Design of Logical Topologies: A Linear Formulation for Wavelength-Routed Optical Networks with No Wavelength Changers

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***Abstract*—We consider the problem of constructing logical topologies over a wavelength-routed optical network with no wavelength changers. We present a general linear formulation which considers routing traffic demands, and routing and assigning wavelengths to lightpaths, as a combined optimization problem. The formulation also takes into account the maximum number of hops a lightpath is permitted to take, multiple logical links in the logical topology, multiple physical links in the physical topology, and symmetry/asymmetry restrictions in designing logical topologies. The objective is to minimize *congestion*. We show by examples how equality and inequality logical degree constraints have a bearing on congestion. We prove that, under certain conditions, having equality degree constraints with multiple edges allowed in the design of logical topologies does not affect congestion. This helps in reducing the dimensionality of the search space and hence speeds up the search for an optimal solution of the linear formulation. We solve the linear formulation for small examples and show the tradeoff between congestion, number of wavelengths available and the maximum number of hops a lightpath is allowed to take. For large networks, we solve the linear formulation by relaxing the integer constraints. We develop topology design algorithms for large networks based on rounding the solutions obtained by solving the relaxed problem. Since the whole problem is linearizable, the solution obtained by relaxation of the integer constraints yields a lower bound on congestion. This is useful in comparing the efficiency of our heuristic algorithms. Following Bienstock and Gunluk, 1995, we introduce a cutting plane which helps in obtaining better lower bounds on congestion and also enables us to reduce the previously obtained upper bounds on congestion.**

***Index Terms*—All-optical networks, linear program, network planning, topology design.**

I. Introduction

A. Optical Networks

OPTICAL networks implemented using wavelength-division multiplexing techniques, called *wavelength-routed optical networks*, are the most promising candidates for backbone high-speed wide area networks [9]. Wavelength-routing optical networks realize the high bandwidth capability of the fiber, provide transparency to bit rates, allow for spatial wavelength reuse (the number of wavelengths available may be limited), provide reliable service (the network can be reconfigured in the event of failures) and are adaptable for building logical topologies on top of physical topologies to reflect the traffic intensities between the various nodes. A physical topology is a graph representing the physical interconnection of the wavelength routing nodes by means of fiber-optic cables [9]. In Fig. 1 is shown a physical topology of a six-node wide-area network. The wavelength routing nodes are numbered from 0 to 5. Node 0 and node 1 are connected by a fiber and so an edge is shown in the physical topology between them. We consider an edge in the physical topology to represent a pair of fibers, one in each direction. A logical topology is a directed graph that results when the lightpaths are setup by suitably configuring the wavelength routing nodes. For example, in Fig. 2 is shown a possible logical interconnection by suitably configuring the wavelength routing nodes of Fig. 1. There is an edge in the logical topology between node 0 and node 2 when the data or packets from node 0 to node 2 traverse the optical network in the optical domain only, i.e., undergo no electronic conversion in the intermediate wavelength routing nodes. Edges in a logical topology are called *logical links*.

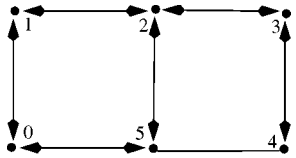

Fig. 1. Six-node network, physical topology.

For example, in Fig. 2 the data from node 3 to node 1 are sent on wavelength f_1 through the wavelength routing node at 2. Simultaneously, we can send a packet from node 3 to node 5 on f_1 through the wavelength routing node at 4. We see that even though in the physical topology there is a fiber connection between node 3 and node 2, to send a packet from node 3 to node 2 we would have to use three logical links (3,1), (1,0) and (0,2). We say that the *hop length* of the logical link (3,1) is two as it traverses two physical edges, (3,2) and (2,1). A logical topology is what is seen by the higher layers of the network, and it is convenient to think of a logical topology as constituting an optical layer [9]. Wavelength changers translate wavelength f_i to f_k. They may be used as components of the wavelength

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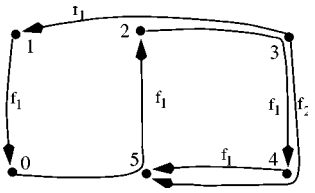

Fig. 2. Logical topology with routing and wavelength assignment.

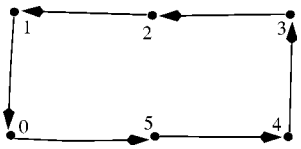

Fig. 3. Degree one logical topology.

routing nodes. The problem in using wavelength changers is primarily that the effective translation of wavelengths in the optical domain is not cost effective. In this paper, we consider logical topology design with no wavelength changers. A linear formulation for the case with wavelength changers has been addressed in [4].

B. Logical Topology Design with No Wavelength Changers

A physical topology is a bidirected graph representing the physical interconnection of the wavelength routing nodes. In a bidirected graph, whenever there is an edge $\overrightarrow{xy}$ where x and y are wavelength routing nodes, there is another edge $\overrightarrow{yx}$. For ease of representation on paper, we represent it by a single edge with arrows at either end, as shown in Fig. 1.

A logical topology is a directed graph that is determined by the lightpaths set up in the physical topology. The lightpaths are setup by configuring the wavelength routing nodes in the physical topology.

A logical topology is said to be *symmetric* if, whenever there is a logical link from node A to node B, there is another logical link from node B to node A passing through the same set of intermediate wavelength routing nodes. A logical topology is *asymmetric* if the above restriction is removed.

In a directed graph, the in-degree of a node is equal to the number of edges terminating on it and the out-degree of a node is equal to the number of edges originating from it. A Δ-regular logical topology is one where all the nodes have the same in-degree (Δ) and out-degree (Δ). We say that a logical topology has multiplicity Q if the maximum number of edges between any node pair is Q.

A logical topology is said to be hop-bound limited if there is a restriction on the maximum number of hops a lightpath is allowed to take. A logical topology is said to be wavelength limited if there is a restriction on the maximum number of wavelengths that can be used while setting up the lightpaths in the corresponding (see Section IV) physical topology. We shall see later that these are important parameters in logical topology design.

We assume that the physical topology, the traffic matrix representing long-term average flows between the end nodes, and the number of transmitters and receivers at each of the nodes are given. We find a logical topology , and also a routing and wavelength assignment for the logical links, such that the resources of the network subject to the given constraints are used optimally. In Fig. 3 is shown a possible logical topology over the physical topology shown in Fig. 1, when each of the nodes is equipped with one transmitter and one receiver, the number of wavelengths the fiber supports is one, and the maximum hop length of the logical links is one. It is reasonable to construct a logical topology which minimizes congestion. *Congestion* is defined as the maximum offered load on any logical link in the logical topology. The number of wavelengths the fiber can support is also an important parameter which has to be taken into account in any logical topology design problem. Wavelengths are scarce and have to be used optimally. The hop length of a logical link is a measure of the number of wavelength routing nodes encountered while setting up a logical link. If the hop length of a logical link is large then there would be degradation of the optical signal by attenuation and crosstalk at the intermediate wavelength routing nodes. Therefore it is important to keep the hop lengths of the logical links small. Symmetry restrictions are also important factors in logical topology design. We shall see from the numerical examples how the congestion is affected when we impose symmetry restrictions on the logical topology. Multiple edges play an important role in the design of logical topologies. We shall illustrate by examples (see Section IV) how congestion varies when we allow multiple edges.

An informal description of the logical topology design problem is as follows (a precise definition as a mixed integer (0–1) linear program is given later in the paper).

Let $TM = (\lambda^{sd})$ be the traffic matrix, i.e., λ^{sd} is the arrival rate of packets at node s that are destined for node d. Let $\Delta_i^{(t)}$ denote the number of transmitters at node i and $\Delta_i^{(r)}$ denote the number of receivers at node i. We seek to create a logical topology, with a routing and wavelength assignment (wavelength changers are not allowed) for the logical links, that minimizes $\lambda_{\max} = \max_{i,j} \lambda_{i,j}$ where $\lambda_{i,j}$ denotes the offered load on logical link (i, j) of the logical topology. $\lambda_{\max}$ is the maximum offered load to any logical link, and is called the *congestion*.

The various parameters included in the mixed-integer linear program are the number of wavelengths the fiber supports, symmetry and asymmetry restrictions, multiplicity restrictions, and hop bound constraints.

The following notation will be used to denote all the above information in compact form: $LT(\Delta, = \ / \ \leq, sym/asym, Q, H, F)$. If the first two arguments are "$\Delta, \leq$" then it denotes a logical topology that has maximum degree [(`maximum` (in-degree, out-degree)] Δ. If the first two arguments are "$\Delta, =$" then it denotes a logical topology that is Δ-regular. The third argument $sym/asym$ denotes that the logical topology is symmetric/asymmetric. The fourth argument Q denotes that the logical topology has at most Q edges between any node pair. The fifth argument H is the hop-bound limit for any logical link in the logical topology. The last argument F denotes that the logical topology requires at most F wavelengths for wavelength assignment. Using the notation stated above Fig. 3 is an example of $LT(1, =, asym, 1, 1, 2)$ logical topology on the physical topology of figure Fig. 1.

We show by small examples in Section IV the relationships between the various parameters.

C. Previous Work

In this section, we position our work in relation to three recent papers, [2], [3], and [1], on the problem of designing optimal logical topologies. A more detailed reference on this topic has been listed in [2].

In [2] the authors formulate the logical topology design problem as a nonlinear optimization problem. The objective considered was either delay minimization or minimizing the maximum offered load. The authors subdivide the problem into four subproblems. They are: 1) determining a logical topology (logical links); 2) routing the logical links over physical links; 3) assigning wavelengths to the routes; and 4) routing packet traffic on the logical topology. The authors only consider subproblems 1) and 4). Simulated annealing (which is NP-hard) has been used to solve subproblem 1) and flow deviation to solve subproblem 4). The drawbacks of the above approach are as follows. 1) If the network is large then using the simulated annealing approach will be computationally very expensive. 2) It is not an integrated approach to solve the four subproblems; rather, it considers subproblems one and four independently.

In [3] the authors formulated the logical topology design problem as a linear program when the nodes were equipped with wavelength changers. The objective of the formulation was to minimize the average hop length of a logical link, with the hope that the number of wavelength changers used could be reduced and therefore this formulation could be approximated to the formulation with no wavelength changers. The shortcomings of this approach are 1) that it works reasonably only if the traffic matrix is balanced (this is because the objective function does not include any traffic variables), 2) that it works well only if the physical topology is dense in the number of edges, and 3) that it only works when the network size (number of nodes) is small because for large networks solving the linear formulation (integer linear program) is computationally expensive. Note that if the physical topology is sparse (it has few edges) then the number of wavelength changers used could increase (fewer alternate routes) and the resulting logical topology would not reflect the traffic intensities between the nodes. This in turn would increase the amount of packet traffic carried on a per-wavelength basis. The wavelength continuity constraint of [2] could not be introduced in [3] as this would make the problem nonlinear.

In [1] the problem of logical topology design is considered but the number of wavelengths the fiber supports is not a constraint. The drawback in this approach is that the physical topology becomes irrelevant for designing a logical topology.

D. Contribution of This Work

In this paper, we take care of all the drawbacks and shortcomings listed above. We present an *exact linear formulation* [integer (0–1) linear program, or ILP] for designing a logical topology with no wavelength changers. This formulation when solved provides an optimal solution for the logical topology design problem. To the best of our knowledge, this is the first time a linear formulation has been stated which provides an optimal solution to the logical topology design problem. This formulation generalizes the formulations of [2], [3], and [1], in the sense that it includes a set of linear constraints which takes care that a lightpath is assigned the same wavelength on every hop (wavelength continuity constraints). Many parameters (explained in Section II-A) could be passed to the ILP to yield the desired logical topology which minimizes congestion. Since the objective of our linear formulation is minimizing the congestion (maximum offered load on any logical link), the resulting logical topology reflects the traffic intensities between the nodes. Due to linearization, the topology design becomes considerably easier. We also see that the relaxation of the ILP yields lower bounds on the congestion and provides a performance reference for any topology design heuristic algorithm. The number of constraints in our formulation grows as O (number of s–d pairs $\times$ number of edges $\times$ number of wavelengths). Though large, this problem formulation could be useful for moderate-sized networks. For larger networks, we develop heuristic algorithms which use as inputs the solutions obtained by relaxing the integer constraints in the integer linear program.

E. Lower Bounds on Congestion

Congestion as defined earlier is the maximum offered load on any logical link. Congestion may be viewed as a function of the various parameters of the network such as the traffic matrix, number of wavelengths the fiber can support, resources at each node (number of transmitters and receivers), the hop lengths of the logical links, the multiplicity restrictions on the logical topology, the multiplicity restrictions on the physical topology, symmetry/asymmetry restrictions, and the propagation delay [1]. Our linear formulation helps us investigate the lower bounds on congestion for different values of the above parameters. In [1] congestion as a function of the traffic matrix, resources at each node, and propagation delay has been studied. Here we consider other parameters: the number of wavelengths available and the hop lengths of the logical links, symmetry/asymmetry conditions, and multiplicity restriction. We do not consider the propagation delay [1] as it makes the formulation nonlinear. We compare the bounds obtained previously with the bounds obtained by inclusion of the above mentioned parameters.

F. Outline of the Paper

In Section II we give a precise formulation of the logical topology design problem with no wavelength changers as a mixed-integer (0–1) linear programming problem (MILP). In Section III we introduce different constraints for the various cases. We introduce the aggregate formulation in Section III-C and a cutting plane in Section III-C-1 for the aggregate formulation to improve the lower bounds on congestion. In Section IV we give examples to show the interrelationship between the various parameters. In Section IV-C we prove that under certain conditions we can use equality logical link constraints instead of inequality logical link constraints without affecting congestion. In Section V we explain the rounding heuristics and the wavelength assignment heuristics that are to be used on the solutions obtained by solving the MILP with the integer constraints being relaxed (relaxed problem). In Section VI we

consider a moderately large network (NSFNET), which has also been studied by previous researchers, and solve the relaxed problem for this case and apply our logical topology design algorithms. This yields upper bounds on congestion and on the number of wavelengths. Numerical values are listed in the tables for two different traffic matrices.

II. Problem Formulation

We now formulate the logical topology design problem as a mixed integer linear program (MILP). This formulation yields a $LT(\Delta, \leq asym, Q, H, F)$ logical topology. We shall see in Section III how we can modify the constraints to account for any desired logical topology. We use the following notation.

- s,d source and destination of a packet, when used as superscripts;
- i, j originating and terminating node of a logical link (lightpath);
- q qth multiple logical link between nodes terminating a logical link;
- l, m endpoints of a physical link;
- k wavelength number, when used as a superscript.

A. Parameters

- N number of nodes in the network;
- TM $= (\lambda^{sd})$ is the traffic matrix, i.e., λ^{sd} is the arrival rate of packets at s that are destined for d;
- P_{lm} existence of a physical link in the physical topology. If $P_{lm} = 1$ then there is a fiber link between nodes l and m, otherwise P_{lm} is 0;
- HM maximum hop matrix. $HM_{i,j}$ denotes the maximum number of hops that a logical link between node i and j is permitted to take. If Y intermediate wavelength routing nodes have to be configured for establishing a logical link between node i and node j then the hop length of that logical link is $Y + 1$. Let $H = \max_{i,j} HM_{i,j}$;
- F number of wavelengths the fiber can support;
- $\Delta_i^{(t)}$, $\Delta_i^{(r)}$ number of transmitters and receivers, respectively, at node i.

B. Variables

1) Logical link variables: $b_q(i, j) = 1$, if there exists a qth multiple logical link or directed edge (i, j), q, in the logical topology; else $b_q(i, j) = 0$.
2) Wavelength assignment variables:
   a) $C^{(k,q)}(i, j) = 1$, if the qth logical link between node i and node j uses wavelength k; else $C^{(k,q)}(i, j) = 0$.
   b) $C_{l,m}^{(k,q)}(i, j) = 1$, if the qth the logical link between node i and node j uses wavelength k and is routed through physical link (l, m); else $C_{l,m}^{(k,q)}(i, j) = 0$.
3) Traffic intensity variables:
   a) $\lambda_{(i,j),q}^{(sd)}$ denotes the traffic intensity on the qth multiple logical link (i, j) for the traffic between source–destination pair (s, d).
   b) $\lambda_{(i,j),q}$ denotes the total offered traffic on logical link (i, j), q. $\lambda_{\max} = \max_{(i,j),q} \lambda_{(i,j),q}$, i.e., $\lambda_{\max}$ is the maximum flow on any logical link and is termed the *congestion* of the network.

C. Objective

$$\min(\lambda_{\max})$$

Remark: The objective here is minimizing congestion. The motivation for choosing this objective is that the electronic processing (switching speed) requirement is proportional to the congestion. If the switching speeds at the nodes are limited, then minimizing congestion would be appropriate as it would enable the traffic carried per wavelength to increase. In the examples we have solved, we have noticed that if there is heavy traffic between some source–destination pair, then there is a logical link between them; this is a desirable property. This happens because of the objective function, i.e., if there is heavy traffic between node i and node j then because of the objective there would tend to be an edge (i, j) in the logical topology. If this is not the case, then the traffic from node i to node j may have to span many logical links before being delivered to its destination, which tends to increase the congestion.

D. Constraints

1) Logical link degree constraints:

$$\sum_{q=1}^{Q} \sum_{j} b_q(i, j) \leq \Delta_i^{(t)}, \qquad \text{for all } i$$

$$\sum_{q=1}^{Q} \sum_{j} b_q(j, i) \leq \Delta_i^{(r)}, \qquad \text{for all } i$$

$$b_q(i, j) \in \{0, 1\} \quad \text{and} \quad i \in \{0, 1, 2, \ldots N-1\}.$$

Remark: The above constraint ensures that the number of logical links originating (out-degree) and terminating (in-degree) at node i is less than or equal to the number of transmitters and receivers at that node. For the six-node network and the NSFNET, we considered the number of transmitters and receivers at each node are equal and are the same for every node. We denote the number by Δ. We also set $Q = 1$, that is, no multiple logical links allowed.

2) Wavelength continuity constraints:
   a) *Unique wavelength constraints*

$$\sum_{k=0}^{F-1} C^{(k,q)}(i, j) = b_q(i, j), \qquad \text{for all } (i, j), \quad \text{and} \quad q.$$

Remark: This ensures that if logical link $b_q(i, j)$ exists, then only one wavelength is assigned to it, among the F possible choices.

Note that $k \in \{0, 1, 2, 3, \ldots, F-1\}$ where F is the number of wavelengths the fiber supports.

$$C_{l,m}^{(k,q)}(i,j) \le C^{(k,q)}(i,j), \qquad \text{for all } (i,j), (l,m), q \quad \text{and} \quad k.$$

Remark: The above equation ensures that only those $C_{l,m}^{(k,q)}(i,j)$ could be nonzero for which the corresponding $C^{(k,q)}(i,j)$ variables are nonzero. Let color k be chosen for a logical link $(i,j), q$. This implies that $C^{(k,q)}(i,j) = 1$. Then for all the other colors, $w \neq k$, $C^{(w)}(i,j) = 0$. Then the above constraint would force $C_{l,m}^{(w)}(i,j) = 0$, for all (l,m) and $m \neq k$.

b) *Wavelength clash constraints*

$$\sum_{q} \sum_{(i,j)} C_{l,m}^{(k,q)}(i,j) \le 1, \qquad \text{for all } (l,m) \quad \text{and} \quad k.$$

Remark: We are summing over all possible logical links $(i,j), q$, at a physical link (l,m) and for a wavelength k. By this we are assured that there is no *wavelength clash* at physical link (l,m), i.e., no two logical links traversing through the physical link (l,m) will be assigned the same wavelength.

c) *Conservation of wavelength constraints*

$$\sum_{k=0}^{F-1} \sum_{l} C_{l,m}^{(k,q)}(i,j) P_{l,m} - \sum_{k=0}^{F-1} \sum_{l} C_{m,l}^{(k,q)}(i,j) P_{m,l} = \begin{cases} b_q(i,j), & \text{if } m = j \\ -b_q(i,j), & \text{if } m = i, \\ 0, & \text{if } m \neq i \end{cases} \quad \text{for all } (i,j), q \quad \text{and } m \quad \text{and } m \neq j.$$

Variables $C^{(k,q)}(i,j)$ and $C_{l,m}^{(k,q)}(i,j) \in \{0, 1\}$.

Remark: The above equation ensures that a wavelength is conserved at every node for a logical link $b_q(i,j)$. We call this the *conservation of wavelength equation* as it is analogous to flow conservation equations in multicommodity flow problems. Let logical link $b_q(i,j)$ use wavelength k. Then by conservation of wavelength constraints there is a path in the physical topology from node i to node j with wavelength k assigned to it.

3) Traffic constraints

a) *Traffic Routing Constraints*

$$\lambda_{(i,j),q}^{(s,d)} \le b_q(i,j) \lambda^{(s,d)}, \qquad \text{for all } (i,j), (s,d) \quad \text{and} \quad q.$$

Remark: This ensures that the variable $\lambda_{(i,j),q}^{(s,d)}$ can have a nonzero value if there exists a logical link $(i,j), q$ ($b_q(i,j) \neq 0$). The traffic on logical link $(i,j), q$ between the source–destination pair (s,d) is upper-bounded by the total flow of traffic $\lambda^{(s,d)}$ between (s,d).

$$\lambda_{(i,j),q} = \sum_{\forall (s,d)} \lambda_{(i,j),q}^{(s,d)}, \qquad \text{for all } (i,j) \quad \text{and} \quad q.$$

$$\lambda_{(i,j),q} \le \lambda_{\max}, \qquad \text{for all } (i,j) \quad \text{and} \quad q.$$

Remark: The above two equations ensure that the load on any logical link is no greater than the maximum load $\lambda_{\max}$, which is being minimized.

b) *Flow Conservation*

$$\sum_{q} \sum_{j} \lambda_{(i,j),q}^{(s,d)} - \sum_{q} \sum_{j} \lambda_{(j,i),q}^{(s,d)} = \begin{cases} \lambda^{(s,d)}, & \text{if } s = i \\ -\lambda^{(s,d)}, & \text{if } d = i, \\ 0, & \text{if } s \neq i \text{ and } d \neq i. \end{cases} \quad \text{for all } (s,d)$$

Remark: The above is a flow conservation equation at each node for the traffic between nodes s and d.

4) Hop Bound Constraints

$$\sum_{l,m} C_{l,m}^{(k,q)}(i,j) \le H_{(i,j),q}, \qquad \text{for all } (i,j), q \quad \text{and} \quad k.$$

Remark: Since we are summing over all the physical links (l,m) this ensures that the number of hops in a logical link is bounded by $H_{i,j}$.

Observation: The MILP is NP-hard. If in the MILP the integer constraints are replaced by their continuous counterparts, i.e., $0 \le b_q(i,j) \le 1$, $0 \le C^{(k,q)}(i,j) \le 1$, $0 \le C_{l,m}^{(k,q)}(i,j) \le 1$, the resulting LP is called the *LP-relaxation* of the MILP. Since our formulation is a minimization problem the (objective) value of the LP-relaxation is a lower bound on the MILP.

III. Formulation for Various Cases

A. Multiplicity in Physical Topology

To account for the multiple fibers in the physical topology, we can modify the constraints of Section II as shown below. Let there be P multiple fibers on link (l,m). We have to modify the variable $C_{l,m}^{(k,q)}(i,j)$ to $C_{(l,m),p}^{(k,q)}(i,j)$, where $p \in \{1, 2, 3, \ldots P\}$. The conservation of wavelength constraints would have to suitably modified to take this additional factor into account.

B. Symmetry Constraints

We have the following set of constraints to impose the symmetry restrictions.

$$C_{l,m}^{(k,q)}(i,j) - C_{m,l}^{(k,q)}(j,i) = 0, \qquad \text{for all } k, (i,j), q, \quad \text{and} \quad (l,m).$$

Remark: The above ensures that if there are n logical links from node i to node j then we will have n logical links from node j to node i. And also that the routing and wavelength assignment for the lightpaths associated with the logical links between node i and node j traverse the same set of physical links and are assigned corresponding wavelengths (if nth logical link from node i to node j is assigned wavelength f then nth logical link from node j to node i is also assigned wavelength f).

C. Aggregate Formulation

In the MILP's stated above, we have considered the traffic $\lambda^{(s,d)}$ between each source–destination (s,d) pair as a commodity. This is usually referred to as a disaggregate formulation [6]. We can get a more tractable aggregate MILP formulation by identifying a commodity with each source, rather than each (s,d) pair as follows. Let $\lambda^{(s)} = \sum_d \lambda^{(sd)}$ be the total traffic

from source s, $\lambda^{(s)}_{(i,j),q}$ be the arrival rate of packets from source s on the qth logical link (i, j), $\lambda_{(i,j),q}$ the arrival rate of packets on the qth multiple logical link (i, j) from all sources, and $\lambda_{\max}$ the maximum load on any logical link, *viz.*, the congestion, which we seek to minimize. Then, in the aggregate MILP formulation, the logical link degree constraints and the wavelength continuity constraints remain the same; only the traffic constraints have to be modified. The modified traffic constraints are as follows.

1) Traffic routing constraints:

$$\begin{aligned}
&\lambda^{s}_{(i,j),q} \le b_q(i,j)\lambda^{(s)}, && \text{for all } (i,j),\ q \text{ and } s\\
&\lambda_{(i,j),q} = \sum_{s} \lambda^{(s)}_{(i,j),q}, && \text{for all } (i,j),\ q\\
&\lambda_{(i,j),q} \le \lambda_{\max}, && \text{for all } (i,j),\ q.
\end{aligned}$$

2) Flow conservation:

$$\sum_{q}\sum_{j} \lambda^{(s)}_{(i,j),q} - \sum_{q}\sum_{j} \lambda^{(s)}_{(j,i),q} = \begin{cases} \lambda^{s} & \text{if } s = i\\ -\lambda^{(s,i)}, & \text{if } s \ne i, \text{ for all } s \text{ and } i. \end{cases}$$

Cutting Plane: Following [6] we add the following cutting plane to the MILP:

$$\lambda_{\max} \ge \sum_{s} \lambda^{(s)}_{(i,j),q} + \lambda^{(L)}_{\max}\left(1 - b_q(i,j)\right), \quad \text{for all } (i,j).$$

Here $\lambda^{(L)}_{\max}$ is any *a priori* lower bound on $\lambda_{\max}$, like the minimum flow tree bound [1].

We note that the above is superfluous in the MILP but (usually) becomes active in the relaxation of the MILP (an LP). We solve the LP and get a lower bound, say, $\lambda^{(L)}_{\max}(1)$. Iteratively, we can then set $\lambda^{(L)}_{\max} = \lambda^{(L)}_{\max}(i)$, $i \ge 1$ and solve the LP-relaxation to get an improved lower bound $\lambda^{(L)}_{\max}(i+1)$. We will refer to these bounds as the *iterative LP-relaxation lower bounds*. It is to be noted that the LP-relaxation lower bounds in [1] are obtained when there is no restriction on the number of wavelengths, and the hop length. We could then use the lower bound of [1] as an *a priori* lower bound on $\lambda_{\max}$ in our LP-relaxed problem.

IV. Examples for Various Cases

We give examples to demonstrate how symmetric/asymmetric restriction, multiplicity, wavelength and hop-bound constraints affect congestion. In the notation for the logical topology, if some of the arguments are dropped, then those arguments are free parameters in the MILP. It is interesting to note that if we drop the last two arguments, that is, H and F, the logical topology design is no longer dependent on the physical topology. We use the notation $\Lambda(., TM)$ to denote the optimal congestion value obtained by solving the MILP. The first argument inside the parenthesis is the logical topology desired and the second argument is the traffic matrix to be routed on the logical topology. These are the parameters passed to the MILP. For example $\Lambda(LT(\Delta, = sym, 1, \infty, \infty), TM)$ is the congestion obtained by solving the MILP, with the restriction that the logical topology is Δ-regular symmetric, has no multiple edges allowed and the traffic matrix to be routed is TM. But there is no restriction on the number of wavelengths and on the number of hops taken by a lightpath.

TABLE I
Traffic Matrix, $T1$

Nodes	0	1	2	3
0	0	0	1	0
1	1	0	0	1
2	0	1	0	1
3	1	1	0	0

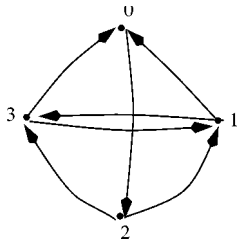

Fig. 4. $\Lambda(LT(2, \le, asym, 1), T1) = 1.00$.

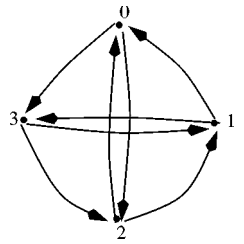

Fig. 5. $\Lambda(LT(2, =, asym, 1), T1) = 1.33$.

In [1], various logical topologies are given which are of the type $LT(\Delta, =, asym, 1)$. We note that the logical link constraints are equality constraints. This forces the restriction that every node should use all its transmitters and receivers even though it may not be required. We shall give examples to show the surprising result that *by using inequality degree constraints, we get lower congestion than that obtained by using equality degree constraints*. By using equality logical link constraints, we enforce that a regular directed graph is the logical topology on which we route packet traffic. Due to the regular nature of the logical topology the packet traffic for a given source–destination pair may traverse more logical links than necessary. This behavior tends to increase congestion. We shall derive the conditions under which we can use equality degree constraints instead of inequality degree constraints, without affecting congestion.

A. Equality/Inequality Constraints

We have to find a logical topology which minimizes congestion. The traffic matrix $(T1)$ between the nodes is given in Table I. On solving the MILP we obtained the logical topology shown in Fig. 4. Thus $\Lambda(LT(2, \le, asym, 1), T1) = 1$. But if we had solved the MILP with equality degree constraints then the congestion value obtained is 1.33. That is $\Lambda(LT(2, =, asym, 1), T1) = 1.33$ (refer to Fig. 5). The above example shows that equality constraints may at times increase congestion. The MILP solution for $\Lambda(LT(2, \le, sym, 1), T1) = \lambda(LT(2, =, sym, 1), T1) = 1.5$ (refer to Fig. 6).

TABLE II
TRAFFIC MATRIX, $T2$

nodes	0	1	2	3
0.	0	1	1	0
1.	1	0	0	1
2.	1	0	0	1/3
3.	0	1	1/3	0

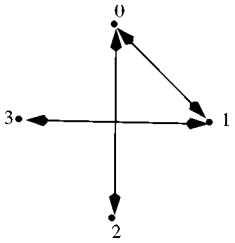

Fig. 6. $\Lambda(LT(2, \leq, sym, 1), T1) = 1.5$.

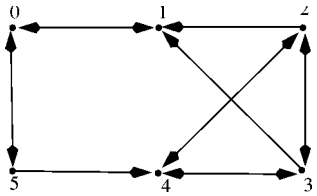

Fig. 7. $\Lambda(LT(3, \leq, sym, 1), T2) = 1.00$.

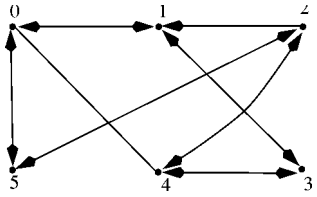

Fig. 8. $\Lambda(LT(3, =, sym, 1), T2) = 1.33$.

Given below are examples when we consider symmetric logical topologies. Consider the traffic matrix $(T2)$ given in Table II. In this case $\Lambda(LT(3, \leq, sym, 1), T2) = 1.0$. The logical topology corresponding to this is shown in Fig. 7. $\Lambda(LT(3, =, sym, 1), T2) = 1.25$ and the logical topology is shown in Fig. 8.

B. Multiplicity Constraints

We can also construct examples where multiplicity constraints affect the congestion value. Assume the traffic matrix shown in Table III is to be routed. We note that $\Lambda(LT(3, =, sym, 1), T3) = 1.0$, in Fig. 9. And $\Lambda(LT(3, =, sym, 2), T3) = 0.667$, in Fig. 10. By permitting multiple edges the congestion reduces by a third. For particular instances we have the following relations: $\Lambda(LT(\Delta, \leq, sym, 1), TM) < \Lambda(LT(\Delta, =, sym, 1), TM)$ and $\Lambda(LT(\Delta, \leq, asym, 1), TM) < \Lambda\ (LT(\Delta, =, asym, 2), TM)$. But in the general case the inequalities are not strict.

C. Two Theorems on Multiplicity and Equality/Inequality Constraints

The equality logical link constraints help in reducing the dimensionality of the search space in finding an optimal solution and hence speed up the MILP. Hence it is important to know when we can use equality constraints without affecting congestion. The two theorems given below give us a criterion for imposing equality constraints.

TABLE III
TRAFFIC MATRIX, $T3$

Nodes	0	1	2	3	4	5
0	0	1	0	0	0	1
1	1	0	1	1	0	0
2	0	1	0	1	1	0
3	0	1	1	0	1	0
4	0	0	1	1	0	1
5	1	0	0	0	1	0

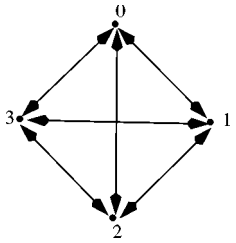

Fig. 9. $\Lambda(LT(3, =, sym, 1), T3) = 1.00$.

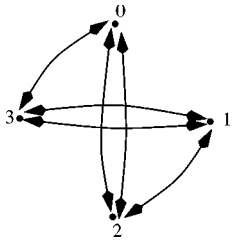

Fig. 10. $\Lambda(LT(3, =, sym, 2), T3) = 2/3$.

Theorem 1: Consider a undirected graph with maximum degree Δ and let its underlying simple graph have minimum degree δ. Let λ be the minimum congestion achieved for a given traffic matrix by routing traffic on the undirected graph. Then we can always construct a regular graph which achieves the same congestion if the maximum allowed multiplicity $Q = \Delta - \delta + 1$, and $N \times \Delta$ is even. N denotes the number of nodes in the graph.

Proof: In the undirected graph, we add edges between deficient degree nodes. We can always do this whenever $Q = \Delta - \delta + 1$. We stop adding edges to deficient degree nodes when we are left with at most one deficient degree node. We note that because Δ and N are not simultaneously odd, the deficiency degree of this node must be even. Let the deficiency in degree of the node be ϕ. We remove $\phi/2$ edges in the graph which do not have the deficient degree node as one of their end points. For each edge removed, we add the following two edges. Let xy be the removed edge and let z be the deficient degree vertex. We add the edges xz and zy. By repeating the aforementioned step for all the removed edges the deficiency of degree is satisfied. The new graph is Δ-regular and has a maximum multiplicity $Q = \Delta - \delta + 1$. The congestion is the same because the traffic on the removed edges (say, xy) now flows through the new paths (edges xz and zy).

Corollary: $\Lambda(LT(\Delta, \leq, sym, \Delta - \delta + 1)) = \Lambda(LT(\Delta, =, sym, \Delta - \delta + 1))$ when $N \times \Delta$ is even, where N is the number of nodes in the network.

Proof: Since the logical topology is symmetric we construct a new graph wherein each bidirectional edge represent a undirected edge. That is the edges $\overrightarrow{xy}$ and $\overrightarrow{yx}$ are replaced by undirected edge xy, where x and y are the nodes in the logical topology. We now apply Theorem 1 to this undirected graph.

Theorem 2: Consider a directed graph with maximum degree Δ and let its underlying simple directed graph have minimum degree δ. Let λ be the minimum congestion achieved for a given traffic matrix on that directed graph. Then we can always construct a directed regular graph which achieves the same congestion if the maximum allowed multiplicity $Q = \Delta - \delta + 1$.

Proof: To make up for the deficiency in degree of the vertices, we can add edges to degree-deficient vertices. From the above step we may assume that we would be left with at most one vertex which has both in-degree deficiency and out-degree deficiency or there may be more than one vertex which all have in-degree or out-degree deficiency. The latter cannot happen because in that situation the condition for a directed graph "sum of in-degrees = sum of out-degrees" would be violated. It also cannot be the case that the deficient degree vertex has out-degree deficiency of the vertex $\neq$ the in-degree deficiency of the vertex for the same reason. It follows that the deficiency of in-degree should be equal to the deficiency of out-degree. Let the in-degree deficiency = out-degree deficiency = ϕ. We adopt the following procedure to satisfy the deficiency of the vertex to make the graph Δ-regular. Pick ϕ edges in the deficient graph such that the picked edges are not originating or terminating in the deficient vertex. For every one of the ϕ edges we add the following two edges. One edge is from the origin node of the picked edge to the deficient degree node. Another edge is from the deficient degree node to the destination node of the picked edge. This procedure constructs a Δ-regular directed graph with multiplicity $Q = \Delta - \delta + 1$ which has the same congestion.

Corollary: $\Lambda(LT(\Delta, \leq, asym, \Delta - \delta + 1)) = \Lambda(LT(\Delta, =, asym\Delta - \delta + 1))$

Proof: By Theorem 2 we can construct an instance of $LT(\Delta\ =, asym, \Delta - \delta + 1)$ with the same congestion obtained on $(LT(\Delta \leq, asym, \Delta - \delta + 1))$.

The import of the two theorems is that if multiple edges is not a constraint in the logical topology design, then it does not matter if we have equality logical link constraints instead of inequality logical link constraints.

D. Wavelengths and Hop Bound Constraints

Here we solve the MILP exactly and show the effect of the number of wavelengths and hop bound constraint on congestion. The parameters passed on to the MILP are $LT(\Delta, =, asym, 1, H, F)$ and the traffic matrix. This configuration was chosen to compare the results presented in [1] with the results presented here. The six-node network and the traffic matrix considered here are the same as that considered in [1, Fig. 4, Table I]. We solve the MILP formulation of Section II for this network by varying the various parameters (refer to Section II-A) and the solutions obtained are given in Table IV. We considered three parameters, the hop length (hop bound), the number of wavelengths, and the degree of the logical topology to be designed. The degree column in Table IV denotes the number of transmitters or receivers present at each of the nodes. The wavelength column denotes the number of wavelengths available. The hop bound column denotes the maximum number of hops any logical link is allowed to take. The lower bound (LB) column denotes the iterative LP-relaxation bound (25 iterations) and is called the LP-bound on congestion. In Table IV "*" indicates no restriction for that particular column parameter. In the wavelength column the first entry for a given degree is the first time the MILP becomes feasible. For example for the degree four case the LP-relaxation is infeasible for $F < 3$ and hence the entry in the wavelength column for the degree 4 case starts with the number of wavelengths being three. We note that since we considered three parameters, the degree, the number of wavelengths and number of hops permissible, many combinations were possible. We present results for some of the combinations. If a row entry has a "*" in the wavelength and in the hop bound column then the LP-bound obtained for congestion in that row will be called the *unconditional congestion*. Similarly the MILP solution will be termed the *unconditional MILP congestion*. For example in the degree 4 case the unconditional congestion is 0.887 and the unconditional MILP congestion is 0.887. The MILP solution is said to be *exact* when either the LP-relaxation solution for congestion and a feasible solution obtained by solving the MILP are equal, or a solution is obtained for the MILP through exhaustive search of the feasible set of integer solutions. For example for the degree one case the unconditional congestion is 5.92 but by exhaustive search the MILP solution obtained is 7.077 for the congestion. In the degree 5 case the unconditional congestion is 0.710 and the branch and bound routine for solving the MILP was terminated after a solution with congestion 0.710 was found (hence this solution is exact).

TABLE IV
RESULTS FOR SIX-NODE NETWORK

Δ	F	Hop Bnd	LB λ_{max}	MILP λ_{max}
1	1	1	5.92	7.36 (Exact)
1	1	2	5.92	7.077 (Exact)
1	*	*	5.92	7.077 (Exact)
2	1	1	2.042	2.34 (Exact)
2	1	2	2.042	2.21 (Exact)
2	2	2	2.042	2.042 (Exact)
2	*	*	2.042	2.042 (Exact)
3	2	*	1.183	1.183 (Exact)
3	*	*	1.183	1.183 (Exact)
4	3	*	0.887	0.887 (Exact)
4	*	*	0.887	0.887 (Exact)
5	4	*	0.710	0.710 (Exact)
5	*	*	0.710	0.710 (Exact)

In Fig. 11 is shown a possible logical interconnection between the wavelength routing nodes of the six-node network. In this figure if node 0 wants to send data to node 1 then the data undergo an optical to electronic and electronic to optical conversion, at nodes five, four, three and two. This topology was obtained by solving the MILP by fixing the number of wavelengths that can be used to one, the in-degree and out-degree of each of

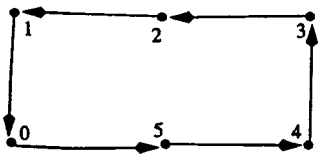

Fig. 11. $\Delta = 1,\ F = 1$, hop-bound ≤ 1, $\lambda_{\max} = 7.36$.

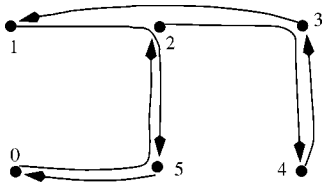

Fig. 12. $\Delta = 1,\ F = 1$, hop-bound ≤ 2, $\lambda_{\max} = 7.077$.

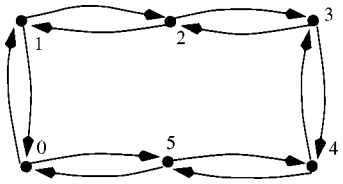

Fig. 13. $\Delta = 2,\ F = 1$, hop-bound ≤ 1, $\lambda_{\max} = 2.340$.

the nodes in the logical topology to one (note that this would imply that each node is equipped with one transmitter and one receiver) and the maximum hop-length to one. In this situation there can only be two possible solutions for the logical topology design, the clockwise ring $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 0$ or the anti-clockwise ring $0 \rightarrow 5 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 0$. For the clockwise ring the congestion is 9.36 and for the anti-clockwise ring the congestion is 7.36. Thus the congestion value obtained by solving the MILP was 7.36 and the logical topology obtained was the anti-clockwise ring.

By changing the maximum hop length from one to two and by fixing all other parameters as in Fig. 11 the logical topology and the congestion obtained by solving the MILP are shown in Fig. 12. We see that by increasing the hop length the congestion is reduced from 7.36 to 7.077. The unconditional MILP congestion for this case is 7.077. Therefore when the number of wavelengths available is one and the maximum hop length permitted is less than or equal to two an *optimal solution* is obtained, i.e., the congestion cannot be decreased further by increasing the number of wavelengths and the number of hops permitted.

In Fig. 13 is shown the logical topology and routes of the logical links obtained by solving the MILP when the degree is fixed at two, the number of wavelengths permitted is one and the hop bound (the maximum hop length permitted) is one. A congestion of 2.340 was obtained by exhaustive search of the search space by a branch and bound routine.

Next we increased the hop bound of the logical link to two, keeping all other parameters fixed. The result obtained is shown in Fig. 14. The congestion is reduced to 2.210. This is because of the two-hop logical links the first between node 3 and node 1, and the second between node 4 and node 2, which reduced the congestion on logical links (3,2) and (4,5) of Fig. 13.

In the next case the hop length permissible was the same (two) but the number of wavelengths available was increased to two.

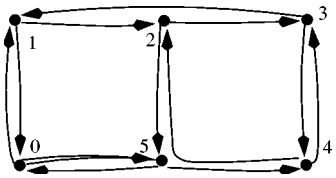

Fig. 14. $\Delta = 2,\ F = 1$, hop-bound $<= 2$, $\lambda_{\max} = 2.210$.

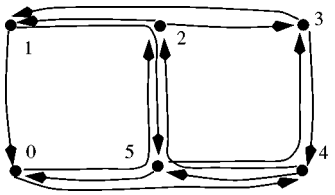

Fig. 15. $\Delta = 2,\ F = 2$, hop-bound $<= 2$, $\lambda_{\max} = 2.042$.

The resulting logical topology with routes is shown in Fig. 15. Since the number of wavelengths permitted was increased to two many more routes were possible thereby reducing the congestion to 2.042 which is the unconditional congestion. Thus the congestion cannot be decreased further by increasing the number of wavelengths and the number of hops permitted.

V. Topology Design Algorithms

A. Rounding Heuristic

In order to obtain a feasible logical topology with routing and wavelength assignment we consider the solutions of the relaxed MILP [LP obtained by relaxing the integer constraints on $b(i, j)$, $C^{(k)}(i, j)$ and $C^{(k)}_{l,m}(i, j)$].

In this solution we first round $b(i, j)$s to 1 or 0.

Various schemes are proposed in [1] for rounding the logical link variables, $b(i, j)$s. We employ the following algorithm to round the logical links variables. The most natural way to construct a 0–1 MILP feasible solution from the LP solution obtained by relaxing the integer constraints is to sequentially set the variables whose value is closer to one to one, and variables whose value is closer to zero to zero, while maintaining feasibility. The rounding algorithms given here essentially do this. List the $b(i, j)$s obtained by iterative LP-relaxation (number of iterations were 25) in decreasing order. Round each successive value of $b(i, j)$ to one if the degree constraints are not violated, and to zero otherwise. Note that this is called LPLDA in [1]. After rounding $b(i, j)$s we now have a logical topology without the routes and wavelength assignment for logical links. We solve the LP with the traffic constraints only (Section II-D) with the objective being to minimize congestion. Thus we get an upper bound on congestion for the logical topology.

To obtain the lightpaths for the logical links we now round the $C^{(k)}(i, j)$ and $C^{(k)}_{l,m}(i, j)$ variables. We use the following *max-round algorithm* for rounding $C^{(k)}(i, j)$.

Max-Round Algorithm: If $b(i, j) = 0$ then $\{C^{(f)}(i, j) = 0;$ for all $f\}$. If $b(i, j) = 1$ then $\max_f C^{(f)}(i, j)$ is set to one and the rest to zeros. A tie is broken by choosing the largest index f.

Let us assume that by the rounding heuristic mentioned above we have set $b(i, j) = 1$ and $C^{(f)}(i, j) = 1$. By the conservation of wavelength equations we are guaranteed that between node i and node j there is at least one path with wavelength assignment f.

Among the possible set of paths from node i to node j with wavelength assignment f we use the following algorithm to pick a path.

- Step 1: Let $\max_m C^{(f)}_{i,m}(i,j)$ be $C^{(f)}_{i,l}(i,j)$
- Step 2: If $l = j$ then stop. Else let $\max_n C^{(f)}_{l,n}(i,j)$ be $C^{(f)}_{l,p}(i,j)$
- Step 3: If $p = j$ then stop. Else assign p to l and continue with step 2.

After this we set $C^{(f)}_{(l,m)}(i,j) = 1$ if it is in the picked path, $C^{(f)}_{(l,m)}(i,j) = 0$, otherwise.

B. Wavelength Assignment Heuristic

We now have the logical topology, the sequence of hops (physical edges the logical links takes), and also a tentative wavelength assignment. But the assignment is still not free of *wavelength clash*, i.e., two logical links may have the same wavelength assigned to it and have a common hop (physical edge in common). There are two approaches for wavelength clash-free assignment. *1:* We develop heuristic algorithms to assign wavelengths to the logical links which have a wavelength clash, and come up with a clash-free wavelength assignment. *2:* Since we know the paths taken by the logical links, we can construct a path-graph as done in [4], and [5]. We note that the chromatic number of the path-graph is the number of wavelengths required. Both approaches were tried and not much difference was found between them. In this paper we have used the second approach. Many heuristics are given in [5] and in [4] for vertex coloring a graph. We note that in [11] a greedy algorithm is described for wavelength assignment. In this paper we present results for one of the four heuristics described in [4] to obtain a valid coloring of the path graph. The results obtained with the other three heuristics were similar. For the sake of completeness we now describe the heuristic from [4] for which we will present the results. We first sort the nodes as follows. Let N denote the number of nodes in a path graph G. *step 0 (Initialization)*: Let $k = N$, and let the current graph be the path-graph $G(.)$. *step 1*: Pick the node with the smallest degree, say node i, in the current graph. Let $\texttt{position}[i] = k$. *step 2*: Replace the current graph with the graph obtained by removing node i. If the remaining graph is a trivial graph (one vertex remains), stop. Else, set $k \leftarrow k-1$ and go to *step 2*.

Once the nodes of the path-graph are sorted, they are colored by sequentially descending the list of nodes and assigning the first available color to each node in the list.

VI. Practical Example

Here we consider a moderately large network, the 14-node National Science Foundation network [1, Fig. 6], shown in Fig. 16. To solve the MILP and the LPs we used IBM's Optimization Subroutine Library (OSL) routines on an IBM 43P/RS6000.

A. NSFNET

The NSFNET, shown in Fig. 16 [1, Fig. 6], is a 14-node network with 21 edges. Each edge represents a pair of directional edges. The pair of directional edges represents a pair of fibers,

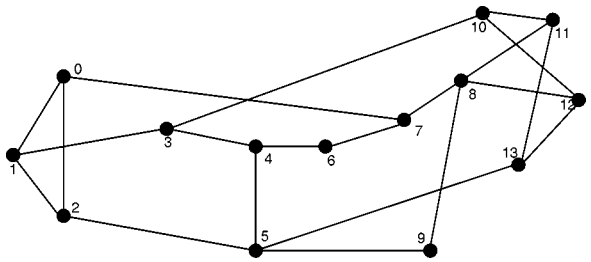

Fig. 16. National Science Foundation network.

one in each direction. The results reported here are for the case where the logical link constraints are satisfied with equality, there is no symmetry requirement, multiplicity is not allowed, and there are no hop bound constraints. The number of wavelengths, the degree of the logical topology to be designed, the traffic matrix, and the physical topology are the parameters considered for the above case. The above configuration was mainly chosen so as to compare our topology design algorithm with that given in [1], [7]. Tables V and VI give the results for two different traffic matrices $P1$ and $P2$. They are the same traffic matrices as used in [1, Tables III and IV, respectively].

1) Feasibility of LP-Relaxation: If we had an MILP with inequality degree constraints, then we would always have a feasible solution if the physical topology has a directed Hamiltonian circuit. (In this case, the edges occurring in the Hamiltonian circuit constitute a logical topology with degree 1.) But since we are insisting on equality degree constraints so as to speed up the MILP by reducing the dimensionality of the search space, it is important to keep track of the feasibility of the MILP. Since the decision problem (feasibility problem) for the MILP is NP-hard and the network sizes are not small, at best we can check for feasibility of the LP-relaxation. An MILP or an LP is feasible if there is an assignment of variables which satisfies the constraints. Let $f_{\min}(\Delta)$ denote the minimum number of wavelengths that are required to make the LP-relaxation feasible. Note that Δ is the degree of the logical topology to be designed. We observe that for N nodes the number of source–destination pairs is $N(N-1)$. In the worst case if we allocate one wavelength for each (s, d) pair then we require $N(N-1)$ wavelengths. The range of F, the number of wavelengths available, for the problem to become feasible, is between 1 and $N(N-1)$. We could do a binary search, taking logarithmic number of steps, in the range $\{1, 2, 3, \ldots N(N-1)\}$, to find $f_{\min}(\Delta)$ for which the LP-relaxation becomes feasible. For example, in Table V we see that $f_{\min}(3) = 2$ and $f_{\min}(13) = 13$.

2) Discussion of Results Obtained: In Tables V and VI under the *LP-Relaxation* heading there are three columns, *viz.*, degree, wavelength , and lower bound on congestion. The degree (Δ) specifies the degree of the logical topology to be designed. Wavelength denotes the number of wavelengths available which is the parameter F (Section II-A). The lower bound (LB) on congestion denotes the solution (the objective value) of the LP-Relaxation. For example, in the degree 2 case with 4 wavelengths, the lower bound on congestion is 126.74. Under the *Heuristic* heading there are two columns, *viz.*, upper bound (UB) on congestion and the number of wavelengths. As explained in Section V we employ the rounding heuristic

TABLE V
RESULTS FOR NSFNET: TRAFFIC MATRIX $P1$

LP-Relax		Heur.		Previous work	
	LB	UB	F	LB	UB
Δ F	λ_{max}	λ_{max} :		λ_{max}:[1] :	λ_{max};[7]/[1]
2 1	141.28	519.251	1		
2 2	126.74	273.896	2		
2 3	126.74	210.593	3		
2 4	126.74	145.738	4		
2 *				126.18	147.679 / 243.43
3 2	84.58	139.478	3		
3 3	84.58	93.838	3		
3 4	84.58	84.582	4		
3 *				84.53	88.650/ 102.82
4 2	71.71	92.658	3		
4 3	63.43	92.658	3		
4 4	63.43	70.025	4		
4 *				63.43	65.908 / 82.03
5 4	50.74	57.835	4		
5 5	50.74	50.936	5		
5 *				50.74	51.85 / 53.49
6 4	42.29	52.565	5		
6 5	42.29	49.220	6		
6 6	42.29	44.391	6		
6 *				42.29	42.660 / 44.45
7 5	36.25	43.194	6		
7 6	36.25	36.432	6		
7 *				36.25	36.45 / 36.55
8 6	31.72	32.129	6		
8 7	31.72	31.772	7		
8 *				31.72	31.75 / 32.27
9 7	28.37	28.376	9		
9 8	28.37	28.376	9		
9 9	28.37	28.376	9		
10 8	25.19	25.687	9		
10 9	25.19	25.645	9		
11 10	23.0	23.509	10		
11 11	23.0	23.068	11		
12 11	21.27	21.387	12		
12 12	21.27	21.387	12		
13 13	*20.24*	*20.246*	13		

TABLE VI
RESULTS FOR NSFNET: TRAFFIC MATRIX $P2$

LP-Relax		Heur.		Previous work	
	LB	UB	F	LB	UB
Δ F	λ_{max}	λ_{max} :		λ_{max};[1] :	λ_{max};[7]/ [1]
2 1	284.26	700.47	2		
2 2	284.26	389.93	2		
2 *				282.50	345.42 / 345.42
3 2	189.76	269.43	2		
3 3	189.76	269.43	2		
3 4	189.76	217.80	4		
3 *				189.62	195.71 / 195.71
4 2	142.33	175.18	3		
4 3	142.33	152.99	3		
4 *				142.32	142.33 / 142.33
5 4	*113.87*	*113.87*	4		
5 *				113.87	113.87 / 113.87
6 4	94.89	94.89	5		
6 5	94.89	94.89	5		
6 *				94.89	94.89 / 94.89
7 5	81.33	81.33	6		
7 6	81.33	81.33	6		
7 *				81.33	81.33 / 81.33
8 6	*71.17*	*71.17*	6		
8 *				71.17	71.17 / 71.17
9 7	62.15	63.26	9		
9 8	62.15	62.15	8		
10 8	56.93	56.93	10		
10 9	56.93	56.93	10		
11 10	*51.75*	*51.75*	10		
12 11	47.44	47.44	13		
12 12	47.44	47.44	13		
12 13	47.44	47.44	13		
13 13	*43.79*	*43.79*	13		

(Section V-A) on the solutions obtained by solving the LP-Relaxation to get a feasible set of integer solutions, i.e., we now have a Δ-regular logical topology and the routes for the logical links of the logical topology. We then obtain an upper bound on congestion by solving an LP (Section V). Since we now have the routes for the logical links we obtain a wavelength-clash free assignment by using the wavelength assignment heuristic algorithm (Section V-B). The number of wavelengths required is tabulated in the wavelength column under the *Heuristic* heading.

Under the *previous work* heading the results mentioned in the lower bound column are from [1]. In [1] and [7] some heuristic algorithms were developed to round the $b(i, j)$ variables to design the logical topology. A shortest path route for the logical links in the logical topology was chosen because the number of wavelengths available was not a constraint in [1] and [7]. The upper bound on congestion (obtained from the heuristic) shown in the last column is from [7] and [1]. We refer to them as *unconditional lower bound* and *unconditional upper bound*. For example, in Table V, we see that for the degree two case the unconditional lower bound is 126.18 and the unconditional upper bound of [7] is 147.679 and of [1] is 243.43. By our heuristic for the degree 2 case we obtain a congestion of 145.738 with four wavelengths which is an improvement over the previous upper bounds on congestion, even though they did not consider the number of wavelengths as a constraint. However, we do not understand intuitively why this is the case.

In Table V for the degree 3 case and with two wavelengths the lower bound on congestion is 84.58. The upper bound for the same is 139.478. The wavelength assignment heuristic required three wavelengths for a wavelength-clash free assignment for the routes of the logical links in the logical topology obtained by the rounding heuristic. It is not always guaranteed that the LP-relaxation parameter F and the number of wavelengths required by the heuristic are the same. Our heuristic gives an upper bound on the number of wavelengths required for designing the required logical topology. In the above case we need at most three wavelengths for designing a degree 3 logical topology for which the congestion obtained is 139.478. In Table V for the degree 5 case with four wavelengths, the lower bound on congestion is 50.74 and the upper bound is 57.835 and the number of wavelengths required by the heuristic is 4 which is equal to the parameter F. We see that in Table VI for the degree 6 case with four wavelengths, the lower bound on congestion is 94.89

and the upper bound on congestion is also 94.89 but the number of wavelengths required by the heuristic is five. In the same table for the degree 8 case, the parameter F is 6, which is the same as that required by the heuristic and the lower bound and the upper bound on congestion both equal 71.17. In such a situation, we say that the heuristic solutions are *optimal*. In the tables, the optimal solutions are italicized.

It was observed that in many cases the lower bound did not change with the number of wavelengths for a fixed degree. But the heuristic reduced the congestion. This is due to the fact that the LP-relaxation tends to use all the wavelengths available. This helps the rounding heuristic in choosing among many alternate paths and so the congestion drops. For example for the degree 2 case (traffic matrix $P1$) the LP-bound is the same when F is equal to 2, 3, and 4, but the congestion achieved by the heuristic dropped from 273.89 to 145.73.

In many cases the heuristic bounds on congestion obtained by us are an improvement over the previous results but in a very few cases they are worse. The heuristic wavelength assignment algorithm in many cases achieves optimal assignment for the higher degree cases. For example, for the degree 13 case for both the traffic matrices the minimum F for which the LP-relaxation become feasible is feasible 13 and 13 wavelengths are used by the wavelength assignment heuristic. It can be observed that for higher degree the congestion obtained by the heuristic and the lower bound are almost equal. From degree 5 onwards, for the $P2$ traffic matrix the lower bound and the congestion obtained by the heuristic are equal.

3) Complexity: In the MILP the number of constraints and the number of variables grow approximately as $O(N^3 \times F \times$ number of edges $\times$ multiplicity factor). On the average the LP solver took around five minutes on an IBM 43P/RS6000 to solve one iteration of the relaxed MILP problem. The running times of the rounding heuristics were less than a minute.

4) Equality Degree Constraints: It was shown by examples in Section IV that the congestion obtained by using inequality degree constraints would be less than the congestion obtained by using equality degree constraints with all other parameters being the same for both cases. The feasibility of the LP with equality degree constraints would force the requirement of a larger F than necessary. This is certainly a disadvantage but at the same time equality constraints would speed up the branch and bound routine method used for solving the mixed integer linear program. It is our computational experience that in all the cases for the NSFNET example the congestion obtained by the solving the LP with inequality degree constraints was the same as that got by solving the LP with equality degree constraints.

VII. Conclusion

An exact linear formulation was presented for the logical topology design problem with no wavelength changers. The wavelength continuity constraints presented here linearized the problem which hitherto was formulated with a set of nonlinear constraints. The cutting plane of Section III-C-1 when incorporated into the aggregate formulation helped in getting good lower bounds on congestion for the six-node network and NSFNET. In Section III, formulations were presented to take care of symmetry constraints (Section III-B), and multiplicity of optical fibers in the physical topology (Section III-A). In Section IV various examples were shown to illustrate how symmetry/asymmetry affects the logical topology design problem. In Section IV-C two theorems were presented which provided a criterion for determining when equality degree constraints can be used without increasing congestion.

We observe that since we are minimizing congestion in the network the electronic processing of traffic per wavelength decreases. If we have some set of nodes in the network which has a heavy traffic flow among its members then having multiple edges between some pairs of nodes in that set may reduce the overall congestion. This would reduce the electronic processing per wavelength in the network. Reduction in electronic processing of traffic would be beneficial in a high-speed network environment.

For the six-node network, we investigated the tradeoffs among the number of wavelengths the fiber can support, the hop lengths of the logical links and the congestion. We note that the LP-bounds for the six-node network did not vary with the hop bound parameter or with the wavelength parameter. For small networks we could solve the MILP exactly but for networks of larger size like the NSFNET we had to use the topology design algorithms developed in Section V. The performance of the topology design algorithms obtained by rounding the solutions of the LP-relaxation problem was very close to the lower bounds when the logical topology to be designed had a high degree. The rounding heuristic of Section V-A tries to find a route for a logical link which has a large utilization of a wavelength on it. This helps in finding a logical topology which will reduce congestion. The wavelength assignment heuristic given in [4] has been used to assign wavelengths for the logical links. We see that the number of wavelengths needed by the wavelength assignment heuristic (refer Section V-B) and the number wavelengths considered for the LP-relaxation (second column of Tables V and VI) are quite close in many cases. In some of the lower degree cases there is a large difference between the upper and lower bounds on congestion. This calls for for some more cutting planes to be added to the LP-relaxation so as to improve the lower bound. It may also be the case that the heuristic has to be improved. This is still under investigation. In general the iterative LP-relaxation bound for congestion with wavelength continuity constraints did not significantly vary with the parameter F but the rounding of the LP-solution gave good results.

Most importantly, the upper bounds obtained for congestion using our heuristic topology design algorithm were very close to the lower bounds, especially for the higher degree cases, which indicates that this a good heuristic for logical topology design.

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