Design and Manufacture of Pinion Cutters for Finishing Gears with an Arbitrary Profile*

Hidehiro YOSHINO**, Ming SHAO*** and Akira ISHIBASHI**

A new method of calculating grinding-wheel profiles for finishing pinion cutters to shape spur and helical gears with an arbitrary profile is developed using the theory of the element removal method (ERM) previously proposed by the authors. In order to show the usefulness of this method, the profiles of the grinding wheel for finishing the pinion cutter ($m_n=3$) for Novikov gears with concave teeth are calculated numerically, and then the profiles of the pinion cutter finished by the grinding wheel are calculated. The difference (calculation error) between the given and the calculated profile of the pinion cutter was less than 0.1 μm before regrinding. Profile errors of the cutting edges after regrinding are shown in terms of the number of teeth, the helix and rake angles and the regrinding stock. The effective clearance angles of cutting edges, which produce some problems in gear finishing, are calculated from changes in the profiles of the cutting edges before and after regrinding.

***Key Words*:** Gear, Cutting Tool, Generating Motion, Pinion Cutter, Relief Grinding, Cutting Edge, Tooth Profile

1. Introduction

Gear shaping with a pinion-type cutter (pinion cutter) is used for cutting internal and external gears when the gears are difficult to hob and also in the case in which noncircular gears are required to produce irregular motion(1). It is supposed that the use of the gears cut only by shaping will increase due to requirements of reducing the size of gears and/or integration in the mechanism of gear drives. Gear shaping with pinion cutters may play a very important role in the future of gear industries. Many studies on pinion cutters for cutting involute gears have been conducted, but few have been conducted in the case of noninvolute gears(2). In this paper, the element removal method(3),(4) previously proposed by the authors is expanded, and an improved method is proposed for calculating the profile of the grinding wheel for finishing pinion cutters to be used for shaping helical gears with an arbitrary profile. Moreover, profile errors of the cutting edges of pinion cutters after regrinding are investigated numerically.

* Received 18th July, 1991. Paper No. 90-0316A
** Faculty of Science and Engineering, Saga University, 1 Honjo-Machi, Saga 840, Japan
*** Xinjiang Institute of Technology, China

2. Cutting Edge of Pinion Cutter and Wheel Profile for Relief Grinding

When tooth profiles of gears are arbitrarily given, two cases are expected. In one of the cases, the tooth profile of the gears is given directly, while in the other case, the basic rack of the gears is given. This paper describes the latter case because it is easy to estimate profile errors of cutting edges after regrinding.

Figure 1 shows an example of an arbitrarily given tooth profile, indicating a basic rack of Novikov gears with the concave teeth. The center of curvature of the tooth profile is near the pitch line of the basic rack to provide a difference between the curvatures of the concave and the mating convex profile. The profile of the basic rack was expressed by a series of points (about 500 points for half of the pitch). When the profile of the basic rack, the number of teeth, and the helix angle of the pinion cutter are given, the transverse profile of the pinion cutter is definitely determined to produce a helical gear to meet the given basic rack. In this case, the transverse profile of the pinion cutter can be calculated by using the element removal method[3] together with the equations expressing the relative motion between the gear blank and the basic rack.

2.1 Theory of profile calculation

Figure 2 shows the coordinate system used in the calculation of the profile of the grinding wheel which can finish a pinion cutter with the required profile by form grinding with relieving motion. The wheel profile can be obtained as follows.

In the first stage, a helical pinion cutter with an arbitrary cutting-edge profile is mounted to the cutter axis of a relief-grinding machine, and a grinding wheel that has not been trued (dressed) is mounted to the wheel axis. Then, the pinion cutter and the wheel are set in generating motion (helical and relieving motions). The cutting edges of the pinion cutter act as a dresser for the wheel. Consequently, the profile of the grinding wheel for finishing the pinion cutter is obtained. The element removal method proposed by the authors can simulate the metal removal process by a tool, and can calculate the shape of the remaining (not removed) wheel material accurately. However, there is a possibility of interference between the cutting edge of the pinion cutter and the profile to be produced on the wheel. In order to determine whether or not interference has occurred, it is necessary to compare the cutting-edge profile of the pinion cutter finished using the grinding wheel with the given one. If a difference between the two profiles is observed, we know that interference has occurred. In this case, the profile calculation must be done again until no interference occurs after the center distance between the cutter and wheel axes and the inclination angle of the wheel axis are changed. If we cannot find conditions under which no interference occurs, we know that it is impossible to finish the pinion cutter with the required profile.

In this investigation, the motion for relief grinding is given by the changes in the center distance between the cutter and wheel axes. When other motions, such as excessive helical motion, between the cutter and the wheel are used as relieving motion, the wheel profiles must be calculated using the equations which express those relieving motions.

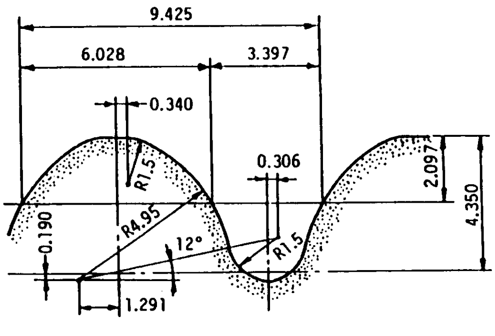


Fig. 1. Basic rack used in calculation of pinion cutter for shaping Novikov gears with concave teeth

2.2 Profile of cutting edge

Table 1 shows the specifications of a pinion cutter being finishied and the conditions used in the calculation of the wheel profile. The cutting edge of the pinion cutter must express a required helicoid when helical motion is given to the cutting edge. When the coordinates of point P on the transverse profile of the pinion cutter are expressed by $(x_0, y_0, 0)$, the coordinates (x', y', z') of point P' on the cutting edge of the pinion cutter are calculated as follows:

$$\left.\begin{aligned} x' &= x_0\cos\phi + y_0\sin\phi \\ y' &= -x_0\sin\phi + y_0\cos\phi \\ z' &= r\phi/\tan\beta \end{aligned}\right\} \qquad (1)$$

where ϕ is a parameter indicating the rotation of the transverse profile, r is the radius of the pitch circle, and β is the helix angle of the pinion cutter. The rake face of the pinion cutter is assumed on a plane, which is expressed by the following equation [refer to Fig. 3 (a)]:

Table 1 Specifications of pinion cutter used in profile calculations and conditions for relief grinding

Normal module	m_n	3 mm
Number of teeth	N_p	30
Helix angle	β	25°R
Radius of pitch circle	r	49.652 mm
Rake angle	δ	5°
Axial clearance angle	ε	6°
Cutting edge angle	γ	25°
Inclination angle of grinding wheel axis	Γ	25°
Radius of grinding wheel	R_w	100 mm
Center distance	a	149.324 mm

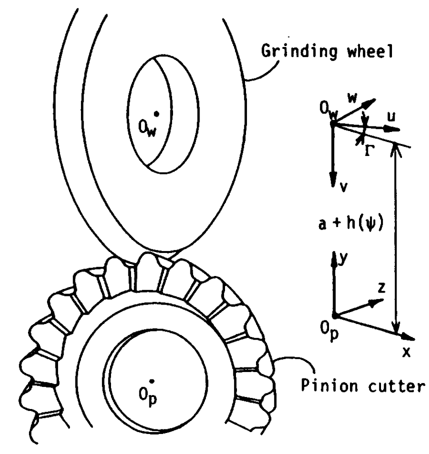


Fig. 2 Coordinate system used to calculate wheel profile for finishing pinion cutter by relief grinding

$$x' \sin \gamma \cos \delta + (y'-r) \cos \gamma \sin \delta + z' \cos \gamma \cos \delta = 0 \qquad (2)$$

where γ is the cutting edge angle, and δ is the rake angle of the pinion cutter. By eliminating the parameter ϕ, the coordinates of point $P'(x', y', z')$ on the cutting edge are calculated numerically using Eqs. (1) and (2). The relationship between the rake angles δ on the transverse plane [refer to Fig. 3(a)] and δ' on the normal plane is expressed by the following equation.

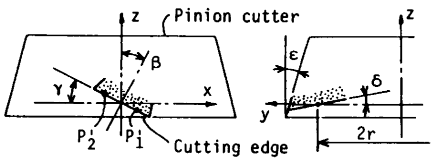
$$\tan \delta' = \cos \gamma \tan \delta . \qquad (3)$$

When both sides of the tooth flank of the pinion cutter are finished at the same time, the cutter must be so located that the grinding wheel is inserted into the tooth space of the pinion cutter. Therefore, the coordinates of point P' obtained by the previous calculation must be rotated about the cutter axis in the left- or right-hand direction by an angle of π/N which is decided by the number of teeth of the pinion cutter. Point P' moves to point $P''(x, y, z)$, as shown in Fig. 3(b).

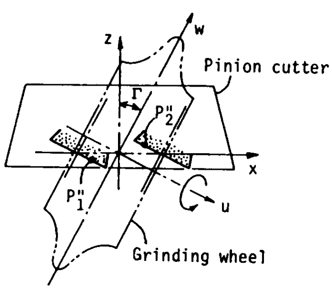
2.3 Wheel profile for relief grinding

Point P'' on the cutting edge participates in dressing of the grinding wheel during the generating motion (helical and relieving motions) which is given between the pinion cutter and the wheel, as shown in Fig. 2. When the pinion cutter is rotated by an angle of ϕ, point P'' moves to point $Q'(u, v, w)$ expressed by the coordinate system fixed to the grinding wheel axis:

$$\left.\begin{aligned} u &= x \cos \Gamma \cos \phi + y \cos \Gamma \sin \phi - z \sin \Gamma \\ &\quad - (r\phi/\tan \beta) \sin \Gamma \\ v &= x \sin \phi - y \cos \phi + a + h(\phi) \\ w &= x \sin \Gamma \cos \phi + y \sin \Gamma \sin \phi + z \cos \Gamma \\ &\quad + (r\phi/\tan \beta) \cos \Gamma \end{aligned}\right\} \qquad (4)$$



(a) Rake face of pinion cutter



(b) Position of cutting edge in relief grinding

Fig. 3 Position of cutting edges of pinion cutter for relief grinding

where Γ is the inclination angle of the wheel axis, a is the standard center-distance between the cutter and wheel axes, which is decided by the radius of the wheel used for relief grinding. It is possible to use several functions of $h(\phi)$ which can express the relieving motion between the cutter and the wheel. Moreover, $h(\phi)$ also decides the shape of the tooth flank of the pinion cutter. In this investigation, the function $h(\phi)$ is selected so that the center distance between the cutter and wheel axes changes in proportion to the traveling distance of the pinion cutter:

$$h(\phi) = (r\phi/\tan \beta) \tan \varepsilon \qquad (5)$$

where ε is the axial clearance angle on the pitch cylinder of the pinion cutter. The coordinates of point $Q(X, Y)$ on the axial plane of the grinding wheel are obtained after revolution of point $Q'(u, v, w)$ about the wheel axis.

$$\left.\begin{aligned} X &= u \\ Y &= \sqrt{v^2 + w^2} \end{aligned}\right\} . \qquad (6)$$

The calculations are conducted for all of the points on the cutting edge of the pinion cutter by changing the rotation angle ϕ in small steps. As a result, many points expressing the required profile on the axial plane are obtained. The series of points on the boundary produced by these calcuated points indicates the required profile of the wheel. The element removal method[3] can be used in the aforementioned calculation efficiently.

Figure 4 shows the grinding-wheel profile calculated under the conditions in Table 1, which can produce the required cutting-edge profile. It is found that the profiles of the wheel for finishing both the right and left flanks of the pinion cutter are not symmetrical in shape. A step is observed at the tip of the grinding wheel. This is ascribed to the inclination of the rake faces of the pinion cutter. By using the grinding wheel with the step, the cutting edges with the exact tooth flanks can be successfully finished at the same time. It seems difficult in trueing to produce the step of the wheel. However, this difficulty does not occur in practice because the tip of the wheel is

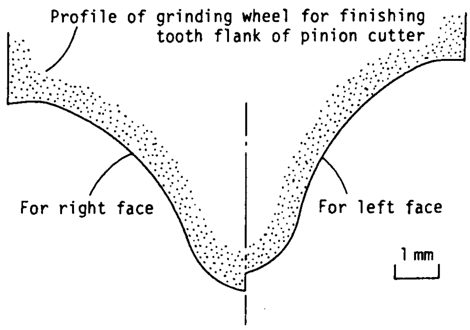


Fig. 4 Wheel profile capable of finishing both flanks of pinion cutter

related to finishing the root of the cutting edge, which is not used in cutting gears in most cases.

3. Profile of Produced Cutting Edge

In order to determine whether or not the pinion cutter with the required cutting edge is successfully finished, a calculation which simulates the metal removal process in relief grinding must be performed.

3.1 Rake face after regrinding

In the case of regrinding of the rake face of the pinion cutter, an infeed must be given in the direction of the tooth helix. The rake face after regrinding is not parallel with that before regrinding, which forms a plane recessed in the axial direction by a regrinding stock and rotated around the axis of the pinion cutter. The rake face after regrinding is expressed by the following equation:

$$x \sin \gamma \cos \delta + (y - r) \cos \gamma \sin \delta + (z - \lambda) \cos \gamma \cos \delta = 0 \quad (7)$$

where λ is the regringding stock measured in the axial direction of the pinion cutter. Since regrinding of the rake face is conducted along the tooth helix, the pinion cutter must be rotated by an additional angle of $\zeta = (\tan \beta / r)\lambda$ corresponding to the regrinding stock.

3.2 Profile of cutting edge after regrinding

When the pinion cutter is finished by relief grinding using a wheel with a calculated profile, the profile of the cutting edge can be obtained from the margin of the cross section produced by the intersection between the rake face and the surface of revolution of the grinding wheel (the equations of the coordinate transformation are omitted). In this case, the cutting edge after regrinding can be calculated when the equation for the rake face after regrinding is used. In order to calculate the points of intersection in the three-dimensional space, a convergence calculation must be performed. This calculation can be efficiently and accurately performed by using the fact that the neighboring points on the wheel profile intersect with the rake face at almost the same axial section of the wheel. The element removal method[3] proposed by the authors can only be used for calculating the envelope curve produced on a plane. Therefore, the profile of the cutting edge was calculated by transferring the intersecting points to the transverse section of the pinion cutter along the tooth helix.

Figure 5 shows the calculated profile of the cutting edge on the transverse section of a new pinion cutter produced by relief grinding using a wheel with the profile shown in Fig. 4. The difference (calculation error) between the calculated and required profiles on the transverse plane is enlarged in the normal direction of the profile curve. Both profiles agree well within errors of less than about 0.1 μm, indicating no interference. Consequently, it is found that the grinding wheel, of which the profile is calculated using the equations introduced in the present investigation, can be used for relief grinding of the pinion cutter.

3.3 Basic rack expressed by cutting edge

It is impossible to evaluate profile errors of a cutting tool after regrinding by using changes in shape of the cutting edge because the cutting edge after regrindig recedes in the radial direction. The profile errors can be obtained when the rack profile expressed by the cutting edge after regrinding is compared with the given basic rack. The changes in the profiles of the gears shaped with the pinion cutter before and after regrinding are evaluated indirectly by comparing the profiles of the two racks. The element removal method[3] is applicable to the calculation of the rack profile expressed by the cutting edge. This corresponds to shaping of the rack using the pinion cutter.

Figure 6 shows the changes in the rack profiles of the pinion cutter for regrinding stocks of $\lambda = \pm 2$ mm. In this figure, the tops of the rack profiles were shifted to the same point to compensate the recession of the cutting edge after regrinding. This procedure corresponds to giving the same depth of cut in gear shaping. Comparatively large profile errors are observed on the right face. However, they can be reduced by shifting the rack profile in the direction of its pitch line. This means that most of these errors cause changes in the tooth thickness, not profile errors of the cutting edge.

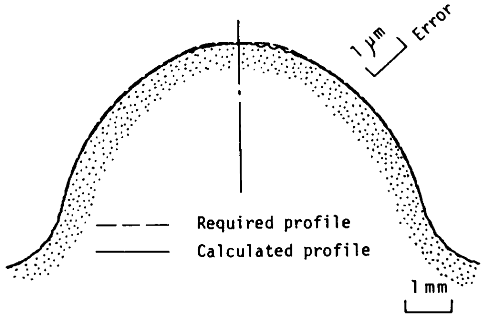


Fig. 5 Cutting-edge profile of pinion cutter finished by grinding wheel with calculated profile

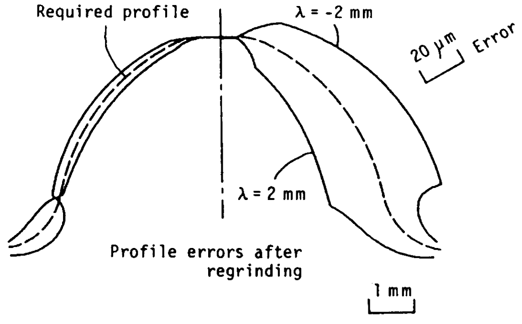


Fig. 6 Profile errors of cutting edge after regrinding ($\lambda = \pm 2$ mm)

For regrinding stocks of $\lambda=\pm2$ mm, it is found that the profiles of the cutting edges after regrinding are sufficient to shape accurate Novikov gears. If a negative regrinding stock is given in the design of pinion cutters, the regrinding stocks can be increased when the allowable limit of the profile errors is the same.

4. Profile Error of Reground Cutting Edge

With the method proposed in this investigation, the profile of the grinding wheel for finishing the pinion cutter with the required profile can be calculated except when interference has occurred. This is true in the case in which specifications of a pinion cutter such as the tooth profile, the number of teeth, the rake angle, etc., arbitrarily given. However, the profile errors of the cutting edge are always produced when the pinion cutter is reground. The profile errors are affected by the specification of the pinion cutter, such as the cutting-edge profile, and also by the conditions for calculating the wheel profile shown in Table 1. This means that there are optimum design conditions which minimize the profile errors after regrinding.

In the following calculation, the effects of the specifications of the pinion cutter, etc., on the profile errors were examined when regrinding was conducted at a regrinding stock of $\lambda=2$ mm. The profile errors were obtained by comparing the rack profile corresponding to the cutting edge after regrinding with the given basic rack. The errors are enlarged and shown in the normal direction of the profile curve.

Figure 7 shows the effects of the numbers of teeth, i.e., the outside diameters of the pinion cutters, on the profile errors after regrinding. The greater the number of teeth, the smaller the profile errors after regrinding. Figure 8 shows the effects of the helix angles on the profile errors after regrinding. In this case, it was assumed that the cutting edge angle γ, the inclination angle Γ, and the helix angle β are equal to each other. The profile errors become slightly smaller when the helix angle increases. Figure 9 shows the effects of the rake angles on the profile errors after regrinding. The effects are very small.

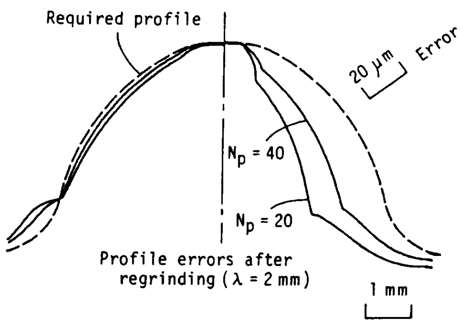


Fig. 7 Effects of tooth numbers on profile error after regrinding

5. Relief Grinding of Pinion Cutter for Novikov Gears

To demonstrate the method for finishing pinion cutters proposed in this investigation, a pinion cutter, of which the basic rack is shown in Fig. 1, was made for shaping Novikov gears with concave teeth. The specifications of the pinion cutter are as follows: the normal module $m_n=3$ mm, the number of teeth $N_p=21$, the helix angle $\beta=17.736°$ RH, the cutting edge angle $\gamma=17.736°$, the rake angle $\delta=5°$, and the clearance angle $\varepsilon=6°$.

The wheel profile used for relief grinding was calculated assuming that the inclination angle Γ is equal to β and that the wheel radius R_w is 70 mm. The radii of the grinding wheel obtained by calculation were $R_{wr}=69.52$ mm for the right face and $R_{wl}=69.81$ mm for the left face. The wheel profile was approximated at an accuracy of about 1 μm using a series of eight arcs for each face. Then, the NC data for trueing the grinding wheel were programed. The grinding wheel (WA 80 J) used for relief grinding was trued by means of a single-point diamond dresser controlled by an NC servomotor. There was no problem in trueing of the step at the tip of the wheel because the tip radius of the diamond dresser was small ($r_f\fallingdotseq0.3$ mm).

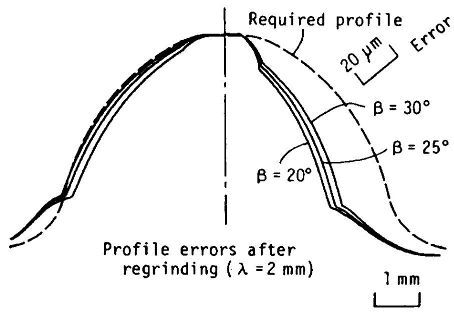


Fig. 8 Effects of helix angles on profile error after regrinding

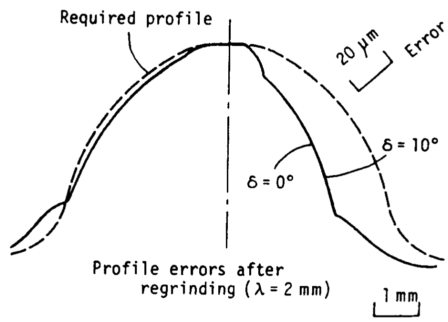


Fig. 9 Effects of rake angles on profile error after regrinding

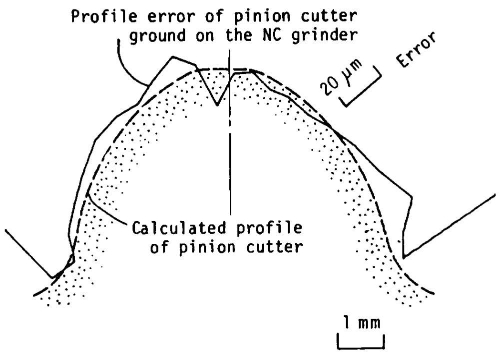


Fig. 10 Profile error of trial pinion cutter measured in axial direction

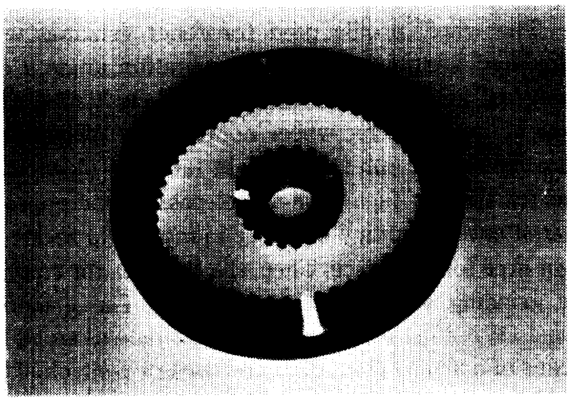


Fig. 11 Trial pinion cutter and shaped internal Novikov gears with concave teeth

A round bar made from a high-speed steel, SKH55, was turned and hobbed, and then quenched to the designed hardness. The rough-finished pinion cutter was ground on the NC gear grinder[4] made by the authors. The rake face of the pinion cutter was ground on a surface grinder by inclining the axis of the pinion cutter.

Figure 10 shows the profile errors of the ground pinion cutter measured with a universal measuring microscope. The profile error is shown in the normal direction of the profile curve of the required cutting edge. It is found that the profiles of a tooth of the pinion cutter are almost the same as those given in the design stage. It is added that the profile error of the pinion cutter is measured in the diagonal direction of the rake face for ease of measurement. Therefore, it is supposed that the actual profile error of the pinion cutter may be smaller than that shown in Fig. 10[3],[4].

Figure 11 shows a trial pinion cutter made in this investigation and the internal Novikov gear (the number of teeth $N=58$, the face width $b=37$ mm) shaped with this pinion cutter. Generally, internal gears are made from a mild or tempered steel for ease in shaping. If internal gears with circular-arc teeth are made from these steels, it is expected that the load-

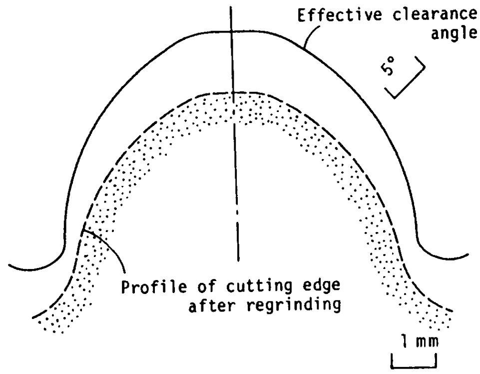


Fig. 12 Calculated result of effective clearance angle of cutting edge

carrying capacity of the gear drives appreciable increases because the contact pressure becomes lower when compared with those of the corresponding involute gears.

6. Discussion

It is important to know whether or not clearance angles are sufficient at tooth flanks of a cutter finished by relief grinding, especially when the profile of the cutting edge is complex. Using the method for calculating profile errors after regrinding, we can accurately calculate an effective clearance angle of the cutting edge. Figure 12 shows the effective clearance angle calculated assuming that the clearance angle in the axial direction of the pinion cutter is $\varepsilon=6°$. The effective clearance angle can be obtained by dividing the difference (clearance in the normal direction) between the two profiles before and after regrinding by the regrinding stock. It was found that the smallest clearance angle of the pinion cutter is about 1.5°, which is nearly equal to the allowable limit.

7. Conclusions

The method for designing and making pinion cutters to be used for shaping helical gears with an arbitrarily given basic rack was investigated. The results obtained are as follows:

(1) Grinding-wheel profiles for finishing (relief-grinding) pinion cutters with an arbitrary profile can calculated accurately by means of the element removal method proposed by the authors.

(2) Moreover, profile errors which occur on the cutting edges of pinion cutters after regrinding can be evaluated in the design stage.

(3) To demonstrate the usefulness of the authors' method, a pinion cutter for Novikov gears with concave teeth was finished on an NC gear grinder. The profile errors of the pinion cutter were less than 20 μm.

Acknowledgments

The authors express their thanks to Mr. K. Fukuda, Kyushu Precision Eng. Ltd. and Mr. N. Shimono and others, Mitsubishi Electric Co. Ltd., Fukuoka Works, for their encouragement and assistance.

References

(1) Yamasaki, T., Design and Manufacture of Noncircular Gears and Their Applications, Machine Design, (in Japanese), Vol. 29, No. 11 (1985), p. 47.

(2) Yoneda, S., On a Profile of Pinion Cutters for Helical Gears, J. Jpn. Soc. Prec. Eng., (in Japanese), Vol. 8, No. 6 (1941), p. 331.

(3) Ishibashi, A. and Yoshino, H., Design and Manufacture of Gear Cutting Tools and Gears with an Arbitrary Profile, JSME Int. J., Vol. 30, No. 265 (1987), p. 1159.

(4) Yoshino, H. and Ishibashi, A., [Continued Report of Ref. (3)], A Method for Calculating Grinding Wheel-Profile for Hob Finishing, JSME Int. J. Ser. III, Vol. 32, No. 1 (1989), p. 124.