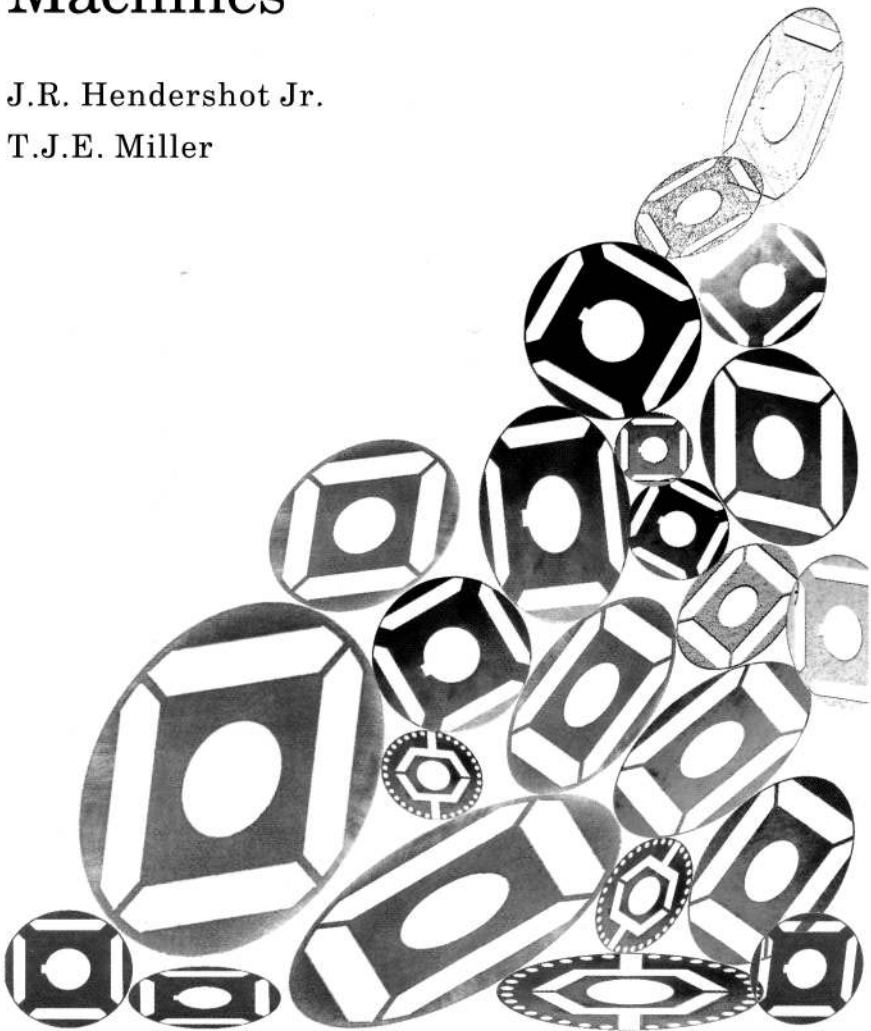


# Design of Brushless Permanent-Magnet Machines

J.R. Hendershot Jr.

T.J.E. Miller



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