2}}\right)$$

$$\theta_4 = \arctan\left(\frac{X_i - X_j - \frac{W}{2}}{Y_j + \frac{H}{2}}\right)$$

2.2.3.2. Inverse Gravity Modeling

Given a set of measured data and an algorithm for the problem, inverse modeling would help us to determine the model parameters**,** position, density and the geometry of a causative body, provided that sufficient prior information about the source is mathematically translated and automatically incorporated by the method. However, the basic difficulties encountered in an inverse problem are not only the lack of a guaranteed solution or the probable existence of many solutions giving the same answer (non-uniqueness) but also it is "ill-posed". That means, small variations in the solution vector **x** can produce large fluctuations in the model response **f**, and small fluctuations in the observed data **y** can produce large fluctuations in the solution vector **x** (Tarantola, 1987; Krzysztof, 2005). Thus, the algorithm to be used for gravity data inversion should handle parameterization, measurement and data processing error to avoid problem of instability. Moreover, it should also be flexible enough to accommodate prior information that can avoid the inherent problem of non-uniqueness. This is achieved by adjusting the solution space, minimization criteria and the mathematical regularization criteria.

In general, most of the inversion algorithms that lead to a solution can be grouped either as Bayesian or deterministic approaches. In the Bayesian approach, one can assumes that a prior density function can be assigned to the model and tries to solve the problem by maximizing the posterior probability of the model (Griffths, 1965; Götze, 1988; Chakravarthi, 2002). On the other hand, the deterministic approach assumes that there is no prior information and tries to solve the problem by using as much information as possible from the measured data.

These approaches try to find a solution that has several local minima, through an iterative application to achieve "global optimization". To attain this global optimal value, the density of a rectangular prism was considered as a weighting matrix (W_m) to get the compact subsurface mass distribution. The inversion procedure converges to a compact model that is not necessarily single density; and mostly it will give a large density model that is not realistic. To overcome this problem, we put a density constraint that can limit the upper boundary of the density. Any block that cross the density barrier (Xo) will then be set to Xo and the algorithm automatically freezes this block in the next iteration by assigning it a very small weight. Using matrix notation the expression is rewritten as follows:

$$\overline{X} = W_m^{-1} A^T \left(A W_m^{-1} A^T + \frac{\delta_m^2}{1+\delta_e^2} \right)^{-1} G \text{ ------- (2)}$$

Where G = The data vector whose elements are the data that are filtered at a point; A = Matrix whose jth element contains the vertical gravitational attrition of the jth block with a unit density on the ith measuring point; A^T = The transpose of matrix A; $\overline{X}$ = A vector that contains M number of unknown parameters; W_m^{-1} = The parameter weighting matrix; δ_m^2 = The variance of the parameter values; δ_e^2 = Variance of the deviation between filtered and computed gravity anomalies.

In summary, forward gravity modeling needs iterative numerical trial and correct error procedures, which in turn are time consuming and difficult for complicated problems. On the other hand, in inverse modeling it is often difficult to quantify all the qualitative knowledge of experts, and we will end up with conceptual geological targets which cannot be adequately described by numerical data. This therefore calls for a possible combination of the methods in order to give a better result.

3. Results and Discussion

3.1. System Design and Implementation

The series of activities that will be performed during gravity data interpretation may be summarized as follows: by means of modeling software, the interpreter creates an initial density model using his knowledge about the region of investigation. The initial model is then sent to the inversion program, the model resulting from the inversion is then analyzed and the eventual corrections are applied and repeated until a satisfactory result is obtained. To this end, during requirement analysis, the system was described completely from the theoretical point of view and does not contain information about the internal structure of the system or how the system should be realized. However, in system design we decompose the system into smaller subsystems that can be realized at the time of implementation, and are organized in a menu format.

3.2. Tests

To set up the forward or inverse model, a station file in ASCII format is developed, that includes two variables: distance and measured gravity anomaly. These must first be available to the system and then the subsurface information is generated automatically. The structures generated by the program have the following features: all blocks at the same depth have the same height, and the width of a block cannot vary with depth or along the profile. The proper size of the block has a direct effect on the quality of the interpretation of relatively small bodies. For example, if the size of the block is too big, the resulting structure will be rough. On the other hand, the use of two small blocks will generate a large number of prisms and makes the modeling process a tedious task .The model resolution and the number of observation points influences the physical size of the model. Hence, one should keep in mind that the size of the model and the time needed to calculate its gravity anomaly is directly proportional to the product of the number of stations and prisms.

3.2.1. Testing the Tool Using Synthetic Gravity Data

To test our system, we generated synthetic data using a forward model as shown in Figure 2. This figure has two active sub-windows: the partitioned subsurface representation being the bottom one, and the graph area, the upper one. The curve in this figure indicates the synthetic gravity data which were produced by taking a random density value. For example, we can assume the existence of some buried body whose density is 0.8 g cm^{-3}. The small rectangles that are arranged in rows and columns indicate the subsurface division. These rectangles were generated from a reference point in the study area along the earth's surface and at a certain depth below this reference point. The list of numbers and color spectra, density contrast, color, and density of substance, that are arranged to the right side of this windows provide a legend which can indicate the possible types of substances in the study area and what types of substances are buried under a given modeling process.

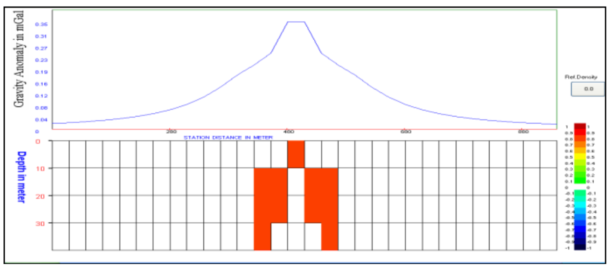

Figure 2: Synthetic data

In order to check the result of an inverse model, the generated artificial data in Figure 2 were put into the inverse modeling model.

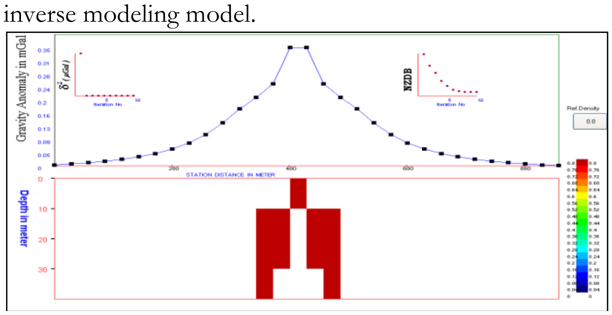

Figure 3: Inverse modeling result for Figure.

Figure 3 above depicts the generated artificial gravity data (dotted line) and the resulting estimated gravity value (solid line) along with the buried body-boundary for a particular iteration. The two graphs which are found on the upper left and right corners of this Figure indicate the criteria that were set in order to know whether each iteration was converging to some value or not. As can be seen from this figure the response obtained by this technique was close to the randomly fabricated data within the predefined root mean square error. Hence, it seems reasonable to accept the inverse modeling as a useful reference for other real gravity data inversion purposes.

3.2.2. Testing the Tool Using Borehole Data

We also used real gravity data around the Filweha area, Addis Ababa, Ethiopia, at localities having different geologic settings and borehole (drilled subsurface) data, as shown in Table 1. The purpose of testing our system in this area of hot springs was to determine the location of faults which have offset the bedrock and which might possibly control the ascending thermal waters of the Filweha geothermal system. In order to start the forward gravity data modeling, the interpreter first selects certain prism(s) and changes the density of these prisms and observes the effect. If there is a significant gap between the forward modeling result and the actual gravity data, the interpreter continues to carry out modifications. Using the data in Table 2 for the subsurface splitting parameters, and after a number of modifications, the fitted model may look like that shown in Figure 4.

Table 1. Measured gravity data in miliGal.

Station No.	*Distance*	*Measured gravity (miliGal)*
1	0	0.575
2	40	0.6
3	80	0.575
4	120	0.475
5	160	0.35
6	200	0.175
7	240	0.1
8	280	0
9	320	-0.05
10	360	-0.075
11	400	-0.1
12	440	0.025
13	480	0.125
14	520	0.2
15	560	0.275
16	600	0.35
17	640	0.4
18	680	0.4
19	720	0.325
20	760	0.35
21	800	0.3
22	840	0.325
23	880	0.4
24	920	0.45
25	960	0.55
26	1000	0.6

Table 2. Subsurface splitting values.

No.	Parameters	Value(in meter)
1	Prism width	10
2	Prism Height	10
3	Depth	80
4	Profile Length	1000

The small black dots represent the actual filtered gravity values at each survey station that is assumed to provide information about the distribution of subsurface structure during inversion. The solid line indicates the estimated gravity anomaly. As can be seen from the above figure, our system showed a subsurface structure that is in acceptable agreement with that of the expected geological structure.

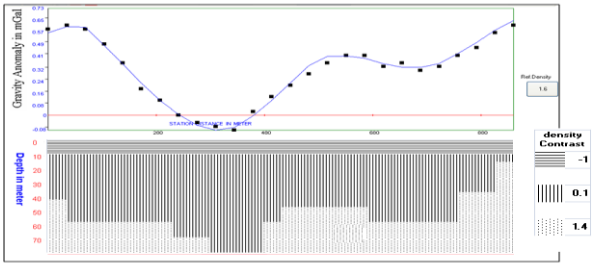

Figure 4: Inferred subsurface structure based on real gravity data.

4. Conclusion

The geophysical interpretation of gravity data is a complicated and often intensive task that requires a lot of experience from an interpreter. On the other hand, interactive modification of model parameters and direct visualization of both computed and measured fields of gravity data enable the subsurface structure to be interpreted as realistically as possible. Towards this end, we have discussed the steps in processing and interpretation of gravity data: parameterization, and a forward and inverse gravity data inversion in terms of the causative bodies. To minimize the parameterization problem, we assumed the subsurface of the earth to be represented by a set of small rectangular blocks. However, limitations of computer power and memory forced us to restrict implementation of the model to 2-D bodies, which prevented its application to the rectangular blocks methods in modeling of high resolution density structures.

Forward modeling is the important technique in gravity interpretation in certain simple cases. However, in order to speed up the interpretation significantly and to overcome the limitation of this modeling technique, we combined the forward and inversion methods. Nevertheless, an inversion needs an appropriate target parameter which totally depends on the interpreter's prior knowledge of the study area. The program was tested on several sets of 2-D synthetic and real gravity profiles data and the misfits analyzed using statistical testing techniques. To this end, the maximum threshold of the mean square error was set to 10μGal, but in most cases after a few iterations the inversion algorithm was able to generate a compact model which satisfied the threshold set at the onset. The 10 μGal threshold is generally set based on the accuracy of measurement. In general the models satisfied the constraints set on the basis of the a priori information and it can be concluded that the tool was able to achieve the desired goal.

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