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Design of Compliant Mechanisms: Applications to MEMS

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Abstract. Compliant mechanisms are single-piece flexible structures that deliver the desired motion by undergoing elastic deformation as opposed to jointed rigid body motions of conventional mechanisms. Compliance in design leads to jointless, no-assembly (Fig. 1), monolithic mechanical devices and is particularly suited for applications with small range of motions. The compliant windshield wiper shown in Fig. 1 illustrates this paradigm of no-assembly. Conventional flexural mechanisms employ flexural joints that connect relatively rigid links as depicted in Fig. 2. Reduced fatigue life, high stress concentration and difficulty in fabrication are some of the drawbacks of flexural joints. Our focus is on designing compliant mechanisms with *distributed compliance* which employs flexural links (see Fig. 3) and have no joints (neither pin nor flexural joints) for improved reliability, performance, and ease of manufacture. Distributed compliant mechanisms derive their flexibility due to topology and shape of the material continuum rather than concentrated flexion at few regions. This paper focuses on the unique methodology employed to design jointless mechanisms with distributed compliance. The paper also illustrates a compliant stroke amplification mechanism that was recently designed, fabricated and tested for MEMS application.

Key Words: MEMS, compliant mechanism, topology synthesis, size and shape synthesis

1. Introduction

Traditionally, engineered artifacts are designed to be strong and stiff. Designs in nature, on the other hand, are strong but not necessarily stiff—they are *compliant*. Compliance in design leads to creation of jointless, no-assembly, monolithic mechanical device [1]. Nature has realized the pivotal role that compliance plays at the realm of microorganisms, the level at which MEMS fit. Nearly 90 percent of living creatures are invertebrates and the percentage of invertebrates increases as we go down the dimension scale where compliant structures reign [1] .

Although simple deformable (compliant) structures such as beams and diaphragms have performed adequately in many micro devices, more sophisticated micromechanical functions can be realized by fully exploiting the preferred uses of elastic deformation via compliant mechanisms. Besides, the small scale and high aspect ratio of micromechanical structures makes them inherently flexible. Therefore, in MEMS, a compliant design that needs no assembly is not merely a prudent choice it is a necessity.

In this paper, we discuss a systematic method of design of compliant micro mechanisms and present a stroke-amplification mechanism for MEMS application as a design example. In particular, we present a brief overview of mathematical procedures employed for design of compliant mechanisms for

(i) Topology synthesis—which involves generation of a functional design in the form of a feasible topology starting from input/output force/motion specifications, and
(ii) Size and shape optimization—to meet performance requirements such as maximum stress, motion amplification or force amplification etc.

2. Design of Compliant Mechanisms

The first step in the design of a compliant mechanism is to establish a kinematically functional design that generates the desired output motion when subjected to prescribed input forces. This is called topological synthesis. Although the size and shape of individual

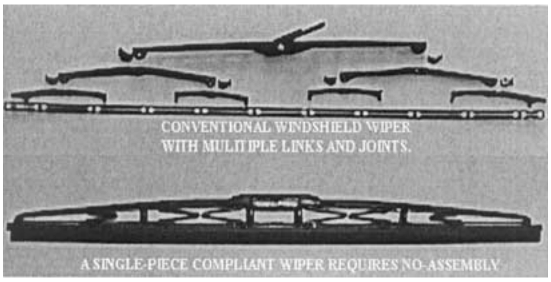

Fig. 1. A conventional rigid wiper blade compared to a compliant wiper blade. The compliant wiper blade can be manufactured in one single step, drastically reducing manufacturing costs.

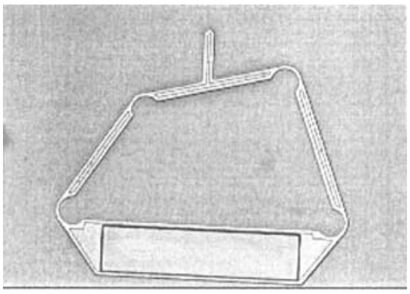

Fig. 2. A micro compliant four-bar mechanism with *lumped* compliance.

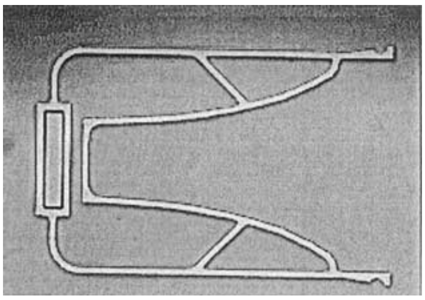

Fig. 3. Micro compliant crimping mechanism with *distributed* compliance.

elements can be optimized to a certain extent in this stage, local constraints such as stress and buckling constraints cannot be imposed while the topology is being determined. Once a feasible topology is established, performance constraints can be imposed during the following stage in which size and shape optimization are performed. Performance constraints may include minimizing the energy loss in the mechanism, obtaining desired motion amplification (geometric advantage) or force amplification (mechanical advantage), or ensuring that none of the elements buckle under the action of applied forces and external loads.

In this section, we briefly explain systematic methods of design of compliant mechanisms starting from functional specifications. First, we describe a method of deriving the topology (configuration) of a compliant mechanism given the desired input forces and output displacements. Next, we describe the method of optimizing the size and shape of various elements of a compliant mechanism in order to satisfy prescribed mechanical or geometric advantage, stress constraints, size constraints etc. The work reported in this paper assumes linear elastic models. Readers who are interested in geometric non-linearities due to large deformation, or dynamic performance of compliant mechanisms should contact the authors for on-going research on these topics.

2.1. Topology Synthesis

The goal of this first stage in compliant mechanism design is to establish a feasible topology (configuration) to meet prescribed input-output force-displacement relationship. Although multiple input forces and multiple output displacements can be prescribed, we will describe only a single input-output case for the sake of simplicity.

Given, a single-force-input and a single-displacement-output design specifications, first, we formulate an objective function that captures the need for (a) compliance to undergo desired deformation (kinematic requirement), and (b) stiffness to resist external loads (structural requirement) once the mechanism assumes the desired configuration. We then employ a formal structural optimization technique to *synthesize* a form, which is an optimal topology, shape and size of a compliant mechanism that performs the intended function [3,4,5].

Problem Formulation

As shown in Fig. 4, to satisfy both the kinematic and structural requirements in compliant mechanism synthesis, a two-part problem is posed. The first part, the “mechanism design,” is where the kinematic requirements are met by maximizing Geometric Advantage (GA) to generate desired kinematic motion under the action of applied input. This is achieved by maximizing

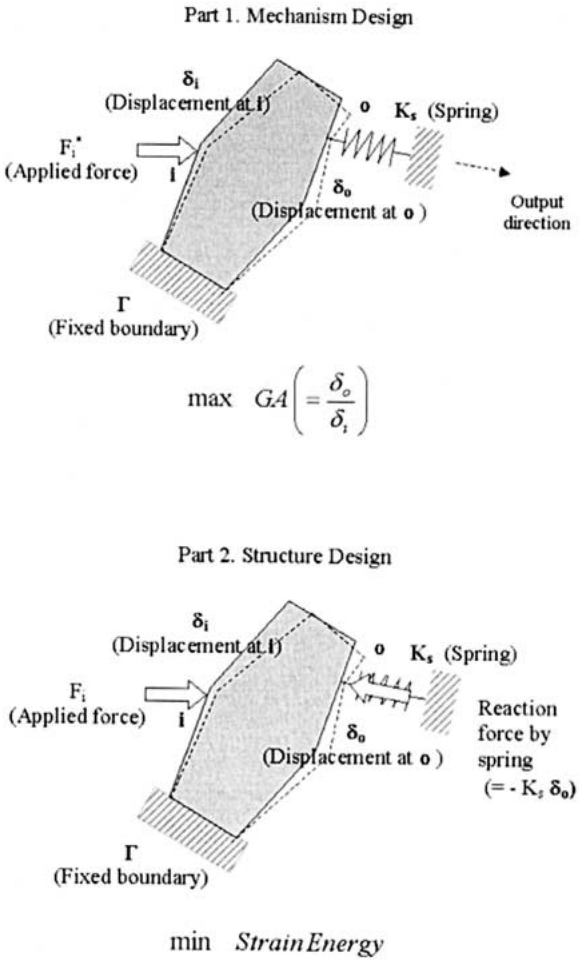

Fig. 4. Illustration of problem formulation [5].

the ratio of the output displacement ($= \delta_o$) over the input displacement ($= \delta_i$).

The second part of the two-part problem is the structure design, where the structural requirements are met by maximizing the stiffness. Maximizing the stiffness is equivalent to minimizing the strain energy. $[K]$ is the global stiffness matrix and cross sectional area of each links (beam elements) is the design variable. The combined problem is shown as equation (1).

Combined Problem

$$\begin{aligned} &\max \left(\frac{\textit{Geometrical Advantage}}{\textit{Strain Energy}} \right) \\ &\textit{subject to} \quad [K]\{d\} = \{f\} \\ &\qquad\qquad\quad \textit{Volume} \leq \textit{Volume}^* \\ &\qquad\qquad\quad \textit{Area}_{lower} \leq \textit{Area} \leq \textit{Area}_{upper} \end{aligned} \tag{1}$$

The mechanism design and structure design objectives are then combined into a single problem via multi-criteria optimization [3]. The design example, shown in Fig. 4, illustrates the ground structure approach using an array of beam elements. Reference [5] discusses an improved problem formulation and its implementation using beam elements.

An Array of Beam Elements

In this method, the prescribed design domain (this is the area within which the mechanism should fit) is first divided into a number of nodes. Each node is connected to several other nodes via modular array of *beam elements*. This serves as an initial guess. Certain nodes are "fixed" to imply the points where the mechanism is anchored to the substrate. The cross sectional area of each beam element serves as the design variable with specified upper and lower bounds. The resource constraint provides less material than the available space. The objective then is to distribute the material in a way that maximizes the objective function. During the optimization process, those beam elements whose cross sectional area reaches the lower bound are removed (deemed unnecessary) leaving only a network of beam elements whose area reached the upper bound. This defines the topology of the compliant mechanism.

The results of the automated synthesis method are illustrated by the following example of a compliant gripper mechanism (Fig. 5). The design specifications are that the applied force, F, causes the motion, D, at the indicated location, which will allow the device to grip some object at that point as shown in Fig. 5(a). The design domain shown in Fig. 5(a) represents the upper-half view since this is assumed to be a symmetric problem without any loss of generality in the solution procedure. The dashed line represents the desired space within which the mechanism should fit.

The initial guess is a modular beam structure (Fig. 5(b)) with a uniform distribution of cross-sectional areas. When the algorithm converges, the solution consists of beam members whose design variable reached (or is close to) the upper bound. The beam members whose design variables reached the lower bound constraint are eliminated. The optimized solution and corresponding finite element model are shown in Fig. 5(c), where the deformed shape is denoted by the dashed lines and the un-deformed shape is denoted by the solid lines. Compliant grippers based on this design were fabricated in nylon using a rapid prototyping machine

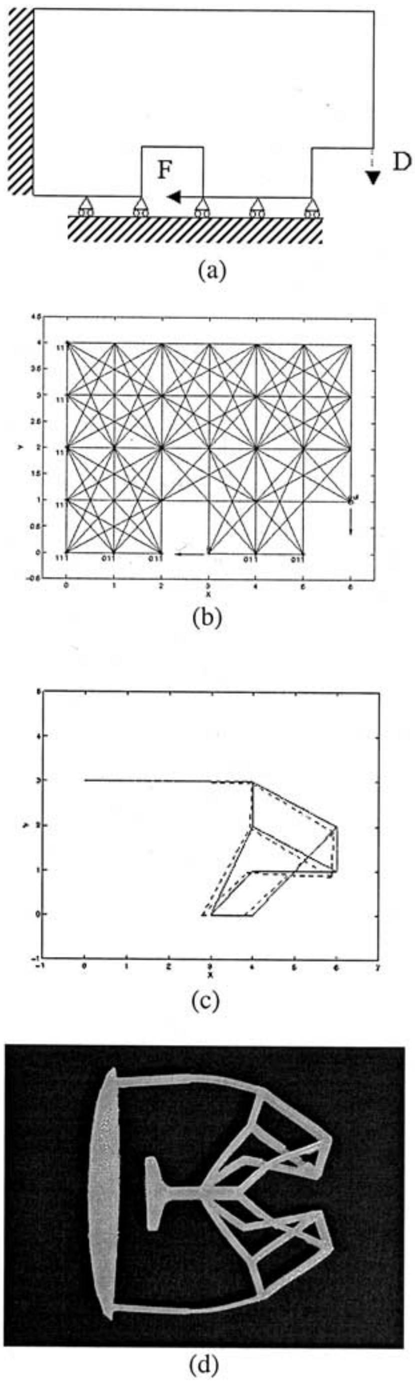

Fig. 5. Compliant design methodology [5]. From (a) or (d) are the steps for synthesizing a 2-D compliant gripper. (a) Define the design problem and desired forces. (b) Create an initial guess of what the structure may look like. (c) Obtain the solution through structure optimization (deformed shape shown with dashed lines). (d) Fabricate design. Note that only half of the structure is modeled, and the design is mirrored to create the physical device.

(Stratasys 3D Modeler). The methodology described here applies for three-dimensional problems as well as design problems with multiple sets of input/output force/displacements [4] requirements particularly for shape-change applications [6].

2.2. *Size and Shape Optimization*

Topology optimization provides qualitative results in that it provides a kinematically functional mechanism. It cannot provide a mechanism with prescribed performance characteristics. Therefore, once the topology is established, the next logical step is to perform a size and shape optimization.

In order to produce practical compliant mechanism designs, the following design criteria must be addressed: (i) required kinematic motion (both magnitude and direction), (ii) required stiffness to an external load, (iii) design space, and (iv) materials properties. (v) Stress limitations, (vi) buckling instabilities, (vi) dynamic considerations, and (v) weight limitations.

For structural optimization of compliant mechanisms, the stiffness of the mechanism must be quantified in order to achieve maximum performance. A compliant mechanism absorbs energy as the mechanism deforms.

Therefore, one way to quantify (and thereby optimize) the performance of a compliant mechanism is to maximize the energy efficiency. This forms the basis for determining the cross-sectional size, and shape of individual beam elements and the location of endpoints of each beam element (geometry).

Energy Efficiency Formulation

Considering a linear elastic body, work can be measured at both the input and output ports by assuming the following boundary conditions:

- The input is “actuated” by controlling the displacement of the input port on the body. The input displacement in effect controls the maximum range of travel of the mechanism.
- Work performed at the output is measured by applying an external resistive load which opposes the desired direction of the output port on the body. The external load can be considered to be a worst case load (on an average load) arising from the environment of the mechanism.

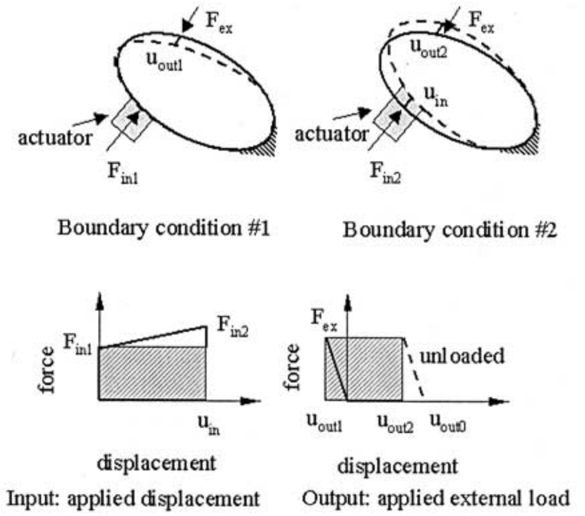

Fig. 6. Energy efficiency formulation used in size and shape optimization [6].

As shown in Fig. 6 above, these boundary conditions are applied in two separate stages. First, the external force is applied to the body while the input is held fixed. Second, the input is actuated a finite distance with the external load applied. Triangular regions in figure above at the input and output illustrate the energy absorbed due to loading and flexure. The shaded areas at both ports represent the reciprocal work or a fixed kinematic relationship between the input and the output. The mechanical efficiency can be formulated as equation (2).

$$\eta_{efficiency} = \frac{F_{ex}\left(u_{out2} + \frac{1}{2}u_{out1}\right)}{\frac{u_{in}}{2}(F_{in1} + F_{in2})} \tag{2}$$

Ref. [4] discusses implementation of the mechanical energy efficiency formulation as an objective function for size and geometry optimization. Alternately, spring formulation method can used to optimize the size and geometry of the mechanism. The goal then is to determine optimum size, shape and geometry of a compliant mechanism to satisfy desired input/output displacement, external loads acting against the mechanism, maximum stress, and geometrical advantage.

Fig. 7 shows a typical spring formulation specification. The input displacement is applied at point I of the mechanism body and the output motion is desired at point O. The external load acting against the mechanism is represented by a spring of stiffness Ks at the output port O. To set up the problem for optimization,

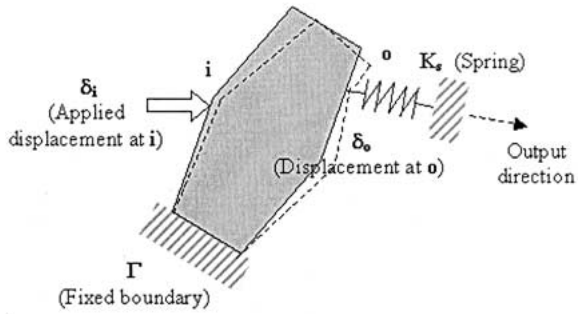

Fig. 7. Spring formulation specification.

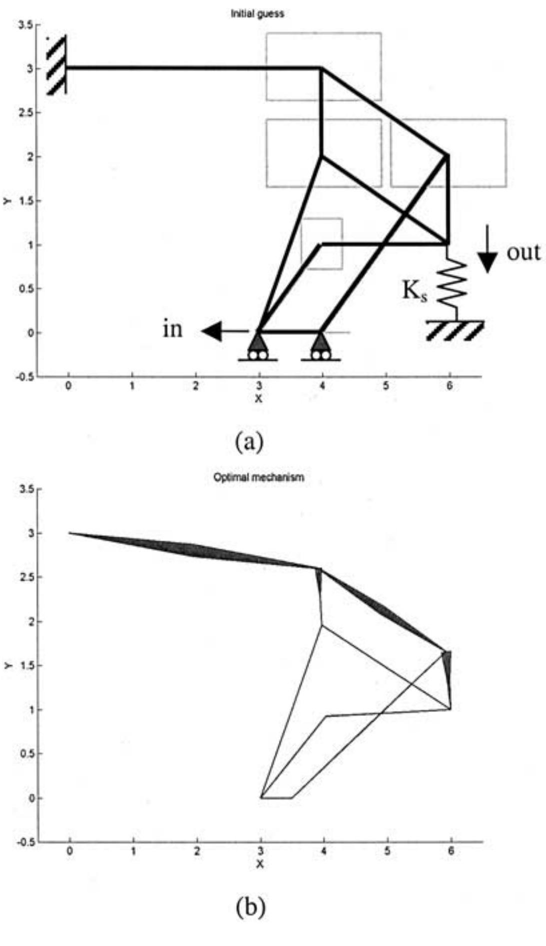

Fig. 8. Size/shape design methodology of compliant mechanism. (a) Take an optimal topology (Fig. 7) as an initial guess of size/shape synthesis. Then, define desired output, wandering range of node and boundary conditions. (b) Obtain the solution through size/shape optimization.

we use tapered beam elements to describe the size and shape of the compliant links. The cross sectional areas of links are the design variables. Additionally, we define a rectangular window around each node of the mechanism and allow the node to wander within the window until optimum location is determined. The objective function is defined as maximizing the geometric advantage of the mechanism. The complete formulation with constraints is described in equation (3).

$$
\begin{aligned}
&\max\ GA\left(= \frac{\delta_o}{\delta_i}\right) \\
&\textit{subject to}\quad Kd = f \\
&\qquad GA = GA^* \\
&\qquad \text{stress constraint} \\
&\qquad \Longrightarrow \begin{pmatrix} \textit{if tension} + \textit{bending} \geq 0 \\ \max(|\text{stress}^{top}|, |\text{stress}^{bottom}|) \leq \sigma_{yield} \\ \textit{if compression} + \textit{bending} < 0 \\ |\text{stress}| \leq \min(\textit{critical load}, \sigma_{yield}) \end{pmatrix} \\
&\qquad f_{in} \leq f^* \\
&\qquad V \leq V^* \\
&\qquad A_{lower} \leq A \leq A_{upper} \qquad (3)
\end{aligned}
$$

3. MEMS Multiplier

As an application of the design methodology described above, we present the design of a stroke-multiplier for MEMS application. Manufactured in Sandia's advanced 5-level surface micromachining technology known as SUMMiT-V [7,8], these computer generated structures provide high work and area efficiency in designs that are highly compatible with the fabrication process (see Fig. 9). The actual devices display outstanding yield, robustness, endurance, and resistance to surface adhesion effects during the final release process. One device has been driven to a 20-μm output displacement at resonance for more than 10^{10} cycles with no apparent fatigue. Even though the SUMMiT-V process provides for batch fabrication of jointed micromechanisms [7], compliant structures can offer superior alternatives in cases where joint play becomes an issue.

For the MEMS amplifier design described here, prescribed performance requirements included (1) desired geometric advantage of 12, (2) maximum permissible stress, (3) minimum in-plane beam-width of 1 μm, and

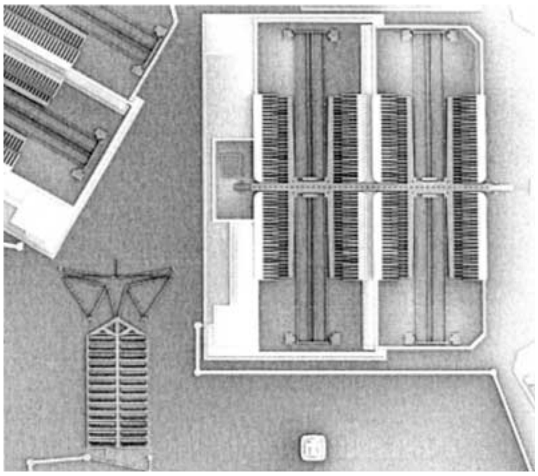

Fig. 9. Compliant based actuation system (short stroke comb drive with stroke amplifier) is considerably smaller than the comb drives currently used (patents pending).

(4) The energy efficiency of the final design is 72.9%. The mechanism amplifies the input motion by a factor of twelve; i.e. the mechanism is said to have a Geometric Advantage (G.A) of 12. Fig. 9 shows an electrostatic comb-drive actuator integrated with a compliant mechanism.

The primary driver for this work was the need to create core actuation components that are much smaller than existing devices, thus saving valuable die area and considerably reducing manufacturing costs. This size reduction will also allow more complex microelectromechanical systems to be realized. A paper presented by one of the collaborators at Transducers '99 [9] highlighted the level of complexity that has already been achieved with 5-level surface micromachining. Not highlighted was the fact that the most complex of the systems presented already requires 2/3 of the available die width, therefore greater complexity cannot be achieved without making the micro-components even smaller (Figs. 10 and 11).

Dynamic Analysis

The energy efficiency method above is based on kinetostatic design specifications and does not consider the dynamic effects. Therefore, for high-speed applications, the dynamic analysis is necessary in order to predict the dynamic characteristics of the mechanism.

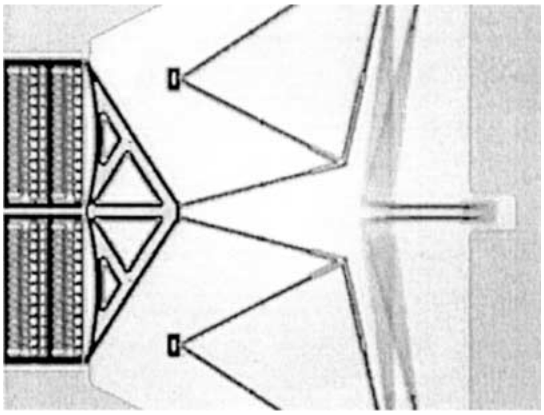

Fig. 10. Optical micrograph showing blur envelope created by the output beams of an actual device that is being driven at resonance. This combination has an overall system resonance of 26.9 kHz. Total output displacement is approximately 20 μm. Higher magnification is required to accurately determine input displacement (patents pending).

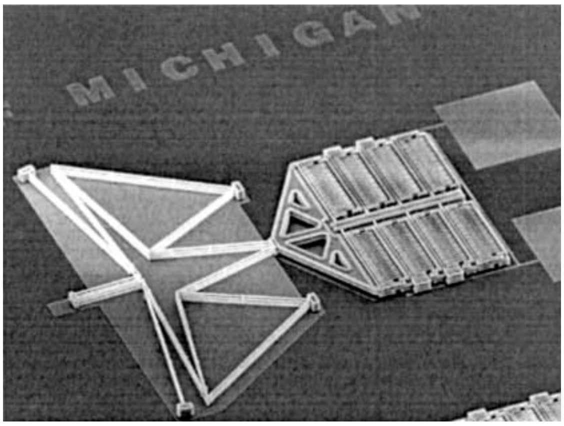

Fig. 11. The electrostatic actuator combined with the stroke multiplier increased the force per unit area by 220 times. Note the size of the actuator relative to the size of the bond pad (patents pending).

The first four natural frequencies of the stroke amplification compliant shown in Fig. 11 mechanism are:

$$f_1 = 3883.24 \text{ (Hertz)},$$
$$f_2 = 124{,}030.12 \text{ (Hertz)},$$
$$f_3 = 155{,}498.66 \text{ (Hertz)},$$
$$f_4 = 182{,}115.04 \text{ (Hertz)}.$$

Fig. 12 shows the characteristic of geometric advantage (GA) of the compliant multiplier based on our dynamic simulation model. The geometric advantage is the ratio

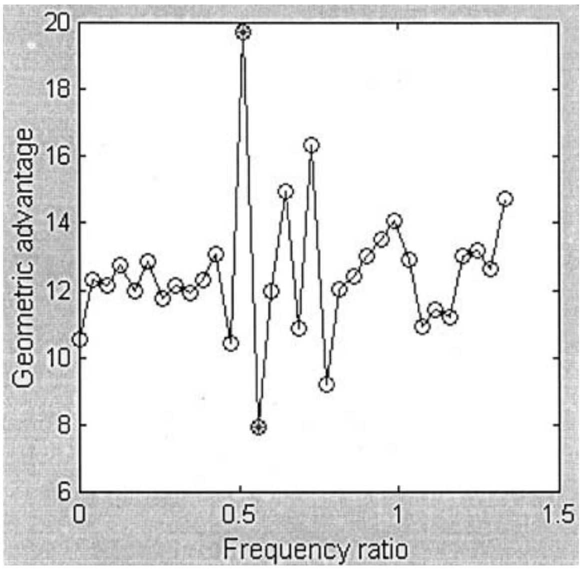

Fig. 12. Dynamic analysis of the stroke amplifier shows the geometric advantage as a function of the frequency ratio.

of the output to the input displacement. The static displacement of the output in this case is approximately 12 times the input displacement, i.e.; the GA is 12 at low frequency. The operating frequency of this mechanism is about 0.5 times the natural frequency, at which point the GA is nearly 20. More advanced designs are now in fabrication.

4. Conclusions

Complaint mechanisms play an important role in the design of micro mechanical structures for MEMS applications. These monolithic mechanical structures can be designed to perform complex mechanical functions and fabricated within the constraints of present day micromachining processes [10]. Based on linear elastic models, we have developed methods of synthesis of compliant mechanisms to meet kinematic and static stiffness requirements. Our future work includes nonlinearities due to large deformation, and dynamic aspects of micromechanical structures. The design methods developed to-date can generate micro mechanism designs for a variety of applications including, motion/force amplification, static shape change, and multiple input/output force-displacements.

Interested readers should refer to the following references for more details on various steps in the design synthesis methodology and different classes of problem formulations. Ref [3] provides multi-criteria formulation for topology synthesis and its implementation using truss ground structures. Ref [4] presents the size and shape optimization method using energy efficiency formulation including design of stroke amplification mechanisms. Ref [5] presents an improved problem formulation for topology synthesis and its implementation using beam elements. Ref [10] provides an overview of some of the early work on design of compliant mechanisms using homogenization method [11] and its applications to MEMS.

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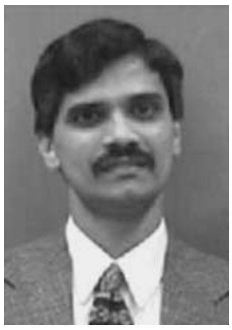

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Prior to MEMX, he was a Distinguished Member of Technical Staff at Sandia National Laboratories, where Dr. Sniegowski was the technical lead responsible directing research and development of MEMS technologies at Sandia's Microelectronics Development Laboratory. Dr. Sniegowski has authored over one hundred publications (book chapters, papers, and patents), and holds 13 MEMS-related patents. Dr. Sniegowski received his B.S., M.S., and Ph.D. in Nuclear Engineering and Engineering Physics from the University of Wisonsin-Madison in 1981, 1983, and 1991, respectively.