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Design with Constructal Theory*

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This paper outlines a new 4th-year undergraduate course on the generation of system configuration (geometry, architecture, and drawing) during the maximization of performance of flow systems. The configuration is free to morph. Real systems are destined to remain imperfect because of finiteness constraints. They are plagued by resistances to the flow of fluid, heat, and electricity. The balancing and distributing of resistances (irreversibility) through the available volume is the mechanism that generates the architecture. The course teaches the generation of configuration, multi-scale and multi-objective hierarchical structures for fluid flow and heat flow, tree-shaped networks for collection and distribution, and strategies for reducing the cost of generating the flow configuration.

Keywords: constructal theory; design; flow; mechanical structures

CONCEPT

IN THIS PAPER we describe the development of an entirely new course on the principle-based generation of flow-system architecture. What is new about this approach is the focus on the principle and method of generating geometry (shape, structure, configuration) for energy systems with flows of fluid, heat, electricity and mass. The course is about design discovery—the maximization of global performance subject to global constraints—in engineered systems and in natural systems as well. We believe that this concept is an ideal mechanism for teaching the connection between engineering and nature, and for demonstrating to the student the real-life importance of core courses (fluid mechanics, heat transfer, thermodynamics) in design. It is also an effective vehicle for reviewing mathematical methods, especially the simplest pencil and paper skills, on which the graduating engineer will have to rely in his or her career.

The approach is based on *constructal theory* [1]—the view that flow configuration (geometry, design) can be reasoned on the basis of a principle of maximization of global flow access in systems that are free to morph under global constraints. The generation of flow configuration is viewed as a phenomenon, and the principle that sums up its repeated occurrence in nature and engineering (the constructal law) is deterministic.

Constructal theory provides a broad coverage of 'design' everywhere, from engineering to geophysics and biology [2]. To see the generality of the method, consider the following metaphor, which we use to teach in the introductory segment of the course. Imagine the formation of a river drainage basin, which has the function of providing flow access from an area (the plain) to one point (the river mouth). The constructal law calls for a configuration with maximum flow access (minimum global flow resistance). The invocation of this law leads to an optimal balancing of all the internal flow resistance, from the seepage along the hill slopes to the flow along all the channels. Resistances (imperfections) cannot be eliminated. They can be 'matched' neighbor to neighbor, and distributed so that their global effect is minimal and the whole basin is the least imperfect that it can be. The river basin 'morphs' en route to its optimal configuration.

The visible and valuable product of this way of thinking is the *configuration*: the river basin, the tree of cooling channels in a package of electronics, etc. The configuration is the big unknown in design; constructal law draws attention to it as the unknown, and guides our thoughts in the direction of discovering it.

In the river basin example, the configuration that the constructal law recommends is a tree-shaped flow, with balances between highly dissimilar flow resistances such as seepage (Darcy flow) and river channel flow. The 'tree' is the theoretical way of providing maximum access between one point (source, sink) and an infinity of points (area, volume). The tree is a *complex* flow structure, which has multiple length scales that are distributed optimally (hence non-uniformly) over the available area or volume. The tree is 'order out of chaos': the order is the stream flow through the tree channels, and the chaos is the diffusion across the interstices between the smallest tree channels.

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All these features, the tree shape and the multiple scales, are found in any other flow system the purpose of which is to provide access between one point and an area or volume. If we think of the trees of electronics, vascularized tissues and city traffic, then we get a sense of the universality of the principle that was used to generate (to discover) the tree configuration.

Trees are not the only class of configurations that result from invoking the constructal law. Straight tubes with round cross-sections are discovered when one maximizes the access for fluid flow between two points. Round tubes are found in many natural and engineered flow systems (blood vessels, piping, etc.). Optimal spacings between solid components are discovered by invoking the constructal law. Examples are the spacings between fins in heat exchangers, and the spacings between heat-generating electronics in a package. Optimal intermittency (rhythm) is discovered in the same way, and, once again, the examples unite nature with engineering, from human respiration (in and out flow), to the periodic shut down and cleaning of heat exchangers in power plants.

COURSE OUTLINE

The current textbook and solutions manual for the course is [2], with [3] as a supplement. We developed this course material during four years at Duke University, where it was just approved as a permanent course (ME166 Constructal Theory and Design). We also taught this method in short-course format at the University of Évora, Portugal (2003), the University of Lausanne, Switzerland (2004), Yildiz University, Istanbul (2004), and Memorial University, St. John's, Canada (2005). We have been invited to teach the short course in June 2006 at the University of Limerick, Ireland.

In the table of contents shown below, a key feature is the continuity between the main segments. Our objective is to develop a cohesive method and to teach a course that flows. Our philosophy is to teach the simplest first, and to progress gradually toward the more complex, the multi-objective and the multi-disciplinary. Each segment is tutorial, with worked-out examples, and original problems proposed as homework.

The flow of fluid, heat and mass

This is a brief review of the main concepts and results of fluid mechanics and heat and mass transfer that find application in the design of flow systems. The main topics are laminar flow, turbulent flow, Moody chart, local pressure losses, conduction, convection, radiation, mass diffusion and convection.

Thermodynamics and design

Here we begin with a structured outline of the concepts and first and second laws of thermodynamics. We then focus on the 'imperfections' of flow systems and establish the connection between imperfections (irreversibility) and flows against resistances. We continue with the basic concepts of design: objectives, constraints, and the global minimization of flow system imperfection, and how this can be achieved through the generation of flow configuration (the constructal law). This segment is essential—it is a tutorial on the thermodynamics and design concepts that the student needs in order to see why it is important to focus on the discovery of flow configuration.

Simple flow configurations

Here we offer an introduction to how to generate optimal flow configuration from the constructal law. We teach the simplest first: duct cross-sections, river cross-sections, spacings between plates in stacks bathed by forced convection or natural convection, mixed convection as a choice between forced and natural convection, and maximization of ionic transport. These lessons are simple and can be taught analytically with simple methods such as scale analysis and the intersection of asymptotes.

Tree networks for distribution and collection

Next, we introduce the student to the optimization of layout and how to fill the available space with tree-shaped architectures. We start with the simplest class of tree-shaped flows: T and Y-shaped constructs. We devote more space to trees that connect a circle with its center, because these quasi-radial structures will be used in applications for heating and cooling later in the course. We also teach methods for decreasing the cost (time) needed to generate tree architectures (e.g. by minimizing channel lengths everywhere, and optimizing the angles of confluence of two branches).

Multi-scale configurations

The generation of tree architectures introduces the student to multiple length scales and optimized complexity. Here we exploit this idea to generate classes of flow configurations that offer maximum compactness (e.g. maximum heat transfer density): stacks of parallel plates with multiple lengths and spacings, surfaces with discrete heat sources (e.g. electronic components in a package), stacks with multiple spacings and non-uniformly distributed heat sources, optimal layout of electrodes for maximum ionic transport during the decontamination of soil and concrete structures, etc.

Multi-objective configurations

The convective structures developed in the preceding segment introduce the student to the idea that one flow configuration serves two functions: low fluid-flow resistance and low thermal resistance. We pursue this idea further. One topic is tree-shaped flow structures for cooling electronics. Another is tree-shaped networks for distributing hot or cold water (i.e. trees for maximum point-area flow access and minimum heat loss from all the pipes to the ambient). We end with heat exchangers and fuel cells that have tree-shaped flow configurations.

Flow and mechanical structures

Flow systems with configuration do not exist alone. They are confined and supported by solid structures that provide strength and rigidity. In this concluding part of the course we teach how to combine thermo-fluid objectives with mechanical objectives in the pursuit of configuration. Examples are cavernous brick walls for maximum thermal insulation and stiffness, and support beams that must be strong and resistant to sudden heating (e.g. fire, thermal attack).

Term papers

Every student is asked to develop an original topic in a term paper, and to defend it orally at the end of the course. This part of the course has been a source of real intellectual growth for all those enrolled in the course, and a source of new directions of inquiry in constructal theory and design. For example, at the end of the spring 2005 semester at Duke University the students presented term papers on constructal-theory topics such as the universal shape of Egyptian pyramids, leaf venation, coral dendrites, hot-water networks for preventing the icing of bridges, river delta evolution and confinement, etc.

In sum, the course does for the design of energy flow systems what the optimization of geometry does for mechanical structures. In the latter, the global objective (maximal strength) is met by moving the material of the morphing structure to spots where material is needed most [2, 4]. The mechanical structure 'evolves' toward an animal-like skeleton with a more uniform distribution of maximal stresses. There is a clear analogy between this and what happens in a flow system that morphs toward its optimal flow architecture. Instead of stresses, a flow system has flow resistances (fluid, thermal, electrical) that are distributed optimally (non-uniformly) through the available volume.

In the next sections, we present in more detail some of the pedagogical highlights of the course.

TREE-SHAPED FLOW CONFIGURATIONS

The simplest problems are the best for teaching lessons that are new. They are also the best for showing other instructors what we are developing. The simplest fluid flow system is the one that must connect two points with minimal flow resistance, $R = \Delta P/\dot{m}$ where ΔP and $\dot{m}$ are the pressure difference and the mass flow rate. In this simplest description [5], a flow system has two constraints, or two global sizes (properties): the external dimension of the system (L), and the internal dimension that is represented by the total duct volume (V). Consequently, a flow system has a new property called *svelteness*,

$$\mathrm{Sv} = \frac{L}{V^{1/3}} \tag{1}$$

There is an infinity of possible designs that have the same L and V. Better than most are the straight ducts with constant cross-sections. Among these, the best is the round pipe—the straight duct with mathematically circular cross-section.

Just like the round pipe, there are many other optimal architectures that deserve to be generated, taught and used. Familiar single-scale examples are the tear-drop shape of bodies with minimal drag (e.g. water droplet, fish, airfoil). Examples of multi-scale architectures are the tree-shaped networks that connect discrete points (sources, sinks) with infinite numbers of points (curves, areas, volumes); e.g. Fig. 1. Tree-shaped flows illustrate the commonality of optimal form in engineering and nature, which is a central theme of this course.

Many more lesson-size examples of tree architectures are provided by the constructal literature. Constructal design grew as a new hierarchical (multi-scale) method for maximizing the access between one point (source or sink) and an infinity of points (volume or area). Flows of several types were considered (e.g. fluid, heat, electricity, and traffic). In every case, the result was an optimized flow architecture with two features: links organized in the shape of a tree, and interstices that inhabit the spaces between the smallest links (branches, tributaries) of the tree.

At least two flow modes (mechanisms) must be available, one with high resistance and the other with significantly lower resistance. The low-resistance mechanism is placed in the tree links, while the high-resistance mechanism occupies the interstitial spaces. In a river basin, for example, the low-resistance mechanism is channel flow (rivers, rivulets) and the high-resistance mechanism is Darcy flow through saturated porous media (wet banks, silt, sand). In urban traffic, the corresponding mechanisms are street traffic and walking.

Loops and grid-shaped flow patterns can be used because they add *robustness* to the tree-shaped flow configurations that they serve. Robustness and redundancy are precious properties in design, and our course teaches how to endow designs with such properties.

We teach that a constructal tree structure represents a perfect balance between the resistance to flow along the branch and the resistance to flow across the interstice. The interstices are as important as the branches. Those who describe the 'tree' as only an assembly of branches are missing most of the picture.

Optimized area and volume elements become the building blocks of larger and/or more complex flow structures that do the same thing as the element: they optimize access between one point

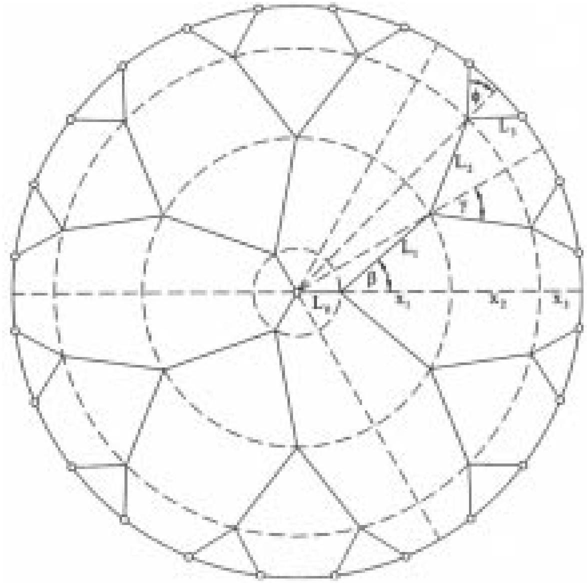

Fig. 1. The optimized tree-shaped structure for tubes with fully developed laminar flow connecting a circle with its center [6].

and an area. Larger areas are accessed through steps of assembly and optimization. Purposeful assembly and optimization means *construction*.

The optimal distributing of flow imperfection is how the geometry of the flow is generated. This thought opens the student's eyes to the *graphic* meaning of flow systems, thermodynamics, engines and refrigerators. Flow systems are not the black or white boxes used in the first course on thermodynamics. A real flow system has configuration, or drawing, which comes from objectives, constraints, and enough time to morph [1–3, 5].

WHERE TEACHING MEETS RESEARCH

As in the work of Turns *et al.* [7], one objective of the course is to use the existing research base, and to bring the latest design ideas into the classroom. We believe that this is the best way to prepare engineers: teach fundamentals and disciplines first, and emphasize freedom, novelty and the interdisciplinary next. In constructal design, interdisciplinary considerations recommend themselves 'naturally' when designers encounter systems with more than one objective.

One example is how to deduce the optimal cavernous structure of a wall made of hollow bricks [8] (see Fig. 2). The wall design has two objectives: maximal mechanical strength, and maximal thermal resistance. These two objectives clash, and from this competition emerges the optimal number and size of wall cavities. The internal structure of the wall (the number of air caverns) can be optimized so that the overall thermal resistance of the wall is maximal, while the mechanical stiffness of the wall is fixed. The maximized thermal resistance increases when the effect of natural convection in the air gaps is weaker, and when the specified wall stiffness decreases. The optimal number of air gaps is larger when the effect of natural convection is stronger, and when the specified wall stiffness is smaller. The optimal structure is such that the volume fraction occupied by air spaces decreases when the natural convection effect (the overall Rayleigh number) increases, and when the prescribed wall stiffness increases.

The example shown in Fig. 2 belongs to a new class of thermal design problems, in which the system architecture is derived from a *combination* of heat transfer and mechanical strength considerations [8, 9]. This class represents an extension of the constructal design method, which until recently [1, 2] has been used only for maximizing thermofluid performance subject to size constraints. A good start is the optimization of shapes in support structures found in civil engineering. One example is the cantilever beam with minimal weight, and with prescribed loading and stiffness. The student can derive with pencil and paper the optimal shape, out of an infinity of candidate shapes.

MULTI-SCALE DESIGN

The traditional approach to the design of convective flow structures, such as heat exchangers and electronics cooling, starts with the channels (ducts) and the spacings between them. Flow channels and wall features (e.g. fins) are first assumed. Later, they are packed into larger constructs that fill the space allocated to the device. The flow is forced to reside in regular spaces with a single length scale. This length scale is distributed *uniformly* through the available space.

In this course we open the students' eyes to the possibility of using *multiple* length scales that are distributed *non-uniformly* through the available

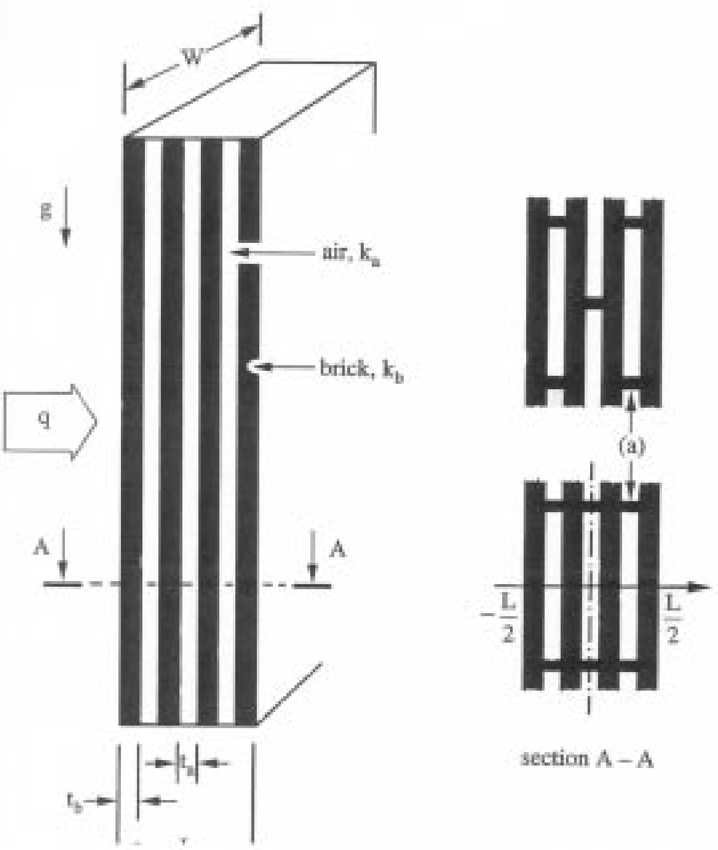

Fig. 2. Vertical insulating wall with alternating layers of solid (brick) and air [8].

volume. The tree-shaped cooling designs that are now invading the research literature [2, 3, 10, 11] are examples of multi-scale structures that are distributed non-uniformly (here 'non-uniformly' means that in a tree design there is a special place for the trunk and the larger branches, and another place for the canopy).

In this section we give two additional examples, which can be taught with chalk on the blackboard. One is the cooling of a two-dimensional volume with laminar forced convection and specified overall pressure drop [3]. The usual approach is to optimize the spacing D_0 in a stack of parallel heat-generating plates of one length: the swept length of the volume, L_0. The new method is illustrated in Fig. 3. It consists of exploiting every available flow volume element for the purpose of transferring heat. In laminar forced convection, the working volume has a thickness that scales with the square root of the length of the streamwise heat transfer surface. Larger surfaces are surrounded by thicker working volumes. The starting regions of the boundary layers are thinner. They are surrounded by fresh flow that can be put to good use: heat transfer blades with smaller and smaller lengths can be inserted in the fresh fluid that enters the smaller and smaller channels formed between existing blades.

The optimal number of scales of the multi-scale flow structure increases slowly as the flow becomes stronger. The flow strength is accounted for by the pressure difference maintained across the structure. The boundary layers become thinner as the pressure difference increases, and this means that more small-scale heat transfer blades can be inserted in the interstitial spaces of the entrance region of the complex flow structure.

Two trends compete as the number of length scales increases. The structure becomes less permeable, and the flow rate decreases. At the same time, the total heat transfer surface increases. The net effect is that the heat transfer *density* increases as the number of length scales increases. This increase occurs at a decreasing rate, meaning that each new (smaller) length scale contributes less to the global heat transfer enterprise than the preceding length scale. In this way we also teach the concept of diminishing returns, and the importance of knowing when to stop the process of refining the features of the complex flow structure.

The second example concerns the augmentation of heat transfer from a wall with discrete heat sources (Fig. 4) [12]. Here we teach analytically how to optimize the spacings between heat sources, and that near the tip of every boundary layer the best spacing is zero. There must be a starting region of length x_0 along which the heat sources are positioned flush next to each other. Even when the heat sources themselves have a single length scale (D_0), the flow architecture has multiple length scales (x_0, S_i) that are distributed

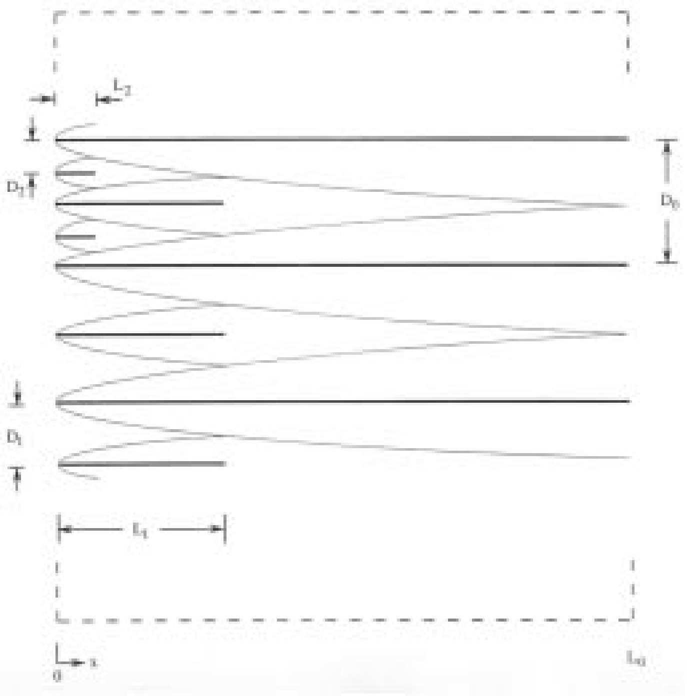

Fig. 3. Optimal multi-scale package of parallel plates [3].

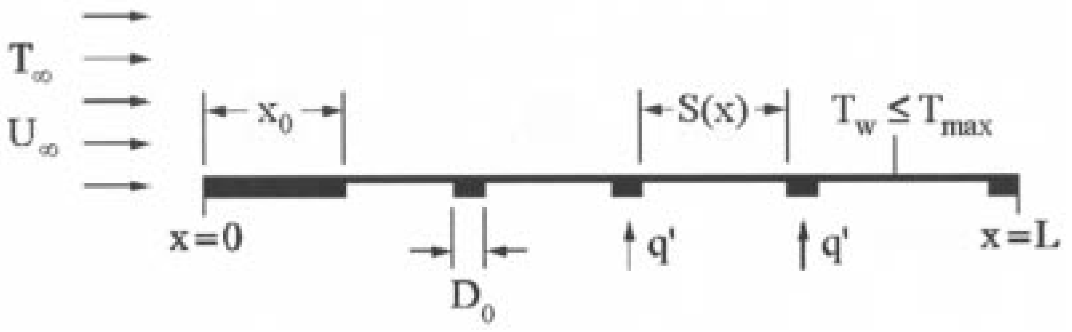

Fig. 4. Wall with non-uniformly distributed heat sources [12].

non-uniformly: x_0 and the smallest spacings must be positioned near the start of the flow region.

THE COST OF GENERATING THE CONFIGURATION

Another important lesson of the course is that the work required for developing the configuration of the flow system can be reduced. Consider the computational cost of optimizing complex multi-scale flow structures for cooling high-density heat-generating volumes. We teach how to reduce the computational cost by devising effective strategies. For example, tree-shaped structures of the type shown in Fig. 1 have been developed rigorously and laboriously by optimizing every free geometric detail (tube lengths and diameters) [6]. An alternative strategy is explained in Fig. 5. The lattice of tubes shown near the rim of the disc in Fig. 1 can be deduced approximately by optimizing the match between each tube and its allocated rectangular area [13].

The rectangular area isolated in Fig. 5 has fixed size (xy) and variable shape (x/y). The tube crosses it from the middle of one side to the opposing corner. This area element is a good approximation of the areas that serve as background for the tubes near the disc periphery in Fig. 1. The tube length is minimal when the rectangle shape is $x/y = 2$. This design has minimal flow resistance and pumping power requirement, in laminar flow and turbulent flow.

Tree designs can be generated with ease by assembling progressively larger elements of the type optimized in Fig. 5. The cost of this alternative design [13] is negligible in comparison with the most rigorous optimization (Fig. 1), but it is approximate. Through such examples we teach the competition between cost and accuracy, which is related to the diminishing returns mentioned earlier.

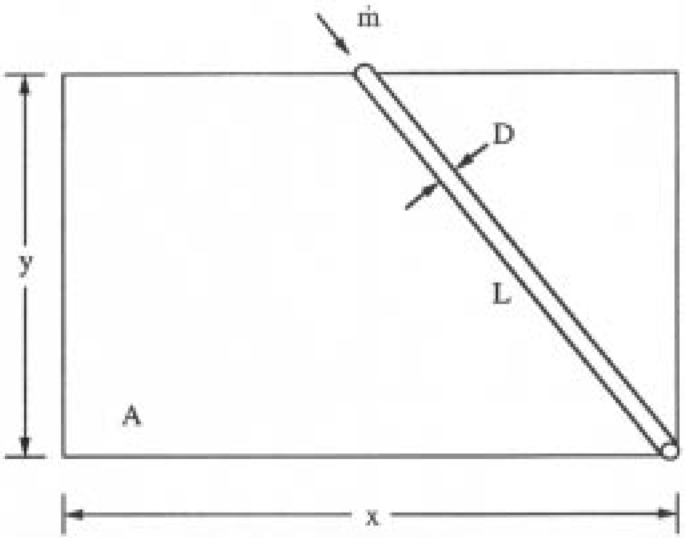

Fig. 5. Duct crossing a rectangular element with fixed area and variable shape.

DESIGN AS SCIENCE: FROM THE CONSTRUCTAL LAW TO CONFIGURATION

In summary, the proposed course method provides the student with a strategy for how to pursue and discover design (configuration, pattern). Constructal theory pushes design thinking closer to science, and tears down the walls between engineering and natural sciences. As in Feland *et al.* [14], it represents a more comprehensive approach, with more balance between art, science and philosophy.

The development of configuration requires a working knowledge of fundamentals, and this gives us a very good opportunity to teach again (this time with purpose) some of the main lessons of the core disciplines. Thermodynamic imperfection is due to currents that flow by overcoming resistances. The balancing and distributing of flow resistances is what generates the configuration. This, the optimal distribution of imperfection, can be pursued for gain in design, not only in mechanical and civil engineering [15], but also in design and manufacturing [16, 17].

This course concept is a necessary addition to the current emphasis on smaller and smaller scales. The reason is that, ultimately, the devices that we touch must possess our scale—they must be macroscopic, no matter how small the smallest components. The more successful we are in making smaller and smaller components, the greater the challenge to install larger and larger numbers of such components, and to connect them with currents (heat, fluid, electricity), to keep them *alive*. The challenge is to *construct*, to assemble and optimize while assembling (i.e. to optimize complexity), and to *deduce* the flow configuration of the macroscopic device. For this, constructal theory provides the key.

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