Design of Non-Uniform Antenna Arrays Using Genetic Algorithm

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Abstract   The performance of a single-element antenna is somewhat limited. To obtain high directivity, narrow beam-width, low side-lobes, point-to-point and preferred-coverage pattern characteristics, etc., antenna arrays are used. An antenna array is an assembly of individual radiating antennas in an electrical and geometrical configuration. Nowadays, antenna arrays appear in wireless terminals and smart antennas, so robust and efficient array design is increasingly becoming nec-essary. In antenna array design, it is frequently desirable to achieve both a narrow beamwidth and a low side-lobe level. In linear antenna arrays, a uniform array yields the smallest beamwidth and hence the highest directivity. It is followed, in order, by the Dolph-Chebyshev and Binomial arrays. In contrast, Binomial arrays usually possess the smallest side-lobes followed, in order, by the Dolph-Chebyshev and uniform arrays.Binomial and Dolph-Chebyshev arrays are typical examples of non-uniform arrays. In this paper we deal only with linear arrays and it is shown that using genetic algorithm it is possible to design a non-uniform array that approximates the beamwidth of a uniform array and having smaller side-lobe level than the Dolph-Chebyshev array. The result is that the designed antenna array exhibits the largest directivity as compared to the uniform, Binomial and Dolph-Chebyshev arrays. In the design, the genetic algorithm is employed to generate the excitation amplitudes of the antenna array.

Keywords   Antenna Arrays, Uniform Arrays, Non-Uniform Arrays, Binomial Arrays, Dolph-Chebyshev Arrays, Ge-netic Algorithm

1. Introduction

The performance of a single-element antenna is somewhat limited. To obtain high directivity, narrow beamwidth, low side-lobes, point-to-point and preferred–coverage pattern characteristics, etc., antenna arrays are used[1]. An antenna array, or simply array, is an assembly of individual radiating antennas in an electrical and geometrical configuration.

The design of an array involves mainly first the selection of elements and array geometry, and then the determination of the element excitations required for achieving a particular performance.

Over the past decades, the theory of antenna array design has been studied in depth and is certainly well docu-mented[1]. But recently the applications of Genetic Algo-rithms (GAs) to engineering electromagnetics[2,4] and an-tenna array design and synthesis[5,9] have attracted much attention.

This paper is organized as follows. The system model is presented in Section 2. Section 3 describes the genetic

algorithm as applied in the context of non-uniform arrays. In Section 4, we provide some representative numerical results. Finally in Section 5 concluding remarks are made.

2. Problem Formulation

Consider a uniformly spaced linear array of N elements places along the z-axis. Supposethe distance between any two adjacent elements is $d$and the array is operated at λ.The array factor is

$$AF(\theta) = \sum_{n=1}^{N} a_n e^{j(2\pi nd \cos\theta)/\lambda} \quad (1)$$

The feeding coefficient a_n can be written as

$$a_n = |a_n| e^{j\alpha_n} \quad (2)$$

In uniform arrays, the elements have equal amplitude and a constant phase difference between adjacent ones, i.e.,

$$a_n = a_o e^{-jn\beta} \quad (3)$$

whereβ is the progressive phase shift.

In binomial and Dolph-Chebyshev arrays, which are typical examples of non-uniform arrays, the amplitude ex-citations are derived from the binomial series and Cheby-shev's polynomials, respectively[1].

As stated in the abstract, in antenna system design it is frequently desirable to achieve both a narrow beamwidth and

a low side-lobe level[10]. A uniform array yields the smallest beamwidth and the highest directivity. It is followed, in order, by the Dolph-Chebyshev and binomial arrays. In contrast, binomial arrays usually possess the smallest side-lobes followed by, in order, by the Dolph-Chebyshev and uniform arrays[1].

The objective of the present paper is to design, using GA, a non-uniform linear array that approximates the beamwidth of uniform arrays and produces smaller side-lobe level than that of Dolph-Chebyshev arrays. The array so designed willposses, as will be shown, the highest directivity than uniform, binomial, and Dolph-Chebyshev arrays.

From Equation (1), the array factor for a non-uniform broadside ($\alpha_n = 0$) array is

$$AF(\theta) = \sum_{n=1}^{N} a_n e^{j(2\pi nd \cos \theta)/\lambda} \quad (4)$$

where, for simplicity, we let $a_n = |a_n|$ since $\alpha_n = 0$ (broadside).

To design the array, first the number of array elements N and the inter-element spacing d are specified. The genetic algorithm generates N excitation coefficients a_n and forms the non-uniform array factor $AF(\theta)$ (Equation 4). By successive generation of a_n's through the GA, the desired array factor is made to approximate a hypothetical array factor $AF_h(\theta)$ in some acceptable sense. The array factor $AF_h(\theta)$ is one which, in the region of the main lobe, is the same as a uniform broadside array with the same N and d, but is zero elsewhere-i.e., with no side-lobes.

$$AF_h(\theta) = \begin{cases} \frac{1}{N} \frac{\sin\left(\frac{\pi Nd}{\lambda}\cos\theta\right)}{\sin\left(\frac{\pi d}{\lambda}\cos\theta\right)}, \text{ for } \cos^{-1}\left(\frac{\lambda}{Nd}\right) \leq \theta . . \\ \quad . . \leq \frac{\pi}{2} + \cos^{-1}\left(\frac{\lambda}{Nd}\right) \\ 0, \text{ otherwise} \end{cases} \quad (5)$$

Since the required non-uniform array $AF(\theta)$ is to approximate $AF_h(\theta)$, the mean-squared error of the difference between $AF(\theta)$ and $AF_h(\theta)$ is passed to the genetic algorithm as an objective or fitness function, $f(\theta; a_1, \cdots, a_N)$, to be minimized, where

$$f(\theta; a_1, \cdots, a_N) = \left\| AF(\theta) - AF_h(\theta) \right\|^2, \text{ or}$$

$$f = \begin{cases} \left\| \sum_{n=1}^{N} a_n e^{j(2\pi nd \cos\theta)/\lambda} - \frac{1}{N} \frac{\sin\left(\frac{\pi Nd}{\lambda}\cos\theta\right)}{\sin\left(\frac{\pi d}{\lambda}\cos\theta\right)} \right\|^2 \\ \text{for } \cos^{-1}\left(\frac{\lambda}{Nd}\right) \leq \theta \leq \frac{\pi}{2} + \cos^{-1}\left(\frac{\lambda}{Nd}\right) \\ \left\| \sum_{n=1}^{N} a_n e^{j(2\pi nd \cos\theta)/\lambda} \right\|^2, \quad \text{otherwise} \end{cases} \quad (6)$$

3. The GA as Applied to Non-UniformLinear Antenna Array Design

The Genetic Algorithm is a method for solving optimization problems that is based on the evolutionists' natural selection concept. The GA repeatedly modifies a population of individual solutions. At each step, the GA selects individuals at random from the current population to be parents and uses them produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. At each step of the iteration, the GA uses the selection, crossover and mutation rules to create the next generation from the current population.

The GA can be applied to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear.

The algorithm begins by creating a random initial population- in our case, a set of random excitation amplitudes in a given range, say, twenty sets of Na_n's. The algorithm then creates a sequence of new populations, or generations. At each step, the algorithm uses the individuals in the current generation to create the next generation. To create the new generation, the algorithm performs the following steps: scores each member of the current population by computing its fitness value. The fitness value of an individual is the value of the fitness function for that individual. The fitness or objective function is the function to be optimized. In the non-uniform array context, the fitness function is the mean-squared error of the difference between the array factors generated by individual members of a generation and the hypothetical side-lobe free uniform array (Equation 6). The algorithm then selects parents with the lowest fitness values. Children are produced from parents either by making random changes to a single parent- mutation, or by combining the vector entries of a pair of parents- crossover. The current population is replaces with the children to form the next generation. Finally the algorithm stops when one of the stopping criteria is met. Usually the stopping criteria is when the number of generations reaches a pre-set maximum number of generations or when the value of the fitness function for the best point in the current population is less than or equal to a pre-set fitness value.

4. Results

As it is well known in optimization problems, the number of local minima gets on exploding as the number of optimization variables increase. This is to be expected in antenna array design, since the number of nulls increase as N increases. Compared to standard optimization algorithms, the genetic algorithm is strong at escaping the local minima[12].

The non-uniform array designed using the genetic algorithm exhibits the highest directivity, will have a narrow beamwidth comparable to uniform array and displays lower side-lobe levels than Dolph-Chebyshev or uniform arrays. In Dolph-Chebyshev arrays, the side-lobe levels are assumed to be 20 dB below the maximum of the major lobe.Some rep-

resentative examples will be considered.

*Example*1: For a linear array with $N = 10$ and $d = 0.25\lambda$, a broadside uniform, binomial, and Dolph-Chebyshev arrays are produced. With the same N and d, a non-uniform array is designed using the genetic algorithm. The result is displayed in Figure 1. The excitation coefficients of the non-uniform array as produced by the genetic algorithm are:

$a_1 = 1.5051$ $a_2 = 0.8778$
$a_3 = -0.4210$ $a_4 = 3.5444$
$a_5 = -0.3286$ $a_6 = 0.9350$
$a_7 = 2.3436$ $a_8 = 0.5822$
$a_9 = 0.2470$ $a_{10} = 1.7174$

The directivities in descending order are shown in Table I.

Table 1. Directivities for Example 1

Array	Directivity
Using GA	5.4382
Uniform	5.1658
Dolph-Chebyshev	4.9253
Binomial	2.6967

As claimed earlier, the array designed using the genetic algorithm exhibits the largest directivity while keeping its beamwidth as narrow as the uniform array.

Example 2: As in Example 1 above, but with $N = 20$ and $d = 0.25\lambda$, the result is displayed in Figure 2. The excitation coefficients are:

$a_1 = 1.4647$ $a_2 = -0.0557$
$a_3 = 0.7709$ $a_4 = 1.6612$
$a_5 = 0.2775$ $a_6 = 1.1205$
$a_7 = 0.9641$ $a_8 = 1.6770$
$a_9 = 0.6786$ $a_{10} = 0.2991$
$a_{11} = 3.1648$ $a_{12} = -0.5235$
$a_{13} = 1.3070$ $a_{14} = 1.6972$
$a_{15} = 1.0998$ $a_{16} = -0.1354$
$a_{17} = 1.7464$ $a_{18} = 1.0624$
$a_{19} = -0.4338$ $a_{20} = 1.4978$

The directivities in descending order are:

Table 2. Directivities for Example 2

Array	Directivity
Using GA	10.1630
Uniform	10.1593
Dolph-Chebyshev	10.0092
Binomial	3.8884

Example 3: When the inter-element spacing d becomes larger, the number of side-lobes proportionally increases. This means the fluctuations between nulls and side-lobe peaks increases and the efficiency of the genetic algorithm decreases. This effect can be illustrated by taking $N = 10$ and $d = 0.5\lambda$. The result is displayed in Figure 3 and the excita-tion coefficients are:

$a_1 = 0.4950$ $a_2 = 0.6905$
$a_3 = 0.7415$ $a_4 = 0.8094$
$a_5 = 0.8379$ $a_6 = 0.8401$
$a_7 = 0.8126$ $a_8 = 0.7458$
$a_9 = 0.6782$ $a_{10} = 0.5018$

The directivities in descending order are:

Table 3. Directivities for Example 3

Array	Directivity
Uniform	9.9980
Using GA	9.7197
Dolph-Chebyshev	9.6203
Binomial	5.3914

Example 4: If the inter-element spacing d becomes smaller, the number of side-lobes proportionally decreases. The array factor becomes smoother. This increases the efficiency of the genetic algorithm. The effect can be illustrated by taking $N = 10$ and $d = 0.15\lambda$. The result is displayed in Figure 4. The excitation coefficients are:

$a_1 = -2.8430$ $a_2 = -1.0364$
$a_3 = -0.5084$ $a_4 = -0.0046$
$a_5 = -3.3524$ $a_6 = -1.5005$
$a_7 = -3.2832$ $a_8 = 1.2169$
$a_9 = -0.0279$ $a_{10} = -4.1288$

The directivities in descending order are:

Table 4. Directivities for Example 4

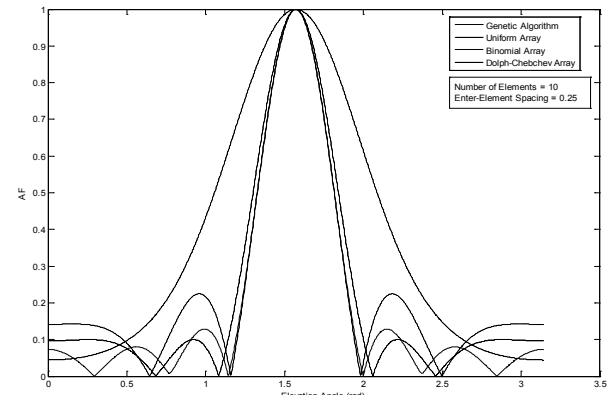| Array	Directivity
Using GA	3.6364
Uniform	3.2045
Dolph-Chebyshev	3.2045
Binomial	1.6826

Figure 1. Magnitude of normalized array factor, |AF|, versus θ for N=10, d= 0.25λ
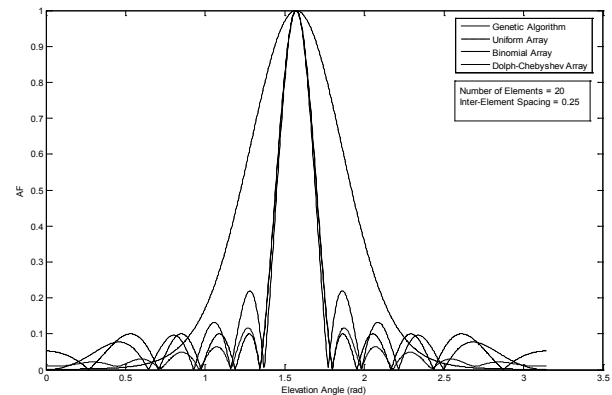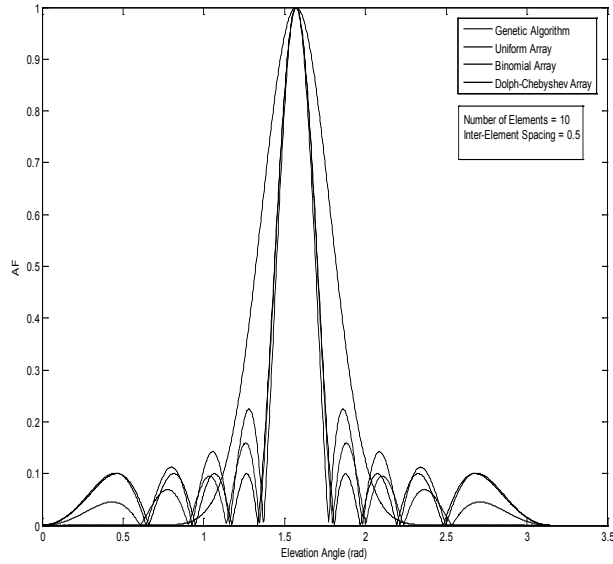

Figure 2. Magnitude of normalized array factor, |AF|, versus θ for N=20, d= 0.25λ

Figure 3.   Magnitude of normalized array factor, |AF|, versus *θ* for *N*=10, *d*= 0.5*λ*
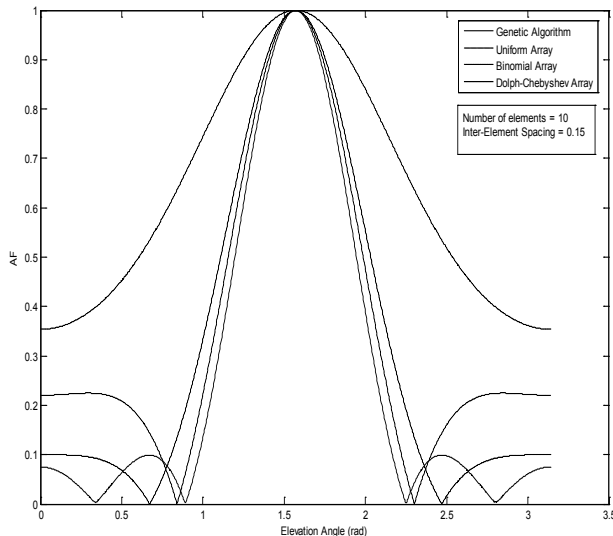

Figure 4.   Magnitude of normalized array factor, |AF|, versus *θ* for *N*=10, *d*= 0.15*λ*5. Conclusions

5. Conclusions

As demonstrated above, genetic algorithm can be used to design an efficient non-uniform array. The designed array exhibits the largest directivity and a beamwidth approximating that of uniform arrays. Besides, the side-lobe level is smaller than that of either uniform or Dolph-Chebyshev arrays.

For inter-element spacing greater than 0.5*λ*, the efficiency of the designed array relatively decreases as opposed to smaller spacing. This is not a problem in practical terms since real arrays will have inter-element spacing less than or equal to 0.5*λ* to avoid large number of side-lobes and perhaps grating lobes.

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