Design competition through multidimensional auctions

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This article studies design competition in government procurement by developing a model of two-dimensional auctions, where firms bid on both price and quality, and bids are evaluated by a scoring rule designed by a buyer. Three auction schemes—first score, second score, and second preferred offer—are introduced and related to actual practices. If the buyer can commit to a scoring rule in his best interest, the resulting optimal scoring rule underrewards quality relative to the buyer's utility function and implements the optimal outcome for the buyer under first- and second-score auctions. Absent the commitment power, the only feasible scoring rule is the buyer's utility function, under which (1) all three schemes yield the same expected utility to the buyer, and (2) first- and second-score auctions induce the first-best level of quality, which turns out to be excessive from the buyer's point of view.

1. Introduction

■ The Department of Defense (DoD) often relies on competitive source selection to procure weapons systems.[1] Unlike most private auctions, this procurement competition involves many performance/quality dimensions other than price; typically, competitors specify in their bids such information as promised technical characteristics, delivery date, and managerial performance, as well as an estimated project cost. DoD's evaluation of bids then involves application of an elaborate scoring system: each individual component of a bid is evaluated and assigned a score, these scores are summed to yield a total score, and the firm achieving the highest total score wins a contract.

Previous studies of defense procurement have generally assumed that the design specification for any weapons system is fixed prior to competitive source selection—hence bidding competition is restricted to the price dimension. While such an approach may be appropriate for auctions of homogeneous goods, or for goods for which design competition is relatively

[1] Two institutions distinctively characterize DoD's weapons system procurement. One is competitive source selection; the other is a set of regulations called "profit policy," which covers noncompetitive, negotiated contracts. Although noncompetitive, negotiated contracts continue to be a large fraction of total procurement dollars, the majority of contracts are awarded on the basis of competitive procedures. In 1986, competitively awarded contracts accounted for 57% of total procurement. (Source: Secretary of Defense, *Annual Report to Congress FY1988.*)

unimportant, in defense procurement design competition frequently dominates price competition (typically, more than 50% of the total score for a submitted bid is assigned for nonprice factors; see Fox (1974)). Therefore, it seems important to take account of the multidimensional nature of procurement competition. In this article I develop a model of two-dimensional auctions in which each firm bids on both quality and price and bids are evaluated according to a scoring rule designed by the buyer. In the model, firms have private information about the costs of improving quality.

A number of other articles have considered a similar procurement problem. Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987) identify an optimal direct revelation mechanism in a competitive environment similar to the one considered here. This article complements their results by focusing on how to implement the optimal mechanism via a score-based system of two-dimensional auctions. Hansen (1988) and Desgagne (1988) have previously considered two-dimensional auctions where bids are evaluated according to an ad hoc rule; in contrast, in this article the auctioneer (buyer) designs an optimal scoring rule. Most closely related to my analysis is Dasgupta and Spulber (1990). They consider implementation by per-unit bid auctions, where firms offer per-unit bids against a given quantity (quality in my framework) schedule. Compared with the approach of Dasgupta and Spulber, the score-based auctions in this article have some attractive features: first, implementation does not require a special form of cost function (per-unit bids implementation requires a cost function to exhibit increasing returns to scale); second, this model fits well with the description of the DoD source-selection procedure by explicitly modelling design competition.

I consider three two-dimensional auctions. In what I call a "first-score" auction, each firm submits a sealed bid and, upon winning, produces the offered quality at the offered price. In other auction rules, labelled respectively "second-score" and "second-preferred-offer" auctions, the winner (highest-scorer) is required to match the highest *rejected* score in the contract. The second-score auction differs from the second-preferred-offer auction in that the latter requires the winner to match the exact quality-price combination of the highest rejected bid while the former has no such constraint. These second-bid auctions can be implemented by a two-stage sealed-bid/negotiation scheme (which will be described in the text), just as a second-price auction can be implemented by an open oral (ascending English) auction. Actual DoD practices seem to be represented by these two auction rules: the sealed-bid procedure of the actual practice appears similar to the first-score auction, and the process of negotiation whereby DoD goes back and forth between firms in an effort to get the best offer is similar to the second-bid auctions.

Several issues arise. The first one is, Which auction rule performs the best for the buyer? This question would have important implications for whether DoD should emphasize the sealed bid aspect or the negotiation aspect of the current procedure. It turns out that the answer depends critically on how the buyer (DoD) designs the scoring rule. This is also related to the buyer's commitment power. Given the difficulties associated with contracting upon complicated quality specifications, it may be realistic to assume that DoD can only credibly commit to a scoring rule that reflects its true preference ordering. With this "naive" scoring rule, I find that all three schemes yield the same expected utility to the buyer. (This is a two-dimensional version of the revenue equivalence theorem (Riley and Samuelson, 1981; Myerson, 1981).)

The second issue concerns the optimality of the winning firm's quality choice under alternative auctions. Compared to the optimal mechanism identified by the revelation principle, the naive scoring rule entails excessive quality under first- and second-score auctions. It does so because it fails to take account of the information costs (the costs the buyer bears due to his inferior informational position) associated with increased quality and thus overrewards quality. This suggests that there may be an incentive for the buyer to deviate from the naive scoring rule. The optimal deviation of the scoring rule from the naive rule

corresponds to an optimal distortion often discussed in the mechanism-design literature (Baron and Myerson, 1982; Laffont and Tirole, 1986). If the buyer is allowed to commit to any scoring rule, we find: (1) the first- and second-score auctions can implement the optimal outcome, correcting the excessive quality, while the second-preferred-offer scheme cannot, and (2) the optimal scoring rule systematically discriminates against quality.

The rest of the article is organized as follows. Section 2 sets up a model and introduces alternative auction rules. In Section 3 I analyze equilibrium under each auction rule. Section 4 discusses how to design a scoring rule to implement the optimal mechanism and studies the implication of the buyer's commitment power. Section 5 concludes by commenting on related issues.

2. A model of multidimensional auctions

■ This section describes a model of two-dimensional bidding on quality and price. A buyer, interpreted as the DoD, solicits bids from N firms. Each bid specifies an offer of promised quality, q, and price, p,[2] at which a fixed quantity of products with the offered level of quality q is delivered. The quantity is normalized to be one. For simplicity, quality is modelled as a one-dimensional attribute. (In reality, however, the quality offer includes technical characteristics, a delivery schedule, and other managerial performance standards.)

The buyer and firms are characterized as follows. The buyer derives utility from a contract,[3] $(q, p) \in \Re_+^2$:

$$U(q, p) = V(q) - p,$$

where $V' > 0$, $V'' < 0$, and $\lim_{q \to 0} V'(q) = \infty$, $\lim_{q \to \infty} V'(q) = 0$ to ensure an interior solution. A firm i, upon winning, earns from a contract (q, p) profits:

$$\pi_i(q, p) = p - c(q, \theta_i),$$

where firm i's cost $c(q, \theta_i)$ is increasing in both quality q and its cost parameter θ_i. I assume $c_{qq} \geq 0$, $c_{q\theta} > 0$, and $c_{qq\theta} \geq 0$. That is, marginal cost is increasing with θ. Finally, it is assumed that the buyer never wishes to split the contract to more than one firm (i.e., the cost function is not too convex in q). These assumptions are satisfied by a cost function with constant unit cost θ, i.e., $c(q, \theta) = \theta q$.

Losing firms earn reservation profits, normalized at zero. Prior to bidding, each firm i learns its cost parameter θ_i as private information. The buyer knows only the distribution function of the cost parameter. It is assumed that θ_i is independently and identically distributed over $[\underline{\theta}, \bar{\theta}]$ $(0 < \underline{\theta} < \bar{\theta} < \infty)$, according to a distribution function F for which there exists a positive, continuously differentiable density f. Because of complete symmetry among firms, the subscript i is dropped in the remainder of the analysis. Throughout, the following assumptions are made:[4]

Assumption 1. (regularity). $Cq + \dfrac{F}{f} c_{q\theta}$ is nondecreasing in θ.

Assumption 2. The trade always takes place (even with the highest cost type $\bar{\theta}$).

By the revelation principle (Myerson, 1982), an optimal outcome can be identified as a direct revelation mechanism. The following proposition is due to Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987).[5]

Proposition 1. In the optimal revelation mechanism, the firm with the lowest θ is selected; the winning firm is induced to choose quality q_o, which for each θ maximizes

$$V(q) - J(q, \theta), \qquad \text{where} \qquad J(q, \theta) \equiv c(q, \theta) + \frac{F(\theta)}{f(\theta)} c_\theta(q, \theta).$$

A standard interpretation applies: in the optimal mechanism, quality is distorted downward to limit the information rents accruing to relatively efficient firms, while competition further curtails the absolute magnitude of the rents. In this article I take this optimal outcome as a benchmark and focus on how it can be implemented through two-dimensional auctions that capture the salient features of the DoD source selection.

I consider the following three auction rules: first-score auction, second-score auction, and second-preferred-offer auction. In all three auction schemes, the buyer selects the winning firm based on a scoring rule; more specifically, a contract is awarded to a firm whose offer achieves the highest score.[6] Let $S = S(q, p)$ denote a scoring rule for an offer (q, p). The rule is assumed to be publicly known to the firms at the start of bidding, and initially it is assumed that the buyer can design and commit to a rule in his best interest. The assumption of commitment is subsequently relaxed. Lack of commitment power on the part of the buyer will imply that the true preference of the buyer must be reflected in the scoring rule: i.e., $S(q, p) = U(q, p)$.[7] In general, I restrict attention to quasi-linear scoring rules:

Assumption 3. $S(q, p) = s(q) - p$, where $s(q) - c(q, \theta)$ has a unique interior maximum in q for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and $s(\cdot)$ is increasing at least for $q \leq$ argmax $s(q) - c(q, \underline{\theta})$.

Many scoring rules belong to this class, including the naive rule $U(q, p)$ and the optimal rule discussed in Section 4.

To understand the differences among the three auction schemes, it is convenient to distinguish between a winning firm's offer and a contract finally awarded to the firm. In a first-score auction, a winning firm's offer is finalized as a contract. Thus, this auction rule can be considered a two-dimensional analogue of the first-price auction. Second-score and second-preferred-offer auction rules represent two variants of the second-price auction. In a second-score auction, a winning firm is required (in the contract) to match the highest rejected score. In meeting this score, however, the firm is free to choose any quality (or quality-price combination). A second-preferred-offer auction rule, suggested by Desgagne (1988), is the same as the second-score auction except that a winner has to match not only the score but also the exact quality choice of the highest losing firm.[8]

These three schemes are not just abstract mechanisms. To some extent, the actual source-selection procedure employed by DoD resembles aspects of all three mechanisms.

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[5] In fact, my model is a special, limiting case of Riordan and Sappington (1987) in which first- and second-period costs are perfectly correlated. It is also a special case of Laffont and Tirole (1987), which also incorporates a moral hazard problem.

[6] If there is more than one firm that achieves the highest score, then a random drawing determines the winner. As is typical with atomless distributions, a particular tie-breaking rule does not affect expected equilibrium payoffs.

[7] As usual, using any monotone transformation of U as a scoring rule will have the same effect.

[8] The following example illustrates the difference among the rules. Suppose two firms A and B offer $(7, 5)$ and $(5, 3)$, and the scoring rule is such that A gets 10 points and B gets 8 points (i.e., $S(7, 5) = 10$, $S(5, 3) = 8$). Then, under all three rules A is declared the winner. However, the final contract awarded to A is: $(7, 5)$ under the first-score auction; any (q, p) satisfying $S(q, p) = 8$ under the second-score auction; and $(5, 3)$ under the second-preferred-offer auction.

Although the initial stage of sealed-bid competition is similar to the first-score auction, sealed-bid competition is often followed by a negotiation phase in which DoD approaches and asks each firm to beat the offers made by the other firms. The second-score auction can be implemented by a two-stage scheme where a winner is selected in the first period through a sealed-bid auction and engages in a negotiation in the second period making a take-it-or-leave-it offer to the buyer. The second-preferred-offer auction can be implemented by a similar two-stage scheme, but with an added restriction on the negotiation that the winning firm should dominate other firms in both price and quality.[9] Therefore, studying these alternative auction schemes will essentially amount to examining alternative aspects of current DoD practice. Comparisons between the auction rules will have policy implications for whether a negotiation phase is desirable and whether the negotiation, if desirable, should involve the added restriction.

## 3. Equilibria under alternative multidimensional auction rules

■ In this section, the equilibrium under each alternative auction scheme is characterized for a general scoring rule that satisfies Assumption 3. Each auction rule can be viewed as inducing a Bayesian game where each firm picks a quality-price combination (q, p) as a function of its cost parameter. Without any loss of generality, the strategy of each firm can be equivalently described as picking a score and quality, (S, q). The following lemma establishes that the equilibrium quality bid can be determined separately from the choice of score in the case of first- and second-score auctions.

Lemma 1. With first- and second-score auctions, quality is chosen at $q_s(\theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta})$, where $q_s(\theta) = \text{argmax } s(q) - c(q, \theta)$.

Proof. See the Appendix.

The simple intuition behind the result is presented in Figure 1. Suppose firm i wants to offer any arbitrary score S, represented by the buyer's iso-score curve (depicted as the concave curve in Figure 1). As long as the firm moves along the curve, its probability of

FIGURE 1
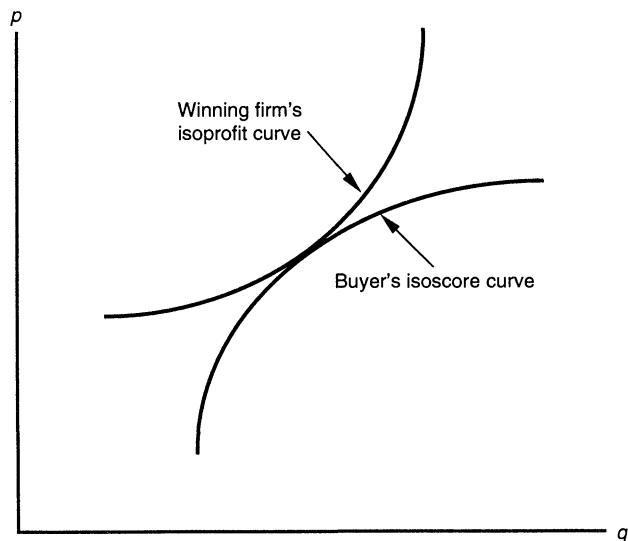
winning remains unchanged. Clearly, the firm's profit is maximized at the tangency point between its iso-profit curve and the buyer's iso-score curve, since that point allows the firm to maximize its profit without lowering its winning probability. Since the scoring rule is additively separable, quality choice is set independently of the score.

As a result of Lemma 1, there is no loss of generality in restricting attention to $q_s(\cdot)$ when searching for equilibria. (Essentially, Lemma 1 reduces a two-dimensional auction to a single-dimensional problem.) Let $S_o(\theta) \equiv \max s(q) - c(q, \theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Then, by the envelope theorem, $S_o(\cdot)$ is strictly decreasing; therefore, its inverse exists. Now, consider the following change of variables:

$$v \equiv S_o(\theta), \qquad H(v) \equiv 1 - F(S_o^{-1}(v)), \qquad b \equiv S(q_s(\theta), p).$$

The problem can then be reinterpreted as one in which each firm, indexed according to its productive potential v with cumulative distribution $H(\cdot)$, proposes to meet the level of score b. In particular, letting $b(\cdot)$ denote an equilibrium bid function of v that is symmetric and increasing, the objective function facing each firm in the first-score auction can be described as

$$\pi(q_s(\theta), p|\theta) = [p - c(q_s(\theta), \theta)] \operatorname{Prob} \{\operatorname{win}| S(q_s(\theta), p)\}$$

$$= [v - b]\{H(b^{-1}(b))\}^{N-1}.$$

The following proposition is immediate:

Proposition 2. (i) A unique symmetric equilibrium of a first-score auction is one in which each firm offers

$$q_s(\theta) = \operatorname{argmax} s(q) - c(q, \theta)$$

$$p_s(\theta) = c(q, \theta) + \int_{\underline{\theta}}^{\bar{\theta}} c_\theta(q_s(t), t) \left[\frac{1 - F(t)}{1 - F(\theta)}\right]^{N-1} dt. \qquad (1)$$

(ii) The second-score auction game has a dominant strategy equilibrium, where each firm with type θ offers

$$q_s(\theta) = \operatorname{argmax} s(q) - c(q, \theta)$$

$$\bar{p}_s(\theta) = c(q_s, \theta). \qquad (2)$$

Proof. (i) follows from the above change of variables and the usual equilibrium result in first-price auctions (for example, Riley and Samuelson, 1981); (ii) is immediate from Vickrey (1961) after applying the change of variables. *Q.E.D.*

Several observations can be made. First, the equilibrium in the first-score auction is reduced to the equilibrium in the first-price auction if $c(q, \theta) = \theta$ (i.e., quality is fixed). Second, the Vickrey auction intuition applies for the second-score auction: Given the score $S_o(\theta)$, if a firm with type θ bids a higher score, it would risk winning at negative profits without increasing its profit conditional on winning; if it bids a lower score, it would forgo some opportunity of winning at positive profits. Finally, implementation of these auctions relies on the invertibility of $S_o(\cdot)$, which is always satisfied; this is in contrast to implementation via per-unit bid auctions, which requires costs to exhibit increasing returns to scale (Dasgupta and Spulber, 1990).[10]

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[10] If a technology exhibits decreasing returns to scale, a more efficient firm may have a higher average cost (since it is induced to produce more quantity) and thus may offer a higher per-unit bid than a less efficient firm. Thus, a bid function may not be monotonic. This problem does not arise in my auction implementation, since the score bid function is always monotone decreasing in θ.

Substituting equilibrium bids from (1) yields the buyer's expected utility under the first-score auction:

$$EU_{FS} = E\{V(q_s(\theta_1)) - J(q_s(\theta_1), \theta_1)\}, \tag{3}$$

where θ_1 is, with a slight abuse of notation, the lowest-order statistic (i.e., $\theta_1 \equiv \min\{\theta_i\}_{i=1}^{N}$).[11] Since in general revenue equivalence may not hold, the expected utility under the second-score auction is separately obtained.

$$EU_{SS} = E\{V(q_s(\theta_1)) - p_s(\theta_1, \theta_2)\}, \tag{4}$$

where $p_s(\theta_1, \theta_2)$ is the price a winning firm actually pays to the buyer; i.e.,

$$p_s(\theta_1, \theta_2) = s(q_s(\theta_1)) - s(q_s(\theta_2)) + c(q_s(\theta_2), \theta_2).$$

Applying the argument used for the second-score auction, one can show that, in the second-preferred-offer auction, each firm will offer a score that will earn the firm zero profit, should the bid be finalized as a contract. However, in the second-preferred-offer auction a winning firm has no control over the quality choice. Therefore, each firm's zero-profit score offer depends on the quality-price bids of other firms. Hence, a dominant strategy equilibrium does not exist. In fact, the following proposition shows that there are many symmetric Bayesian-Nash equilibria in which each firm offers a zero-profit score.

Proposition 3. In the second-preferred-offer auction, any $(q(\cdot), p(\cdot))$ is a symmetric Bayesian-Nash equilibrium strategy for each firm if $p(\theta) = c(q(\theta), \theta)$, and $S(q(\theta), p(\theta))$ is decreasing in θ.

Proof. Suppose all the firms except for the one under consideration follow the suggested equilibrium strategy, $(q(\cdot), p(\cdot))$. Define the equilibrium score offer $S_e(\theta) = S(q(\theta), p(\theta))$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$, and let S_2 denote the highest score of all the scores offered by the remaining $N - 1$ firms. Then, there exists a distribution of S_2, G, induced by the equilibrium strategy such that $G(S_2) = F(S_e^{-1}(S_2))^{N-1}$. (The inverse function is well defined since $S_e(\cdot)$ is decreasing.)

The firm under consideration (with cost type c) faces the following problem:

$$\max_{q,p} E_{S_2}[(p(S_e^{-1}(S_2)) - c(q(S_e^{-1}(S_2)), \theta)1_{\{S(q,p) > S_2\}}]$$

$$= \max_{q,p} \int_{-\infty}^{S(q,p)} (p(S_e^{-1}(S_2)) - c(q(S_e^{-1}(S_2)), \theta)dG$$

$$= \max_{q,p} \int_{-\infty}^{S(q,p)} [c(q(S_e^{-1}(S_2)), S_e^{-1}(S_2)) - c(q(S_e^{-1}(S_2)), \theta)]dG.$$

Since S_e is decreasing, the integrand is positive if and only if $S_e(\theta) < S_2$. Therefore, the optimal strategy is any (q, p) such that $S(q, p) = S_e(\theta)$. In particular, the firm can do no better than offer the suggested $(q(\theta), p(\theta))$.   *Q.E.D.*

From the set of equilibria identified by this proposition, consider the equilibrium in which each firm offers the highest score that does not incur losses, (i.e., $(q_s(\theta), \bar{p}_s(\theta))$). This equilibrium is the most score-efficient in that the offered score is the highest (among all the equilibria) for a given price offer. From now on, I shall restrict attention to this particular equilibrium.

In this equilibrium, each firm offers $q_s(\theta)$, just as for the two other auction rules. But since the winning firm is obligated to produce the quality offered by the highest losing firm,

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[11] Analogously, I use θ_2 for the second-lowest-order statistic.

the equilibrium quality *produced* by the winning firm will be $q_s(\theta_2)$, which is lower than its offer $q_s(\theta_1)$.

The buyer's expected utility under this equilibrium is given by

$$EU_{SPO} = E\{S_o(\theta_2)\} = E\{V(q_s(\theta_2)) - c(q_s(\theta_2), \theta_2)\}. \tag{5}$$

4. Optimal scoring rule and the buyer's commitment power

■ The previous section establishes the equilibrium outcomes under alternative auctions given a general scoring rule that satisfies Assumption 3. Two questions can be raised. First, what is the optimal scoring rule if the buyer has full commitment power? Second, does the optimal scoring rule allow any auction scheme to achieve the optimal outcome identified in Proposition 1? It turns out that these two questions can be answered simultaneously.

Consider the following scoring rule: $\tilde{S}(q, p) = V(q) - p - \Delta(q)$. Here, $\Delta(q)$ is chosen to be

$$\Delta(q) = \int_k^q \frac{F(q_o^{-1}(s))}{f(q_o^{-1}(s))} c_{q\theta}(s, q_o^{-1}(s)) ds \qquad \text{for} \qquad q \in [q_o(\bar{\theta}), q_o(\underline{\theta})],$$

and $\Delta(q) = \infty$ for $q \notin [q_o(\bar{\theta}), q_o(\underline{\theta})]$, where k is any arbitrary real number. Recall that $q_o(\cdot)$ is the optimal quality defined in Proposition 1. Note that this scoring rule differs from the true utility function by the term $\Delta(q)$. Roughly speaking, the rule subtracts additional points from a firm for an incremental increase in quality according to the function $\Delta(q)$. The following proposition demonstrates that this modified scoring rule can work remarkably well for first- and second-score auctions but not for the second-preferred-offer auction.

Proposition 4. Under the scoring rule $\tilde{S}(q, p)$, first- and second-score auctions implement the optimal mechanism; the second-preferred-offer auction cannot implement the optimal mechanism under any scoring rule.

Proof. See the Appendix.

Proposition 4 demonstrates a potential advantage of the two-dimensional auctions: with an appropriate scoring rule, first- and second-score auctions can implement the optimal outcome. The optimal scoring rule involves systematic discrimination against quality. The intuition behind this is well known. As Proposition 1 shows, an optimal mechanism induces a downward distortion of quality from the first-best level, to internalize the information costs of the buyer. This optimal downward distortion can be implemented by a scoring rule that penalizes quality relative to the buyer's actual valuation of quality.

One should note that implementation of the optimal scoring rule warrants a caveat. The optimal scoring rule may sometimes require the buyer to choose an *ex post* unappealing proposal. Figure 2 shows one example of this possibility. In Figure 2, a scoring rule is represented by an iso-score curve that is less steep than the buyer's indifference curve. According to the scoring rule, bid A should be preferred to bid B. But after the two bids are received, the buyer may renege on the scoring rule and select B over A.

Thus, for the optimal scoring rule to be implementable, the buyer needs strong commitment power to support the scoring system. In the defense procurement context, such commitment power is often absent. First, it is practically very difficult for DoD to communicate its preference over complicated technical tradeoffs, especially in a way that is verifiable to a third party. This makes quality almost impossible to contract upon. Second, the government often abstains from disclosing its evaluation procedure so as to avoid becoming entangled in costly bid disputes, as public access to the evaluation documents makes it easy for losing firms to bring lawsuits.

These arguments suggest that it may be more appropriate to consider a case where the buyer is unable to commit to the optimal scoring rule. If the buyer lacks commitment

FIGURE 2
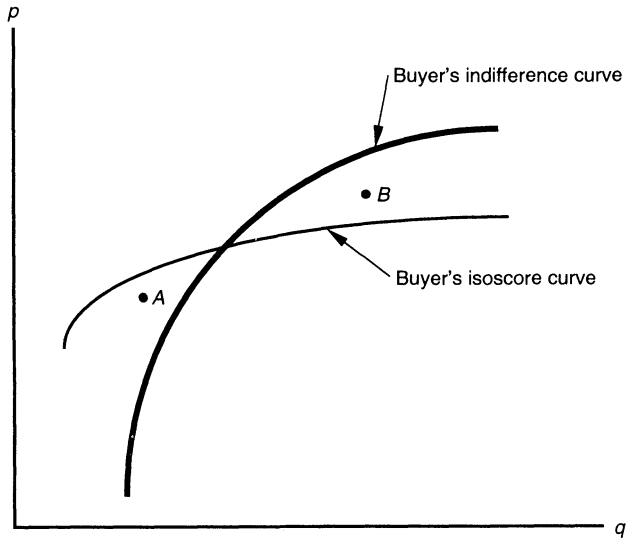
power, the only feasible scoring rule is one that reflects the buyer's preference ordering. The following proposition shows that, in this noncommitment case, all three schemes yield the same expected utility to the buyer—a two-dimensional extension of the revenue equivalence theorem.

Proposition 5. (equivalence). If the buyer is unable to commit to a scoring rule differing from $U(\,\cdot\,, \,\cdot\,)$, all three auction schemes yield the same expected utility to the buyer, equal to

$$E\{V(q^*(\theta_1)) - J(q^*(\theta_1), \theta_1)\},$$

where $q^*(\theta_1) \equiv \operatorname{argmax} V(q) - c(q, \theta_1)$, the first-best quality level.

Proof. See the Appendix.

Several remarks are in order. First, in first- and second-score auctions, the winning firm produces the first-best level of quality, which the buyer finds excessive (relative to the optimal quality in Proposition 1). The reason that the lack of commitment power induces more-than-optimal quality must be clear from Proposition 4; i.e., the true utility function fails to internalize the informational costs associated with increasing quality. Second, the equivalence property has an interesting policy implication. Recalling that the second-preferred-offer auction can be implemented with a phase of restricted negotiation, the above result can be interpreted as suggesting that the added restriction on negotiation does not harm DoD. Third, even though all three auction rules are equivalent from the buyer's standpoint, the same equivalence may not hold for firms. Since in the second-preferred-offer auction a winning firm picks the highest losing firm's first-best level of quality, $q^*(\theta_2)$ (which is lower than its own first-best level, $q^*(\theta_1)$), the total surplus under the second-preferred-offer auction is strictly smaller than under the other two auction rules. Since the expected utility for the buyer is the same under all auction rules, we obtain the following corollary.

Corollary. Expected profits of the winning firm are the same in the first- and second-score auctions but strictly smaller in the second-preferred-offer auction.

5. Some concluding remarks

■ This article has studied alternative multidimensional auction rules to describe DoD's source-selection procedure. The basic framework of the model can be further extended to shed light on some relevant features of defense procurement. The following are some examples.

Quality standard, quality ceiling. Given the potential difficulty associated with implementing the optimal scoring rule, one may wonder if there are simpler rules to which procurement officers can commit. Quality standards that fix quality before competition begins, and a quality ceiling that provides a cap on quality while allowing for downward flexibility in quality choice are two obvious instruments. Quality standards are actually used in some sequential procurement competitions, in which quality is determined first and then price is determined in a second round. One might also hope that a quality cap could correct the excessive quality entailed by DoD's lack of commitment. However, using our multidimensional auction framework it is easy to show that these instruments cannot improve upon the naive scoring rule (that results in the noncommitment case). This is true because a rigid quality regulation does not ameliorate the incentive compatibility of firms but does aggravate technological flexibility; Dasgupta and Spulber (1990) find a similar result.

Misrepresentation of the scoring rule by an agent. Often, a buyer delegates the evaluation of bids to an agent with the expertise to understand the technical characteristics of a procured system. In fact, DoD can be regarded as an agent that procures on behalf of Congress, the eventual buyer. For many reasons, the agent may have an incentive to misrepresent the buyer's interest. Rogerson (1990) identifies one such circumstance in which the agent induces more quality than the buyer desires. If, in fact, the agent systematically overvalues or undervalues quality (relative to the buyer's true preference), the equivalence of the alternative auction schemes will break down. Moreover, since the second-preferred-offer auction induces a lower equilibrium quality than the other two schemes, one can easily verify that when the agent systematically overvalues quality, the buyer may prefer the second-preferred-offer auction to the other auctions.[12]

Prebidding R&D investment by firms. Firms usually undertake R&D investment before entering into source selection competition. Even though such R&D investment has a very significant and lasting effect on the overall procurement performance, the government cannot easily control this investment because it cannot usually be directly contracted upon. Several authors have observed that production contracts can indirectly serve as incentives for prebidding R&D investment, noting that firms will try to improve their design specifications to increase the probability of winning (Rogerson, 1990; Lichtenberg, 1988). In this regard, Riordan and Sappington (1989) found that the incentives for precontractual investment are positively related to the informational rents a firm expects to earn upon winning a contract. Similarly, it can be argued that the investment incentives depend on the way the scoring rule is designed. If the investment is to reduce the cost of quality improvement, the weight a scoring rule gives to quality will matter: in particular, rewarding quality through a higher weight will motivate firms to engage in more investment. In such a case, the optimal scoring rule obtained in Proposition 4 ought to be modified to provide a greater reward for quality. Furthermore, this may imply that the excess quality that results from the buyer's lack of commitment may not be as serious a problem as it appears.

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[12] In fact, the buyer may want to do more than select an auction scheme in responding to a conflicting incentive of the agent. Laffont and Tirole (1991), for example, study a contractual mechanism through which a buyer can respond to potential favoritism on the part of the auctioning agent.

There are several ways to extend the model. The multidimensional bidding model discussed in this article does not address distinctive features of quality: q in the model could well be interpreted as quantity. In the context of defense procurement, however, quality is distinguished from quantity by the sequential decision making: quality is determined first and quantity is determined afterwards as a result of the budgetary decisions of Congress. Another feature of quality is its imperfect verifiability. In practice, evaluation of bids is based on signals of quality rather than actual quality. An interesting extension will be to incorporate these features of quality in the framework of multidimensional bidding.

Appendix

■   Proofs of Lemma 1 and Propositions 4 and 5 follow.

Proof of Lemma 1. Suppose to the contrary an equilibrium bid (q, p) has $q \neq q_s$ at least for one firm with $\theta < \bar{\theta}$. A contradiction is derived by showing that the bid is strictly dominated by an alternative bid (q', p') where $q' = q_s$ and $p' = p + s(q_s) - s(q)$. Notice that $S(q, p) = S(q', p')$. Also, Prob $\{$win$|$ $S(q, p)\} > 0$. This can be proved as follows: First let $\underline{S} = \inf \{S|$ Prob $\{$win$|$ $S\} > 0\}$ and $S_o(\theta) = \max_q s(q) - c(q, \theta)$. Then, $\underline{S} \leq S_o(\bar{\theta})$. To prove this claim, suppose, to the contrary, the probability of winning for some type $\theta < \bar{\theta}$ is zero. Then, the choice of the score S must be that $S \leq \underline{S}$. But since S_o is decreasing in c, there is an alternative score $S' \in (\underline{S}, S_o(\theta))$, which allows positive profits for the type θ, contradicting the optimality of the score S.

Now,

$$\pi(q', p'|\theta) = [p' - c(q', \theta)] \text{ Prob} \cdot \{\text{win} | S(q', p')\}$$

$$= [p - c(q, \theta) + \{V(q_s) - c(q_s, \theta) - (V(q) - c(q, \theta))\}] \text{ Prob} \{\text{win} | S(q, p)\}$$

$$> [p - c(q, \theta)] \text{ Prob} \{\text{win} | S(q, p)\}$$

$$= \pi(q, p|\theta),$$

as desired.   *Q.E.D.*

Proof of Proposition 4. First I show that the new scoring rule implements the optimal quality schedule $q_o(\cdot)$ under first- and second-score auctions. Under both schemes, quality is chosen to maximize $V(q) - \Delta(q) - c(q, \theta)$. Notice

$$\frac{d\tilde{S}(q, c(q, \theta))}{dq} = V'(q) - c_q(q, \theta) - \Delta'(q)$$

$$= V'(q) - c_q(q, \theta) - \frac{F(q_o^{-1}(q))}{f(q_o^{-1}(q))} c_{q\theta}(q, q_o^{-1}(q))$$

$$= 0 \text{ if } q = q_o(\theta).$$

To check the second-order condition, let $y \equiv \dfrac{F(\theta)}{f(\theta)} c_{q\theta}(q, \theta)$. By Assumption 1, y is increasing in θ. Then, for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and all $q \in [q_o(\bar{\theta}), q_o(\underline{\theta})]$,

$$\frac{\partial^2 \tilde{S}(q, c(q, \theta))}{\partial q^2} = V''(q) - c_{qq} - \frac{F}{f} c_{qq\theta} - \frac{\frac{\partial y}{\partial \theta}}{q'_o}$$

$$= \left(V''(q) - c_{qq} - \frac{F}{f} c_{qq\theta}\right) \frac{c_{q\theta}}{c_{q\theta} + \frac{\partial y}{\partial \theta}}$$

$$< 0.$$

The second equality follows from the definition of $q_o(\cdot)$, and the last inequality uses Assumption 1. Thus, I have proved that the optimal quality schedule is implemented by the modified scoring rule $\tilde{S}(q, p)$.

Now, to show

$$EU_{FS}^{\tilde{S}} = EU_{SS}^{\tilde{S}} = E\{V(q_o(\theta_1)) - J(q_o(\theta_1), \theta_1)\},$$

I use a method similar to that in Milgrom (1989). Consider an abstract mechanism M. The expected profit of a firm with cost θ under M is given by

$$\pi(x_M, p_M, q_M | \theta) = x_M(p_M - c(q_M, \theta)),$$

where x_M denotes the probability of winning. Define the optimized value of the expected profit

$$\pi_M^*(\theta) \equiv \pi(x_M^*, p_M^*, q_M^* | \theta).$$

Then, using the envelope theorem,

$$\pi_M^{*'}(\theta) = \frac{\partial \pi_M^*}{\partial \theta} = x_M^* c_\theta(q_M^*, \theta).$$

From this it follows that

$$\pi_M^*(\theta) = \int_\theta^{\bar{\theta}} x_M^*(t) c_\theta(q_M^*(t), t) dt + \pi_M^*(\bar{\theta}).$$

Now, notice that $x_{FS}^* = x_{SS}^* = [1 - F]^{N-1}$ and $\pi_{FS}^*(\bar{\theta}) = \pi_{SS}^*(\bar{\theta}) = 0$. Since equilibrium quality is the same under both schemes equal to q_o, $\pi_{FS}^* = \pi_{SS}^* = \int_\theta^{\bar{\theta}} c_\theta(q_o(t), t)[1 - F(t)]^{N-1} dt$ and hence

$$E\pi_{FS}^* = E\pi_{SS}^* = \int_\theta^{\bar{\theta}} \int_\theta^{\bar{\theta}} c_\theta(q_o(t), t)[1 - F(t)]^{N-1} dt d\theta = E\left\{ c_\theta(q_o(\theta_1), \theta_1) \frac{F(\theta_1)}{f(\theta_1)} \right\}.$$

Since the expected total surplus is the same in the two schemes and equal to $E\{V(q_o(\theta_1)) - c(q_o(\theta_1), \theta_1))\}$, subtracting above-expected profits from the expected total surplus yields expected utilities under the two schemes:

$$EU_{FS}^{\hat{S}} = EU_{SS}^{\hat{S}} = E\{V(q_o(\theta_1)) - J(q_o(\theta_1), \theta_1)\}.$$

Finally, that the second-preferred-offer auction implements only a random quality schedule proves that it cannot implement the optimal mechanism. *Q.E.D.*

Proof of Proposition 5. When $S(\cdot, \cdot) = U(\cdot, \cdot)$, from Propositions 2 and 3, firms propose the first-best quality, and $p_s(\theta_1, \theta_2) = V(q^*(\theta_1)) - V(q^*(\theta_2)) + c(q^*(\theta_2), \theta_2)$. Substituting this into (3) yields that

$$EU_{SS} = E\{V(q^*(\theta_2)) - c(q^*(\theta_2), \theta_2)\} = EU_{SPO}.$$

The rest of the proof follows, since

$$EU_{SS} = EU_{SPO} = E\{V(q^*(\theta_2)) - c(q^*(\theta_2), \theta_2)\}$$

$$= N(N - 1) \int_\theta^{\bar{\theta}} [V(q^*) - c(q^*, \theta)] F f (1 - F)^{N-2} d\theta$$

$$= N \int_\theta^{\bar{\theta}} [V(q^*) - c(q^*, \theta)] F \left(-\frac{d(1 - F)^{N-1}}{d\theta} \right) d\theta$$

$$= N \int_\theta^{\bar{\theta}} [(V(q^*) - c(q^*, \theta)) f - c_\theta(q^*, \theta) F] (1 - F)^{N-1} d\theta$$

$$= N \int_\theta^{\bar{\theta}} [V(q^*) - (c(q^*, \theta)) + \frac{F}{f} c_\theta(q^*, \theta)] f (1 - F)^{N-1} d\theta$$

$$= E\{V(q^*(\theta_1)) - J(q^*(\theta_1), \theta_1)\}$$

$$= EU_{FS}.$$

The third equality uses integration by parts and the envelope theorem. *Q.E.D.*

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