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DESIGN OF A RECONFIGURABLE ANTENNA ARRAY WITH DISCRETE PHASE SHIFTERS USING DIFFERENTIAL EVOLUTION ALGORITHM

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Abstract—The reconfigurable design problem is to find the element that will result in a sector pattern main beam with side lobes. The same excitation amplitudes apply to the array with zero-phase that should be in a high directivity, low side lobe pencil shaped main beam. Multi-beam antenna arrays have important applications in communications and radar. This paper presents a new method of designing a reconfigurable antenna array with quantized phase excitations using a new evolutionary algorithm called differential evolution (DE). In order to reduce the effect of mutual coupling among the antenna-array elements, the dynamic range ratio is minimized. Additionally, compared with the continuous realization and subsequent quantization, experimental results indicate better performance of the discrete realization of the phase-excitation value of the proposed algorithm.

1. INTRODUCTION

Reconfigurable antenna arrays that are capable of radiating multiple patterns using a single power-divided network are desirable in many applications. Many methodologies have been proposed to obtain the multi-pattern arrays in the literatures [1–3]. Among these methods, evolutionary algorithms may be one of the most successful methods and have been successfully applied to antenna array synthesis problems,

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such as null steering in phased arrays by positional perturbations [4–27]. Compared with other algorithms, evolutionary algorithms can generate a nearly optimal solution in a reasonable computational time. In evolutionary algorithm-based antenna-array synthesis procedures, phased excitations are usually represented by continuous values; however, discrete phase shifters are used to realize the phase excitation sometimes. Accordingly, Baskar et al. [21] proposed a mixed-integer optimization for the first time in an evolutionary search method, namely, the generalized generation-gap model GA (G3-GA). Akdagli et al. proposed a method based on the clonal selection algorithm (CLONALG) to design a reconfigurable dual-beam linear antenna array with excitation distributions differing only in phase [22]. From the practical implementation viewpoints, the proposed method takes discrete phase shifters into account during synthesis. However, GA and CLONALG can trap into the local minima easily. Differential evolution (DE) [28] is a method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. The basic idea of DE is to create new candidate solutions by combining the parent individual and several other individuals of the same population, and a candidate replaces the parent only if it has better fitness. Compared with GA and CLONALG, DE is easier to implement and has been applied to many problems with both discrete and continuous design parameters. In this paper, we will use the differential evolution algorithm to perform reconfigurable antenna array optimization with quantized phase excitations. In order to demonstrate the advantages of the proposed design, the results obtained using continuous-phase excitations followed by quantization are compared with other algorithms. Experimental results show that our algorithm is both effective and efficient.

The rest of this paper is organized as follows. In Section 2, we introduce the problem formulation. Section 3 describes the fitness function. Section 4 describes the differential evolution. Corresponding experimental results are given in Section 5. In the last section we conclude this paper and point out some future research directions.

2. PROBLEM FORMULATION

In order to design a reconfigurable dual-beam antenna array, an amplitude distribution can generate either a pencil-shaped or a sector power pattern, when the phase distribution of the array is modified appropriately. All excitation phases are set at 0° for the pencil-shaped beam and are varied in the range $-180^\circ \leq \phi \leq 180^\circ$ for the sector pattern [7]. If the excitation is symmetrical about the centre of the linear array, the array with even number of uniformly spaced isotropic elements (2N) can be written as [10]:

$$F(\theta) = 2\sum_{k=1}^{N} \left(a_{kR}\cos\phi_k - a_{kl}\sin\phi_k\right) \tag{1}$$

with

$$\phi_k = \frac{2\pi}{\lambda} d_k \sin\theta \tag{2}$$

where d_k is the distance which is between the position of the kth element and the center; θ is the scanning angle from broadside; a_{kR} is the real parts of the kth element excitation; a_{kI} is the imaginary parts of the kth element excitation; and a_{kR} and a_{kI} are set within the range $[0, 1]$ and $[-1, 1]$, respectively. N excitation amplitude and phase coefficients are to be chosen to optimize the desired pattern. The pencil and sector patterns should have high directivity, low side lobe pencil shaped main beam and wide sector beam.

3. FITNESS FUNCTION EVALUATION

For the reconfigurable dual-beam optimization, the objective of the fitness function must qualify the entire array radiation pattern. The calculated pattern can be described in terms of the criteria of the desired pattern. The fitness function for the dual-beam optimization can be described as follows [7]:

$$E(P) = \sum_{i=1}^{3} \left(P_{i,d}^{(p)} - P_i^{(i)}\right)^2 + \sum_{i=1}^{4} \left(P_{i,d}^{(s)} - P_i^{(s)}\right)^2, \tag{3}$$

where the superscript p is the design specification for the pencil pattern, and the superscripts s are the design specification of the sector pattern. The superscript d indicates the desired value of the design specification, and P indicates the applicable fitness factor in Table 1. The first part of this fitness function is summarized over the first column of Table 1, and the other part of this function is summarized over the second column. Different from the fitness function of the pencil beam pattern, the pattern ripple needs to be calculated for the sector pattern.

In order to decrease the effect of coupling between elements, an additional term is included in the objective function Equation (4) [21]. The ratio is used to minimize the coupling effect between the maximum and minimum excitation amplitudes. The minimization of the amplitude-excitation dynamic range (ARD) can reduce the mutual

Table 1. Design specifications.

Design Parameters	Pencil Pattern	Sector Pattern
Side-lobe level (SLL)	−30 dB	−25 dB
Half-power bandwidth (HPBW)	6.8°	24°
Bandwidth at SLL	20°	40°
Ripple	NA	0.5 dB

coupling problem [29, 30]. The objective function can be expressed as follows:

$$Ec(P)=\sum_{i=1}^{3}\left(P_{i,d}^{(p)}-P_i^{(i)}\right)^2+\sum_{i=1}^{4}\left(P_{i,d}^{(s)}-P_i^{(s)}\right)^2+ADR, \quad (4)$$

where ADR is the amplitude-dynamic ratio. The ADR is defined as the ratio between the maximum excitation amplitude to the minimum excitation amplitude. The differences between the excitation amplitudes are minimized by minimizing the ADR; therefore, the effect of coupling can be minimized.

4. DIFFERENTIAL EVOLUTION ALGORITHM

Differential Evolution (DE) is an Evolutionary Algorithm first introduced by Storn and Price [28]. Similar to other evolutionary algorithms particularly genetic algorithm, DE uses some evolutionary operators as selection recombination and mutation operators. Different from genetic algorithm, DE uses distance and direction information from current population to guide the search process. The crucial idea behind DE is a scheme for producing trial vectors according to the manipulation of target vector and difference vector. If the trail vector yields a lower fitness than a predetermined population member, the new trail vector will be accepted and be compared in the following generation. Different kinds of strategies of DE have been proposed based on the target vector selected, and the number of difference vectors is used. In this paper, we use two strategies, DE/rand/1/bin, described as follows.

For each target vector $x_i(t)$, trail vector $v_i(t)$, $i = 1, \ldots, NP$, let N be the dimension of target vector and G be the G generation. The mutant vectors are generated in these DE/rand/1/bin strategies respectively:

For DE/rand/1/bin

$$v_{i,G}=x_{a,G}+F(x_{b,G}-x_{c,G}) \quad (5)$$

where a, b, c, $d \in [1, \ldots, NP]$ are randomly chosen integers and $a \neq b \neq c \neq d \neq i$. F is the scaling factor controlling the amplification of the differential evolution.

The cross-over operator implements a recombination of the trial vector and the parent vector to produce offspring. This operator is calculated as:

$$u_{j,i,G} = \begin{cases} v_{j,i,G}, & (rand_j[0,1] \leq CR) \text{ or } (j = j_{rand}) \\ x_{j,i,G}, & \text{otherwise} \end{cases} \tag{6}$$

where $j = [1, \ldots, D]$; $rand_j \in [0, 1]$; $j_{rand} = [1, \ldots, D]$ is the randomly chosen index. CR is the crossover rate. $v_{j,i,G}$ is the difference vector of the jth particle in the ith dimension at the Gth iteration, and $u_{j,i,G}$ denotes the trail vector of the jth particle in the ith dimension at the Gth iteration. Selection operator is used to choose the next population between the trail population and the target population:

$$x_{i,G+1} = \begin{cases} u_{i,G}, & f(u_{i,G}) < f(x_{i,G}) \\ x_{i,G}, & \text{otherwise} \end{cases} \tag{7}$$

The standard differential evolution algorithm can be described as the followings:

```
procedure Algorithm description of DE algorithm
begin
Step 1:   Set the generation counter G=0; and randomly initialize a
population of NP individuals X_i . Initialize the parameter F, CR
Step 2:  Evaluate the fitness for each individual in P.
Step 3:  while stopping criteria is not satisfied do
     for i= 1 to NP
          select randomly a ≠ b ≠ c ≠ d ≠ i
            for j=1 to D
              j_rand = ⌊rand (0,1) * D⌋
              If rand (0,1) ≤ CR or j== j_rand then
                   u_i,j = x_a,j + F × (x_b,j − x_c,j)
              Else
              u_i,j = x_i,j
              end if
            end for
    end for
     for i=1 to NP do
       Evaluate the offspring u_i
        If  u_i is better than P_i then
            P_i = u_i
       end if
     end for
      Memorize the best solution achieved so far
Step 4: end while
end
```

5. EXPERIMENTAL RESULTS

To evaluate the performance of the differential evolution algorithm, the reconfigurable antenna-array design with the discrete phase shifters is considered. In [21], two separate experiments (I and II) are designed. In experiment I, 20 design parameters are expressed by continuous values, and the results of the excitation phases are not usable and approximate to the nearest values for an n-bit phase. In experiment II, 10-phase excitations are indicated as quantized values corresponding to the n-bit phase shifter. Therefore, the values of the phase excitation are quantized between $-180°$ and $180°$ with $5.6265°$ per step. In practice, discrete phase shifters are used to implement the phase excitations at every element of the array. Similar to [21], we also consider experiments I and II in this paper. For simulating differential evolution algorithm and generalized generation gap GA (G3-GA), the population size NP is 20. The maximum function evaluations are 20000. The crossover rate CR is 0.9. The scale factor F is 0.5. In G3-GA, the number of the offspring $\lambda = 6$. The maximum function evaluations are 20000. Population size NP is 500, and $\sigma_\alpha = \sigma_\beta = 0.25$.

5.1. Optimization without ARD

In this section, we will use differential evolution for the reconfigurable antenna-array design without coupling effects using the objective function (3). The results of the excitation amplitude and phase are listed in Table 2. Table 2 shows the best of optimal results for experiment I, experiment I after quantization of the phase excitations, and experiment II. The table also gives the ADR of the optimized

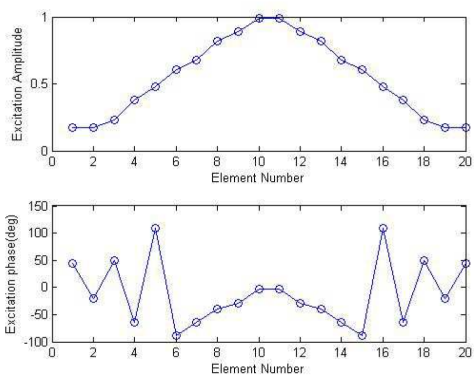

Figure 1. Amplitude and phase excitation (experiment I).

excitation amplitudes and fitness function value. The optimized excitation patterns and dual-beam patterns are shown in Figure 1 and Figure 2, respectively. Figure 2 illustrates the satisfaction of designing parameters simultaneously for both pencil and sector beam.

For experimental I, the best dual-beam pattern using differential

Table 2. Optimum results of experimental I and experiment II without ADR.

Element Number	Experiment I		Experiment I After Quantization		Expt-II	
	Amplitude	Phase [deg.]	Amplitude	Phase [deg.]	Amplitude	Phase [deg.]
1/20	0.173	−3.3	0.173	−2.9	0.122	−14.3
2/19	0.172	−28.4	0.172	−25.7	0.126	−25.7
3/18	0.229	−39.9	0.229	−37.1	0.266	−48.6
4/17	0.380	−64.6	0.380	−60.0	0.388	−54.3
5/16	0.478	−89.5	0.478	−88.6	0.416	82.9
6/15	0.604	108.6	0.604	111.4	0.565	111.4
7/14	0.674	−63.7	0.674	−60.0	0.707	−77.1
8/13	0.819	48.8	0.819	54.3	0.844	77.1
9/12	0.890	−20.5	0.890	−20.0	0.890	2.85
10/11	0.991	44.4	0.991	48.6	0.939	48.6
ADR	5.77		5.77		7.69	
Fitness value	0.16		4.84		0.36	

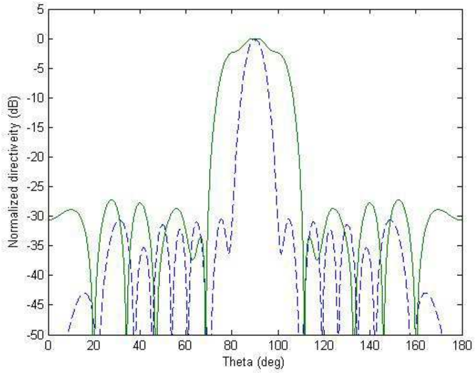

Figure 2. Dual-beam array patterns (experiment I) (imaginary line represents pencil_beam. Straight line represents sector_beam).

evolution is 0.16. From Table 2, we can find that the fitness value is increased to 4.84 after quantizing the optimum phase values to the nearest 6-bit phase-shifter values. The sector beam increases most of the fitness values. The quantization of the optimum result obtained in experiment I may not be optimum for the discrete case. Hence, in the evolution process, discrete values represent the phase excitation that can eliminate the error arising due to quantization.

Table 3 gives the deviation between the desired and computed design specifications of the optimized results in experiment I and experiment II. Compared with the quantization of the phase values in continuous formulation, we can find that the direct discrete phase

Table 3. Effects of quantization on different design specifications.

	Pencil beam			Sector Beam				Fitness
	HPBW	SLLBW	SLL	HPBW	SLLBW	SLL	Ripple	
Continuous phase excitation (Expt-I)	0.4	0	0	0	0	0	0	0.16
After quantization	0.4	0	0	1.8	1.2	0	0	4.84
Optimization with discrete variable (II)	0.6	0	0	0	0	0	0	0.36

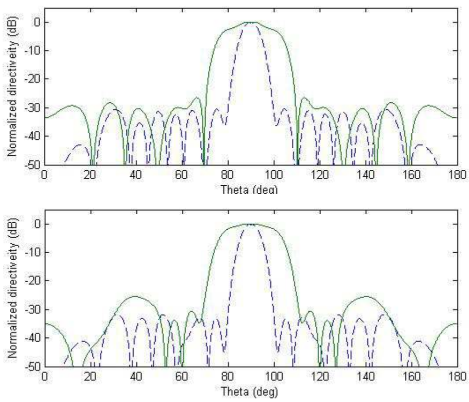

Figure 3. Dual-beam array pattern: experiment I (top) after quantization and experiment II (imaginary line represents pencil_beam. Straight line represents sector_beam).

excitation formulation can give a better fitness value. Experimental I after quantization and experimental II for the dual-beam patterns are shown in Figure 3. The difference between experiment I and experimental II is clearly shown. The best amplitude and phase excitations with discrete values are given in Figure 4.

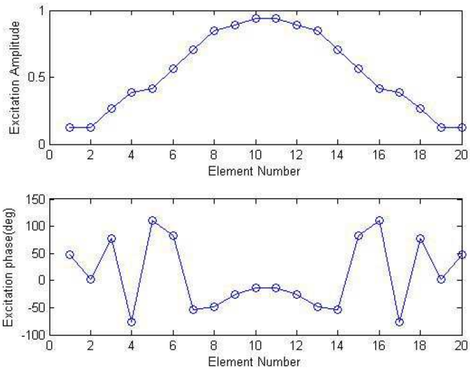

Figure 4. Amplitude and phase excitation (experiment II).

Table 4. Optimum results of experimental I and experiment II with ADR.

Element Number	Experiment I		Experiment I After Quantization		Expt-II	
	Amplitude	Phase [deg.]	Amplitude	Phase [deg.]	Amplitude	Phase [deg.]
1/20	0.224	−10.2	0.224	−8.57	0.193	−14.3
2/19	0.224	−24.9	0.224	−20	0.194	−20.0
3/18	0.250	−29.0	0.250	−25.7	0.249	−54.3
4/17	0.365	−68.7	0.365	−65.7	0.356	−60.0
5/16	0.502	−80.2	0.502	−77.1	0.445	−82.9
6/15	0.591	93.1	0.591	100	0.566	100
7/14	0.774	−92.8	0.774	−94.3	0.705	77.1
8/13	0.826	−97.5	0.826	−94.3	0.801	94.2
9/12	0.963	82.9	0.963	88.6	0.912	−25.7
10/11	0.975	77.7	0.975	82.9	0.912	71.4
ADR	4.35		4.35		4.72	
Fitness value	0.04		20.17		0.16	

5.2. Optimization with ADR

In this section, we use differential evolution for the reconfigurable antenna-array design with coupling effects using objective function (4). The results for experiment I and experiment II are listed in Table 4. The table also gives the ADR and fitness values. The best fitness is less than previous known results in this case. Furthermore, in experiment I, the ARD is reduced from 5.7689 to 4.3470. In experiment II, the ADR is reduced from 7.6928 to 4.7160. Therefore, in practice, we

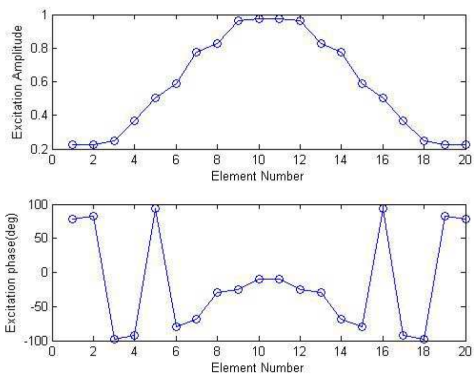

Figure 5. Amplitude and phase excitation (experiment I) with coupling effect.

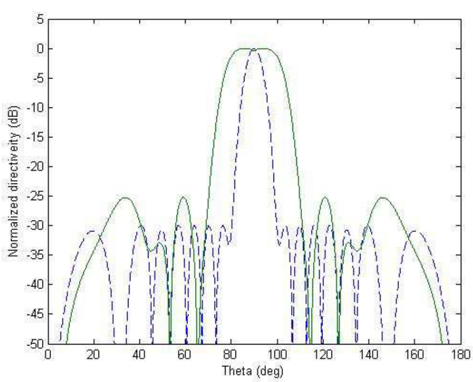

Figure 6. Dual-beam array pattern (experiment I) with coupling effect (imaginary line represents pencil_beam. Straight line represents sector_beam).

can reduce the coupling effects by minimizing the dynamic rang ratio. Figure 5 and Figure 6 show the excitation pattern and dual-beam pattern obtained in experiment I. Figure 7 and Figure 8 show the excitation pattern and dual-beam pattern obtained in experiment II.

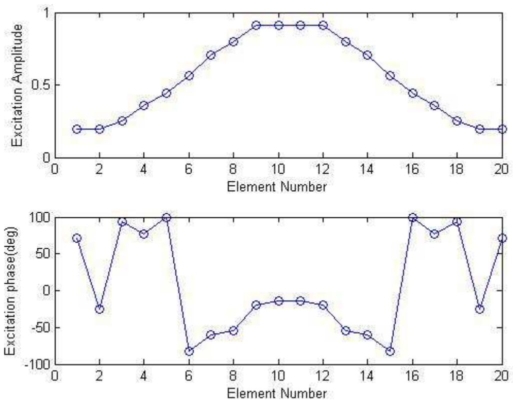

Figure 7. Amplitude and phase excitation (experiment II) with coupling effect.

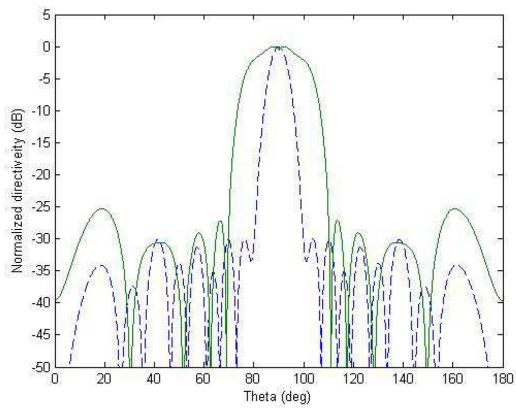

Figure 8. Dual-beam array pattern (experiment II) with coupling effect (imaginary line represents pencil_beam. Straight line represents sector_beam).

Table 5. Comparison of G3-GA with DE.

	Exp-I without ADR	Exp-II without ADR	Exp-I with ADR		Exp-II without ADR	
	fitness	fitness	ADR	fitness	ADR	fitness
G3-GA	**0.16**	0.619	4.4137	0.1028	5.8026	0.2630
DE	**0.16**	**0.36**	**4.3470**	**0.04**	**4.7190**	**0.16**

5.3. Comparison with DE and G3-GA [10]

In order to study the effect of the differential evolution, we carry out a scalability study comparing with the generalized generation gap genetic algorithm. The experiments are conducted for the determination of amplitude and phase excitation patterns for the dual beam optimization with quantization. The best fitness is reported in Table 5. Form Table 5, we can find that the differential evolution can obtain better solutions for experiment I and experimental II. Especially, for the dual beam optimization with quantization, DE can perform better than G3-GA. The DE can obtain the value of 0.36 better than the G3-GA's value 0.618. By minimizing the dynamic ratio, we can find that the DE can provide the values of 4.7190 (ARD) and 0.16 (fitness) better than those of the G3-GA. This demonstrates differential evolution is better suitable to solve the dual beam optimization problem.

6. CONCLUSION

Application of differential evolution for the reconfigurable antenna array with quantized phase shifter is discussed in this paper. The effectiveness of the proposed algorithm is demonstrated on the design of a reconfigurable array antenna without and with the quantized phase excitations. The effect of the quantization in the continuous formulation of phased excitation is presented. In order to reduce the effect of mutual coupling between the antenna-array elements, the dynamic range ratio is minimized. Experimental results clearly indicate superior performance of differential evolution to the generalized generation gap model genetic algorithm (G3-GA). In this paper, we consider only the differential evolution, so our future work will focus on adding some other algorithms for this problem.

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