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Design Considerations for Refrigeration Cycles

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ABSTRACT

The Carnot COP, which assumes a thermodynamically ideal cycle in which no irreverrsibilities exist is often considered to be a design goal for actual cycles. However, the Carnot COP does not consider heat transfer mechanisms. Heat transfer at a finite rate is necessarily an irreversible process and unavoidable in a refrigeration cycle. The lack of consideration of rate processes reduces the usefulness of the Carnot COP as a realistic design goal. In this paper, the limitations of both thermodynamics and heat transfer are considered to identify a more realistic design goal for the COP of refrigeration cycles. The consideration of heat transfer limitations leads to a design rule for the optimum distribution of heat exchange area between the low- and high-temperature heat exchangers.

NOMENCLATURE

- C — sum of effectiveness capacitance rate products for both heat exchangers, defined in equation (9) [kW/°K]
- $\dot{C}_H$ — capacitance rate (mass flow rate – specific heat product) for the external high temperature heat transfer fluid [kW/°K]
- $\dot{C}_L$ — capacitance rate (mass flow rate – specific heat product) for the external low temperature heat transfer fluid [kW/°K]
- COP — coefficient of performance for a refrigeration cycle
- f_H — ratio of the condenser effectiveness capacitance rate product to the sum of the effectiveness capacitance rate products for the condenser and evaporator.
- h_j — specific enthalpy of refrigerant at state point j [kJ/kg]
- $\dot{m}$ — refrigerant mass flowrate [kg/sec]
- $\dot{Q}_L$ — rate of heat transfer from the refrigerated space at temperature T_L to the refrigeration cycle [kW]
- $\dot{Q}_H$ — rate of heat transfer from the refrigeration cycle to the thermal sink at temperature T_H [kW]
- T_j — temperature of refrigerant at state point j [°K]
- T_h — temperature at which heat is transferred from the internally reversible refrigeration cycle to the sink [°K]
- T_ℓ — temperature at which heat is transferred to the internally reversible refrigeration cycle from the refrigerated space [°K]
- T_H — entering temperature of the high temperature external fluid [°K]
- T_L — entering temperature of the low temperature external fluid [°K]
- $(UA)_H$ — overall heat transfer coefficients for the condenser [kW/°K]
- $(UA)_L$ — overall heat transfer coefficients for the evaporator [kW/°K]
- $\dot{W}$ — power required to operated refrigeration cycle (excluding fan power) [kW]
- $\dot{W}_s$ — power required for isentropic compression [kW]
- ΔT — temperature difference defined in equation (7) [°K]

ε_L effectiveness of the low temperature (evaporator) heat exchanger

ε_H effectiveness of the high temperature (condenser) heat exchanger

η compressor isentropic efficiency

INTRODUCTION

The coefficient of performance (COP) for a refrigeration cycle is defined as the ratio of the rate of heat transfer from the refrigerated space to the power supplied to the refrigeration cycle. The Second Law of Thermodynamics places an upper limit on the COP which is often called the Carnot COP.

$$COP_{Carnot} = \dot{Q}_L/\dot{W} = T_L / (T_H - T_L) \quad (1)$$

where

$\dot{Q}_L$ is the rate at which thermal energy is transferred from the refrigerated space

$\dot{W}$ is the power supplied to the refrigeration cycle

T_L is the temperature of the refrigerated space

T_H is the sink temperature.

The Carnot COP, which assumes a thermodynamically ideal cycle in which no irreversibilities exist, is often considered to be a design goal for actual cycles. However, the Carnot COP does not consider heat transfer mechanisms. Heat transfer at a finite rate is necessarily an irreversible process and unavoidable in a refrigeration cycle. The lack of consideration of rate processes reduces the usefulness of the Carnot COP as a realistic design goal. In this paper, the limitations of both thermodynamics and heat transfer are considered to identify a more realistic design goal for the COP of refrigeration cycles. The consideration of heat transfer limitations leads to a design rule for the optimum distribution of heat exchange area between the low and high temperature heat exchangers.

MAXIMUM COP FOR A REFRIGERATION CYCLE PROVIDING A SPECIFIED COOLING CAPACITY

The criterion of reversible heat transfer requires infinitesimal temperature differences between the machine and external streams. For finite heat exchange areas, the infinitesimal temperature differences result in zero heat transfer rates. As a consequence, the cooling capacity supplied by a finite-sized machine operating at the Carnot COP is zero.

The heat transfer processes occurring in a refrigeration cycle are major sources of thermodynamic irreversibility. In the following analysis, a refrigeration cycle is considered in which the heat transfer processes are the only irreversibilities. The results of this analysis provide a relation between the maximum COP of a refrigeration cycle and its cooling capacity for specified heat exchange conditions.

Consider the internally-reversible refrigeration cycle shown in Figure 1 which is designed to provide a specified cooling capacity $\dot{Q}_L$ to a flowing stream which enters at temperature T_L and flowrate $\dot{m}_L$. In this process, heat is rejected to a second fluid entering at temperature T_H and flowrate $\dot{m}_H$. In order to provide a finite cooling capacity, heat must be transferred to the refrigeration cycle at temperature T_l which is lower than T_L. The difference between T_L and T_l is dependent upon characteristics (e.g., surface area, external fluid flowrate) of the low temperature heat exchanger. Similarly, the cycle must reject heat at temperature

T_h which is higher than T_H by an amount dependent on characteristics of the high temperature heat exchanger. The effects of irreversibilities resulting from heat transfer to and from the cycle are considered in this section. The refrigeration cycle itself is assumed to be internally reversible, operating between uniform temperatures T_l and T_h, as in the Carnot cycle and, approximately, in actual vapor compression cycles in which heat transfer occurs during constant temperature vapor/liquid phase changes. The system shown within the dotted line box in Figure 1 is essentially a Carnot cycle.

The heat transfers to and from the cycle occur by convection to flowing fluid streams having finite mass flowrates and specific heats. The rate of heat transfer to the cycle at the low temperature is

$$\dot{Q}_L = \varepsilon_L \dot{C}_L (T_L - T_l) \tag{2}$$

where

ε_L is the low temperature heat exchanger effectiveness defined by Kays and London (1964)

$\dot{C}_L$ is the capacitance rate (mass flowrate – specific heat product) of the external low temperature heat transfer fluid.

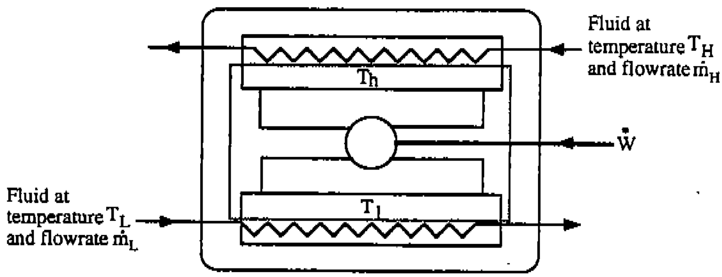

Figure 1. Schematic of an internally reversible refrigeration cycle which irreversibly transfers heat to external streams.

Similarly, the rate of heat transfer between the refrigeration cycle and the sink is

$$\dot{Q}_H = \varepsilon_H \dot{C}_H (T_h - T_H) \tag{3}$$

where

ε_H is the high temperature heat exchanger effectiveness

$\dot{C}_H$ is the capacitance rate of the external high temperature heat transfer fluid

The coefficient of performance of the refrigeration cycle is defined

$$COP = \frac{\dot{Q}_L}{\dot{W}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} \tag{4}$$

Because the refrigeration cycle is assumed to have no internal irreversibilities, it operates as a Carnot refrigeration cycle so that,

$$COP = \frac{T_l}{T_h - T_l} \quad (5)$$

Equations (2) - (5) can be algebraically manipulated to eliminate T_h and T_l, resulting in

$$COP = \frac{(T_L - \Delta T)}{T_H - (T_L - \Delta T)} \quad (6)$$

where

$$\Delta T = \dot{Q}_L \frac{(\varepsilon_L \dot{C}_L + \varepsilon_H \dot{C}_H)}{\varepsilon_L \dot{C}_L \varepsilon_H \dot{C}_H} \quad (7)$$

The significance of ΔT is that it is a measure of the differences between the external fluid inlet and refrigeration cycle heat exchange temperatures. This significance can easily be observed for the case in which the thermal resistance of the high temperature heat exchanger is eliminated by allowing its capacitance rate and surface area to be very large. In this case, ΔT in equation (7) reduces to

$$\lim_{\varepsilon_H \dot{C}_H \to \infty} \Delta T = \frac{\dot{Q}_L}{\varepsilon_L \dot{C}_L} = (T_L - T_l)$$

If instead, the thermal resistance of the low temperature heat exchanger is eliminated, then

$$\lim_{\varepsilon_L \dot{C}_L \to \infty} \Delta T = \frac{\dot{Q}_L}{\varepsilon_H \dot{C}_H} = \frac{COP}{COP+1} (T_h - T_H)$$

Equations (6) and (7) provide a relation between the maximum COP of a refrigeration cycle and its cooling capacity for specified capacitance rates and inlet temperatures of the external streams. The COP in Equation (6) is a maximum COP in that there are no irreversibilities other than those resulting from the unavoidable heat transfers to and from the cycle. A plot of COP/COP_{Carnot} versus ΔT is shown in Figure 2 for several combinations of T_L and T_H. The major point of Figure 2 is that as ΔT (and thus $\dot{Q}_L$ for fixed external stream conditions and heat exchanger sizes) is increased, the maximum COP must decrease. The Carnot COP can be achieved only when ΔT is 0, which occurs only if $\dot{Q}_L$ is zero when external flowrates are finite.

The heat exchanger effectiveness factors appearing in equations (2) and (3) are, in general, functions of the heat exchanger design and surface area, as described by Kays and London (1964). When one fluid undergoes heat exchange isothermally, as for example during phase change, the effectiveness is independent of the flow arrangement. In this case:

$$e_L = 1 - \exp\left(-(UA)_L/\dot{C}_L\right) \quad e_H = 1 - \exp\left(-(UA)_H/\dot{C}_H\right) \quad (8a \text{ and } b)$$

where $(UA)_L$ and $(UA)_H$ are the overall heat transfer coefficients for the low and high temperature heat exchangers, respectively.

The design of a refrigeration cycle requires a selection of the external stream capacitance rates and heat exchanger parameters which determine the physical size and cost of the heat exchangers. Increasing the effectiveness - capacitance rate product of a heat exchanger generally increases its size and cost, but not in a linear manner. However, increasing the effectiveness - capacitance rate product also decreases parameter

ΔT in equation (7) which means the refrigeration cycle can operate at a higher COP in providing a specified cooling capacity. An engineering compromise must be reached between the first cost of the refrigeration equipment and its operating costs. A problem involved in this optimization is how to allocate the total heat exchanger area between the two heat exchangers so as to minimize the power required to provide a specified cooling capacity for a fixed first cost. A reasonable postulate is that a larger fraction of the total heat exchanger UA should be allocated to the high temperature heat exchanger since it must transfer more energy than the low temperature heat exchanger. However, this is not the case.

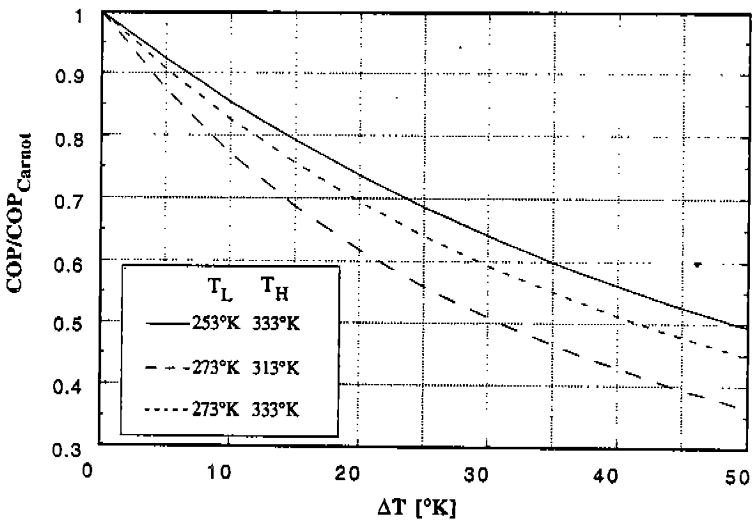

Figure 2. COP/COP$_{Carnot}$ as a function of ΔT (defined in equation 7).

OPTIMUM DISTRIBUTION OF HEAT EXCHANGE AREA

Consider a situation in which a refrigeration cycle is to supply a specified cooling capacity, $\dot{Q}_L$, operating with external streams which enter at specified temperatures T_L and T_H. Equations (2) – (5) and (8) provide six equations involving 10 undetermined variables, namely, T_l, T_h, COP, $\dot{Q}_H$, ε_L, $\dot{C}_L$, ε_H, $\dot{C}_H$, $(UA)_L$, and $(UA)_H$, resulting in four degrees of freedom. Equations (2) and (3) demonstrate that only the products of the effectiveness and capacitance rates, not their individual values, enter into the refrigeration cycle performance calculations. As a result, it is possible to reduce the number of degrees of freedom from four to two in the following optimization study by considering the effectiveness - capacitance rate product for each heat exchanger to be a single variable. In this case, equations (8a and b) and $(UA)_L$, and $(UA)_H$ are eliminated resulting in four equations with six unspecified variables. (The UA and capacitance rate values which result in the optimum effectiveness-capacitance rate product can be determined in separate optimizations for each heat exchanger in which equations 8a and b, in addition to pumping power, duct noise, and comfort considerations are considered.) Two additional relations between the remaining six variables are required in order to fix the design. One relation results by placing a limit on the total heat exchanger investment such that

$$\varepsilon_L \dot{C}_L + \varepsilon_H \dot{C}_H = C \tag{9}$$

where C is a specified constant. Equation (9) produces an effect similar to setting the sum of UA_H and UA_L to a fixed constant, but it is algebraically simpler and more general in that it directly considers the effects of changes in the external fluid capacitance rates. A second relation is obtained by setting the derivative of the COP with respect to any one of the 5 other undetermined variables (such as $\varepsilon_L \dot{C}_L$) to zero so as to maximize the COP.

Although the algebra is somewhat complicated, this set of six equations can be solved analytically. A remarkable conclusion obtained from the solution is that, when the COP is at a maximum,

$$\varepsilon_L \dot{C}_L = \varepsilon_H \dot{C}_H \tag{10}$$

Equation (10) is valid for any specified values of C, $\dot{Q}_L$, T_L and T_H. If the capacitance rates of the external streams are equal, equation (10) indicates that the optimum COP will be attained when $UA_H = UA_L$.

ACTUAL VAPOR COMPRESSION CYCLE PERFORMANCE

Equation (10) indicates the heat exchangers in a refrigeration cycle should be treated equally in order to achieve optimum performance, even though the high temperature heat exchanger must transfer energy at a larger rate. However, equation (10) was derived for a refrigeration cycle with no irreversibilities other than those due to heat transfers to and from the cycle. In this section, the validity of this result is investigated for vapor compression machines which have additional irreversibilities due to vapor compression and expansion and perform in a manner dependent on the thermodynamic properties of the refrigerant.

The standard vapor compression cycle shown in Figure 3 is investigated with R-12 as the refrigerant. Control of the cycle is assumed such that, during steady operation, saturated vapor enters the compressor at state 1 and saturated liquid exits the condenser at state 3. The compressor operation is described in terms of an isentropic efficiency, η, so that its power requirement is given by

$$\dot{W} = \dot{m}\,(h_2 - h_1) = \dot{W}_s/\eta \tag{11}$$

where

$\dot{m}$ is the mass flow rate of the refrigerant

h_1 and h_2 are the specific enthalpies of the refrigerant at state points 1 and 2

$\dot{W}_s$ is the power required for isentropic compression.

The refrigerant enters the condenser at state 2 in a superheated state. However, the majority of the heat transfer takes place while the refrigerant is condensing. This heat transfer process can be described, approximately, in terms of a heat exchanger effectiveness based on the condensation temperature, T_3, as suggested by Stoecker and Jones (1982).

$$\dot{Q}_H = \dot{m}\,(h_2 - h_3) = \varepsilon_H \dot{C}_H\,(T_3 - T_H) \tag{12}$$

The throttling process is assumed isenthalpic.

$$h_4 = h_3 \tag{13}$$

The low temperature heat transfer occurs while the refrigerant vaporizes at temperature T_1.

$$\dot{Q}_L = \dot{m}\,(h_1 - h_4) = \varepsilon_L \dot{C}_L\,(T_L - T_l) \tag{14}$$

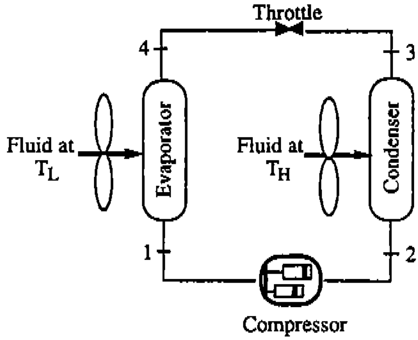

Figure 3. Schematic of a vapor compression refrigeration cycle showing state point locations.

The COP of the cycle (neglecting the power required to move the external fluids) is

$$COP = \frac{\dot{Q}_L}{\dot{W}} \tag{15}$$

To investigate the effect of relative heat exchanger size, a factor f_H is defined as

$$f_H = \frac{\varepsilon_H \dot{C}_H}{C} \tag{16}$$

where C is defined by equation (9). f_H is related to the fraction of the total heat exchanger area (for the condenser and evaporator) which is used by the condenser but the relation between heat exchanger area and f_H is not linear.

Equations (11) – (16) have been solved numerically for a range of values of C, η, T_L and T_H using thermodynamic property data for several different refrigerants. The equation solving program, EES (Klein and Alvarado (1991)) which incorporates refrigerant property data algorithms, was used for this purpose. Representative results are shown in the following figures.

Figure 4 shows how the COP varies with f_H for a range of values of C with $\dot{Q}_L$=10 kW, T_L=0°C, T_H=40°C and η=0.60 with R-12 as the refrigerant. The refrigerant mass flow rate varies as necessary to achieve the specified capacity. Increasing C increases the total (condenser and evaporator) heat exchange capability and, as a result, the COP increases. The COP approaches an asymptotic value for values of C greater than about 500. At any specified value of C, the COP is at a maximum when f_H is approximately (but not exactly) 0.5. The results in Figure 4 can be scaled for any cooling capacity since they depend on the ratio of $\dot{Q}_L$/C. Similar results to those in Figure 4 were obtained for other values of T_L and T_H and for other refrigerants.

In Figure 5, the value of f_H which maximizes the COP of the cycle is plotted as a function of compressor isentropic efficiency. The results in this figure are nearly independent of the value of $\dot{Q}_L$/C.

Figure 5 shows that, as the compressor efficiency decreases, the fraction of total heat exchange area needed by the condenser to achieve a maximum COP increases. This additional compressor area is needed to dissipate the additional required compressor power. The optimum value of f_H is slightly affected by the values of T_L and T_H. However, the optimum value of f_H is still quite close to 0.5. Results similar to those in Figures 5 were found for other refrigerants.

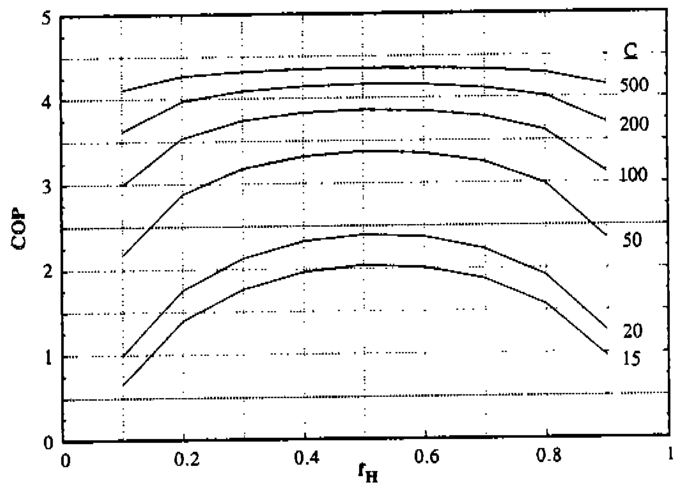

Figure 4. COP versus the heat transfer factor f_H (equation (16)) for a range of values of C (equation (9)) with η=0.60, T_L=0°C, T_H=40°C and refrigerant R-12.

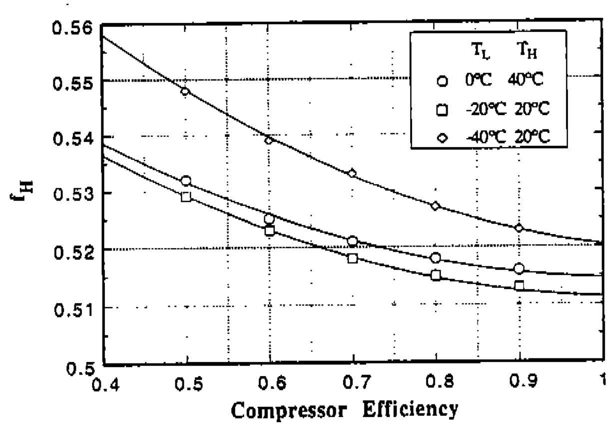

Figure 5. Optimum values of f_H versus compressor isentropic efficiency for various values of T_L and T_H and refrigerant R-12.

CONCLUSIONS

The Carnot analysis provides an upper bound on the COP of a refrigeration cycle, but this upper bound can only be attained as the cooling capacity approaches zero. A more useful design goal for the COP is provided by equation (6) for a refrigeration cycle which provides a specified cooling capacity with no irreversibilities other than those resulting from heat exchange between the cycle and external streams. The analysis presented here assumes that the heat transfers occur to and from the refrigeration cycle isothermally, as in the Carnot and the ideal vapor compression cycles. The COP given in equation (6) could be exceeded by a refrigeration cycle which reduces heat transfer irreversibilities by transferring heat over a temperature range, as in a cycle using a non-azeotropic refrigerant.

Analysis of the internally reversible refrigeration cycle indicates that its COP is maximized for a specified cooling capacity when the product of the heat transfer effectiveness and external fluid capacitance rate is the same for both heat exchangers. The result is found to be true, approximately, for actual vapor compression cycles as well, despite the irreversibilities of throttling and non-isentropic compression and the effects of refrigerant thermodynamic properties. This result is directly useful in identifying economic optimum refrigeration cycle designs.

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