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# Design Optimization of Gas Turbine Blades with Geometry and Natural Frequency Constraints

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## ABSTRACT

In this paper an automated procedure is presented to obtain the minimum weight design of gas turbine blades with geometry and multiple natural frequency constraints. The objective is achieved using a combined finite element-sequential linear programming, FEM-SLP technique. Thickness of selected finite elements are used as design variables. Geometric constraints are imposed on the thickness variations such that the optimal design has smooth aerodynamic shape. Based on the natural frequencies and mode shapes obtained from finite element analysis an assumed mode reanalysis technique is used to provide the approximate derivatives of weight and constraints with respect to design variables for sequential linear programming. The results from SLP provide the initial design for the next FEM-SLP process. An example is presented to illustrate the interactive system developed for the optimization procedure.

## NOMENCLATURE

| | |
|---|---|
| $A_j$ | element area |
| C | blade cross sectional chord length |
| $f_i$ | natural frequency |
| K | global stiffness matrix |
| $K_j$ | element stiffness matrix |
| $K^*_j$ | element stiffness matrix in modal coordinate |
| $\bar{K}$ | stiffness matrix of modified system in modal coordinate |
| M | global mass matrix |
| $M_j$ | element mass matrix |
| $M^*_j$ | element mass matrix in modal coordinate |
| $\bar{M}$ | mass matrix of modified system in modal coordinate |
| $q_i$ | modal coordinate vector |
| $t_j$ | element thickness |
| $T_{ij}$ | element kinetic energy |
| $T^*_{ij}$ | element kinetic energy in modal coordinate |
| Tmax | maximum thickness of each blade cross section |
| $V_{ij}$ | element strain energy |
| $V^*_{ij}$ | element strain energy in modal coordinate |
| W | total structural weight |
| $W_j$ | element structural weight |
| $\alpha_j$ | element thickness ratio |
| $\beta$ | ratio of Tmax to C |
| $\Lambda$ | eigenvalue matrix |
| $\lambda_i$ | ith mode eigenvalue |
| $\rho_j$ | element mass density |
| $\Phi$ | eigenvector matrix |
| $\phi_i$ | ith mode eigenvector |

Subscripts and superscripts

| | |
|---|---|
| i | mode number |
| j | element number or design variable number |
| $\ell$ | constraint lower limit |
| NP | number of design variables |
| NF | number of frequency constraints |
| o | original design |
| u | constraint upper limit |
| * | modal coordinate system |

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## INTRODUCTION

In order to avoid resonances of natural frequencies and external excitation forces, structures must be designed properly to obtain desired distribution of natural frequencies. In this paper, an automated procedure is presented for the optimal design of gas turbine blades [1-5]. The objective of the procedure is to achieve the minimum weight design of gas turbine blades subject to geometry and multiple natural frequency constraints. The blades are modeled by thin shell elements. Thickness of selected elements are used as variables to control the optimization process. Thickness of each element may vary independently from others or the thickness of several elements may vary accordingly with the same scale to maintain the smoothness of structural contour shape.

The optimal design problem is treated as one of the shape optimization and is solved by a combined finite element method and sequential linear programming [5-9], FEM-SLP technique. The general purpose finite element program MSC/NASTRAN is used to perform the free vibration model analysis. Based on the calculated eigenvalues and eigenvectors an assumed mode reanalysis technique [5,10,11] is used to perform approximate eigenvalue sensitivity analysis. Using the approximate eigenvalue sensitivities the SLP iterative procedure is then applied to redesign the blades. After the SLP procedure converges finite element analysis is performed for the updated design to calculate eigensolutions and to check the validity of the optimized blades. The FEM-SLP procedure will be repeated until all design constraints are satisfied.

## FORMULATIONS

The problem of the blade shape optimization can be stated as minimization of the blade weight, W, subject to the frequency constraints

$$f_i^\ell \leq f_i \leq f_i^u \qquad i = 1, \ldots, NF \tag{1}$$

and geometric side constraints

$$\alpha_j^\ell \leq \alpha_j \leq \alpha_j^u \qquad j = 1, \ldots, ND \tag{2}$$

with

$$\alpha_j = \frac{t_j}{t_j^o} \tag{3}$$

where NF is the number or frequency constraints, ND is the number of design variables, $f_i$ is the natural frequency of the ith mode, $t_j$ and $\alpha_j$ are the thickness and the thickness magnification factor of element j, the superscript o denotes the state of original design and the superscripts $\ell$ and u denote the lower and upper limits of the associated items.

The weight of the blade is

$$W = \sum_j W_j \alpha_j \tag{4}$$

where

$$W_j = \rho_j A_j t_j^o \tag{5}$$

$\rho_j$ and $A_j$ are density and area of element j.

Therefore, W is a linear combination of $\alpha_j$, the weight sensitivities with respect to $\alpha_j$ are constants

$$\frac{\partial W}{\partial \alpha_j} = W_j \qquad j = 1, \ldots, ND \tag{6}$$

The equation of free vibration for the structure modeled by finite elements is

$$K \phi_i = \lambda_i M \phi_i \tag{7}$$

where K and M are global stiffness and mass matrices, $\lambda_i$ and $\phi_i$ are eigenvalue and eigenvector of the ith natural mode of the structure.

The eigenvectors are normalized such that

$$\phi_i^T K \phi_i = \lambda_i \tag{7a}$$

$$\phi_i^T M \phi_i = 1 \tag{7b}$$

Since $\lambda_i = (2\pi f_i)^2$, the natural frequency constraints specified in equation (1) can be replaced by

$$\lambda_i^\ell \leq \lambda_i \leq \lambda_i^u \tag{8}$$

Taking derivatives of equation (7) then premultiplying the results by $\phi_i^T$ and applying equations (7a) and (7b) yield the eigenvalue derivatives shown in the reference [12].

$$\frac{\partial \lambda_i}{\partial \alpha_j} = \phi_i^T \left( \frac{\partial K}{\partial \alpha_j} - \lambda_i \frac{\partial M}{\partial \alpha_j} \right) \phi_i \tag{9}$$

Since the bending modes are mostly affected by the thickness variations, the stiffness and mass matrices [5] for the jth design variable can be written as

$$K_j = \alpha_j^3 K_j^o \tag{10}$$

$$M_j = \alpha_j M_j^o \tag{11}$$

Therefore, equation (9) can be reduced to

$$\frac{\partial \lambda_i}{\partial \alpha_j} = 6 \alpha_j^2 V_{ij} - 2 T_{ij} \tag{12}$$

where $V_{ij}$ and $T_{ij}$ are strain and kinetic energies associated with the ith mode and the jth design variable

$$V_{ij} = \tfrac{1}{2} \phi_i^T K_j^o \phi_i \tag{13}$$

$$T_{ij} = \tfrac{1}{2} \lambda_i \phi_i^T M_j^o \phi_i \tag{14}$$

Using the first order Taylor series expansion W and $\lambda_i$ of the current design can be approximated as

$$W = W^0 + \sum_j \frac{\partial W}{\partial \alpha_j} \Delta\alpha_j \tag{15}$$

$$\lambda_i = \lambda_i^0 + \sum_j \frac{\partial \lambda_i}{\partial \alpha_j} \Delta\alpha_j \tag{16}$$

With equations (6), (12), (15) and (16) the objective function W and frequency constraints can be linearized to form a linear programming problem.

It has been shown that assumed mode reanalysis formulation [10,11] can use results obtained from finite element analysis and LP problem to calculate approximate frequency sensitivities for another LP operation. Therefore, a sequence of LP, SLP, can be performed after the finite element analysis of an initial design. When the SLP converges finite element analysis will be applied to confirm the results and provide data for the follow up SLP procedure. The expensive free vibration analysis for the full structure model thus can be avoided for each LP procedure.

Using an assumed mode reanalysis method the eigenvalue problem solved for each LP step becomes

$$\bar{K}\, q_i = \lambda_i\, \bar{M}\, q_i \tag{17}$$

where

$$\bar{K} = \Lambda^0 + \sum_j (\alpha_j^3 - 1)\, K_j^* \tag{18}$$

$$\bar{M} = I + \sum_j (\alpha_j - 1)\, M_j^* \tag{19}$$

$$K_j^* = \Phi^T K_j^0\, \Phi \tag{20}$$

$$M_j^* = \Phi^T M_j^0\, \Phi \tag{21}$$

where $\Phi$ is a truncated orthonormal modal matrix and $\Lambda^0$ is the diagonal eigenvalue matrix of the original design and I is an unitary matrix.

From the solution of the reduced eigenproblem, the eigenvalue derivatives of the updated system can be approximated as

$$\frac{\partial \lambda_i}{\partial \alpha_j} = 6\, \alpha_j^2\, V_{ij}^* - 2\, T_{ij}^* \tag{22}$$

where

$$V_{ij}^* = \tfrac{1}{2}\, q_i^T K_j^* q_i \tag{23}$$

$$T_{ij}^* = \tfrac{1}{2}\, \lambda_i\, q_i^T M_j^* q_i \tag{24}$$

Equation (22) is used to calculate $\partial\lambda_i/\partial\alpha_j$ in the SLP problem until the iterations converge. The updated design then becomes the initial design of the next FEM-SLP procedure.

## SOFTWARE IMPLEMENTATION

MSC/NASTRAN is used for finite element analysis. A program, BLADEOPT1 [5], written in FORTRAN has been developed for the SLP procedure. The system flowchart is shown in Figure 1. One input file contains data for both NASTRAN and BLADEOPT1. NASTRAN provides the structural weight of each finite element and modal analysis results for BLADEOPT1 to formulate and solve the SLP problem. Users can run the whole system interactively to adjust the input data for BLADEOPT1 and check the results at the end of NASTRAN or BLADEOPT1 run. Therefore, constraint limits can be adjusted such that a feasible solution can be easily obtained for the SLP problem. As the designs improve from one FEM-SLP to another, constraints can be moved toward desired limits to obtain an optimal design.

## DESIGN EXAMPLE

A gas turbine blade is used as an example to illustrate the system developed. Young's modulus, Poisson's ratio and weight density of the blade are $1.5\times10^7$ psi, 0.3 and 0.16924 $lb/in^3$ respectively. The blade is modeled by 192 quadratic shell elements, NASTRAN CQUAD8 elements, with 633 nodes. The finite element model is shown in Figure 2. All the degrees of freedom along the hub line are fixed as boundary conditions.

Initially, the total weight of the blade is 0.0634 pounds, and the first three natural frequencies of the blade are 1524 Hz, 2863 Hz, and 4439 Hz. The objective is to find a minimum weight design such that

$$f_1 \leqq 1400 \text{ Hz}$$
$$f_2 = 2800 \text{ Hz}$$
$$f_3 \geqq 4450 \text{ Hz}$$

with upper and lower limits imposed on the design variables.

Since critical speeds may occur at multiples of the blade rotating speed, according to Figure 4, natural frequency constraints of this example are selected so as to increase the margin from critical speeds at 100 percent blade operational speed.

In order to maintain smooth aerodynamic shape at each cross section chordwise of the blade, a single thickness magnification factor for each cross section is used as an independent design variable. Thickness variations in the longitudinal direction are less restrictive. However, these variations must be kept smooth and monotonic spanwise in the

$$\beta = \frac{Tmax}{C} \tag{25}$$

longitudinal direction. This ratio is the maximum thickness, Tmax, to the chord length, C, of each cross section chordwise.

For an aerodynamically well-designed blade it is desired that the variations of $\beta$ for a modified design do not exceed 10 percent of those of the initial design. Since chord length of each cross section is fixed, the variations of Tmax or $\alpha$ should be less than 10 percent of their initial values. Therefore, the geometric constraints of the design variables are

$$0.9 \leq \alpha_j \leq 1.1 \qquad j = 1, \ldots, ND \tag{26}$$

For the 192 finite element model shown in Figure 2, there are 16 elements per cross section chordwise and 12 sections spanwise. One design variable is used for each cross section, thus 16 elements in each cross section vary in the same scale.

Initial values of the 12 design variables used for the model are all one. The initial and final values of the maximum thickness at each cross section and final values of the design variables are shown in Table 1. The location of the maximum thickness is measured from the leading edge of each cross section and is presented as the element number in Table 1.

Table 2 presents the iteration history of natural frequencies and blade weight. Data for design cycle Number 0 are the analytical results for the original design. Data for cycle Numbers 1, 2, and 3 are the results from the first, second and third FEM-SLP operations. Using IBM 3090 each FEM-SLP requires 157.8 CPU seconds of which 141.0 CPU seconds are for NASTRAN (FEM) and 16.8 CPU seconds are for BLADEOPT1 (SLP). Total blade weight is reduced by 3.12 percent. All of the three frequency constraints are satisfied within one percent of design limits. The first 10 modes are presented to illustrate the affect of the geometry and frequency constraints on the overall characteristics of the blade design.

Figure 3 indicates that $\beta$ of the optimal design is distributed spanwise as a smooth and monotonically decreasing function of air flow rate.

Finally, a Campbell diagram associated with the blade and its environment is presented in Figure 4. Clearly, at engine full speed of 50000 RPM resonance of the natural frequencies and excitation forces are avoided with the new design.

## CONCLUSION

The automated procedure presented in this paper is very effective to obtain the optimal design of gas turbine blades with geometry and natural frequency constraints. Since the use of geometry constraints and thickness magnification factors can keep the overall aerodynamic shape within an acceptable range, the number of design iterations between structural dynamics and aerodynamics can be greatly reduced. In the future aerodynamic performance optimization ought to be included in the automated procedure to further reduce the design effort.

In order to reduce the effort of selecting frequency constraints to obtain feasible solutions from SLP procedure, adaptive constraint algorithms should be developed. With the adaptive constraint algorithms SLP may even be able to provide better designs for the next FEM-SLP procedure and further reduce the computation effort.

At present shell elements are used to model thin blades. Further research is required for the optimal design of thick blades with other types of elements.

Table 1. Distribution Of Maximum Thickness And Design Variables.

| Section No. | Element No. | Maximum Thickness Initial | Maximum Thickness Final | $\alpha$ Final |
|---|---|---|---|---|
| 1 | 9 | 0.21308 | 0.22800 | 1.07002 |
| 2 | 9 | 0.20767 | 0.22013 | 1.06000 |
| 3 | 9 | 0.20058 | 0.20585 | 1.02627 |
| 4 | 9 | 0.19128 | 0.18489 | 0.96659 |
| 5 | 9 | 0.18038 | 0.16415 | 0.91002 |
| 6 | 9 | 0.16923 | 0.15400 | 0.91000 |
| 7 | 9 | 0.15586 | 0.14105 | 0.90498 |
| 8 | 9 | 0.13276 | 0.11949 | 0.90005 |
| 9 | 9 | 0.10443 | 0.09959 | 0.95365 |
| 10 | 9 | 0.08435 | 0.08857 | 1.05003 |
| 11 | 10 | 0.07501 | 0.08101 | 1.08000 |
| 12 | 10 | 0.07116 | 0.06930 | 0.97386 |

Table 2. Iteration History Of Natural Frequencies And Weight.

| Design Cycle No. | Natural Frequencies (Hz) $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | Weight (lbs) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1524 | 2863 | 4439 | 5023 | 6403 | 6473 | 8467 | 9552 | 10200 | 11251 | 0.06340 |
| 1 | 1424 | 2808 | 4462 | 4972 | 6404 | 6457 | 8306 | 9503 | 10218 | 11230 | 0.06118 |
| 2 | 1410 | 2800 | 4449 | 4965 | 6412 | 6440 | 8322 | 9528 | 10208 | 11191 | 0.06105 |
| 3 | 1400 | 2795 | 4447 | 4959 | 6420 | 6458 | 8393 | 9627 | 10231 | 11184 | 0.06142 |

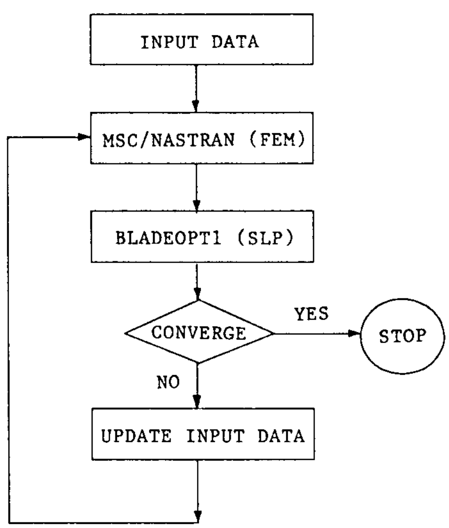


Figure 1. Blade Optimal Design System Flow Chart.

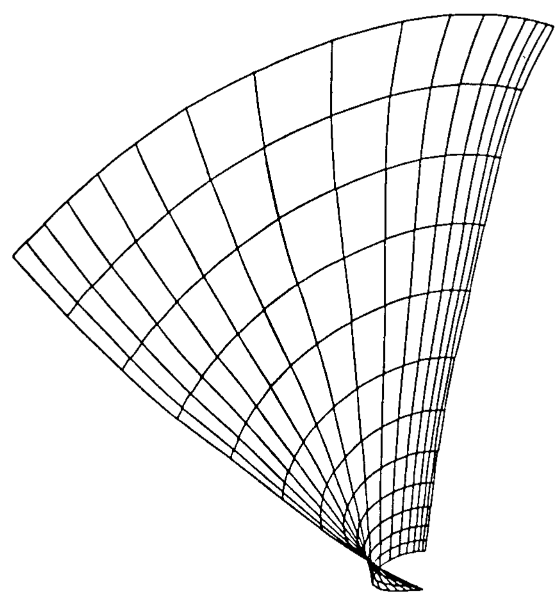

Figure 2. Blade Modeled by Shell Finite Elements.

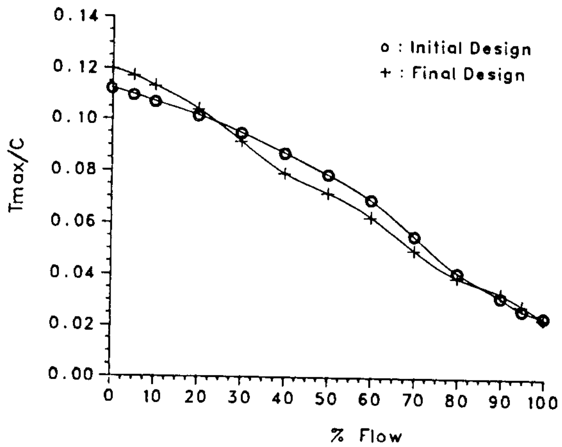


Figure 3. Distribution of Tmax/C in Longitudinal Direction.

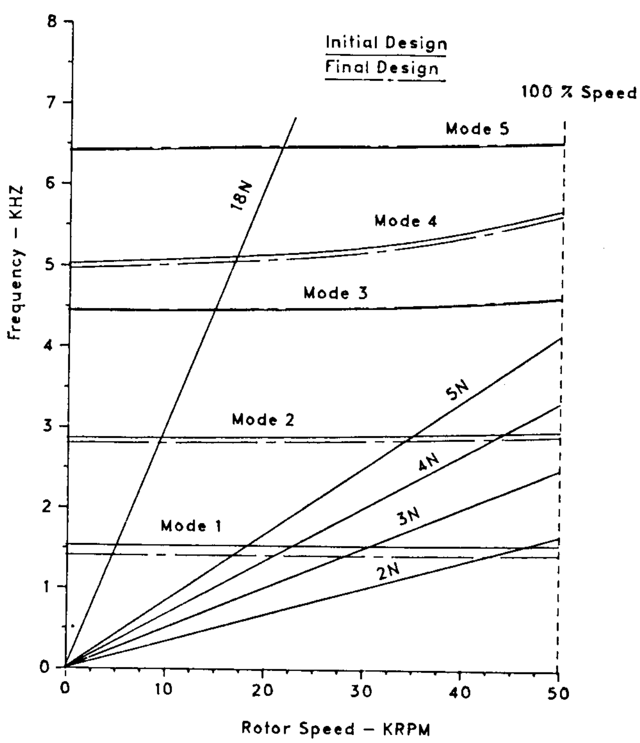


Figure 4. Campbell Diagram.

## REFERENCES


1. DeSilva, B.M.E, Negus, B., and Worster, B., "Penalty Function Type Optimal Control Methods for the Design of Turbine Blade Profiles," NASA Report N76 11464, 1975.

2. DeSilva, B.M.E., Grant, G.N.C., and Pierson, B.L., "Conjugate Gradient Optimal Control Methods for the Design of Turbine Blades," NASA Report N76 11463, 1975.

3. DeSilva, B.M.E, and Grand, G.N.C., "Comparison of Some Optimal Control Method for the Design of Turbine Blades, " ASME Paper No. 77 Det. 43, 1987.

4. Queau, J.P., and Tompette, P., "Optimal Shape Design of Turbine and Compressor Blades," Proceedings of International Symposium on Optimum Structural Design, Oct. 19-22, 1981, Tucson, Arizona, pp. 5-9 to 5-14.

5. Wang, B.P., Cheu, T.C., and Chen, T.Y., "Optimal Design of Compressor Blades with Multiple Natural Frequency Constraints", Proceedings of ASME Design Automation Conference, Sept. 27-30, 1987, Boston, Massachusetts.

6. Pederson, P., "The Integrated Approach of FEM-SLP for Solving Problems of Optimal Design", In Optimization of Distributed Parameter Structures, Hange and Cea, Edited by Sijthoff and Nourdhoff, 1981, pp. 757-780.

7. Wang, B.P., "Synthesis of Structures with Multiple Frequency Constraints," AIAA Paper No. 0951-CP, Proceedings of AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics and Materials Conference, Volume 1, pp. 394-397.

8. Vanderplaats, G.N., Numerical Optimization Techniques for Engineering Design: With Applications, McGraw-Hill, 1984.

9. Zienkiewicz, O.C., and Campbell, J.S., "Shape Optimization and Sequential Linear Programming," Chapter 7 of Optimum Structural Design Theory and Applications, Edited by Gallagher, R.H., and Zienkiewicz, O.C., John Wiley and Sons, 1973.

10. Wang, B.P., Pilkey, W.D., and Palazzola, A., "Reanalysis, Modal Synthesis and Dynamic Design", Chapert 8, State-Of-The-Art Survey on Finite Element Technology, edited by Noor and Pilkey, ASME 1983.

11. Wang, B.P., "On Computing Eigensolution Sensitivity Data Using Free Vibration Solutions," NASA Conference Publication 2457, National Aeronautics and Space Administration, 1987, pp. 233-246.

12. Fox, R.L., and Kapoor, M.P., "Rate of Change of Eigenvalues and Eigenvectors," AIAA Journal, Volume 6, No. 12, December 1968, pp. 337-351.