DESIGN OF COMPOSITE-MATERIAL PLATES FOR MAXIMUM UNIAXIAL COMPRESSIVE BUCKLING LOAD

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Using the theory for buckling of laminated, anisotropic, thin plates presented by Whitney and Leissa, optimal designs for simply supported rectangular plates, laminated of composite material and subjected to uniaxial compressive loading, are investigated. Numerical results are presented for optimal-design plates laminated of glass/epoxy, boron/epoxy, and carbon/epoxy composite materials.

INTRODUCTION

A major potential advantage claimed for fibrous composite materials in structural applications is that the material can be "tailored" by proper orientation of the fibers in the various layers so as to optimize the desired structural behavior. Yet the very nature of fibrous composites with their anisotropic elastic behavior has kept such optimal tailoring beyond the reach of most design engineers.

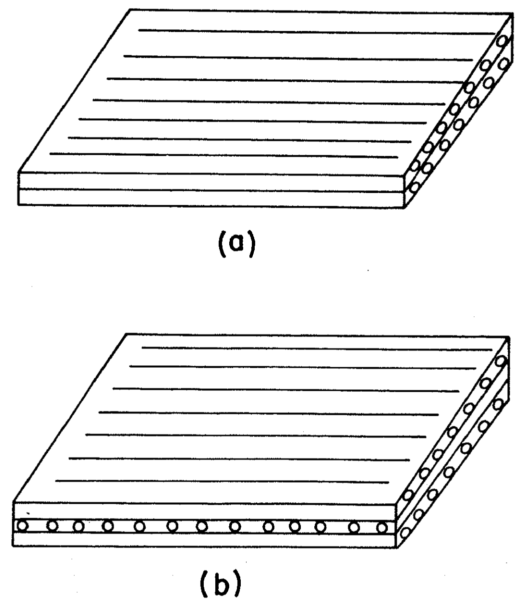

FIGURE 1. Two simple types of laminates: (a) parallel-ply (b) cross-ply.

In comparison to the numerous analyses available in the literature on buckling of anisotropic plates, very few optimal-design syntheses have been published for this problem. In 1960, Gerard (1) gave a synthesis for uniaxially compressed structurally orthotropic plates, but it was devoted only to longitudinally, transversely, or grid stiffened plates. For composite-material plates, Rothwell (2) presented a crude synthesis using netting analysis for the composite layers. Such an analysis completely neglects the contribution of the matrix material and is known to be in considerable disagreement with experimental results for laminated composite-material plates (3). Recently Hayashi (4) synthesized for maximum uniaxial buckling load in both symmetric cross-ply and symmetric angle-ply plates. However, his synthesis was based on the treatment of the number of axial half waves as a continuous rather than discrete variable.

The present synthesis considers both symmetric and unsymmetric laminates subjected to uniaxial compressive loading parallel to two of the plate edges.

GENERAL THEORY AND CLASSIFICATION OF SIMPLE LAMINATION TYPES

The theory used is that of Whitney and Leissa (5), which is generally recognized as the classical, linear, thin-plate theory for arbitrarily laminated anisotropic plates. Thus, both geometric and material nonlinearities, as well as thickness shear flexibility and thickness normal stress effects, are neglected. These simplifications are considered to be sufficiently accurate for most practical composite-material panels as pointed out recently by Ashton (6), for example.

The following lamination layup arrangements (see Figures 1 and 2) are most widely used and most simply analyzed:

1. Parallel-ply (all plies at the same arbitrary orientation θ
2. Unsymmetric cross ply (even number of plies oriented at θ and 90 degrees)
3. Symmetric cross ply (all plies oriented at either θ or 90 degrees and arranged symmetrically about the midplane)
4. Alternating balanced angle ply (even number of plies oriented alternately at +θ and -θ)
5. Symmetric balanced angle ply (even number of plies arranged symmetrically about the midplane, with an equal number of plies oriented at +θ and at -θ).

It can easily be shown that Layup 1 induces undesirable anisotropic in-plane shear coupling unless θ is either zero, as shown in Figure l (a), or 90 degrees.

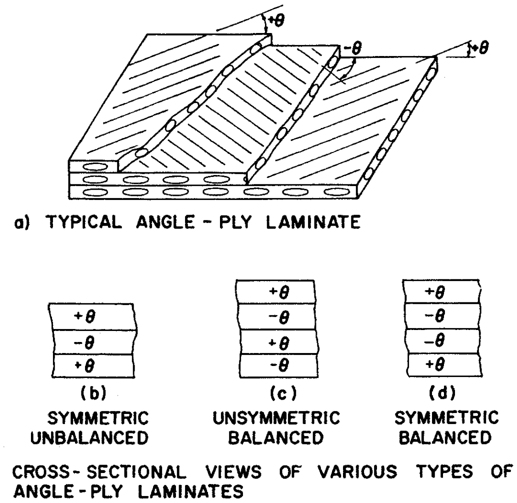

FIGURE 2. Angle-ply laminates.

BUCKLING UNDER UNIAXIAL COMPRESSION

The analysis of Jones (7) is applicable to calculation of the uniaxial buckling load of simply supported plates with Layups 1, 2 and 3. Using individual layer property data typical of unidirectionally reinforced layers of E-glass/epoxy (GFRP), boron/epoxy (BFRP) and carbon/epoxy (CFRP) as listed in Table 1, the dimensionless uniaxial-compression buckling loads tabulated in Table 2 were obtained by use of Jones' analysis. For comparative purposes, for isotropic material ($\nu = 1/4$), the dimensionless buckling load is 3.51 at m = 3. It

TABLE 1. *Individual-ply material properties.*

Property	Composite Material GFRP	BFRP	CFRP
Major Young's modulus, E_L (msi)[a]	7.8	30.0	30.0
Minor Young's modulus, E_T (msi)	2.6	3.0	0.75
Major Poisson's ratio, ν_{LT}[b]	0.25	0.30	0.25
Shear modulus, G_{LT} (msi)	1.3	1.0	0.375
Specific weight, γ (pci)	0.072	0.068	0.055

[a]The unit msi denotes millions of psi.
[b]It is assumed that the minor Poisson's ratio ν_{TL} is given by: $\nu_{TL} = (E_T/E_L)\,\nu_{LT}$

TABLE 2. *Compressive buckling loads for four-ply rectangular plates of parallel-ply and cross-ply layups.*[a]

Layup designation	Detailed angles (deg.)	Dimensionless buckling load[b] (m = no. of axial half waves) GFRP	BFRP	CFRP
1	0/0/0/0	5.08 (m = 2)	7.15 (m = 2)	13.1 (m = 1)
1	90/90/90/90	4.98 (m = 4)	6.89 (m = 5)	12.5 (m = 8)
2	0/0/90/90	4.70 (m = 3)	5.88 (m = 3)	12.6 (m = 3)
2	90/0/90/0	5.24 (m = 3)	9.51 (m = 3)	30.0 (m = 3)
3	90/0/0/90	5.23 (m = 4)	8.87 (m = 4)	25.9 (m = 5)
3	0/90/90/0	5.42 (m = 3)	8.84 (m = 2)	35.8 (m = 3)

[a]The plates have an aspect ratio, a/b, of 3 and are simply supported on all four edges.
[b]The dimensionless buckling is $K \equiv (\bar{N}_1)_{cr} b^2/E_T h^3$. In every case there is only one transverse half wave, *i.e.* n = 1.

is clear that the best orientation for Layup 1 is zero degrees, that considerable improvement over Layup 1 can be obtained by using the best Layup 2 (90 /0/90/0), and that the best layup of all three is the optimal Layup 3 (0/90/90/0).

The analysis of Whitney and Leissa (5), as corrected here in the Appendix, is applicable to Layups 4 and 5. Dimensionless uniaxial-compression buckling loads, using individual-ply properties from Table 1 and the Appendix analysis, are tabulated in Table 3. It is interesting to note that when the results are expressed in the dimensionless form used here, they are independent of the *number* of plies. It is clear that Layup 5 always gives a higher buckling load than Layup 4. Further, the Layup 5 results suggest that there is an optimal orientation angle θ in the vicinity of 45 degrees. More detailed calculations show that the optimal angle is exactly 45 degrees, thus confirming Hayashi's synthesis (4) for practical composites and plate geometry. This is an important result because in Hayashi's work the number of axial half waves was assumed to be continuous while in the present work it is permitted to remain at discrete integer values.

Another layup, originated by Werren and Norris (8) and generally known as quasi-isotropic, has three or more plies, each ply oriented by an angle of 180°/n from each adjacent ply, where n is the number of plies. Thus, a four-ply quasi-isotropic layup would have plies oriented at 0, 45, 90 and 135 degrees. Although its in-plane stiffness is isotropic, its bending stiffness is not and it exhibits bending-stretching coupling. It can be shown that the resulting buckling load for a quasi-isotropic plate is always lower than that for a symmetric 45-degree angle-ply plate made from the same composite material.

It is important to note that Layup 5 always gives a higher dimensionless buckling load than *any* other four-ply laminate considered (*i.e.* Layups 1-4 and quasi-isotropic). Thus, Layup 5, with θ = 45 degrees, can be considered to be *the* optimal four-ply laminates.

CONCLUDING REMARKS

For four-ply, simply supported rectangular plates having an aspect ration of 3 and subjected to uniaxial compressive loading, the following conclusions may be drawn:

1.The symmetric angle-ply design (Layup 5) with θ = 45 degrees was found by direct calculation to be the optimal design. This verifies the approximate synthesis presented by Hayashi (4).

2. Both the strength-weight figure of merit and the percentage increase in buckling load over that of the best parallel-ply design were ordered as follows: (a) CFRP, (b) BFRP, and (c) GFRP (Table 4).

TABLE 3. *Compressive buckling loads for rectangular plates of various angle-ply layups.*[a]

Layup designation	Detailed angles (deg.)	Dimensionless buckling load[b] (m = no. of axial half waves) GFRP	BFRP	CFRP
4	15/−15/15/−15	5.47 (m = 2)	8.97 (m = 2)	23.3 (m = 1)
	30/−30/30/−30	6.14 (m = 3)	13.4 (m = 2)	44.1 (m = 2)
	45/−45/45/−45	6.40 (m = 3)	16.0 (m = 3)	56.1 (m = 3)
	60/−60/60/−60	6.14 (m = 4)	13.4 (m = 4)	44.0 (m = 5)
	75/−75/75/−75	5.32 (m = 4)	8.82 (m = 5)	21.7 (m = 7)
5	15/—15/—15/15	5.55 (m = 2)	9.61 (m = 2)	26.1 (m = 1)
	30/−30/−30/30	6.40 (m = 3)	15.2 (m = 2)	52.7 (m = 2)
	45/−45/−45/45	6.72 (m = 3)	18.3 (m = 3)	67.5 (m = 3)
	60/−60/−60/60	6.37 (m = 4)	14.3 (m = 4)	52.6 (m = 5)
	75/−75/−75/75	5.41 (m = 4)	9.45 (m = 5)	24.6 (m = 7)

[a]The plates have an aspect ration, a/b, of 3 and are simply supported on all four edges.
[b]The dimensionless buckling load is $K \equiv (\overline{N}_1)_{cr} b^2/E_T h^3$. In every case there is only one transverse half wave, *i.e.* n = 1.

TABLE 4. *Strength-weight characteristics of optimal symmetric angle-ply rectangular plates (simply supported and with aspect ratio of 3).*

	GFRP	BFRP	CFRP
Strength-weight figure of merit K′ (in.)[a]	242 x 10⁶	805 x 10⁶	921 x 10⁶
Increase in buckling load over that of the best parallel-ply design of same plate dimensions, %	32.5	155	414

[a]Buckling load per unit weight $= (\overline{N}_1)_{cr}/\nu ah = (K'/a)(h/b)^2$, where K′ is the figure of merit given by $K' = KE_T/\gamma$.

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APPENDIX: EQUATIONS FOR BUCKLING LOADS OF ANGLE-PLY, SIMPLY SUPPORTED RECTANGULAR PLATES

The corrected versions of the pertinent equations of Ref. (5) are

$$\begin{aligned}\bar{N}_1 = (\pi/ma)^2 \Big\{ & D_{11}m^4 + D_k m^2 n^2 R^2 + D_{22} n^4 R^4 \\ & - (1/J_6)\Big[J_4 m^2 (B_{16}m^2 + 3\,B_{26}n^2R^2) \\ & + J_5 n^2 R^2\,(3\,B_{16}m^2 + B_{26}n^2R^2)\Big]\Big\}\end{aligned} \qquad \textbf{Eq. A-1}$$

where

$$\begin{aligned}J_4 \equiv\; & \left(A_{11}m^2 + A_{66}n^2R^2\right)\left(B_{16}m^2 + 3\,B_{26}n^2R^2\right) \\ & - \left(A_{12} + A_{66}\right)\left(3B_{16}m^2 + B_{26}n^2R^2\right) n^2R^2\end{aligned}$$

$$\begin{aligned}J_5 \equiv\; & \left(3\,B_{16}m^2 + B_{26}n^2R^2\right)\left(A_{66}m^2 + A_{22}n^2R^2\right) \\ & - \left(A_{12} + A_{66}\right)\left(B_{16}m^2 + 3\,B_{26}n^2R^2\right) m^2\end{aligned}$$

$$\begin{aligned}J_6 \equiv\; & \left(A_{11}m^2 + A_{66}n^2R^2\right)\left(A_{66}m^2 + A_{22}n^2R^2\right) \\ & - \left(A_{12} + A_{66}\right)^2 m^2\, n^2\, R^2\end{aligned}$$

$$D_k \equiv 2\left(D_{12} + 2\,D_{66}\right) \quad ; \quad R \equiv a/b$$

$$\left[A_{ij},\, B_{ij},\, D_{ij}\right] \equiv \int_{-h/2}^{h/2} \left[1,\, z,\, z^2\right] Q_{ij}(z)\, dz$$

m, n = axial and transverse half-wave numbers