

# Design Rules for Maximally Flat Wavelength-Insensitive Optical Power Dividers Using Mach-Zehnder Structures

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**Abstract**—A cascade of two directional couplers with a relative phase shift between them, serves as an optical power divider. Simple analytic universal design rules for selecting the appropriate net coupling strength in each device, and for the phase shift, yield maximally flat response in terms of deviations in wavelength, polarization, or uniform fabrication errors, and are applicable to any power division ratio.

**Index Terms**— Directional coupler, optical power splitter/divider.

## I. INTRODUCTION

A GUIDED-WAVE structure that divides an input signal into two output ports with a specified excitation ratio is a useful element in integrated optics. Performance which is insensitive to changes in parameters such as wavelength, polarization or uniform fabrication errors are the desired traits in such power dividers. Examples of 50% (3 dB) dividers requiring high tolerances are the Sagnac loop mirror [1], and the grating circulator filter [2]. The realization of parameter insensitive optical power dividers is possible using adiabatic couplers [3]. Wavelength-flattened devices have also been proposed by attempting to balance the coupling and material dispersions [4], or by finding dispersionless coupling coefficients [5], [6]. The foregoing methods depend on the details of the actual structures and indexes, and thus, lack universality. Alternatively, one may try to balance deviations in parameters by cascading two similar devices with an appropriate phase-shift between the two. In this configuration, the purpose of the second device is to undo deviations introduced in the former. Cascaded directional couplers have been proposed for this purpose [7], [8]. Such configurations have simple analytic universal solutions governing the appropriate selection of net coupling coefficients and phase shift yielding maximally flat response, which are applicable to arbitrary splitting ratios. These universal design rules are the subject of this Letter, and are now highlighted.

The balanced optical power divider is shown in Fig. 1(a) [7], [8]. It is comprised of two synchronous directional couplers

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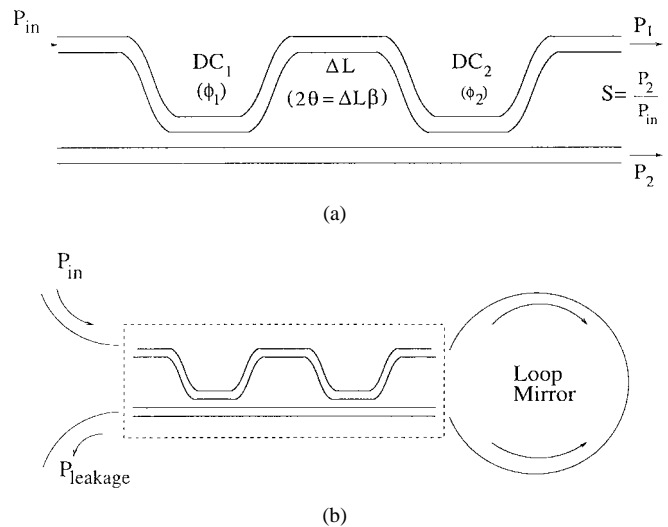


Fig. 1. (a) The balanced optical power divider, comprised of two directional couplers  $DC_1$  and  $DC_2$ , separated by an uncoupled section in which a phase delay  $2\theta$  is introduced. (b) The Sagnac loop mirror utilizing the balanced power divider in order to reduce fabrication, wavelength, and environmental sensitivities.

( $DC_1$  and  $DC_2$ ) of different coupling lengths, separated by an uncoupled region in which a relative phase-shift ( $2\theta$ ) is introduced. The phase shift in Fig. 1(a) is produced by unequal path lengths, but can also be realized by unequal propagation constants. The power transfer within each of the two couplers, along with the relative phase shift, gives three design parameters. One might expect that the output could then be accordingly adjusted to give a specific splitting ratio at three different wavelengths. Although the response is not polynomial, this expectation turns out to be justified, and indeed, the response can be made to appear cubic about selected operating points. The response can, thus, be designed to be maximally flat over a certain wavelength range, or this range may be extended by foregoing maximally flat response in favor of equal-ripple.

The response of the power splitter at the output of one waveguide, to a signal applied at the input of the adjacent waveguide is [7]

$$S = \cos^2 \theta \sin^2[\phi_1 + \phi_2] + \sin^2 \theta \sin^2[\phi_1 - \phi_2] \quad (1a)$$

$$\phi_i = \int_0^{L_i} K(\lambda, z) dz, \quad 2\theta = \beta(\lambda)\Delta L \quad (1b)$$

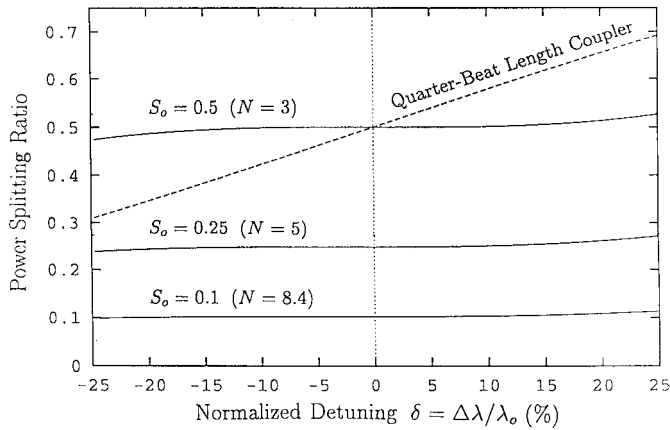


Fig. 2. Typical maximally flat response of the balanced power dividers, for splitting ratios  $S_o$  of 0.5, 0.25, and 0.1, solid curves.  $\delta$  is the fractional change in coupling strength arising from deviations in wavelength, polarization, or fabrication. The response of a quarter-beat length coupler used to achieve 50% power division is also shown.

where  $S$  is the ratio of coupled power at the output port to the input power. It is also the dividing ratio.  $\phi_i$  is the accumulated or net coupling over the length  $L_i$  of directional coupler  $i$ , and  $K(\lambda, z)$  is the  $z$  and wavelength dependent coupling strength in either of these couplers.  $2\theta$  is the relative phase delay introduced in the uncoupled section by the path length difference  $\Delta L$  and  $\beta$  is the propagation constant of the waveguide mode. Deviations that are due to a change in wavelength, polarization, or due to uniform fabrication errors, cause the net coupling  $\phi_i$  to change by  $\phi_i = \phi_i^o(1 + \delta)$ , where  $\phi_i^o$  is some ideal value to be determined below, and  $\delta$  is the fractional change in this value. The normalized response of the splitters is here defined as the change in the output due to the fractional change  $\delta$ . In particular, for the case of wavelength dependent changes, one may use the usual form for the coupling coefficients [9], to show that in first order,  $\delta = -\Delta\lambda/\lambda_o$ , where  $\Delta\lambda$  is the deviation in wavelength about the central wavelength  $\lambda_o$ .

Let us choose the ideal accumulated coupling factors, [and hence, the device lengths in accordance with (1b)] in the following manner:

$$\phi_1^o = \frac{3\pi}{8} \left(1 + \frac{1}{N}\right), \quad \phi_2^o = \frac{3\pi}{8} \left(1 - \frac{1}{N}\right) \quad (2)$$

where  $N$  is a real number greater than 3, to be determined below. It may be shown that with this choice, near maximally flat response is achieved when the phase difference  $\theta$  is selected to be

$$\cos^2 \theta = \sin \left( \frac{3\pi}{2N} \right) \left( N + \sin \left[ \frac{3\pi}{2N} \right] \right)^{-1} \quad (3)$$

The power division ratio in such an ideally constructed device is then

$$S_o = \frac{1}{4} \sin \left( \frac{3\pi}{N} \right) \left[ N + \sin \left( \frac{3\pi}{2N} \right) \right]^{-1} + \sin^2 \left( \frac{3\pi}{4N} \right) \quad (4)$$

at the center wavelength,  $\delta = 0$ . Equations (2)–(4), characterize the balanced optical power divider by defining the splitting

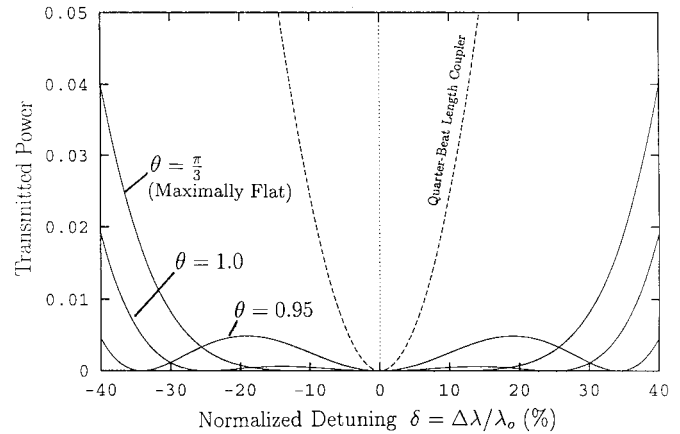


Fig. 3. The response of the loop mirror, Fig. 1(b), using a balanced 50% power divider, solid curves. The responses are defined as the power in the leakage port ( $P_{\text{leakage}}$  Fig. 1(b)). Transmitted powers that are not zero here represent unwanted leakage or crosstalk. The phase delay  $\theta$  is used to control the insensitivity bandwidth, and degree of insensitivity. The response of the quarter-beat length couplers commonly used in these devices is also shown.

ratio  $S_o$ , the directional coupler lengths  $\phi_1^o$  and  $\phi_2^o$ , and the phase delay  $2\theta$ , respectively, in terms of the parameter  $N$ .

Typical normalized splitter responses are shown in Fig. 2, for the cases of  $S_o = 0.5$  ( $N = 3$ ),  $S_o = 0.25$  ( $N = 5$ ), and  $S_o = 0.1$  ( $N = 8.4$ ), as evaluated by (1a), solid curves. Clearly, the responses are maximally flat over a range in which the coupling parameters deviate by as much as  $\pm 20\%$ . The response of a single quarter-beat length coupler ( $\phi_1 = \pi/4$ ) commonly used to achieve 50% splitting is also plotted for comparison, dashed curve. [The response of this coupler goes as  $S = \sin^2 \pi/4(1 + \delta)$ .]

The phase delay  $2\theta$  is also subject to deviations. However, this is primarily due to fabrication errors because  $\Delta L$  is always less than half a wavelength, and so  $\theta$  is a weak function of wavelength and polarization. The roll of changes in  $\theta$  about the operating point in (3) is to change the response from maximally flat to equal-ripple. Hence, a degree of insensitivity is also built into this choice of  $\theta$ .

A 50% power divider is an important special case. Its operation is considered a little more closely. Here, the net coupling values are to be selected according to  $\phi_1^o = \pi/4$ ,  $\phi_2^o = \pi/2$ . Maximally flat response is achieved when  $\theta = \pi/3$ , while other (smaller) values of  $\theta$  give equal-ripple response. Specifically, precisely equal power division occurs at three values of  $\delta$ , given by

$$\delta_{3 \text{ dB}} = \{0, \pm 2\pi \cos^{-1}[(2 \cos \theta)^{-1}]\}. \quad (5)$$

The balanced optical power dividers may serve as building blocks in other devices which require a reduced sensitivity to wavelength, fabrication, and polarization. One such device is the Sagnac loop mirror shown in Fig. 1(b). This device requires a 50% power divider. The response of the Sagnac loop is taken at the transmission port [ $P_{\text{leakage}}$  in Fig. 1(b)] when the loop is stationary. An output that is not zero represents unwanted leakage or crosstalk. The response of the loop mirror is highlighted in Fig. 3. The three solid curves represent the response for three values of  $\theta$ :  $\theta = \pi/3$ , 1.0, and 0.95. As  $\theta$  decreases from its maximally flat value of

$\pi/3$ , the insensitivity-bandwidth increases at the expense of larger inband ripple. For comparison, the response of the loop incorporating the commonly used quarter-beat length directional coupler is also shown, dashed curve. (The loop response in this case is  $S = \cos^2[\pi/2(1 + \delta)]$ .) Because of the remarkably insensitive nature of the balanced splitters, the response of the loop using the quarter-beat length coupler quickly goes off scale at the magnification displayed. (For instance, over a normalized detuning of  $\pm 10\%$  the output of the loop using the maximally flat device deviates by a factor of 3000 times less than a similar loop using a quarter beat length coupler.)

## II. CONCLUSION

Simple analytic design rules for wavelength, polarization and fabrication insensitive optical power dividers have been developed. These structures are comprised of a pair of synchronous directional couplers connected by an uncoupled section in which a phase shift is introduced. The operation may be viewed as one of error cancellation. The second directional coupler subtracts out deviations introduced by the former, if these deviations are similar in both couplers. Other maximally flat solutions exist besides those given here. However, those given here keep the phase delay  $2\theta$  as small as possible, and the directional couplers 1 and 2 as identical as possible. Equation

(2) shows that as  $N$  increases (the splitting ratio decreases), the couplers do become more identical. Thus, the insensitivities of these solutions should be superior to other choices.

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