Design of Two-Dimensional Recursive Filters Using Genetic Algorithms

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Abstract—In this paper, we examine a new design method for two-dimensional (2-D) recursive digital filters using genetic algorithms (GAs). The design of the 2-D filter is reduced to a constrained minimization problem the solution of which is achieved by the convergence of an appropriate GA. Theoretical results are illustrated by a numerical example. Also, comparison with the results of some previous design methods is attempted.

Index Terms—Constrained optimization, genetic algorithm (GA), multidimensional systems, two-dimensional (2-D) recursive filters, 2-D systems.

I. INTRODUCTION

DURING the last three decades, two-dimensional (2-D) filter design has received growing attention by researchers and practitioners working in image processing, biomedical imaging, seismic data processing, X-rays enhancement, satellite image processing, astronomy, etc. An overview is given in [1]–[3]. The design approaches for 2-D filters can be broadly classified into two categories:

1) those based on appropriate transformation of 1-D filters [2], [3];
2) those based on appropriate optimization techniques [3]–[10].

However, most of the existing algorithms [3]–[10] may result in an unstable filter. Various techniques have been proposed in order to overcome these instability problems, but the outcome is likely to be a system that has a very small stability margin and therefore may not be of practical importance [11].

In this paper, an optimization procedure is proposed by using the so-called evolutionary computation, or genetic algorithm (GA). The desired stability of 2-D filter yields appropriate constraints for the minimization problem. GAs are computational models inspired from biological evolution, which is based on natural selection. Thus, GAs are search algorithms based on the mechanics of natural genetics and natural selection. The backbone of every GA is the reproduction of an original population,

the performance of crossover and mutation and the selection of the best. Artificial reproduction schemes were first developed in the seventies [12] and more were extended during the eighties [13], [14]. The search area for the GAs is very wide and it usually converges to a point near the global optimum. Fine tuning operations like hill climbing can be used to further refine the near-optimum solution. Simulating the survival of the most suitable among string structures, the optimal string (solution) is searched by randomized information exchange. Since the GA utilizes the coded discrete information of the artificial strings, it can be applied to ill-structured discrete optimization problems as well as to continuous optimization problems. They are often used in optimization problems, since they are capable of finding solutions to complex problems without carrying out an exhaustive search. Therefore, GAs are very often employed in nonlinear problems and multi-objective optimizations.

II. PROBLEM FORMULATION

For design purposes, we consider the following 2-D transfer function for our filter:

$$H(z_1, z_2) = H_0 \frac{\sum_{i=0}^{K} \sum_{j=0}^{K} a_{ij} z_1^i z_2^j}{\prod_{k=1}^{K} (1 + b_k z_1 + c_k z_2 + d_k z_1 z_2)}, \quad a_{00} = 1. \tag{1}$$

Let us consider M_d, the desirable amplitude response of the 2-D filter as a function of the frequencies ω_1 and ω_2, $(\omega_1, \omega_2 \in [0, \pi])$. The design task at hand amounts to finding a transfer function $H(z_1, z_2)$ as in (1) such that the function $H(e^{j\omega_1}, e^{j\omega_2})$ approximates the desired amplitude response $M_d(\omega_1, \omega_2)$. This approximation can be achieved by minimizing [11]

$$\begin{aligned} J &= J(a_{ij}, b_k, c_k, d_k, H_0) \\ &= \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} [|M(\omega_1, \omega_2)| - M_d(\omega_1, \omega_2)]^p \end{aligned} \tag{2}$$

where

$$M(\omega_1, \omega_2) = H(z_1, z_2) \Big|_{\substack{z_1 = e^{-j\omega_1} \\ z_2 = e^{-j\omega_2}}} \tag{3}$$

and

ω_1 $(\pi/N_1)n_1$;
ω_2 $(\pi/N_2)n_2$;
p even positive integer (usually $p = 2$ or $p = 4$).

Equation (2) is equivalent to

$$J = \sum_{n_1=0}^{N_1} \sum_{n_2} \left[\left| M\left\{ \frac{\pi n_1}{N_1}, \frac{\pi n_2}{N_2} \right\} \right| - M_d\left\{ \frac{\pi n_1}{N_1}, \frac{\pi n_2}{N_2} \right\} \right]^p.$$
(4)

Hence, the aim is to minimize the difference between the actual and desired amplitude response of the filter at $N_1 N_2$ points. Since we are dealing with only first-degree factors in the denominator, it is known that the stability conditions are given by [1]–[3]

$$|b_k + c_k| - 1 < d_k < 1 - |b_k - c_k|, \qquad k = 1, 2, \ldots, K$$

or

$$|b_k + c_k| - 1 < d_k, \qquad k = 1, 2, \ldots, K$$
$$d_k < 1 - |b_k - c_k|, \qquad k = 1, 2, \ldots, K.$$
(5)

Thus, the design of 2-D recursive filters is equivalent to the following constrained minimization problem:

Minimize J
$$= \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \left[\left| M\left\{ \frac{\pi n_1}{N_1}, \frac{\pi n_2}{N_2} \right\} \right| - M_d\left\{ \frac{\pi n_1}{N_1}, \frac{\pi n_2}{N_2} \right\} \right]^p \quad (6a)$$

subject to the constraints

$$|b_k + c_k| - 1 < d_k, \qquad k = 1, 2, \ldots, K$$
$$d_k < 1 - |b_k - c_k|, \qquad k = 1, 2, \ldots, K.$$
(6b)

where p is an even positive integer (usually $p = 2$ or $p = 4$), and N_1, N_2, and K are given positive integers. In [11], this problem has been tackled using neural networks. In this paper, we give a solution using GAs. We will show the present method yields a better solution.

III. PROBLEM SOLUTION

For the purpose of illustration, without loss of generality, we consider the case of $K = 2$. Then, $H(z_1, z_2)$ from (1) is as shown in (7) at the bottom of the page. For a successful implementation of the GA, we need to express $|M(\omega_1, \omega_2)|$ and the constraints given by (5) in appropriate forms. From (7) and (3), we obtain (8a) shown at the bottom of the page, where

$$c_{pq} = c_{pq}(\omega_1, \omega_2) = \cos(p\omega_1 + q\omega_2)$$
$$s_{pq} = s_{pq}(\omega_1, \omega_2) = \sin(p\omega_1 + q\omega_2),$$
$$p, q = 0, 1, 2$$
(8b)

and

$$D = [(1 + b_1 c_{10} + c_1 c_{01} + d_1 c_{11})$$
$$- j(b_1 s_{10} + c_1 s_{01} + d_1 s_{11})]*$$
$$* [(1 + b_2 c_{10} + c_2 c_{01} + d_2 c_{11})$$
$$- j(b_2 s_{10} + c_2 s_{01} + d_2 s_{11})].$$
(8c)

In a compact form, $M(\omega_1, \omega_2)$ can be written as

$$M(\omega_1, \omega_2) = H_0 \frac{A_R - jA_I}{(B_{1R} - jB_{1I})(B_{2R} - jB_{2I})}$$
(9)

where

$$A_R = a_{00} + a_{01}c_{01} + a_{02}c_{02} + a_{10}c_{10} + a_{20}c_{20}$$
$$+ a_{11}c_{11} + a_{12}c_{12} + a_{21}c_{21} + a_{22}c_{22}$$
$$A_I = a_{01}s_{01} + a_{02}s_{02} + a_{10}s_{10} + a_{20}s_{20} + a_{11}s_{11}$$
$$+ a_{12}s_{12} + a_{21}s_{21} + a_{22}s_{22}$$
$$B_{1R} = (1 + b_1 c_{10} + c_1 c_{01} + d_1 c_{11})$$
$$B_{1I} = (b_1 s_{10} + c_1 s_{01} + d_1 s_{11})$$
$$B_{2R} = (1 + b_2 c_{10} + c_2 c_{01} + d_2 c_{11})$$
$$B_{2I} = (b_2 s_{10} + c_2 s_{01} + d_2 s_{11}).$$
(10)

Therefore, we have

$$|M(\omega_1, \omega_2)| = H_0 \frac{\sqrt{A_R^2 + A_I^2}}{(B_{1R}^2 + B_{1I}^2)(B_{2R}^2 + B_{2I}^2)}.$$
(11)

A continuous differentiable form of the constraints can be obtained from (5) in the form

$$-(1 + d_k) < (b_k + c_k) < (1 + d_k)$$
$$-(1 - d_k) < (b_k - c_k) < (1 - d_k)$$
$$(1 + d_k) > 0$$
$$(1 - d_k) < 0.$$
(12)

Let us suppose that the desired amplitude response is given by

$$M_d(\omega_1, \omega_2) = \begin{cases} 1, & \text{if } \sqrt{\omega_1^2 + \omega_2^2} \leq 0.12 \\ 0.5, & \text{if } 0.08\pi \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 0.12 \\ 0, & \text{otherwise.} \end{cases}$$
(13)

$$H(z_1, z_2) = H_0 \frac{a_{00} + a_{01}z_2 + a_{02}z_2^2 + a_{10}z_1 + a_{20}z_1^2 + a_{11}z_1z_2 + a_{12}z_1z_2^2 + a_{21}z_1^2z_2 + a_{22}z_1^2z_2^2}{(1 + b_1z_1 + c_1z_2 + d_1z_1z_2)(1 + b_2z_1 + c_2z_2 + d_2z_1z_2)}$$
(7)

$$M(\omega_1, \omega_2) = H_0 \left[\frac{[a_{00} + a_{01}c_{01} + a_{02}c_{02} + a_{10}c_{10} + a_{20}c_{20} + a_{11}c_{11} + a_{12}c_{12} + a_{21}c_{21} + a_{22}c_{22}]}{D} \right.$$
$$\left. - \frac{j[a_{01}s_{01} + a_{02}s_{02} + a_{10}s_{10} + a_{20}s_{20} + a_{11}s_{11} + a_{12}s_{12} + a_{21}s_{21} + a_{22}s_{22}]}{D} \right]$$
(8a)

Choosing the values $p = 2$, $N_1 = 50$, and $N_2 = 50$, the corresponding constrained optimization problem (6) becomes

$$\text{Minimize } J = \sum_{n_1=0}^{50} \sum_{n_2=0}^{50} \left[\left| M\left(\frac{\pi}{50}n_1, \frac{\pi}{50}n_2\right) \right| - M_d\left(\frac{\pi}{50}n_1, \frac{\pi}{50}n_2\right) \right]^2$$

(14)

subject to the constraints given by (12) with $k = 1, 2$. The same problem has been tackled in [11] using neural networks. In this paper, we give a solution using GAs, which has the advantage of producing a smaller error. To start implementing a GA, an initial population is considered. This is represented by "bit" strings that evolve randomly through successive generations in order to obtain an optimum value for a particular fitness function [13]. In each generation, a new set of artificial strings is created using bits and pieces of the most suitable of the old ones. Solutions with high suitability are mated with other solutions by crossing over parts of solution strings. Strings may also mutate. Solutions with poor fitness are improved by crossover using highly fit solutions.

A simple GA relies on the processes of reproduction, crossover and mutation to reach the global or "near-global" optimum. To start the search, GAs require an initial set of points. This set is called population, analogous to the biological system. It has a population size P_s. A random number generator creates the initial population. This initial set is converted to a binary system and is considered as chromosomes, actually sequences of "0" and "1". The next step is to form pairs of these points that will be considered as parents for a reproduction. Parents come to reproduction and interchange N_p parts of their genetic material. This is achieved by crossover. After the crossover, there is a very small probability P_m for mutation. Mutation is the phenomenon where a random "0" becomes "1" or a "1" becomes "0". Mutation is necessary because although reproduction and crossover efficiently search and mix existing strings, occasionally they may result in loss of some potentially useful "genetic" material. Assume that each pair of "parents" gives rise to N_c children. Thus, the GA generates the initial layouts and obtains the objective function values. The above operations are carried out and the next generation with a new population of strings is formed. By the reproduction, the population of the "parents" is enhanced by the "children", increasing the original population since new members are added. The parents always belong to the population under consideration. The new population has now $P_s + N_c \cdot P_s/2$ members. Then the process of natural selection is applied. According to this process only P_s members survive out of the $P_s + N_c \cdot P_s/2$ members. These P_s members are selected as those members with the highest values of J, if we are attempting to maximize J, (or with the lowest values of J, if we are attempting to minimize J). By repeated iterations of reproduction under crossover and mutation, and natural selection, we can find the maximum (or minimum) of J as the point to which the best values of our population converge. The termination criterion is fulfilled if

either the mean value of J in the P_s-members population is no longer improved (maximized or minimized), or the number of iterations is greater than the maximum number of iterations $N_{\max}$, which is defined by us.

In the present problem under consideration, our aim is to minimize the vector x, where

$$x = (a_{00}, a_{02}, a_{01}, a_{11}, a_{12}, a_{20}, a_{21}, a_{22}, b_1, b_2, c_1, c_2, d_1, d_2, H_0)^T.$$

Each coordinate of the vector is converted to the binary system and is considered as a part of a big chromosome. The search starts with a randomly generated population of such P_s chromosomes. Each coefficient is converted to a t-bits binary number. Then, $2t$ bits are required for the "chromosome" of each element with an absolute value less than 3. Our GA is summarized as follows.

- `Step 1 : Find randomly the` P_s `initial t bits binary numbers`
- `Step 2 : Convert the initial binary numbers to the initial population of` P_s `members`
- `Step 3 : Increase the number of iterations`
- `Step 4 : Select randomly the` $P_s/2$ `pairs from the population`
- `Step 5 : From each pair of parents and take` N_c `children by crossover. Each bit of each child has probability` P_m `for a mutation`
- `Step 6 : Find the new population` $P_s + N_c \cdot P_s/2$ `( parents + children)`
- `Step 7 : Delete the members that do not satisfy the stability conditions (5)`
- `Step 8 : From the new population select the` P_s `members with the lowest values of` J`.`
- `Step 9 : If the absolute value of the difference between the mean value of` J `in the population of the present generation and the mean value of` J `in the population of the previous generation is lower than` ε `(defined by us), or the number of iterations is greater than the maximum number of iterations, say,` $N_{\max}$`, (which is defined by us), then STOP; otherwise, go to STEP 3.`
- `Stop.`

It is observed that the designed filter is stable because in Step 7 of the above GA the Stability conditions (12) are checked.

For better results, the above GA was run P_s times. The P_s best result of each time was kept and was the initial population for a new GA. The parameters of the GA are presented in Table I.

TABLE I
PARAMETERS OF THE GA

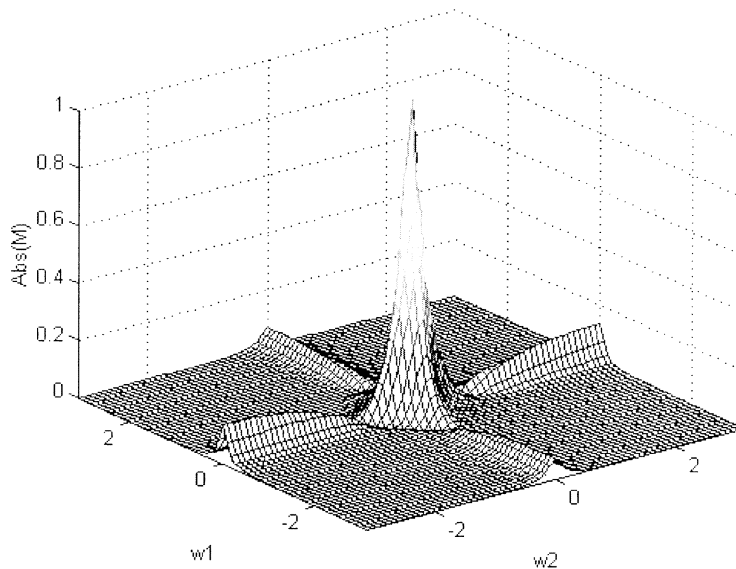| P_s	t	N_c	P_m	N_p	N_{max}
500	32	10	5 %	12	200

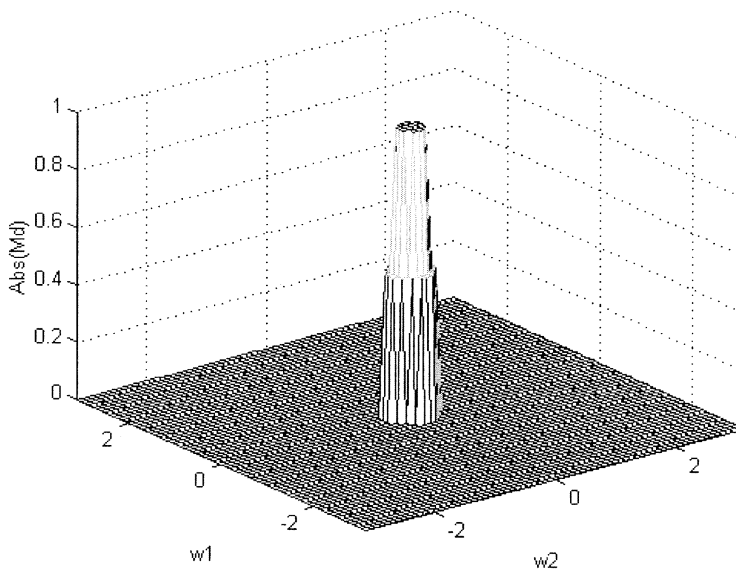Fig. 1.   Amplitude response $|M(\omega_1, \omega_2)|$ of the considered 2-D filter using GAs.

Fig. 2.   Desired amplitude response $|M_d(\omega_1, \omega_2)|$ of the 2-D filter.

The vector x obtained through the use of the GA described above is

$$x = [1.8162, -1.1060, 0.0712, -2.5132, 0.4279,$$
$$0.5926, -1.3690, 2.4326, -0.8662, -0.8907,$$
$$-0.8531, -0.8388, 0.7346, 0.8025, 0.0009]^T.$$

Therefore, we get the equation shown at the bottom of the page. The corresponding amplitude response $|M(\omega_1, \omega_2)|$ is shown in Fig. 1, while Fig. 2 shows the desired amplitude response $|M_d(\omega_1, \omega_2)|$. Furthermore, for comparison, the result of the method given in [3] and [4] is presented in Fig. 3 and that of the method given in [11] is presented in Fig. 4.

$$H(z_1, z_2) = 0.009 \frac{1 + 0.0712z_1 + 0.5926z_1^2 - 2.5132z_1z_2 + 0.4279z_1z_2^2 + 1.8162z_2 + 0.4279z_1^2z_2 + 2.4326z_1^2z_2^2 - 1.1060z_2^2}{(1 - 0.8662z_1 - 0.8531z_2 + 0.7346z_1z_2)(1 - 0.8907z_1 - 0.8388z_2 + 0.8025z_1z_2)}$$
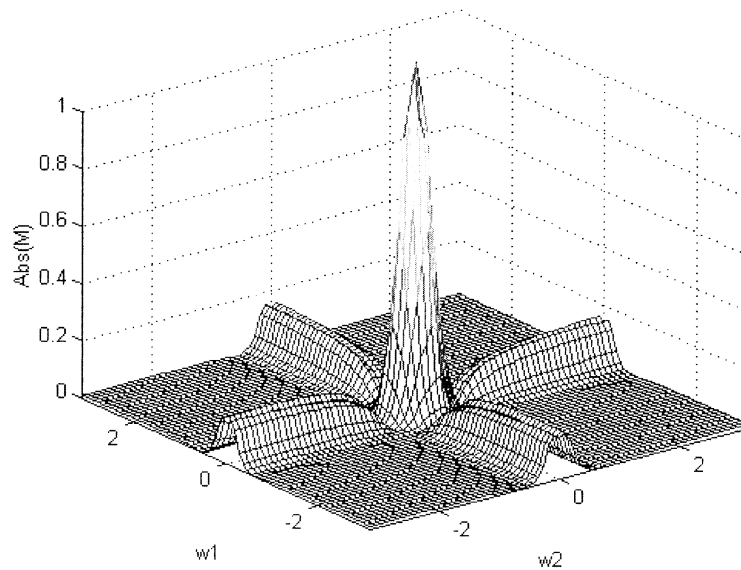
Fig. 3. Amplitude response $|M(\omega_1, \omega_2)|$ of the considered 2-D filter using the method in [3], [4].
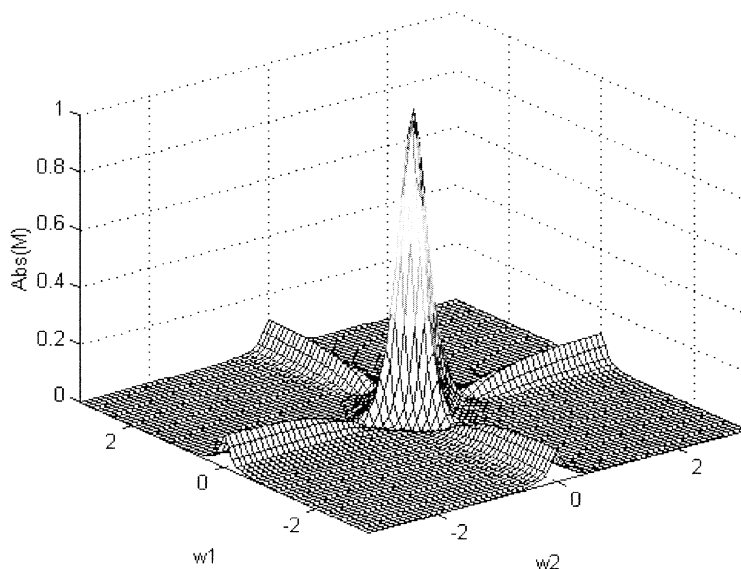

Fig. 4. Amplitude response $|M(\omega_1, \omega_2)|$ of the considered 2-D filter using the method in [11].

IV. CONCLUSION

In this study, the design of 2-D recursive filters is attempted by using an appropriate evolutionary computational scheme. One of the main advantages of the present method over the method of [3] and [4] is that we can check the stability of the designed filter from the beginning of the procedure, since we introduce the desired stability as appropriate constraints. On the contrary, the previous methods, which are based more or less on a trial-and-error approach, cannot always guarantee the stability of the filter. Further, our implementation leads to a simpler filter since, in practice, we have to realize a factorable denominator and in particular of first-order filters, which obviously are simpler than those of [3], [4]. For the same numerical example, the present method finds a better approximation of the transfer function than the method of [11].

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Dr. Swamy is a Fellow of the Institute of Electrical Engineers (U.K.), the Engineering Institute of Canada, the Institution of Engineers (India), and the Institution of Electronic and Telecommunication Engineers (India). Presently, he is President-Elect of the Circuits and Systems (CAS) Society. He has served the IEEE in various capacities such as the Vice-President of the CAS society in 1976, Vice-President (Publications) of the CAS Society during 2001-2002, Editor-in-Chief of the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: FUNDAMENTAL THEORY AND APPLICATIONS during 1999–2001, Associate Editor of the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS during 1985–1987, Program Chair for the 1973 IEEE CAS Symposium, General Chair for the 1984 IEEE CAS Symposium, Vice-Chair for the 1999 IEEE CAS Symposium, and a member of the Board of Governors of the CAS Society. He is the recipient of many IEEE-CAS Society awards including the Education Award in 2000, Golden Jubilee Medal in 2000, and the 1986 Guillemin–Cauer Best Paper Award. In August 2001, he was awarded the Doctor of Science in Engineering (Honoris Causa) by Ansted University "In recognition of his exemplary contributions to the research in Electrical and Computer Engineering and to Engineering Education, as well as his dedication to the promotion of Signal Processing and Communications Applications". He is a member of Micronet, a National Network of Centers of Excellence in Canada, and also its coordinator for Concordia University.

Comments and Corrections_____

Corrections to "Design of Two-Dimensional Recursive Filters Using Genetic Algorithms"

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The paper [1] contains a few typing errors, because of which three of the equations given therein are not correct.

The first error is the absence of the square root in the denominator of (11). The correct form of this equation is

$$|M(\omega_1, \omega_2)| = H_0 \frac{\sqrt{A_R^2 + A_I^2}}{\sqrt{(B_{1R}^2 + B_{1I}^2)(B_{2R}^2 + B_{2I}^2)}}. \qquad (11)$$

The second error is in the inequality sign in the fourth line of (12). The correct inequalities are given by

$$
\begin{aligned}
-(1 + d_k) &< (b_k + c_k) < (1 + d_k) \\
-(1 - d_k) &< (b_k - c_k) < (1 - d_k) \\
(1 + d_k) &> 0 \\
(1 - d_k) &> 0.
\end{aligned}
\qquad (12)
$$

The last typographical error is in (13). The correct expressions are as given

$$
M_d(\omega_1, \omega_2) = \begin{cases} 1, & \text{if } \sqrt{\omega_1^2 + \omega_2^2} \le 0.08\pi \\ 0.5, & \text{if } 0.08\pi \le \sqrt{\omega_1^2 + \omega_2^2} \le 0.12\pi \\ 0, & \text{otherwise.} \end{cases}
\qquad (13)
$$

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