Design of switching junctions for two-dimensional discrete soliton networks

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The performance of switching junctions in two-dimensional discrete-soliton networks is analyzed theoretically by coupled-mode theory. Our analysis can be used for the design of routing junctions with specified operational characteristics. Appropriately engineering the intersection site can further improve the switching efficiency of these junctions. Our analytical results are verified by numerical simulations. © 2001 Optical Society of America

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In recent years, discrete solitons in nonlinear lattices have received considerable attention in many branches of science. In optics, nonlinear waveguide arrays provide an excellent system where these entities can be experimentally studied and possibly used for all-optical applications. In the latter context, discrete solitons (DSs) are self-trapped states that exist as a result of the balance between linear coupling effects and material nonlinearity. 4–8

Recently we investigated the propagation behavior of DSs in two-dimensional networks of nonlinear waveguide arrays. More specifically, we have shown that this family of solitons can be employed to realize intelligent functional operations such as routing, blocking, logic functions, and time gating. These DSs can be navigated anywhere in the network along preassigned array pathways that act as soliton wires. Even more importantly, DSs can be routed at array intersections by use of vector/incoherent interactions with other discrete solitons. In essence, these intersections behave as DS switching junctions. Clearly, because switching junctions are important elements in such two-dimensional DS networks, their analysis and design are issues that need to be addressed.

In this Letter, by employing coupled-mode theory, we analyze the performance of switching junctions in two-dimensional DS networks. The properties of these junctions, i.e., routing efficiency, reflection, and leakage losses, are obtained as a function of the DS phase speed and the intensity of the solitons that control the routing process. Our theoretical analysis can be used to design routing junctions with specific operational characteristics. In addition, we show that, by appropriate engineering of the intersection site, the switching efficiency of these junctions can be further improved. Our analytical results are verified by numerical simulations.

Let us consider an array of nonlinear weakly guiding waveguides made from identical elements separated by distance D. Each waveguide is of the step-index type with a radius ρ and is single mode at the operating wavelength. For illustration, let $\lambda_0=1.5~\mu\text{m}$, $n_2=1.5,~\Delta=2\times10^{-3},~\rho=5.3~\mu\text{m}$, and $D=15.9~\mu\text{m}$, where $\Delta=(n_1-n_2)/n_1$ and where n_1 and n_2 are the core and the cladding refractive indices, respectively. For this set of values, the coupling coefficient between adjacent waveguides is $c=0.279~\text{mm}^{-1}$. The array is

taken to be Kerr nonlinear. In addition, we assume that two different optical waves are interacting incoherently or vectorially in this array system. ^{10,11} In this case, these two mutually incoherent fields evolve according to

$$\begin{split} i\,\frac{\partial U}{\partial z}\,+\,\frac{1}{2k_0n_2}\Big(\frac{\partial^2 U}{\partial x^2}\,+\,\frac{\partial^2 U}{\partial y^2}\Big)\,+\,k_0n_1\Delta f(x,y)U\\ \\ &+\,k_0\gamma I_0(|U|^2\,+\,|V|^2)U\,=\,0\,,\,\,\,\text{(1a)}\\ \\ i\,\frac{\partial V}{\partial z}\,+\,\frac{1}{2k_0n_2}\Big(\frac{\partial^2 V}{\partial x^2}\,+\,\frac{\partial^2 V}{\partial y^2}\Big)\,+\,k_0n_1\Delta f(x,y)V\\ \\ &+\,k_0\gamma I_0(|V|^2\,+\,|U|^2)V\,=\,0\,,\,\,\,\text{(1b)} \end{split}$$

where f(x,y) represents the normalized two-dimensional index distribution of the waveguide array, $k_0=2\pi/\lambda_0$, and γI_0 is the maximum nonlinear index change induced by the soliton at the input. Moreover, for simplicity, the self- and cross-phase modulation coefficients in Eqs. (1) were taken here to be equal (Manakov interaction). The discrete soliton states of each field [in the absence of the other, say, $U\neq 0$ and V=0 in Eqs. (1)] are found numerically by relaxation methods. As an initial condition of the iterative scheme we use the DS solutions obtained from coupled-mode theory. 12

The operation of a DS switching junction involves two different soliton families: the so-called "signals" and "blockers." These two families serve as the basic building elements in this system. A signal DS is a moderately confined state, extending over five to seven sites in a nonlinear waveguide array. In this limit, a DS is known to be highly mobile within the lattice, and its envelope profile is approximately described by a hyperbolic secant function, i.e., $\Psi(n) =$ $\Psi_0 \operatorname{sech}(nD/x_0) \exp(i\alpha nD)$. x_0 is related to the spatial width of this DS, $n = 0, \pm 1...$ is the number of waveguide sites, and α describes the phase tilt that is necessary for this soliton to move along the chain in the transverse plane (x, y). A blocker DS is a strongly confined state that resides almost entirely in one waveguide. This class of soliton is highly immobile. Figures 1(a) and 1(b) depict a typical signal and a typical blocker DS, respectively. Let us now assume that a

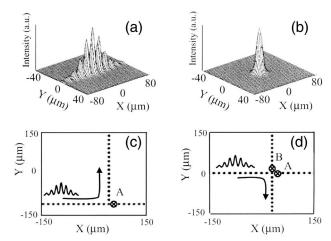


Fig. 1. (a) Moderately confined signal DS; (b) strongly confined DS blocker; (c) routing a signal DS at a T junction when a blocker is positioned at site A; (d) routing a signal DS at an X junction when a blocker is positioned at A and another at B.

blocker is placed at the entry of a particular pathway in the vicinity of a junction, as shown in Figs. 1(c) and 1(d). In that case, a signal DS (mutually incoherent to the blocker), when it is approaching the junction from the left, is effectively rerouted to the unblocked branch, as shown schematically in Figs. 1(c) and 1(d). More importantly, this happens elastically; i.e., the output signal DS remains virtually unaffected after it goes through the junction, whereas the blocker remains in its preassigned position during this interaction. It is important to note that, had the blocker(s) not been positioned at the junction, the signal DS would have been totally disintegrated into reflected and transmitted dispersive waves.

To analyze the switching efficiency of this junction, we use the coupled-mode theory or the tight-binding approximation. A 120° junction is shown in Fig. 2 with a blocker placed at the input of the lower branch. The DS enters the junction from the left arm. The presence of the blocker soliton nonlinearly induces a change $\Delta\beta_1$ in the propagation constant at the site m=1, as can be computed from 11

$$\Delta\beta_1 = \frac{k_0 \int \int \gamma I_0 |E_b(x,y)|^2 |E_0(x,y)|^2 \mathrm{d}x \mathrm{d}y}{\int \int |E_0(x,y)|^2 \mathrm{d}x \mathrm{d}y}, \qquad (2)$$

where $E_b(x,y)$ is the normalized mode profile of the blocker and $E_0(x,y)$ is the field mode of the unperturbed waveguide. Let us also assume for generality that the waveguide at the intersection is different, i.e., that its propagation constant deviates from the rest of the waveguides by $\Delta\beta_0$. In addition, we consider the finite coupling effects κ among the three sites $n=\pm 1$ and m=1. For the set of values given above, $\kappa/c=0.035$ for a 120° junction. From coupled-mode theory, the field amplitudes a_n at every discrete site (except at $n=0,\pm 1$ and m=1) obey $i(\mathrm{d} a_j/\mathrm{d} z)+c(a_{j+1}+a_{j-1})=0$. At these four sites, the fields evolve according to

$$i\frac{\mathrm{d}a_{-1}}{\mathrm{d}z} + c(a_{-2} + a_0) + \kappa a_1 + \kappa a_{m=1} = 0$$
, (3a)

$$i\frac{\mathrm{d}a_0}{\mathrm{d}z} + c(a_{-1} + a_1 + a_{m-1}) + \Delta\beta_0 a_0 = 0,$$
 (3b)

$$i\frac{\mathrm{d}a_1}{\mathrm{d}z} + c(a_0 + a_2) + \kappa a_{-1} + \kappa a_{m=1} = 0$$
, (3c)

$$i \frac{\mathrm{d}a_{m=1}}{\mathrm{d}z} + c(a_0 + a_{m=2}) + \Delta \beta_1 a_{m=1} + \kappa a_{-1} + \kappa a_1 = 0,$$
 (3d)

where the subscript of a_i refers to the n line, whereas that of $a_{m=j}$ refers to the lower m branch. By considering an incident and a reflected component for $n \leq -1$, we can write $a_n = \exp(i\mu z) \times$ $[\exp(i\alpha nD) + r \exp(-i\alpha nD)]$, where r is the field reflection coefficient. The transmitted wave for $n \geq 1$ obeys $a_n = t \exp(i\mu z) \exp(i\alpha nD)$, whereas the wave leaking from the blocker (for $m \ge 1$) is described by $a_m = l \exp(i\mu z) \exp(i\alpha mD)$, where t and l are the transmission and the leakage coefficients, respectively. At the junction (n = 0), the field varies according to $a_0 = Q \exp(i\mu z)$. By substituting the appropriate forms into $i(da_n/dz) + c(a_{n+1} + a_{n-1}) = 0$ (except at $n = 0, \pm 1$ and m = 1), we find that the propagation eigenvalue is given by $\mu = 2c \cos(\alpha D)$. Using this result in Eqs. (3), we can then solve this linear system for the four unknowns, r, t, l, and Q.

Figure 3(a) depicts the power transmission coefficient $T = |t|^2$ of a 120° junction as function of the normalized detuning parameter $\Delta \beta_1/c$ (induced by the blocker) for two phase speeds αD . T was obtained by solution of Eqs. (3) under the condition that $\Delta \beta_0 = 0$. In addition, for the same junction, Fig. 3(b) shows the power reflection and leakage coefficients $R = |r|^2$ and $L = |l|^2$ as a function of the same parameters when again $\Delta \beta_0 = 0$. In all cases, $T + \dot{R} + L = 1$. Note that, in the absence of a blocker ($\Delta \beta_1 = 0$), Figs. 3(a) and 3(b) indicate that T = L = 34% and R = 32% when $\alpha D = 0.6$ (and $\Delta \beta_0 = 0$). In other words, in this case the junction is extremely lossy. However, these losses are dramatically reduced when the amplitude of the blocker DS (or $\Delta \beta_1$) is increased, as is clearly shown in Fig. 3(a). For a blocker state that induces a maximum

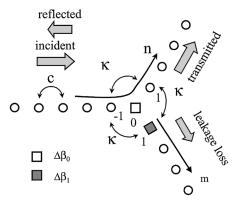


Fig. 2. 120° Y junction with a blocker placed at the entry of the lower arm (m=1). The signal DS travels along the n line.

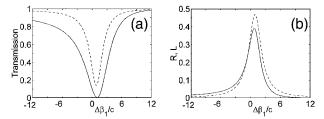


Fig. 3. (a) Transmission coefficient T of a 120° junction as a function of blocking parameter $\Delta\beta_1/c$ when $\alpha D=0.2$ (solid curve) and $\alpha D=0.6$ (dashed curve). (b) Reflection losses R (solid curve) and leakage losses L (dashed curve) as a function of $\Delta\beta_1/c$ for $\alpha D=0.6$. For both figures, $\Delta\beta_0=0$.

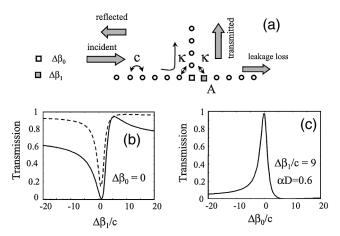


Fig. 4. (a) A T discrete soliton switching junction with a blocker positioned at site A; (b) the transmission coefficient of this T junction as a function of $\Delta\beta_1/c$ when $\alpha D=0.2$ (solid curve), $\alpha D=0.6$ (dashed curve), and $\Delta\beta_0=0$; (c) the transmission coefficient of the T junction as a function of $\Delta\beta_0/c$ for $\alpha D=0.6$ and $\Delta\beta_1/c=9$.

nonlinear index change of the order of $\gamma I_0=1.44\times 10^{-3}$, Eq. (2) gives that $\Delta\beta_1/c=12$. As Figs. 3(a) and 3(b) show, for $\Delta\beta_1/c=12$ the switching junction is highly efficient, with T=99% and R=0.06%. To verify our results, we numerically solved Eqs. (1), using a beam propagation scheme. More specifically, we simulated the rerouting of a signal DS by this 120° junction when $\alpha D=0.6$ and $\Delta\beta_1/c=12$. In this case, we found that the signal DS after the junction had been transmitted (virtually unchanged) to the upper branch with an efficiency of 98.5%. Therefore our theory can be used to design DS switching junctions with specified transmission efficiencies.

Similarly, a T-switching junction such as that depicted in Fig. 4(a) can also be analyzed. Again we use Eqs. (3) to obtain r, t, l, Q, except that in this case we ignore the small coupling coefficient between the two sites around the intersection (along the horizontal branch). For this T junction, and for the set of parameters used above, $\kappa/c=0.15$. Figure 4(b) shows the transmission efficiency of this T junction as a function of $\Delta\beta_1/c$ when $\alpha D=0.2$, 0.6 and $\Delta\beta_0=0$. For a blocker DS with $\gamma I_0=1\times 10^{-3}$, we find from Eq. (2) that $\Delta\beta_1/c=9$. For this blocking state and for $\alpha D=0.6$, Fig. 4(b) suggests that T=97%. What is also interesting is the fact that the performance of these

junctions can be further improved by appropriate engineering of the waveguide at the intersection. This waveguide engineering can be accomplished by alteration of either the junction's core refractive index or its core diameter, which in turn modifies its propagation constant by an amount $\Delta \beta_0$. Figure 4(c) shows the junction transmission coefficient as a function of $\Delta \beta_0/c$ when $\alpha D = 0.6$ and $\Delta \beta_1/c = 9$. A close inspection of Fig. 4(c) reveals that for these values the transmission coefficient attains a maximum (98.5%) when $\Delta \beta_0/c = -0.18$. In other words, we expect that, if we reduce the core refractive index by an amount $\Delta n_L = -0.18c/k_0 = -1.2 \times 10^{-5}$, the transmission efficiency of this junction will improve. The improvement is verified numerically by solution of Eqs. (1), for which we found that indeed the transmission coefficient increased from 94.35% to 97.73% after engineering of the intersection. We note that the small discrepancy between the numerical results and those predicted analytically is attributed to the fact that the DS itself is a wave packet that involves an ensemble of spatial frequencies. We can find a better agreement between theory and numerical simulations by considering the effective transmission coefficient, i.e., $T_{\rm eff} = \int \mathrm{d}\alpha T(\alpha) |\Phi(\alpha)|^2$, where $\Phi(\alpha)$ is the normalized Fourier transform of the DS envelope. Along the same lines, we also analyzed and optimized a T configuration in which the blocker is positioned on the first site of the vertical branch in Fig. 4(a). Again, our theoretical results were in agreement with numerical simulations.

In conclusion, we have theoretically analyzed the performance of switching junctions in two-dimensional discrete-soliton networks.

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