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# Design of Fractional Particle Swarm Optimization Gravitational Search Algorithm for Optimal Reactive Power Dispatch Problems

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**ABSTRACT** In fact, optimal RPD is one of the most critical optimization matters related to electrical power stability and operation. The minimization of overall real power losses is obtained by adjusting the power systems control variables, for instance; generator voltage, compensated reactive power and tap changing of the transformer. In this search, a new heuristic computing method named as fractional particle swarm optimization gravitational search algorithm (FPSOGSA) is presented by introducing fractional derivative of velocity term in standard optimization mechanism. The designed FPSOGSA is implemented for the optimal RPD problems with IEEE-30 and IEEE-57 standards by attaining the near finest outcome sets of control variables along with minimization of two fitness objectives; active power transmission line losses ( $P_{loss}$ , MW) and voltage deviation ( $V_D$ ). The superior performance of the proposed FPSOGSA is verified for both single and multiple runs through comparative study with state of art counterparts for each scenario of optimal RPD problems.

**INDEX TERMS** Optimal power flow (OPF), optimal reactive power dispatch (ORPD), particle swarm optimization (PSO), gravitational search algorithm (GSA), fractional calculus (FC).

## NOMENCLATURE

$F_1, F_2$	Objective Functions
$P_{loss}$	Transmission line losses (MW)
$G_{k(ij)}$	Transfer conductance of k branch
$V_i, V_j$	Voltage magnitudes
$\delta_i, \delta_j$	Voltage angles at ith and jth bus
$\delta_{ij}$	Difference in Voltage Angle between i-th and j-th bus
$P_{PD}^i, Q_{PD}^i$	Active and Reactive power demand
$P_{PG}^i, Q_{PG}^i$	Active and Reactive power generation
$P_{Vi}, P_{Ti}, P_{Qi}$	Penalty Multipliers for bus Voltages, Transformers Tap settings and Reactive power violations

$G_{ij}, B_{ij}$	Transfer conductance & Susceptance of i-th and j-th bus
$P_{GE,i}^{\min}, Q_{GE,i}^{\min}$	Minimum Active and Reactive power generation
$P_{GE,i}^{\max}, Q_{GE,i}^{\max}$	Maximum active and reactive power generation
$V_i^{\min}, V_i^{\max}$	Minimum and Maximum bounds of generator bus voltages
$T_i^{\min}, T_i^{\max}$	Minimum and Maximum limit of transformer tap settings
$Q_{Ci}^{\min}, Q_{Ci}^{\max}$	Minimum and Maximum limit of Shunt VAR compensators
$N_C, N_{GE}$	Number of Shunt VAR compensators, Generators
$N_{TF}, N_l$	Number of Transformers, Load Buses
R	Number of branches (Transmission Lines) in the network
$V_{lp}$	Bus Voltage of the p-th load bus

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$F_{ij}^{\text{dim}}$	Total Forces from i-th to j-th agent with d-th dimension
$R_{ij}(t_i)$	Euclidian distance i-th and j-th agents at i-th iteration
$M_{ac,i}$	Active Mass of i-th agent
$M_{pv,i}$	Passive Mass of i-th agent
$M_{im,i}$	Inertial Mass of i-th agent
$\text{Iteration}_{\max,i}$	Maximum Iterations
$G_c$	Gravitational Constant

## I. INTRODUCTION

The electric power networks are intricate networks which consists of transmission, distribution and generation sub-systems aiming to operate with lowest consumption of resources while providing minimum losses, voltage deviation, operational cost, highest reliability and security [1]–[3]. These objectives are achieved in electric power networks by ORPD [4] which consist of tuning the operational variables, for instant; generator voltages, shunt reactive VAR compensators and tap settings of transformers changer while meeting the load demand. However, the optimal RPD is a complex problem due to nonlinear, multi modal and non-convex nature of optimization problem which contains discrete and continuous variables.

In last few years, myriad of numerical methods have been adopted to solve the ORPD problems such as minimization of power transmission losses ( $P_{loss}$ ) and voltage deviation ( $V_D$ ). We can refer to the classical optimization methods like as; quadratic programming, gradient-based approach, interior point, linear and non-linear programming [5]–[9]. However, these techniques have certain limitations such as premature convergence, trapping of local minima and complexity. Later, these shortcomings were overcome with the development of meta-heuristic algorithms which are widely used to solve the ORPD problems are discussed in [10]–[16].

A number of hybrid methodologies integrating a global and local search algorithm are presented to solve the optimal RPD problems. For instance, the PSO hybridized with DE, fuzzy logic, Pareto optimal set and GSA are the recently developed competitive hybrid strategies with ability to evade local trapping and premature convergence [17]–[20]. While, the new variant of PSO and other hybrid solution mechanisms by relating these concepts are studied in [21]–[27].

The traditional PSO algorithm is mostly suffered with the premature convergence problems and trapped into the local optima [16]. While, GSA usually requires a long computational time for some optimization problems to find the optimum solution [15]. PSO has a tendency to rapid convergence for resolving a multi-variable optimization problem while the GSA global exploration performance is predominantly conspicuous. Hence, both algorithms have their own perspectives and inspired us to develop an efficient hybridization technique of different meta-heuristic algorithms to overcome the weakness of the existed algorithms.

Afterwards, the development of fractional calculus (FC) has attracted the attentions of the research community and was applied in plethora of fields including engineering, fluid mechanics and computational mathematics [28]–[30]. Specifically, the concept of fractional calculus (FC) is exploited in metaheuristic evolutionary techniques and applied effectively in variety of applications such as the image processing, feature selection, design of discretized fractional-order filters, viscoelastic theory and stochastic fractal dynamics [31]–[35]. Moreover, FC has been a fertile field of research in science and engineering [36], [37]. In fact, various scientific areas are paying attention to implement the concept of FC while its adoption is recommended to different fields of science and engineering such as; electromagnetism, biology, electronics, robotics, signal processing, traffic systems, heat transfer, modeling and identification, telecommunications, irreversibility, physics, chemistry and control systems [38]–[41]. However, the fractional calculus-based optimization mechanisms have not yet been explored in field of energy and power sector, specifically in ORPD.

By inspiring the aforementioned ideas and further decreasing the drawbacks of both algorithms by using the concept of FC, a novel hybridization strategy integrates PSO and GSA including fractional properties into the internal structure of PSOGSA to make a novel meta-heuristic design of Fractional PSOGSA. The actual concept of alteration inside the mathematics of the algorithm to improve its characteristics such as convergence rate. We can refer the integration of fractional calculus (FC) concept inside the velocity update equation of the PSO, constituting fractional particle swarm optimization i.e. FPSO and is further hybridized with GSA to develop FPSOGSA.

In this research, the novel meta-heuristic design of FPSOGSA is used to solve the optimal RPD problems namely, minimization of power transmission losses and voltage deviation in IEEE standards such as IEEE-30 (13 and 19 variables) and IEEE-57 (25 variables). The FPSOGSA is designed to tune operational variables such as generators output voltage, transformers tap setting and shunt reactive VAR compensators within allowable limits to meet load demand. The salient features of this study are as follows:

- 1) A new fractional hybrid methodology namely FPSOGSA is designed to solve ORPD problems such as transmission line loss and voltage deviation minimization in the IEEE-30 and 57 bus system.
- 2) The improved performance of proposed FPSOGSA is demonstrated by comparing the yielded results with counter part algorithms reported in the literatures.
- 3) The effectiveness of FPSOGSA is ascertained through a detail statistical analysis in terms of minimum fitness evaluation in multiple autonomous trials, box plots, histograms and cumulative distribution function to endorse the stability, reliability and consistency of FPSOGSA.

4) Wide scale applications in sciences and engineering sector, simple design and reliability are other valued perks of proposed FPSOGSA [38]–[41].

In the research, the special tool of MATPOWER software [42] is used to find the two fitness objectives such as; minimization of transmission line losses (MW) and voltage deviation ( $V_D$ ). The utilization of MATPOWER applied here to ensures that detailed outcomes can be achieved by running the load flow analysis (LFA).

The rest of the paper is set as follows: Section.2, formulates the fitness objectives for optimal RPD (ORPD), Section.3, provides a detail overview of proposed FPSOGSA with graphical abstract, procedural steps or pseudocode, Section.4, is discussing the simulation outcomes and comparison, while Section.5, summarizes the conclusions.

## II. OPTIMAL RPD (ORPD) PROBLEM FORMULATION

### A. REAL/ACTIVE POWER LOSSES ( $P_{Loss}$ , MW)

The first fitness objective adopted is the real power losses minimization by tuning the control variables. The mathematical expression is given as follows.

$$F_1 = \sum_{r=1}^R G_{k(ij)} \left[ V_i^2 + V_j^2 - 2 \times V_i \times V_j \cos(\delta_i - \delta_j) \right] \quad (1)$$

#### 1) EQUALITY CONSTRAINTS

Usually, real and reactive power flow must be balanced during the operation of power system. It is equality constraints in ORPD and expressed as follows.

$$P_{PG}^i = P_{PD}^i + V_i \sum_{j=1}^N V_j \left[ G_{ij} \cos(\delta_{ij}) + B_{ij} \sin(\delta_{ij}) \right] \quad (2)$$

$$Q_{PG}^i = Q_{PD}^i + V_i \sum_{j=1}^N V_j \left[ G_{ij} \sin(\delta_{ij}) - B_{ij} \cos(\delta_{ij}) \right] \quad (3)$$

#### 2) INEQUALITY CONSTRAINTS

The inequality constraints include the voltages of the generator buses, shunt reactive VAR compensator rating, transformer tap setting and security limits associated with the electrical power networks.

##### a: GENERATOR CONSTRAINTS

$$P_{GE,i}^{\min} \leq P_{GE,i} \leq P_{GE,i}^{\max}, \quad i = 1, 2, \dots, N_{GE} \quad (4)$$

$$Q_{GE,i}^{\min} \leq Q_{GE,i} \leq Q_{GE,i}^{\max}, \quad i = 1, 2, \dots, N_{GE} \quad (5)$$

##### b: GENERATION BUS CONSTRAINTS

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \quad i = 1, 2, \dots, N_{GE} \quad (6)$$

##### c: TRANSFORMER TAP CONSTRAINTS

$$T_i^{\min} \leq T_i \leq T_i^{\max}, \quad i = 1, 2, \dots, N_{TF} \quad (7)$$

##### d: SHUNT COMPENSATOR CONSTRAINTS

$$Q_{Ci}^{\min} \leq Q_{Ci} \leq Q_{Ci}^{\max}, \quad i = 1, 2, \dots, N_C \quad (8)$$

The inequality constraints are restricted within their allowable limits by adding penalty factor in the fitness function. The penalty factor is generalized as follows [47]:

$$F_{Penalty} = F_{1,2} + P_{Vi} \sum (V_i - V_i^{\lim})^2 + P_{Ti} \sum (T_i - T_i^{\lim})^2 + P_{Qi} \sum (Q_i - Q_i^{\lim})^2 \quad (9)$$

where, the limits of  $V_i^{\lim}$ ,  $T_i^{\lim}$  and  $Q_i^{\lim}$  are as follows:

##### e: GENERATOR VOLTAGE

$$V_i^{\lim} = \begin{cases} V_i^{\min}, & V_i > V_i^{\max} \\ V_i^{\max}, & V_i < V_i^{\min} \end{cases} \quad i = 1, 2, \dots, N_{GE} \quad (10)$$

##### f: TRANSFORMER TAP CHANGER SETTINGS

$$T_i^{\lim} = \begin{cases} T_i^{\min}, & T_i > T_i^{\max} \\ T_i^{\max}, & T_i < T_i^{\min} \end{cases} \quad i = 1, 2, \dots, N_{TF} \quad (11)$$

##### g: REACTIVE POWER LIMITS

$$Q_i^{\lim} = \begin{cases} Q_i^{\min}, & Q_i > Q_i^{\max} \\ Q_i^{\max}, & Q_i < Q_i^{\min} \end{cases} \quad i = 1, 2, \dots, N_{GE} \quad (12)$$

### B. VOLTAGE DEVIATION ( $V_D$ )

The 2<sup>nd</sup> objective considered is the voltage deviation ( $V_D$ ), which is related to the voltage quality in the electrical power network and measured as sum of voltage deviation of load bus compared from the reference voltage i.e. 1 p.u. The voltage deviation is mathematically expressed as:

$$F_2 = \left( \sum_{p=1}^{Nl} |V_{lp} - 1| \right) \quad (13)$$

## III. METHODOLOGY

The proposed strategy is based on FPSOGSA to solve the optimal RPD (ORPD) problem in 30 bus with 13 and 19 control variables while in 57 bus with 25 control variables. The design approach is described in the following steps:

- A brief overview of PSO, GSA and FC.
- The pseudocode of the proposed FPSOGSA.
- The graphical illustration of overall workflow.

**Algorithm 1** Pseudocode of Designed FPSOGSA for Solving Optimal RPD Problem

**Inputs:** Set number of iterations, swarm size, set limits of control variable as in Table 4 and Load Case data on IEEE-30, IEEE 57 Standards.

**Output:** Minimization of power losses (1) and voltage deviation (15).

**Start FPSOGSA**

**Step 1: Initialization:** Randomly generated population with n particles

$$S = [V_1, V_2, \dots, V_{nV}, \quad T_1, T_2, \dots, T_{nT}, \quad Q_1, Q_2, \dots, Q_{nQ}]$$

Give I/p to each particle according to the IEEE Bus variable dimension

For each particle of the swarm

For the dimension based on control variables

Randomly initialize x and v with permissible real entries

End

The Swarm values are based on random generation within control Variables limits. Mathematically,  $i^{\text{th}}$  member of swarm is set as:

$$\text{Swarm}_{i,j}(0) = \text{Swarm}_j^L + \text{rand}(0, 1) \times (\text{Swarm}_j^U - \text{Swarm}_j^L)$$

Here, rand signifies random real numbers restraints 0 and 1.

**Step 2: Evaluate fitness** for every particle of Swarm using (1) and (15). While, in case of penalty count by (11) and run power flow.

**Step 3:** Stop the execution based on the following factors

a) Total number of iterations executed

b) Tolerance limit attains, i.e., Saturation

If termination criteria satisfy then go to step 5

**Step 4: Computing Parameters:** computing of GSA parameters by (22), (27) and (28).

**Step 5: Updating Velocity:** The velocity in FPSOGSA is updated by (39):

$$v_i^{t+1} = \alpha v_i^{k-1} + \frac{1}{2} \alpha v_i^{k-1} + \frac{1}{6} \alpha (1 - \alpha) v_i^{k-2} + \frac{1}{24} \alpha (1 - \alpha) (2 - \alpha) v_i^{k-3} + C_1 \times \text{rand}(0,1) \\ \times ac_i^t + C_2 \times \text{rand}(0,1) \times (G_{\text{BEST}} - x_i^t)$$

Update Gbest for each particle of swarm and go to Step 2.

**Step 6: Storage:** Save parameters of GBEST particle on basis of minimization of transmission power line losses (Ploss, MW) and voltage deviation (VD).

**Step 7: Analysis:** Repeat step 1 to step 5 for different values of fractional order alpha ( $\alpha$ ) in the algorithm for detailed analysis of the results.

**Step 8: Replication:** Repeat the steps 1 to 6 for IEEE 30 standard with 13 and 19 control variables, and IEEE 57 Standard with 25 control variables.

**End FPSOGSA**

**Statistics:** Repeat from step 1 to 7 for sufficient large number of trials to analyze the performance of FPSOGSA for optimal RPD.

**A. PARTICLE SWARM OPTIMIZATION (PSO)**

PSO is the swarm-based method that is initially expressed by Eberhart and Kennedy in 1995 [43]. This method is built on swarm intelligence in which each candidate solution is known as a particle and represented by two vectors  $x_i^{r+1}$  and  $v_i^{r+1}$ . In swarm, each particle updates its velocity and position based on local best and global best.

$$v_i^{r+1} = w \times v_i^r + C_1 \times r_1 \times (P_{\text{best}} - x_i^r) \\ + C_2 \times r_2 \times (G_{\text{best}} - x_i^r) \quad (14)$$

$$x_i^{r+1} = x_i^r + \chi \times v_i^{r+1} \quad (15)$$

Here,  $v_i^{r+1}$  is the velocity of i-th particle at iteration  $(r + 1)^{\text{th}}$ , w denotes the weight of inertia,  $v_i^r$  is the velocity of

i-th particle at the iteration  $r^{\text{th}}$ ,  $C_1$  and  $C_2$  are the coefficients for global best and personal best positions,  $r_1$  and  $r_2$  represents the randomly generated variables between  $[0, 1]$ ,  $P_{\text{best}}$  and  $G_{\text{best}}$  represents the local best and global best positions. The  $x_i^{r+1}$  represents i-th particle position at iteration  $(r + 1)^{\text{th}}$  and  $x_i^r$  represents i-th swarm position at iteration  $r^{\text{th}}$  while  $\chi$  is the constriction factor. While, the  $w_{\text{inertia}}$  provides better stability is defined as follows:

$$w_{\text{inertia}} = w_{\text{max},i} - \frac{w_{\text{max},i} - w_{\text{min},i}}{\text{Iteration}_{\text{max},i}} \times \text{Iteration} \quad (16)$$

Here,  $w_{\text{max},i}$  is the inertia value at the start of the iteration while  $w_{\text{min},i}$  is the inertia value at the end of the iterations.

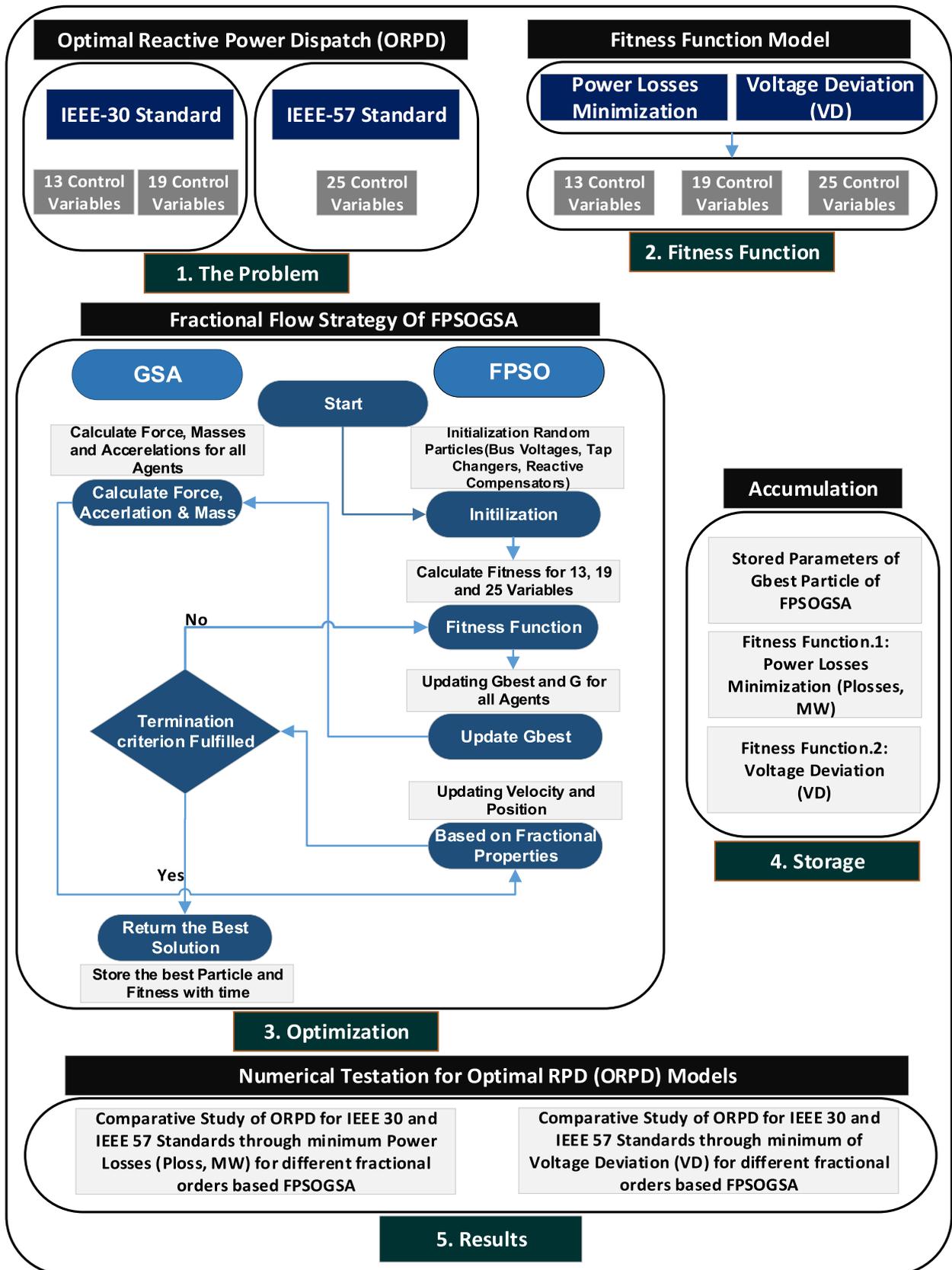


FIGURE 1. Graphical abstract of fractional FPSOGSA model for ORPD solution.

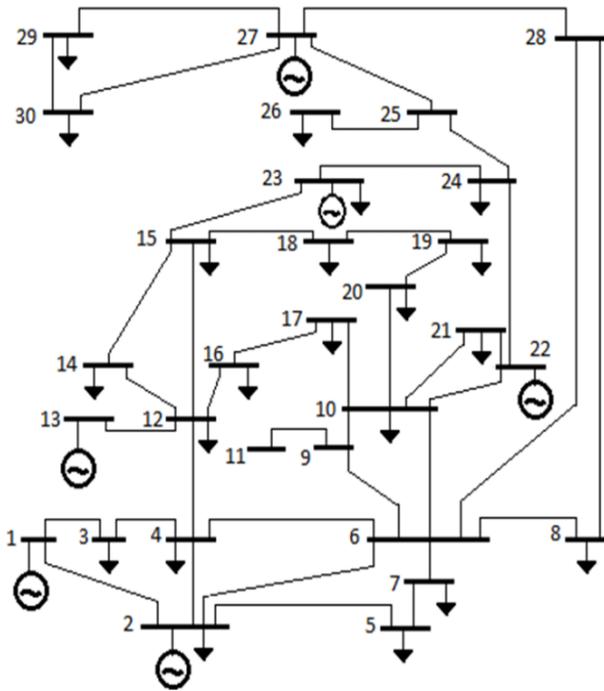


FIGURE 2. Single-Line diagram of IEEE 30 standard bus.

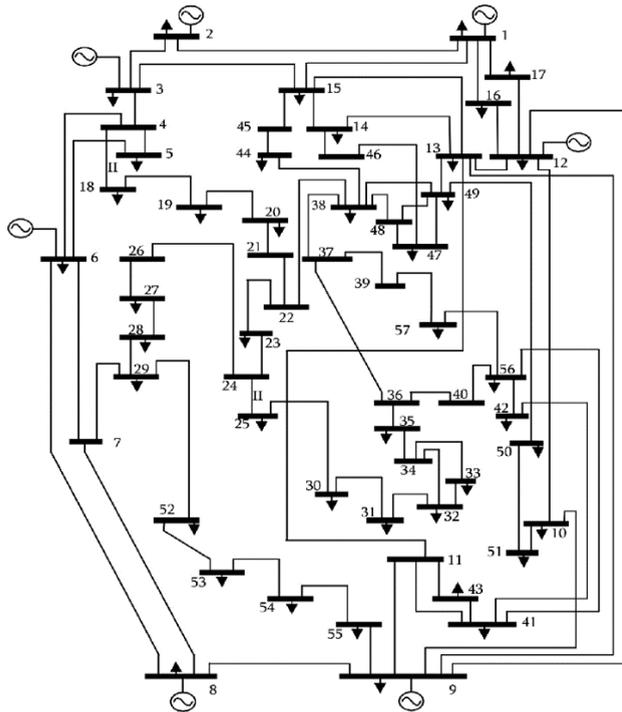


FIGURE 3. Single-Line diagram of IEEE 57 standard bus.

**B. GRAVITATIONAL SEARCH ALGORITHM (GSA)**

GSA is the novel nature inspired technique proposed by E. Rashedi in 2009 [44]. The basic concept of the traditional GSA, it is motivated from the Newton’s Law. This approach can be measured as the gathering of agents who have masses proportionate to the value of the fitness objective. The initial

**TABLE 1. Description IEEE 30 standard systems [58].**

Characteristic	IEEE 30
	13,19 Control Variables
No. of Buses	30
No. of Load Buses	24
No. of Generators	6
No. of Transformers	4
No. of Capacitors	9
No. of reactors	0
No. of branches	41
No. of control variables	13, 19

**TABLE 2. Description of IEEE 57 standard systems [58].**

Characteristic	IEEE 57
No. of Buses	57
No. of Load Buses	50
No. of Generators	7
No. of Transformers	15
No. of Capacitors	3
No. of reactors	0
No. of branches	80
No. of control variables	25

location of N number of agents in search space is given as follows:

$$X_i = (x_i^1 \dots x_i^{dim} \dots x_i^{no}) \quad \text{for } i = 1, 2 \dots N \quad (17)$$

Here,  $x_i^{dim}$  represents the i-th agent position in d<sup>th</sup> dimension while best/worst for every agent at every iteration is given as follows:

$$b_{best}(t_{it}) = \underset{j \in \{1, \dots, m\}}{\min} \text{fit}_j(t_{it}) \quad (18)$$

$$w_{worst}(t_{it}) = \underset{j \in \{1, \dots, m\}}{\max} \text{fit}_j(t_{it}) \quad (19)$$

Here,  $G_c$  which is computed at the iteration  $t_{it}$  is given as follows:

$$G_c(t_{it}) = G_e e^{\alpha t/T} \quad (20)$$

Here,  $G_c$  and  $\alpha$  are initialized in the start and reduced with time (t) to regulate the accuracy of GSA. The  $G_e$  is 1,  $\alpha$  is adjusted to 23, while T signifies the total iterations. The inertial and the gravitational masses are computed as follows.

$$M_{ac,i} = M_{pv,i} = M_{im,i} = M_i \quad i = 1, 2 \dots N \quad (21)$$

$$m_i(t_{it}) = \frac{\text{fit}_i(t_{it}) - w_{worst}(t_{it})}{b_{best}(t_{it}) - w_{worst}(t_{it})} \quad (22)$$

$$M_i(t_{it}) = \frac{m_i(t_{it})}{\sum_{j=1}^N m_j(t_{it})} \quad (23)$$

In a search space of d-th dimension, the total acting force on agent/particle ‘i’ is as follows:

$$F_i^{dim}(t_{it}) = \sum_{i=1, j \neq i}^N \text{rand}_j(t_{it}) \times F_{ij}^{dim}(t_{it}) \quad (24)$$

Here,  $F_i^{dim}$  represents the gravitational forces from j-th agent on i-th agent at the specific time t and is computed as

**TABLE 3. Parameter selection /settings of proposed FPSOGSA For IEEE 30 and IEEE 57 standards [47].**

Parameter	IEEE 30		IEEE 57
	13 (Variables)	19 (Variables)	25 (Variables)
Population Size	20	20	20
Iterations	50	50	50
Local Fractional Accerlation (alpha)	0.1-0.9	0.1-0.9	0.1-0.9
Global Fractional Accerlation (alpha)	0.1-0.9	0.1-0.9	0.1-0.9
Inertia	0.8-1.5	0.8-1.5	0.8-1.5
Best Fractional Order for $P_{loss}$	0.9	0.9	0.7
Best Fractional Order for $V_D$	0.9	0.7	0.2

**TABLE 4. Restraints of variables for IEEE 30 and IEEE 57 standards.**

IEEE	Variable	$V_G^{MAX}$	$V_G^{MIN}$	$T_I^{MIN}$	$T_I^{MAX}$	$Q_c^{MAX}$	$Q_c^{MIN}$
30	13	1.1	0.95	0.9	1.05	-30	30
	19	1.1	0.95	0.9	1.05	-30	30
57	25	1.1	0.95	0.9	1.1	-30	30

**TABLE 5. Comparison of control variables yielded by FPSOGSA For IEEE 30 bus (13 variables).**

Control Variables	IWO [48]	DE [11]	MICA-IWO [48]	C-PSO [49]	MFO [12]	GWO [14]	FODPSO [47]	FPSOGSA (Proposed)
$V_{GT1}$	1.06965	1.095319	1.07972	1.1000	1.1000	1.1	1.01	1.1000
$V_{GT2}$	1.06038	1.085946	1.07055	1.1000	1.0946	1.096149	1.04231	1.0945
$V_{GT5}$	1.03692	1.062628	1.04836	1.0747	1.0756	1.080036	1.0401	1.0752
$V_{GT8}$	1.03864	1.065076	1.04865	1.0867	1.772	1.080444	1.0956	1.0771
$V_{GT11}$	1.02973	1.0266	1.07518	1.1000	1.0868	1.093452	1.0110	1.1000
$V_{GT13}$	1.05574	1.014253	1.07072	1.1000	1.1000	1.1	1.0491	1.1000
$Tc_{6-9}$	1.05	1.017796	1.03	0.99	1.04110	1.04	1.0610	1.0417
$Tc_{6-10}$	0.96	0.979277	0.99	1.05	0.95007	0.95	0.9295	0.9000
$Tc_{4-12}$	0.97	0.9797843	1	0.99	0.95541	0.95	0.9665	0.9752
$Tc_{27-28}$	0.97	1.008938	0.98	0.96	0.95754	0.95	0.9555	0.9650
$QC3$	8	20.22359	-7	9.00	7.1032	12	8.4272	3.8974
$QC10$	35	9.584327	23	30.0	30.796	30	25.1542	4.7813
$QC24$	11	13.02992	12	8.00	9.8981	8	9.2331	4.7105
$P_{loss} (MW)$	4.92	4.888081	4.846	4.6801	4.5865	4.5984	4.606	4.5342
$VD (p.u)$	NR	NR	NR	NR	0.12154	0.12604	NR	0.1025

**TABLE 6. Comparison of percentage line losses reduction in IEEE-30 bus.**

Comparison	Based Case	IWO [48]	DE [11]	MICA-IWO [48]	C-PSO [49]	MFO [12]	GWO [14]	FODPSO [47]	FPSOGSA (Proposed)
Ploss, MW	5.663	4.92	4.888081	4.846	4.6801	4.5865	4.5984	4.606	4.5342
%	-	13.1202	13.6839	14.4270	17.3565	19.0093	18.7992	18.6650	19.9329

follows:

$$F_{ij}^{dim}(t_{it}) = G_c(t_{it}) \times \frac{M_{pv,i}(t_{it}) \times M_{ac,j}(t_{it})}{R_{ij}(t_{it}) + \epsilon} \left( x_j^{dim}(t_{it}) - x_i^{dim}(t_{it}) \right) \quad (25)$$

Here,  $G_c(t_{it})$  represents the computed gravitational constant for the similar iteration while  $\epsilon$  indicates a small constant. Conferring to the act of motion, the acceleration of an

agent/particle is as follows:

$$ac_i^{dim}(t_{it}) = \frac{F_i^{dim}}{M_{im,i}(t_{it})} \quad (26)$$

The new velocity and position are computed as follows:

$$v_i^t(t_{it} + 1) = rand_i \times v_i^{dim}(t_{it}) + ac_i^{dim}(t_{it}) \quad (27)$$

$$x_i^{dim}(t_{it} + 1) = x_i^t(t_{it}) + v_i^{dim}(t_{it} + 1) \quad (28)$$

In GSA, the optimizer starts with the initialization of all masses with random values [0,1] where every initialized mass

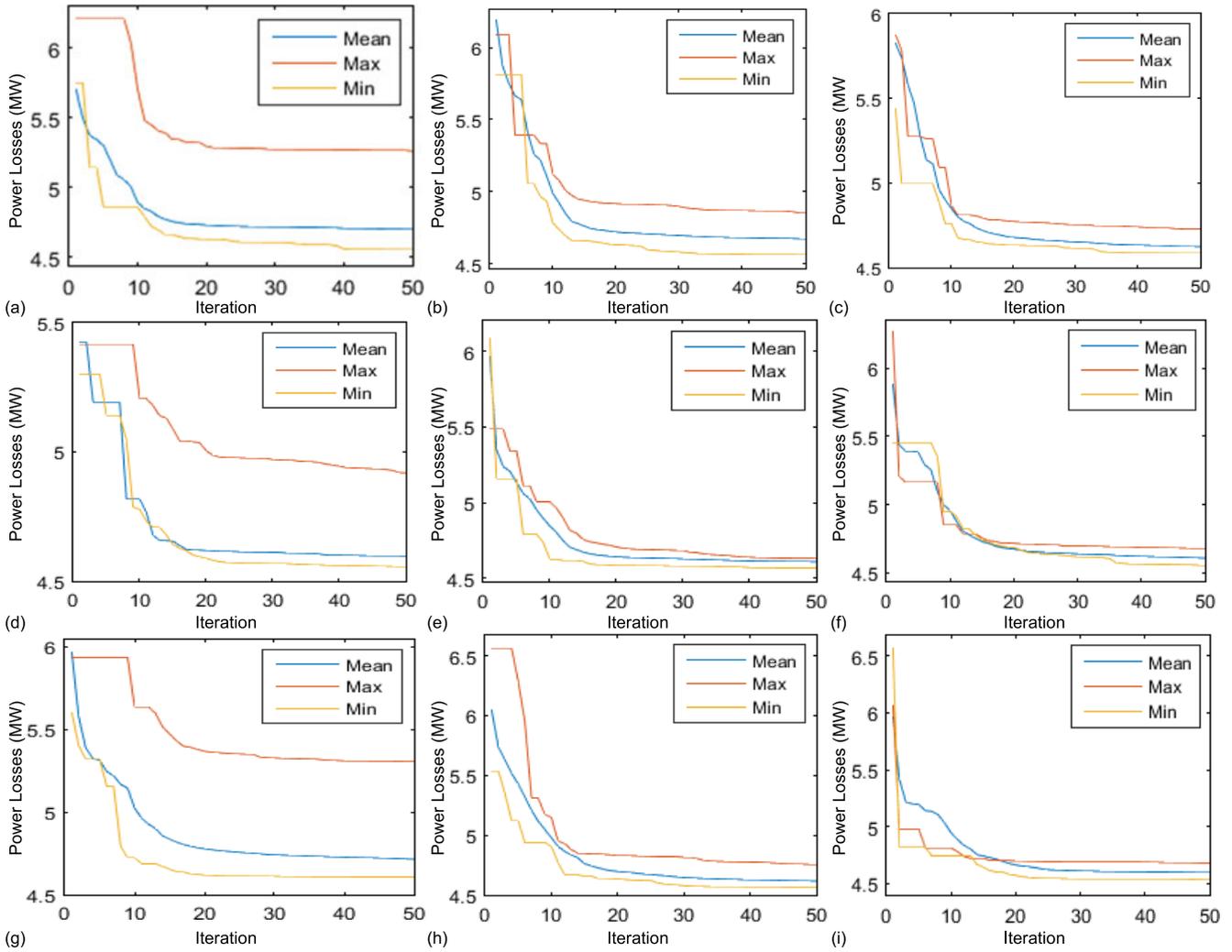


FIGURE 4. FPSOGSA convergence curve of power losses for IEEE30 standard (13 variables) at different fractional alpha orders ( $\alpha = [0.1 - 0.9]$ ).

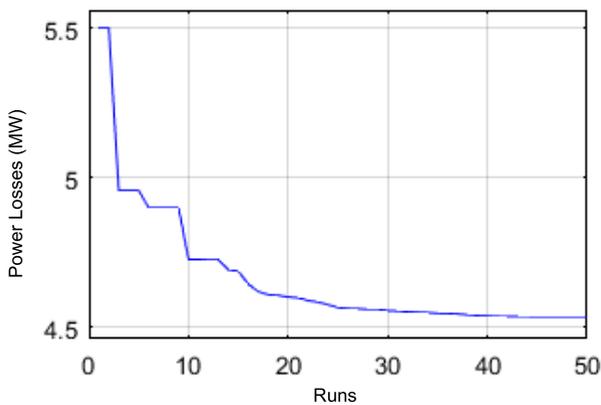


FIGURE 5. FPSOGSA approach to minimization of power losses on IEEE30 standard (13 variable).

is considered as an entrant solution. Then the velocities for the entire masses are computed by (27). Besides, the gravitational constant, resultant forces, and the accelerations are computed

by (20), (25), and (26), respectively, while, the position of the masses are computed by (28).

### C. FRACTIONAL CALCULUS (FC)

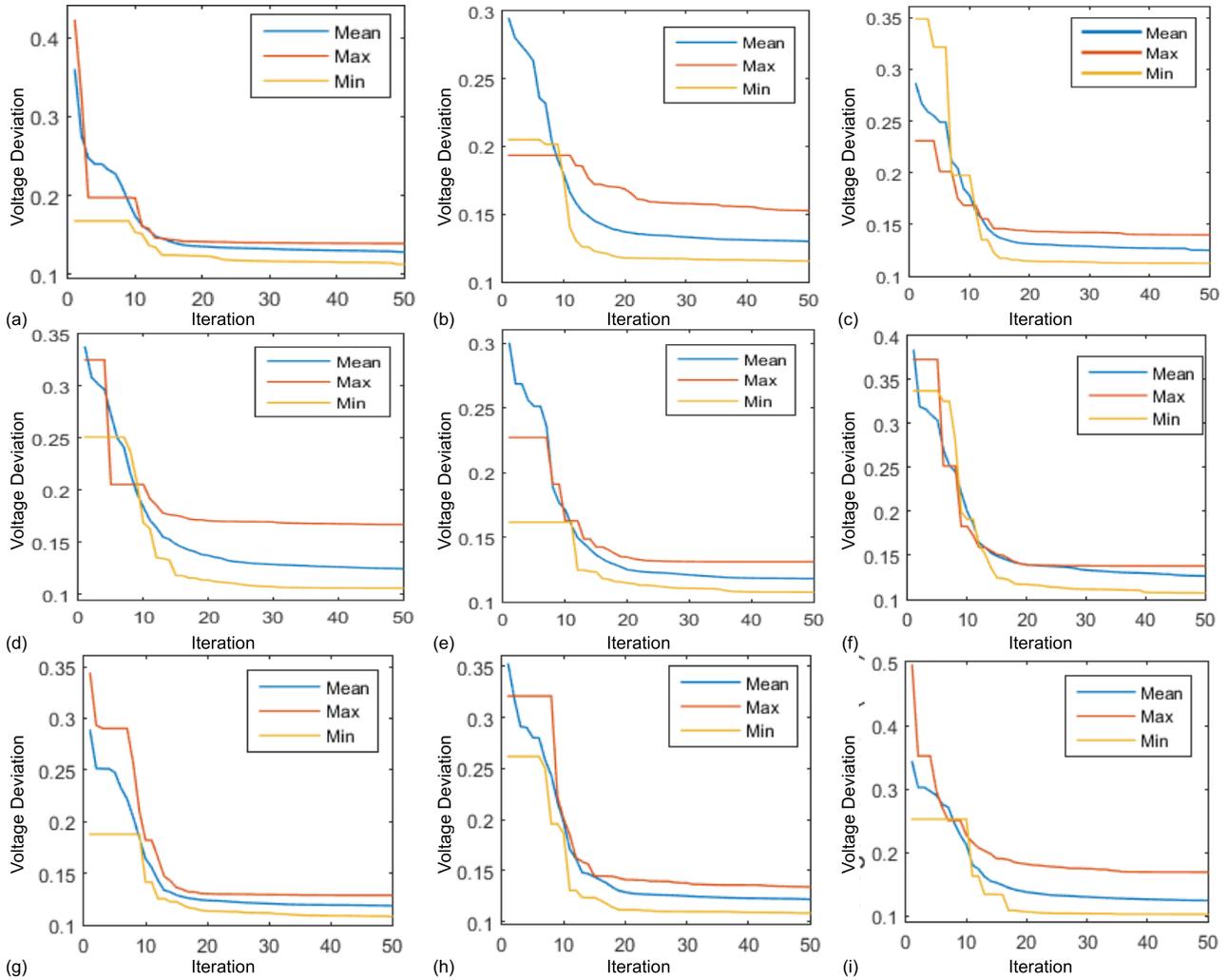
The concept of fractional calculus (FC) is an important mathematical tool for enhancing the performance of algorithms applied in filtering, modeling, pattern recognition, observability, controllability, curve fitting, edge detection, robustness stability, and identification [32]. In literature we find several different interpretations of FC. For instance, the Grünwald–Letnikov [45] interpretation of fractional differential with order  $\alpha \in \mathbb{C}$  for any signal  $x(t)$  is expressed by the following definition:

$$D^\alpha [x(t)] = \lim_{h \rightarrow 0} \left[ \frac{1}{h} \sum_{k=0}^{+\infty} \frac{(-1)^k \Gamma(\alpha + 1) x(t - kh)}{\Gamma(k + 1) \Gamma(\alpha - k + 1)} \right] \quad (29)$$

here,

$$\Gamma(k) = (k - 1)!, \quad (30)$$

defines the Euler gamma function.



**FIGURE 6.** FPSOGSA convergence curve of voltage deviation ( $V_D$ ) tested on IEEE30 Standard (13 variable) at different fractional orders ( $\alpha = [0.1 - 0.9]$ ).

A significant property of Grünwald–Letnikov is that the fractional order derivatives are needed number of infinite terms while a simple integer-order just implies a finite series. Therefore, the fractional derivatives have implicitly of memory effect for all past event which will be decreased over time. Due to inherent memory property of fractional calculus, make this model suited to describe the phenomena of irreversibility and chaos [60].

The discrete time interpolation of signal  $D^\alpha(x[t])$  is as follows [46].

$$D^\alpha(x[t]) = \frac{1}{T^\alpha} \sum_{k=0}^r \frac{(-1)^k \Gamma[\alpha + 1] x[t - kT]}{\Gamma(k + 1) \Gamma(\alpha - k + 1)} \quad (31)$$

Here,  $r$  and  $T$  are representing the truncation order and sampling period, respectively.

At first, the canonical velocity update expression (32) is reshuffled to amend the velocity derivative order, that is as:

$$v_i^{k+1} = v_i^k + r_1(p_{best,i} - x_i^k) + r_2(g_{best} - x_i^k) \quad (32)$$

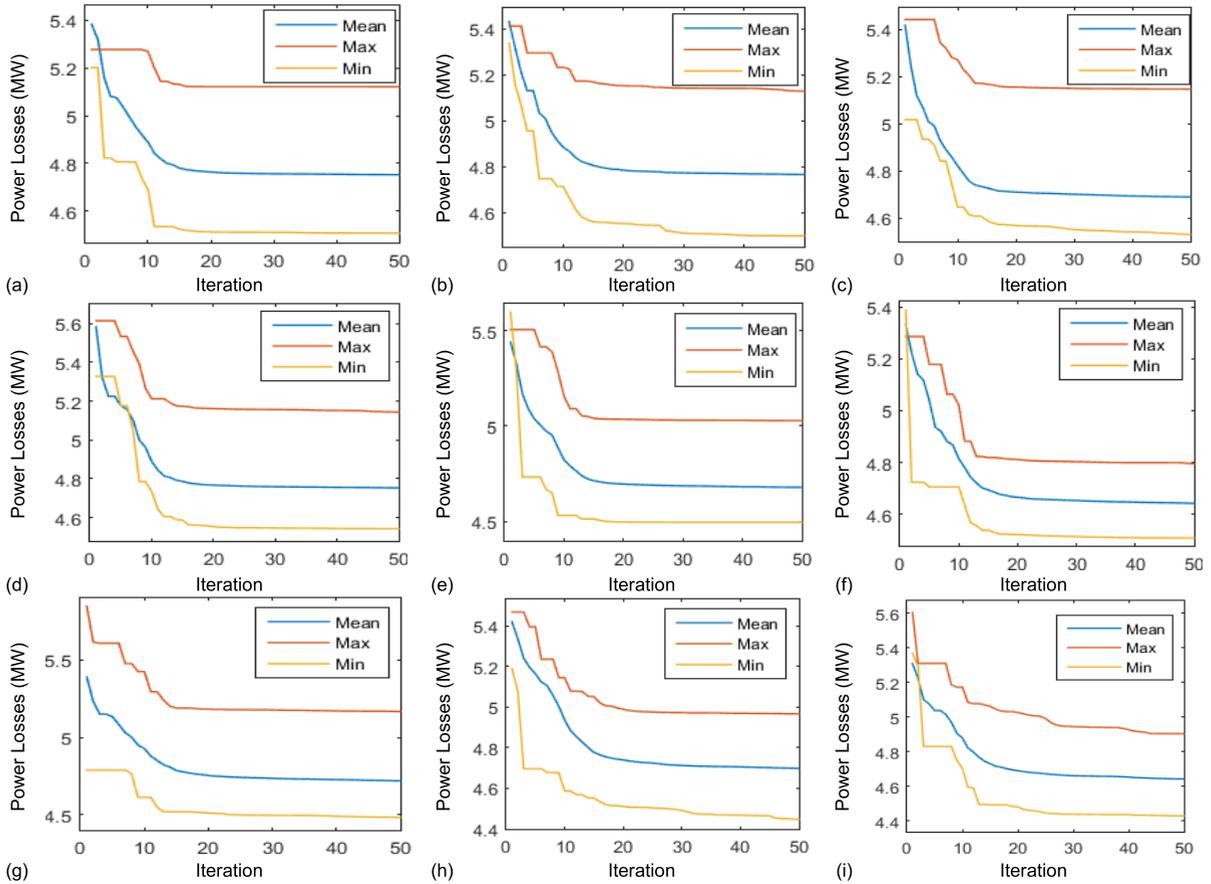
The equation can be redefined as:

$$v_i^{k+1} - v_i^k = r_1(p_{best,i} - x_i^k) + r_2(g_{best} - x_i^k) \quad (33)$$

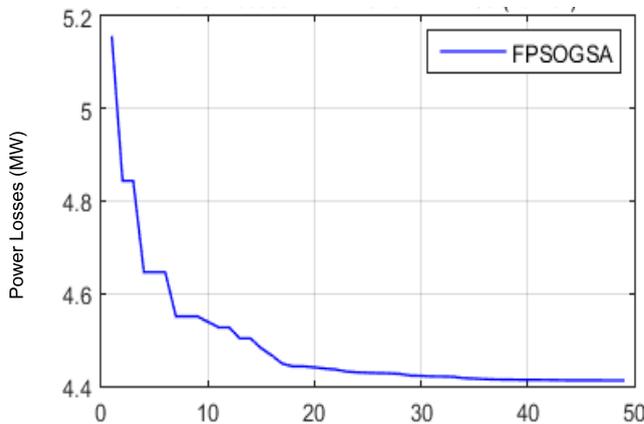
Considering  $T = 1$  in (31), the relation (34) can be rewritten as:

$$v_i^{k+1} = - \sum_{k=1}^r \frac{(-1)^k \Gamma[\alpha + 1] x[t - kT]}{\Gamma(k + 1) \Gamma(\alpha - k + 1)} + r_1(p_{best,i} - x_i^k) + r_2(g_{best} - x_i^k) \quad (34)$$

The order of the velocity derived can be approximated to a real number by restraints  $0 \leq \alpha \leq 1$ , if the fractional calculus perception is considered, an extended memory effect with leading to a smoother variation. To learning the behavior of this novel fractional optimization mechanism, a set of imitations are carried on testing the values of alpha ( $\alpha$ ) reaching between  $\alpha = 0$  to  $\alpha = 1$ , with incrementation of  $\Delta\alpha = 0$  to  $\alpha = 1$ , with increments of steps  $\Delta\alpha = 0.1$ .



**FIGURE 7.** FPSOGSA convergence curve for power losses tested on IEEE30 standard (19 variables) at different fractional orders ( $\alpha = [0.1 - 0.9]$ ).



**FIGURE 8.** FPSOGSA approach to minimization of power losses of IEEE30 standard (19 variables).

Consequently, using  $r = 4$  in (34), yields a new velocity update equation as:

$$\begin{aligned}
 v_i^{t+1} = & \alpha v_i^{k-1} - \frac{1}{2} \alpha v_i^{k-1} + \frac{1}{6} \alpha (1 - \alpha) v_i^{k-2} \\
 & + \frac{1}{24} \alpha (1 - \alpha) (2 - \alpha) v_i^{k-3} \\
 & + r_1 (p_{best_i} - x_i^k) + r_2 (g_{best} - x_i^k) \quad (35)
 \end{aligned}$$

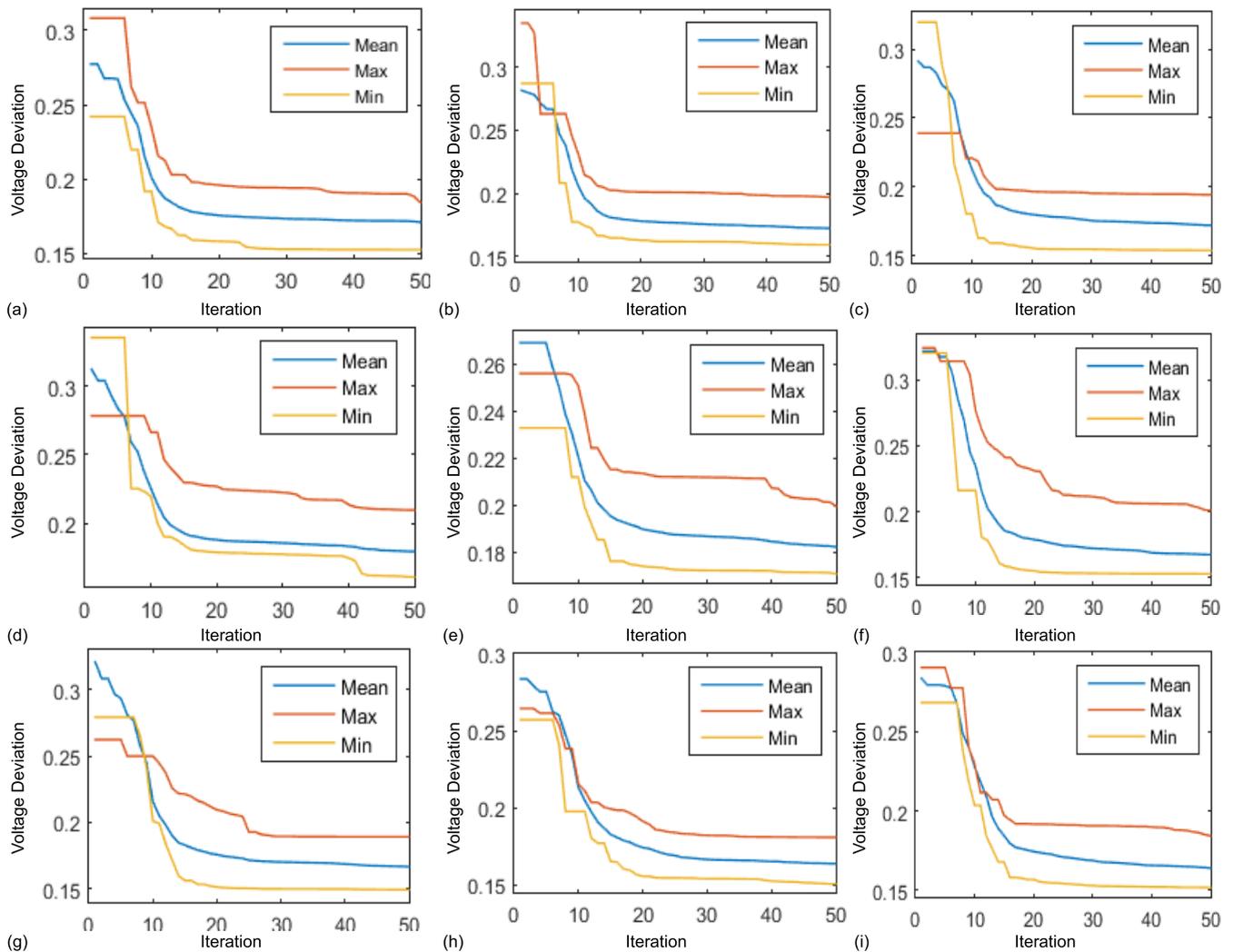
#### D. FRACTIONAL PARTICLE SWARM OPTIMIZATION GRAVITATIONAL SEARCH ALGORITHM (FPSOGSA)

In this section, a new mechanism to control the convergence rate of the PSO algorithm by incorporating the derived fractional velocity inside the mathematical model of algorithm is introduced and denoted as FPSOGSA. The newly designed co-evolutionary heterogeneous approach combines the optimization strength of both algorithms i.e., PSO and GSA, to increase the exploration while the fractional derivatives improves the convergence rate along the algorithm evolution. The PSO algorithm updated its velocity for every iteration is given as follows [20]:

$$\begin{aligned}
 v_i^{t+1} = & w_{inertia} \times v_i^t + C_1 \times rand_{(0,1)} \times ac_i^t \\
 & + C_2 \times rand_{(0,1)} \times (G_{BEST} - x_i^t) \quad (36)
 \end{aligned}$$

while, novel FPSOGSA algorithm updates its fractional velocity by using (37).

$$\begin{aligned}
 v_i^{t+1} = & \alpha v_i^{k-1} + \frac{1}{2} \alpha v_i^{k-1} + \frac{1}{6} \alpha (1 - \alpha) v_i^{k-2} \\
 & + \frac{1}{24} \alpha (1 - \alpha) (2 - \alpha) v_i^{k-3} \\
 & + C_1 \times rand_{(0,1)} \times ac_i^t \\
 & + C_2 \times rand_{(0,1)} \times (G_{BEST} - x_i^t) \quad (37)
 \end{aligned}$$



**FIGURE 9.** FPSOGSA convergence curve of voltage deviation ( $V_D$ ) for IEEE30 standard (19 variables) at different fractional orders ( $\alpha = [0.1 - 0.9]$ ).

here, the new position for FPSOGSA is updated as follows:

$$x_i^{t+1}(t + 1) = x_i^t + v_i^{t+1} \quad (38)$$

The procedural steps of proposed FPSOGSA are given in pseudocode in algorithm 1, while the overall workflow diagram is depicted in Fig. 1.

#### IV. RESULT AND DISCUSSION

The proposed strategy of FPSOGSA is tested on 6 different cases adopting minimization of the transmission line losses and voltage deviation ( $V_D$ ) as objectives of the ORPD in IEEE 30 (13, 19 control variables) and 57 (25 control variables) bus system. The single line diagrams of the IEEE 30 and 57 standard systems are depicted in Fig. 2 and 3, respectively, while the system description is provided in Table 1 and 2, respectively. The effectiveness of designed FPSOGSA is verified for the minimization of transmission line loss and voltage deviation with initial parameter settings documented in Table 3 while considering the following test systems.

- Test system 1: IEEE-30 bus with 13 control variables
- Test system 2: IEEE 30 bus with 19 control variables
- Test system 3: IEEE 57 bus with 25 control variables

The parameters of FPSOGSA i.e., velocity bounds, number of flights/Iterations, number of particles, size of swarm, inertia weight, social, cognitive acceleration vector and fractional coefficient are selected based on experience, knowledge of optimization problem, knowledge of the optimizer, experimentations, and extensive care.

It is necessary to mention that the selection of the parameters is a big challenging task not only for the proposed FPSOGSA approach but for all other meta-heuristic techniques as well. In this study, the selection of control parameters including the inertia weight, population size, iterations and fractional orders is performed through extensive trials and monitoring the best results.

The minimum and maximum restraints for the control variables such as the bus data, generator data and line data have been adapted from [47] for justified comparisons and is documented in Table 4.

**TABLE 7. Optimum Control Variables Setting OF THE IEEE30 bus (19 variables) for  $P_{loss}$  and  $V_D$ .**

Control Variable	Base Case	TS [51]	CLPSO [52]	WOA [10]	BBO [50]	MFO [12]	MSFS [53]	A-CSOS [55]	ALC-PSO [56]	PSOGSA [54]	FPSOGSA (Proposed)
$V_{GT1}$	1.05	1.0835	1.1000	1.1	1.1	1.1000	1.1000	1.1000	1.0500	1.1000	1.0100
$V_{GT2}$	1.04	1.0567	1.1000	1.0963	1.0944	1.0943	1.0939	1.09430	1.0384	1.0944	1.0066
$V_{GT5}$	1.01	1.0671	1.0795	1.0789	1.0749	1.0747	1.0739	1.07470	1.0108	1.0749	0.9903
$V_{GT8}$	1.01	1.0944	1.1000	1.0774	1.0768	1.0766	1.0764	1.07660	1.0210	1.0767	0.9883
$V_{GT11}$	1.05	0.9873	1.1000	1.0929	1.0999	1.1000	1.1000	1.10000	1.0500	1.1000	1.0100
$V_{GT13}$	1.05	1.0863	1.1000	0.9936	1.0999	1.1000	1.1000	1.10000	1.0500	1.1000	1.0099
$T_{C6-9}$	1.078	1.0745	0.9154	0.9867	1.0435	1.0433	1.0473	1.04320	0.9521	1.0452	1.0285
$T_{C6-10}$	1.069	0.9960	0.9000	1.0214	0.90117	0.9000	0.9000	0.90000	1.0299	0.9000	0.9030
$T_{C4-12}$	1.032	0.9678	0.9000	0.9867	0.98244	0.97912	0.9790	0.97905	0.9721	0.9794	0.9283
$T_{C27-28}$	1.068	1.0267	0.9397	3.1695	0.96918	0.96474	0.9634	0.96472	0.9657	0.9651	0.9182
$Q_{C10}$	0	1.4600	4.9265	3.1695	4.9998	5.0000	5.0000	5.00000	0.9000	5.0000	29.9905
$Q_{C12}$	0	3.7600	5.0000	2.0477	4.9870	5.0000	5.0000	5.00000	1.2600	5.0000	23.8555
$Q_{C15}$	0	0.0000	5.0000	4.2956	4.9906	4.8055	5.0000	4.80690	2.0900	5.0000	13.2088
$Q_{C17}$	0	3.3500	5.0000	2.6782	4.9970	5.0000	5.0000	4.99990	5.0000	5.0000	14.3632
$Q_{C20}$	0	0.1900	5.0000	4.8116	4.9901	4.0263	5.0000	4.03010	0.3100	3.9792	0.2464
$Q_{C21}$	0	2.4200	5.0000	4.8163	4.9946	5.0000	4.9990	5.00000	2.9300	5.0000	0.0683
$Q_{C23}$	0	3.0700	5.0000	3.5739	3.8753	2.5193	2.3155	2.51700	2.2600	2.4583	0.3625
$Q_{C24}$	0	2.9400	5.0000	4.1953	4.9867	5.0000	5.0000	5.00000	5.0000	5.0000	8.3715
$Q_{C29}$	0	3.9900	5.0000	2.0009	2.9098	2.1925	2.0180	2.19760	1.0700	2.1865	0.1874
$P_{loss}, MW$	5.811	4.9203	4.5615	4.5943	4.5435	4.5128	4.5143	4.51279	4.4793	4.5309	4.4121
$V_D, pu$	1.1501	0.1540	0.4773	NR	2.0662	2.0316	NR	2.05630	NR	2.0504	0.1468

**TABLE 8. Reduction percentage of losses minimization of IEEE30 standard (19 control variables).**

Comparison	TS [51]	CLPSO [52]	WOA [10]	BBO [50]	MFO [12]	MSFS [53]	A-CSOS [55]	ALC-PSO [56]	PSOGSA [54]	FPSOGSA (Proposed)
Power Loss	4.9203	4.5615	4.5943	4.5435	4.5128	4.5143	4.51279	4.4793	4.5309	4.4121
%	15.3278	21.5023	20.9379	21.8121	22.3404	22.3145	22.3406	22.9168	22.0289	24.0733

**A. TEST SYSTEM 1: IEEE 30 BUS (13 VARIABLES)**

This system contains 6 generator ( $V_{GT}$ ) units on bus 1, 2, 5, 8, 11, 13, four taps changing transformer ( $T_c$ ) at line number 6-9, 6-10, 4-12, 27-28 and three shunt reactive VAR compensators are connected to the bus 3,10 and 24 while the active and reactive power demand is  $P_{load} = 2.832pu$  and  $Q_{load} = 1.262pu$  respectively [48].

**1) POWER LOSSES MINIMIZATION AT DIFFERENT FRACTIONAL ORDERS**

The FPSOGSA is applied to minimize the real power losses considering the set of fractional order  $\alpha = [0.1, 0.2, \dots, 0.9]$  and corresponding learning curves including best, average and worst iterative updates are plotted in Fig. 4. This experiment is performed with an archive size of 20 and 50 iterations for 10 independent trails on each fractional order  $\alpha$  to get the minimum fitness. The sub Fig. 4(i) demonstrated the best minimum fitness achieved to 4.5459 MW at  $\alpha = 0.9$  while the sub Fig. 4(g) is observed as the worst case reported at  $\alpha = 0.7$  with the minimum losses to 4.6068 MW.

The Fig. 5 illustrates the best minimum fitness reported at  $\alpha = 0.9$  with 100 autonomous trails that is 4.5342 MW. The setting of control variables and corresponding losses yielded by FPSOGSA along with those computed by other counterpart algorithms are documented in Table 5.

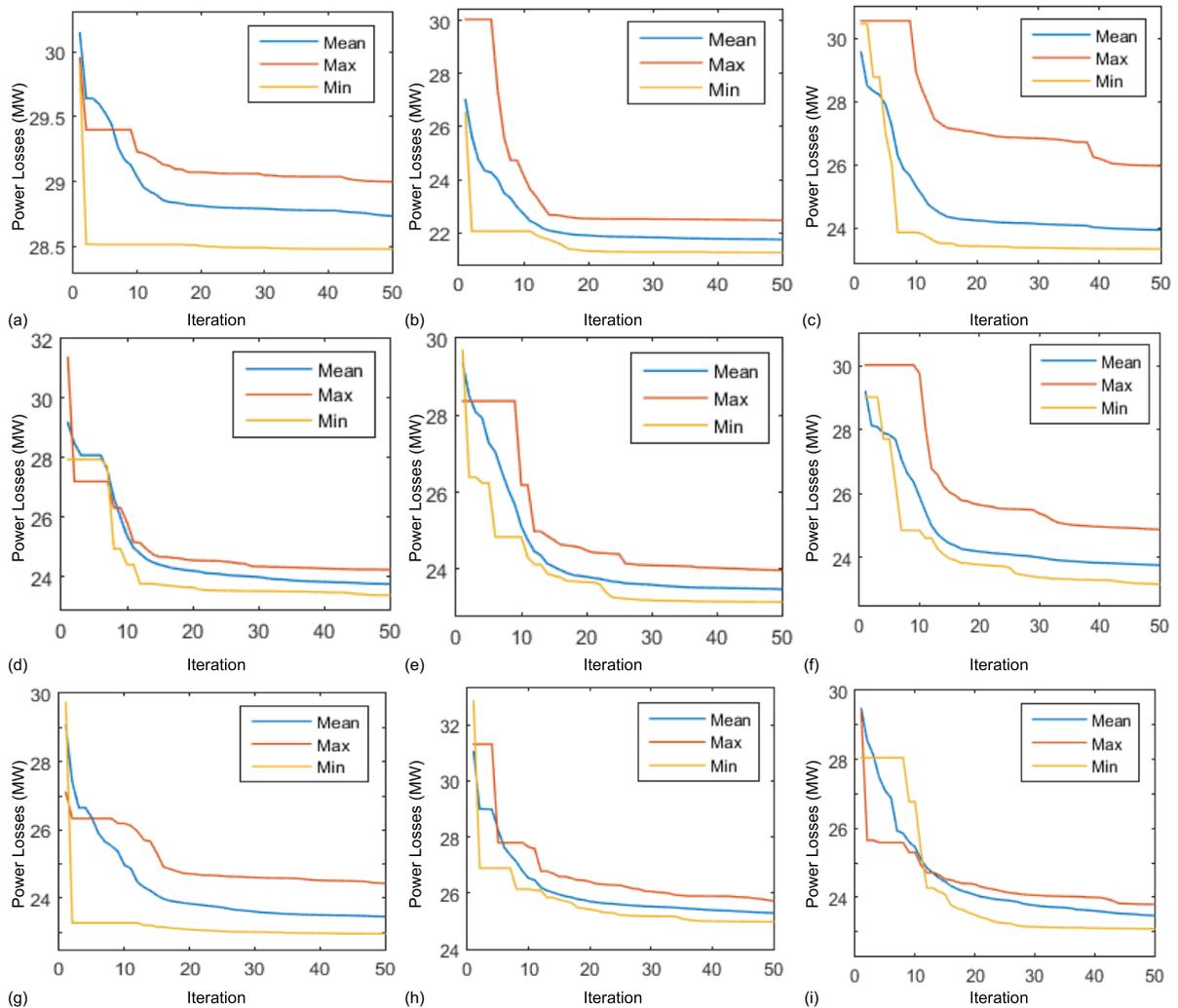
The comparison of line loss reduction with the other well-known algorithms is presented in Table 6 where it can be seen that loss reduction achieved by IWO, DE, MICA-IWO,

C-PSO, MFO, GWO and FODPSO is 13.1202%, 13.6839%, 14.4270%, 17.3565%, 19.0093%, 18.7992%, and 18.6650% respectively. While, the results getting from FPSOGSA is reported to 19.9329% as compared to the based case and other techniques which indicated towards the best performance of the proposed algorithm.

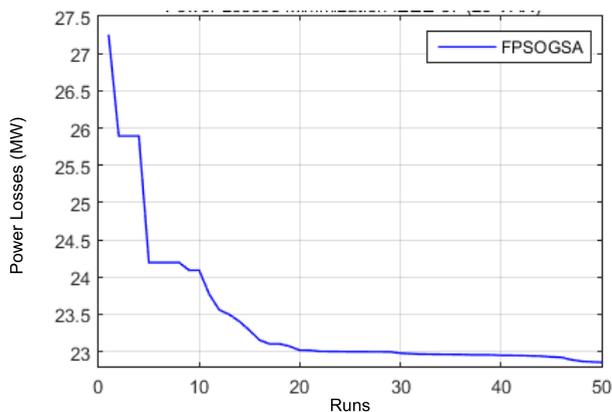
**2) VOLTAGE DEVIATION ( $V_D$ ) AT DIFFERENT FRACTIONAL ORDERS**

The 2<sup>nd</sup> objective adopted is the minimization of the voltage deviation ( $V_D$ ) from the reference voltage. The parameter setting for the designed FPSOGSA and boundaries of operational variables can be seen in Table 3 and 4, respectively. The fitness is again evaluated for all the fractional orders  $\alpha$  and best coefficient is selected based on minimum value of objective function. The learning curves of FPSOGSA are obtained between  $\alpha = [0.1, 0.2, \dots, 0.9]$  with archive size of 20 and 50 for 10 independent trails for getting the minimum voltage deviation and are demonstrated in Fig. 6. The sub Fig. 6(i) gives the best minimum values to 0.1072pu at  $\alpha = 0.9$  while the worst case is reported to 0.1153pu at  $\alpha = 0.2$ .

The FPSOGSA is further run for 100 autonomous tails on the best fractional order to find the global solution. In Table 5, the best result for voltage deviation is achieved to 1025pu which is far better than recently developed MFO and GWO. Hence the effectiveness of FPSOGSA is again endorsed.



**FIGURE 10.** FPSOGSA convergence curve for minimum power losses for IEEE57 standard (25 variables) at different fractional orders ( $\alpha = [0.1 - 0.9]$ ).



**FIGURE 11.** FPSOGSA approach to minimization of power losses of IEEE57 standard (25 variables).

**B. TEST SYSTEM 2: IEEE 30 BUS (19 VARIABLES)**

This system consists of six generator units at bus 1, 2, 5, 8, 11, 13, four tap changing transformers ( $T_c$ ) on line number 6-9, 6-10, 4-12, 27-28 and nine shunt reactive VAR compensators

( $Q_c$ ) connected to the bus 10, 12, 15, 17, 20, 21, 23, 24, 29 [47] while the parameter setting is the same as provided in Table 3. The proposed FPSOGSA is tested for both fitness functions following the limits of the operational variables as given in Table 4.

**1) POWER LOSSES MINIMIZATION AT DIFFERENT FRACTIONAL ORDERS**

In this case, FPSOGSA is applied to achieve the minimum line losses in IEEE 30 bus with 19 control variables using different fractional orders  $\alpha$ . The learning curves plotted with  $\alpha = [0.1, 0.2, \dots, 0.9]$  are shown in Fig. 7 indicating the best, average and worst iterative updates generated by the FPSOGSA. Initially, for learning the behavior, FPSOGSA at each fractional order  $\alpha$  given in Fig. 7 is run for 10 autonomous trails in case of minimum power losses. The sub Fig. 7(i) demonstrated the best minimum fitness is achieved at  $\alpha = 0.9$  with 4.4309 MW while sub Fig. 7(c) is the worst case reported at  $\alpha = 0.3$  with 4.5428 MW. The best order is further run for 100 independent trails to get

**TABLE 9. Best control setting of variable for power loss minimization of IEEE57 standard (25 variables) for fitness objective ( $P_{loss}$  and  $V_D$ ).**

Control Variables	Base Case	SOA [57]	PSO-cf [58]	CLPSO [52]	MFO [12]	SGA (Ff1) [58]	GSA [59]	FPSOGSA (Proposed)
$V_{GT-1}$	1.0400	1.0600	1.06	1.0541	1.06000	1.0600	1.060000	1.1000
$V_{GT-2}$	1.0100	1.0580	1.0586	1.0529	1.05870	1.0594	1.060000	1.0987
$V_{GT-3}$	0.9850	1.0437	1.0464	1.0337	1.04690	1.0490	1.060000	1.0871
$V_{GT-6}$	0.9800	1.0352	1.0415	1.0313	1.04210	1.0418	1.008102	1.0807
$V_{GT-8}$	1.0500	1.0548	1.06	1.0496	1.06000	1.0600	1.054955	1.1000
$V_{GT-9}$	0.9800	1.0369	1.0423	1.0302	1.04230	1.0435	1.009801	1.0845
$V_{GT-12}$	1.0150	1.0336	1.0371	1.0342	1.03730	1.0396	1.018591	0.9600
$TC_{4-18}$	0.9700	1	0.98	0.9900	0.95011	1.0190	1.100000	1.0071
$TC_{4-18}$	0.9780	0.96	0.98	0.9800	1.00760	0.9130	1.082634	1.0847
$TC_{21-20}$	1.0430	1.01	1.01	0.9900	1.00630	1.0320	0.921987	0.9959
$TC_{24-26}$	1.0430	1.01	1.01	1.0100	1.00760	1.0070	1.016731	0.9908
$TC_{7-29}$	0.9670	0.97	0.98	0.9900	0.97523	0.9410	0.996262	0.9964
$TC_{34-32}$	0.9650	0.97	0.97	0.9300	0.97218	0.9780	1.100000	1.0072
$TC_{11-41}$	0.9550	0.9	0.9	0.9100	0.90000	0.9100	1.074625	0.9900
$TC_{15-45}$	0.9550	0.97	0.97	0.9700	0.97186	0.9380	0.954340	0.9906
$TC_{14-46}$	0.9000	0.95	0.96	0.9500	0.95355	0.9250	0.937722	1.0028
$TC_{10-51}$	0.9300	0.96	0.97	0.9800	0.96736	0.9350	1.016790	0.9900
$TC_{13-49}$	0.8950	0.92	0.93	0.9500	0.92788	0.9030	1.052572	1.0027
$TC_{11-43}$	0.9580	0.96	0.97	0.9500	0.96406	0.9260	1.100000	1.0844
$TC_{40-56}$	0.9580	1	0.99	1.000	0.99980	1.0140	0.979992	1.0023
$TC_{39-57}$	0.9800	0.96	0.96	0.9600	0.96060	0.9740	1.024653	0.9900
$TC_{9-55}$	0.9400	0.97	0.98	0.9700	0.97899	0.9430	1.037316	1.0951
$QC_{18}$	0.0000	9.984	9.984	9.8800	9.99680	5.1000	7.8254	4.9846
$QC_{25}$	0.0000	5.904	5.904	5.4200	5.90000	5.7000	0.5869	4.9992
$QC_{53}$	0.0000	6.288	6.288	6.2800	6.30000	6.3000	4.6872	4.3653
$P_{loss}$ , MW	27.86	24.26739	24.28022	24.89	24.25293	23.836	23.461194	22.9185
$VD$ (p.u)	4.1788	NR	NR	1.0929	NR	2.7021	NR	0.8017

**TABLE 10. Percentage of reduction in line losses of IEEE 57 standard comparing by different Algorithms.**

Comparison	Based Case	SOA [57]	PSO-cf [58]	CLPSO [52]	MFO [12]	SGA (Ff1) [58]	GSA [59]	FPSOGSA Proposed
Power Loss, MW	27.86	24.26739	24.28022	24.89	24.25293	23.836	23.461194	22.9185
%	-	12.8952	12.8491	10.6604	12.9471	14.4436	15.7889	17.7674

the best global solution that is reported to 4.4121 MW and demonstrated in Fig. 8.

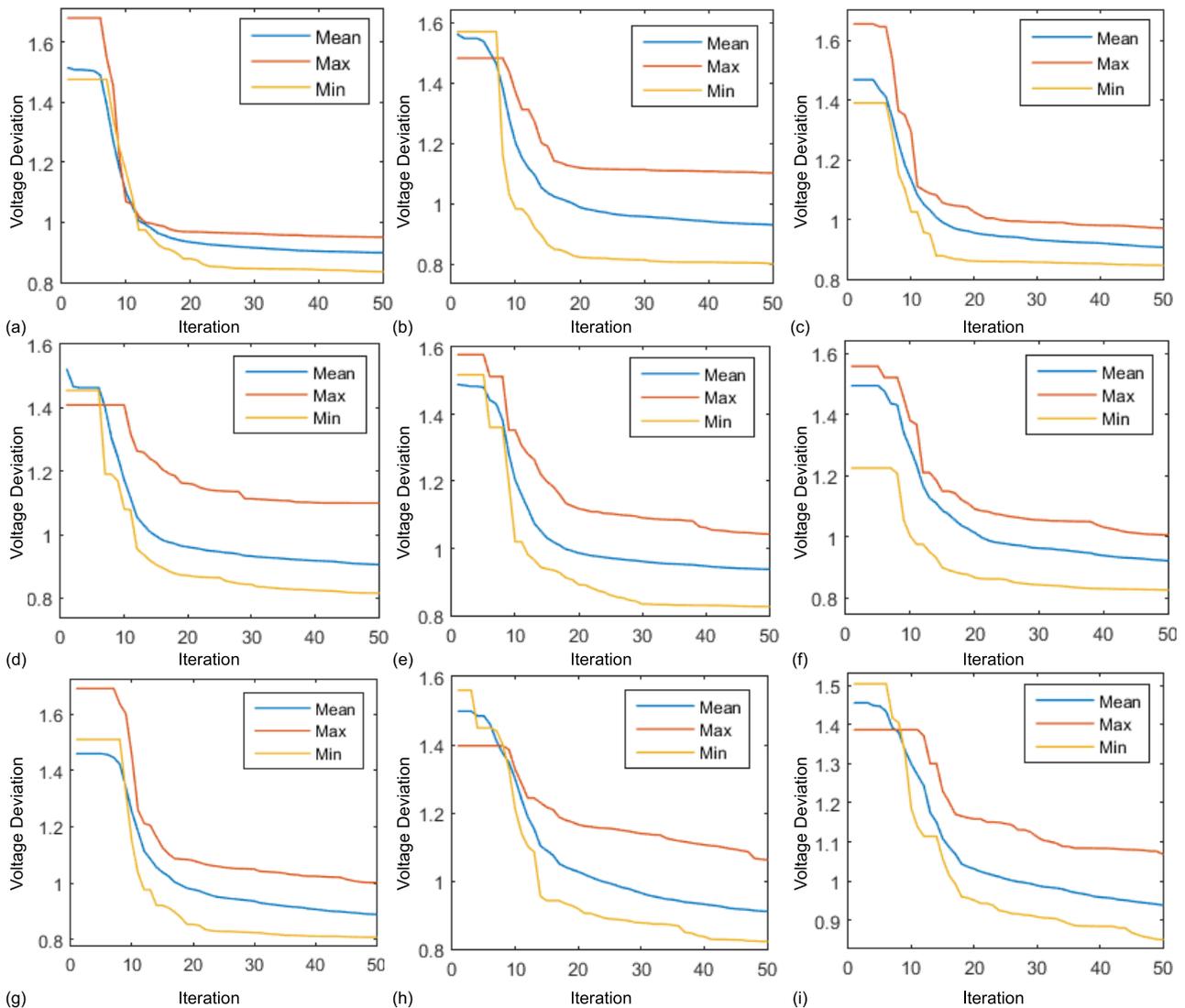
The results reported from other algorithms and the one generated by the FPSOGSA are listed in Table 7 along with the information of the control variables. The comparison of loss reduction as a percentage of base case, is provided in Table 8 where a 24.0733% improvement is achieved by the FPSOGSA in comparison with TS, CLPSO, WOA, BBO, MFO, MSFS, A-CSOS, ALC-PSO and PSOGSA which are reported as 15.3278%, 21.5023%, 20.9379%, 21.8121%, 22.3404%, 22.3145%, 22.3406%, 22.9168% and 22.0289% respectively. So, Fig. 8, Tables 7 and 8 establish the efficacy of FPSOGSA in this case as well.

2) VOLTAGE DEVIATION (VD) AT DIFFERENT FRACTIONAL ORDERS

The convergence curves for all the fractional order  $\alpha$  depict the best, average and worst iterative updates during

minimization of  $V_D$  in present case. The learning curve of FPSOGSA is obtained for 10 autonomous trails with the given range of fraction order  $\alpha = [0.1, 0.2, \dots, 0.9]$  in Fig. 9. The sub Fig. 9(h) illustrates the best minimum fitness in case of voltage deviation that is reported to 0.1493p.u at  $\alpha = 0.7$ , while sub Fig. 9 (e) demonstrates the worst case reported to 0.1707p.u at  $\alpha = 0.5$ . The best fractional order  $\alpha = 0.7$  is further run for 100 autonomous trails to get the global solution for this case. In Table 7, the best minimum fitness achieved by FPSOGSA is reported to 0.1468p.u.

The comparison of results computed by FPSOGSA and counterpart algorithms is provided in Table 7, where one may see that the designed strategy has computed the minimum value of the objective function in comparison with the TS, CLPSO, BBO, MFO, A-CSOS and PSOGSA which has generated 0.1540, 0.4773, 2.0662, 2.0316, 2.05630 and 2.0504p.u previously. Hence, the performance of FPSOGSA is superior to the reported algorithms.



**FIGURE 12.** FPSOGSA convergence curve for voltage deviation (VD) for IEEE57 standard (25 variables) at different fractional orders ( $\alpha = [0.1 - 0.9]$ ).

**C. TEST SYSTEM.3: IEEE 57 BUS (25 VARIABLES)**

The optimization strength of proposed fractional hybrid mechanism is further tested on large scale power system i.e., 57 bus system. This system contains 7 generators units on bus 1, 2, 3, 6, 8,9 and 12, with 15 branches connected to tap changing transformers while shunt reactive compensators are connected to the bus 18, 25 and 53 [48].

**1) POWER LOSSES AT DIFFERENT FRACTIONAL ORDERS**

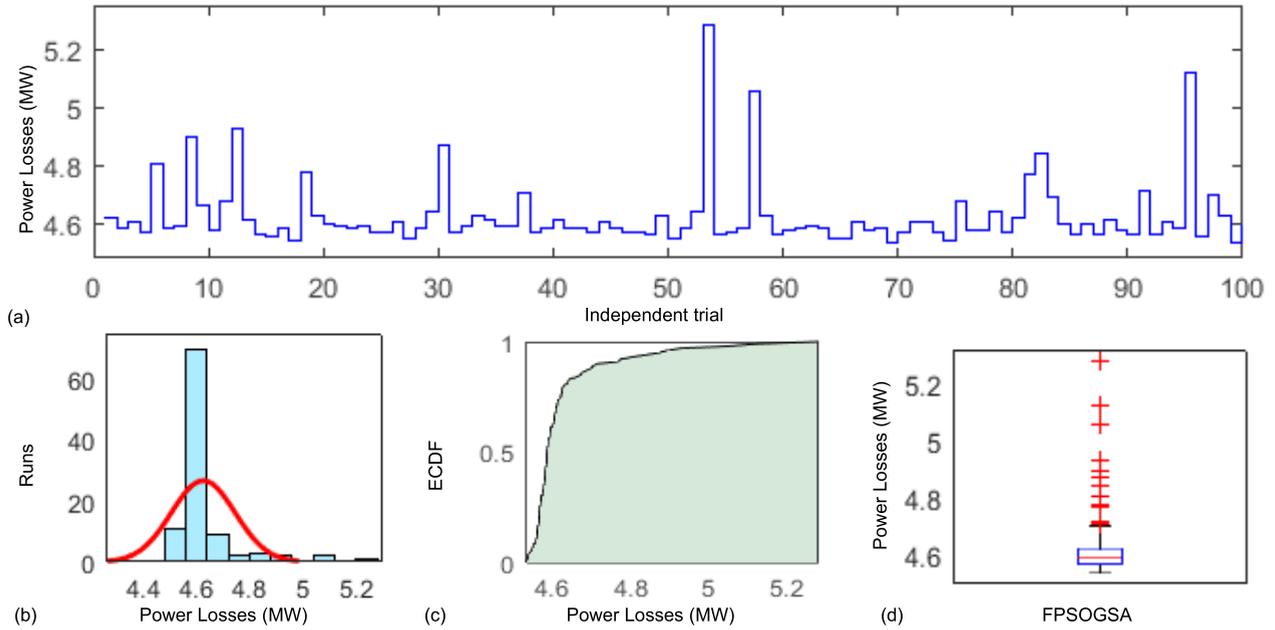
The optimum setting of the control variables and corresponding minimum losses as yielded by the FPSOGSA and other state of the art mechanisms are given in Table 9 while the convergence characteristics can be observed in Fig. 10. To demonstrate the better performance, FPSOGSA is run for 10 independent trails between  $\alpha = [0.1, 0.2, \dots, 0.9]$ . The sub Fig. 10(g) illustrates the best minimum fitness achieved to 22.9638 MW at  $\alpha = 0.7$  in term of power losses minimization

while sub Fig. 10(a) is the worst case reported to 28.4793 MW at fractional order  $\alpha = 0.1$ . The FPSOGSA is further run for 100 independent trails at  $\alpha = 0.7$  for getting the best minimum fitness which is finally reported to 22.9185 MW and given in Table 9.

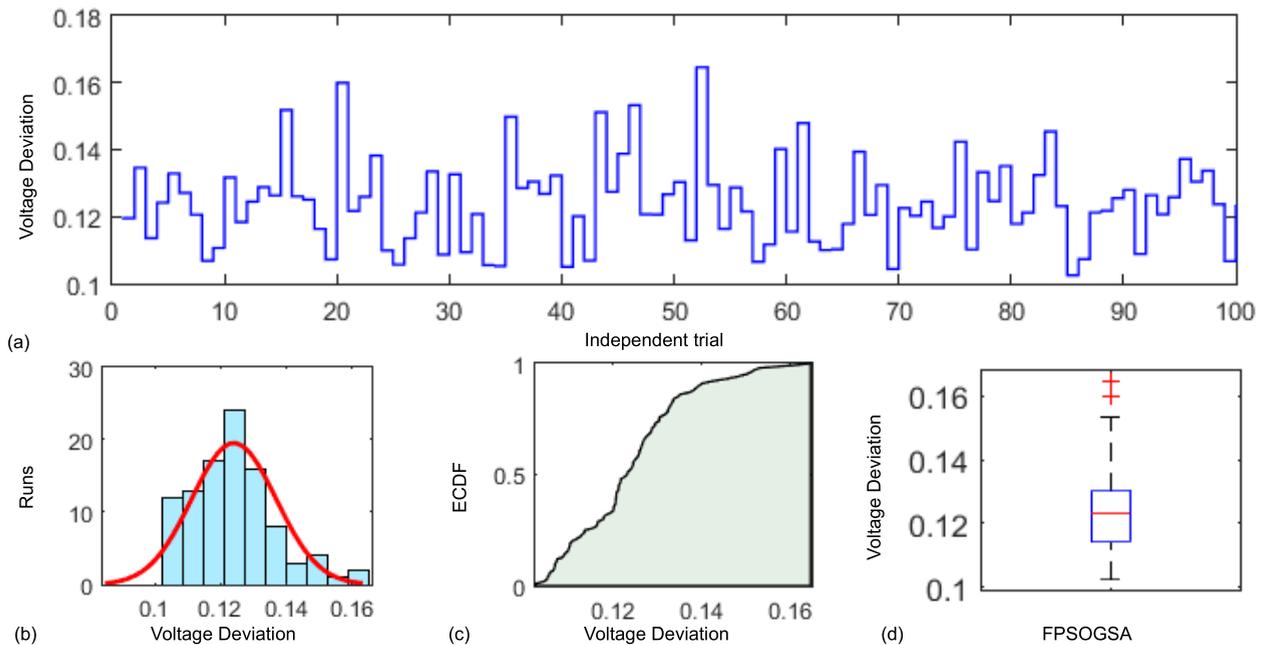
The percentage power loss minimization by the different algorithms such as; SOA, PSO-cf, CLPSO, MFO, SGA( $F_{fl}$ ), GSA and proposed FPSOGSA is 12.8952 %, 12.8491%, 10.6604%, 12.9471%, 14.4436%, 15.7889 and 17.7674%, respectively, as given in Table 10. The result indicates towards the better accuracy and performance of the proposed algorithm for the ORPD problems.

**2) VOLTAGE DEVIATION ( $V_D$ ) AT DIFFERENT FRACTIONAL ORDERS**

The convergence curves for all fractional order  $\alpha$  depicting the best, average and worst iterative updates during



**FIGURE 13.** Statistical analysis for power losses on IEEE30 standard (13 variables) at best alpha order ( $\alpha = 0.7$ ). (a) Fitness comparison. (b) Histogram analysis. (c) ECDF plot analysis. (d) Boxplot analysis.

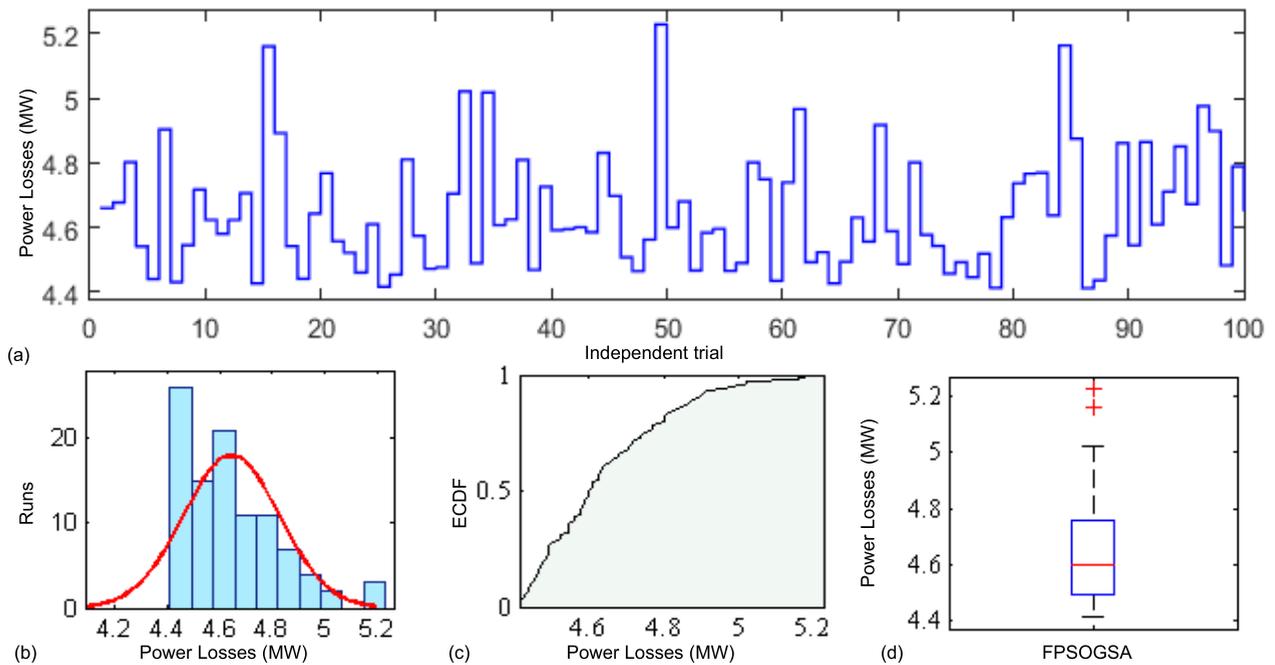


**FIGURE 14.** Statistical analysis for voltage deviation on IEEE30 standard (13 variables) at best alpha order ( $\alpha = 0.9$ ). (a) Fitness comparison. (b) Histogram analysis. (c) ECDF plot analysis. (d) Boxplot analysis.

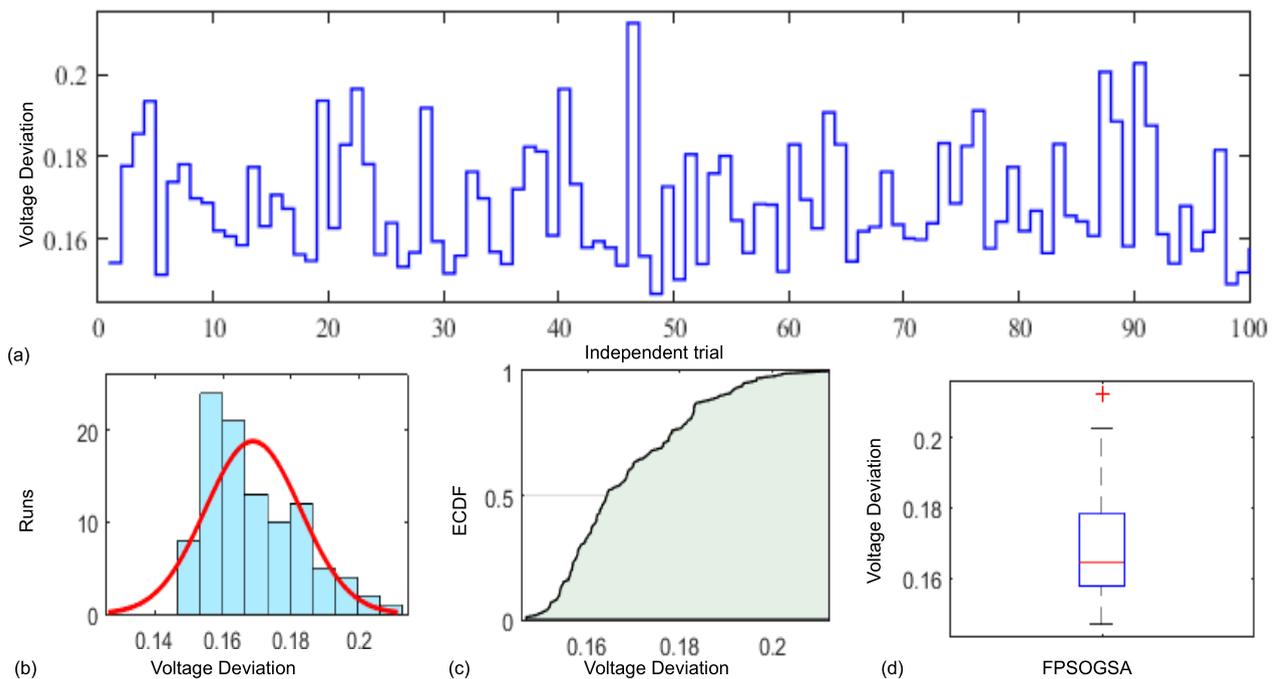
minimization of  $V_D$  in present case are shown in Fig. 12. The Fig. 12 demonstrates the performance of FPSOGSA at different fractional orders for 10 autonomous trails. The sub Fig. 12(b) illustrates the best value reported to 0.8175p.u at  $\alpha = 0.2$  while sub Fig. 12(i) indicates towards the worst case reported to 0.8506p.u at  $\alpha = 0.9$ . The best minimum fitness is further executed for 100 independent trails which is finally reported to 0.8017p.u at the best fractional order  $\alpha = 0.2$ . The comparison of results computed by FPSOGSA

and counterpart algorithms is provided in Table 9 where one may see that the designed strategy has computed the minimum value of the objective function in comparison with the CLPSO [52] and SGA ( $F_{f1}$ ) [58] previously. Hence, the performance of FPSOGSA is superior to the reported algorithms and base case.

In brief, in all the scenarios of ORPD, the newly designed fractional variant of hybrid PSOGSA optimization methodology has proved its effectiveness by evaluation the optimum



**FIGURE 15.** Statistical analysis for power losses on IEEE30 standard (19 variables) at best alpha order ( $\alpha = 0.9$ ). (a) Fitness comparison. (b) Histogram analysis. (c) ECDF plot analysis. (d) Boxplot analysis.



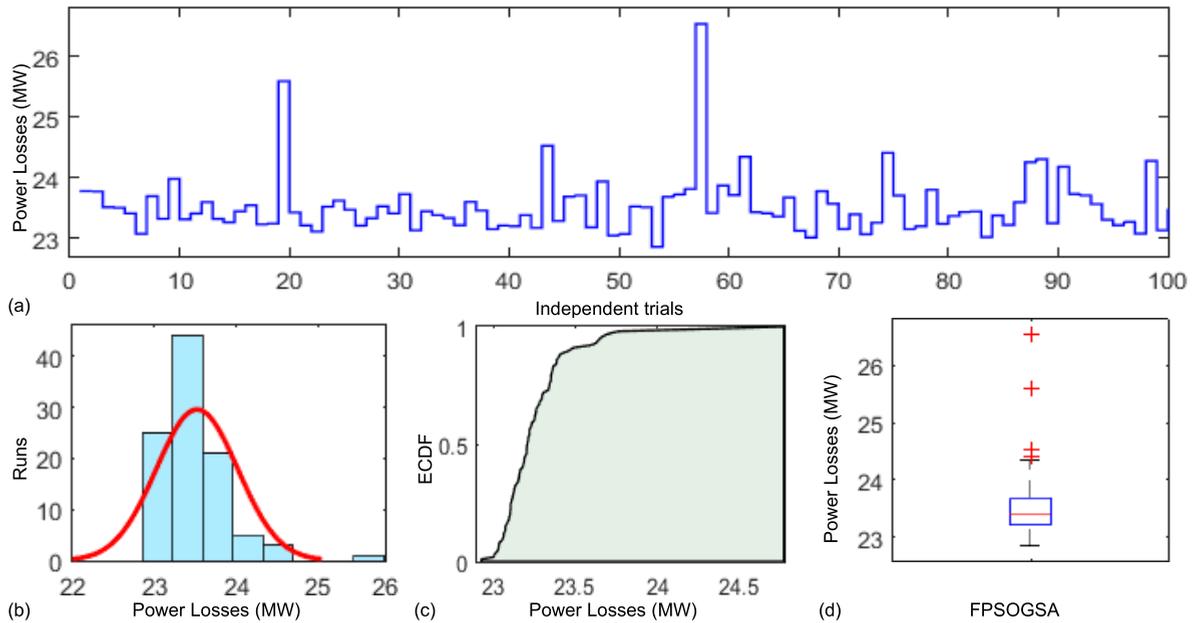
**FIGURE 16.** Statistical analysis for voltage deviation on IEEE30 standard (19 variables) at best alpha order ( $\alpha = 0.8$ ). (a) Fitness comparison. (b) Histogram analysis. (c) ECDF plot analysis. (d) Boxplot analysis.

value of fitness functions as compared those well-known optimization mechanisms.

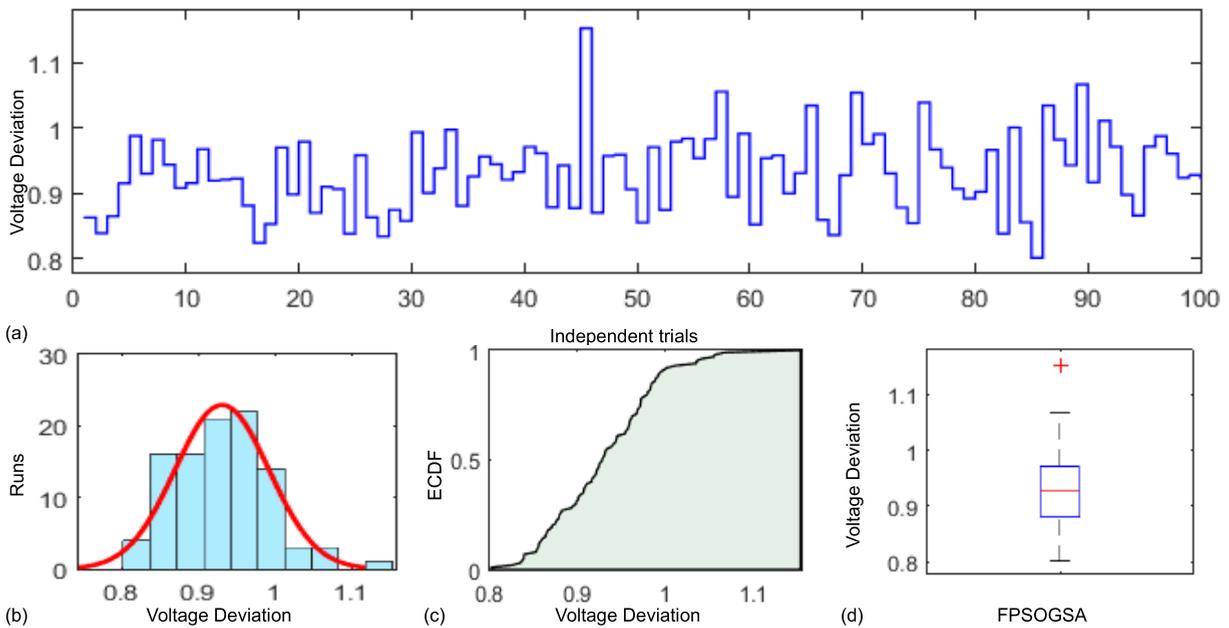
**V. STATISTICAL ANALYSIS**

In this section, the performance of designed FPSOGSA is further established by comparative study through statistics for all the test cases considering the best fractional order of

the respective case. Due to stochastic nature of FPSOGSA, the yielded results are always different from one another, hence hundred independent trials are conducted to draw reliable inferences on FPSOGSA performance during solution of optimal RPD problems in standard power systems. The conducted statistical analysis is based on the minimum fitness evaluation in each independent simulation, histogram



**FIGURE 17.** Statistical analysis for power losses on IEEE57 standard (25 variables) at best alpha order ( $\alpha = 0.7$ ). (a) Fitness comparison. (b) Histogram analysis. (c) ECDF plot analysis. (d) Boxplot analysis.

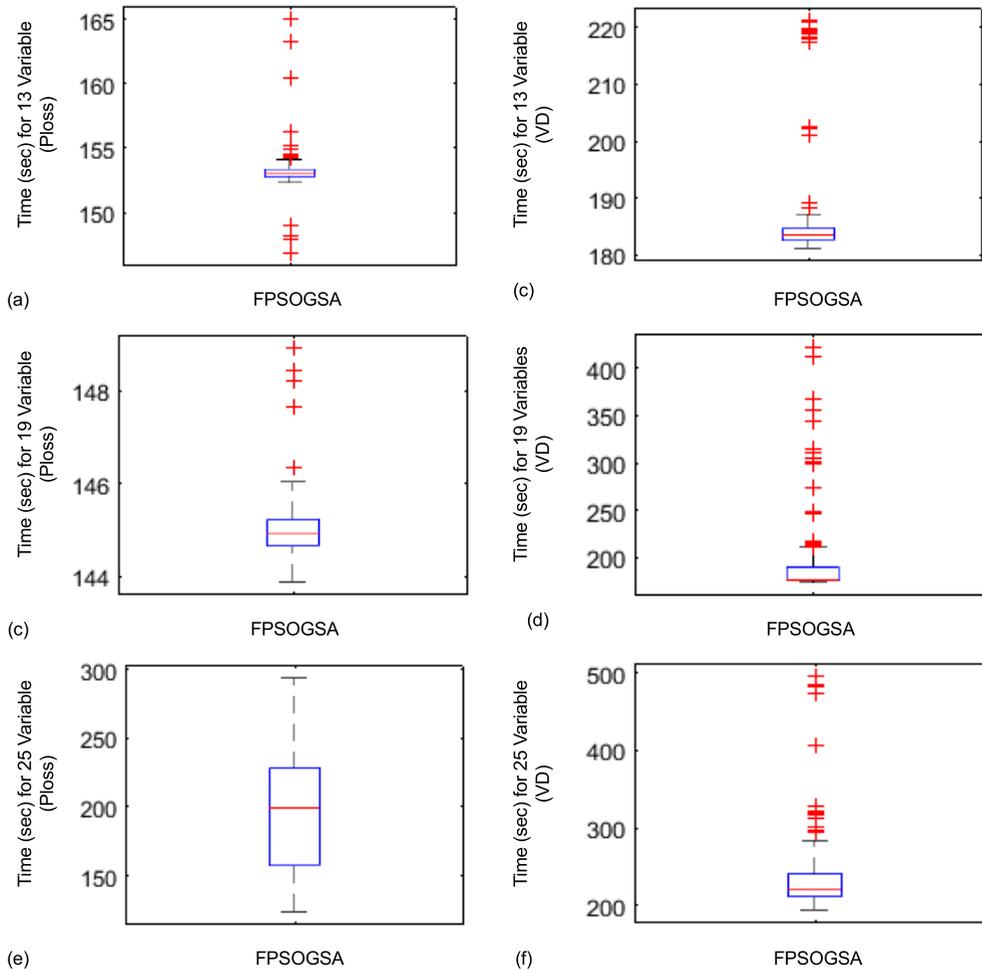


**FIGURE 18.** Statistical analysis for voltage deviation on IEEE57 Standard (25 variables) at best alpha order ( $\alpha = 0.2$ ). (a) Fitness comparison. (b) Histogram analysis. (c) ECDF plot analysis. (d) Boxplot analysis.

curves, cumulative distribution probability charts and box plots.

The results are depicted in Figs.13 and 14 for  $P_{loss}$  and  $V_D$  minimization in 30 bus with 13 variables, respectively, Figs. 15 and 16 for  $P_{loss}$  and  $V_D$  minimization in 30 bus with 19 variables, respectively, and Figs.17 and 18 for  $P_{loss}$  and  $V_D$  minimization in 57 bus with 25 variables, respectively. The minimum fitness values shown in sub-figures 13(a)-18(a) reveal the very small variations in all test cases which

ascertain a considerable accuracy of the FPSOGSA in each autonomous trial. Similarly, one may also see that in all plots, the fitness value is less than the base case value for all the independent trials. The histograms in sub-figures 13(b)-18(b) illustrate that majority of the autonomous simulations of FPSOGSA yielded least gauges of the fitness. The empirical CDF probability graphs depicted in sub figures 13(c)-18(c) reveal that approximately 100 percent of the independent simulation yields fitness values less than the base case value



**FIGURE 19.** Time complexity of FPSOGSA Algorithm for IEEE 30 (13, 19 variables) and IEEE 57 (25 variables) standard buses. Test system 1. (a) Power losses. (b) Voltage deviation, Test system 2. (c) Power losses. (d) Voltage deviation, Test system 3. (e) Power losses. (f) Voltage deviation.

which shows an effective iterative optimization process. The box plots in sub figures 13(d)-18(d) show the spreading of the data where the values are close to each other and even the outliers are very nearer to the median gauge which further recognizes the accurate optimization of the FPSOGSA.

Summarizing, all these graphical descriptions of the statistics demonstrate the stability, efficacy, robustness, reliability and consistency of FPSOGSA as an efficient and reliable optimization solution algorithm for optimal RPD problems. While, some limitations of FPSOGSA are observed such as; computational inefficiencies, dependency on input parameter including fractional order and suboptimal solutions.

The simulations in presented work are conducted using MATLAB 2015, on Window 10, Lenovo-E480 model Professional Intel®Core™i7-8550U CPU @ 1.80 GHz 8GB RAM. The boxplots illustrating the median of execution time for all the adopted fitnesses during 100 autonomous trials can be seen in Fig. 19. One may observe in Fig. 19 that measured time of the algorithm execution in terms of median

gauge for standard IEEE 30 bus with 13 and 19 variables and IEEE 57 (25 variables) considering power loss minimization as fitness are computed as 153.3392s, 145.0689s and 195.0752s, respectively, while considering voltage deviation it is 189.1632s, 200.3447s and 247.3561s, respectively. The data spread is very close in each quartile during the independent trials i.e., which endorse the precision, consistency and smoothness of FPSOGSA evolution.

## VI. CONCLUSION

A novel hybrid meta-heuristic optimization technique FPSOGSA is proposed and applied effectively to solve the ORPD problems including the transmission line loss and voltage deviation minimization in IEEE 30 bus with 13 and 19 variables and IEEE 57 with 25 variables. The introduction of fractional derivative in the velocity update mechanism of the traditional PSO has improved the convergence rate of the optimizer while hybridization of GSA with fractional PSO has increased the ability of finding the global best solution. By using FC concept to such algorithms can help to improve

the convergence properties, enhancing the memory effect [60] with increasing stability, reliability and consistency.

To demonstrate preeminence of the proposed FPSOGSA algorithm, the simulation results were compared with THE various techniques such as IWO, DE, MICA-IWO, C-PSO, MFO, WOA, GWO, FODPSO, TS, CLPSO, BBO, MSFS, A-CSOS, ALC-PSO, SOA, SGA(Ff1), PSO-cf, GSA and PSO-GSA. The minimum fitness for three given test systems are reported such as; power losses 4.5342 MW with reduction of 19.0329% and voltage deviation 0.1025p.u for Test system 1, power losses 4.4121 MW with reduction of 24.0733% and voltage deviation 0.1468p.u for test system 2, while the power losses 22.9185 MW with reduction of 17.7674% and voltage deviation 0.8017p.u for test system 3. Hence, the performance of FPSOGSA algorithm is superior to the reported algorithms in all cases.

In future, such techniques of FC can be implemented to all variants of PSO [21], [58] and other hybrid algorithms to enhance the memory effect and convergence rate. The designed FPSOGSA looks further promising to be exploited/explored for finding the solution of stiff/non-stiff nonlinear models arising in broad application of applied science and technology such as intelligent systems, electromagnetism, electronics, modeling and identification, telecommunications, irreversibility, physics, control systems [38]–[41], fluid dynamics [61], [62], astrophysics models [63], [64], differential equation based electric circuit theory [65], [66], bioinformatics studies [67], [68] and atomic physics models [69], [70].

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