Online Supplement for

Designing Fee Tables for Retail Delivery Services by Third-party Logistics Providers

by

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Validity of inequalities

Proof of Proposition 1 (*cumulative range width forcing constraints*)
Constraint (19) *is valid*: If the [*FTD*] solution does not select a range k that spans intervals j_1 through j and ends at interval j, then $\sum_{j' \le j_1} X^k_{j',j} = 0$, and constraint (19) follows from the non-negativity of the range selection variables. If $\sum_{j' \le j_1} X^k_{j',j} = 1$, then the width of range k is at least $\left(d_j - d_{j_1 - 1}\right)$. In this case, the [*FTD*] must choose a range k + 1 beginning with interval j + 1 (to satisfy range contiguity constraints (3)) and have a width of at least $\left(d_j - d_{j_1 - 1}\right)$ (from the increasing range width constraints (13)); consequently, the solution must select an ending interval j_3 for range k + 1 such that $(d_{j_3} - d_j) \ge (d_j - d_{j_1 - 1})$, implying (19).
The proof of the validity of constraints (20) follows a similar logic. ■

Proof of Proposition 3 (*block-based variable lower bound constraints*)
Constraint (22) *is valid*: Suppose the [*FTD*] solution selects some ending interval $j' \ge j$ for the distance range that includes interval j (from (2) and (3), we know that interval j must belong to exactly one of the K distance ranges). Similarly, suppose weight interval $i' \ge i$ is the ending interval for a range that spans weight interval i. In this case, the fee lower bound constraints (9) and the range-to-fee linkage constraints (4) and (7)ensure that P_{ijt} must be at least as much as the highest lower bound ($l_{i'j't}$) in the block containing cell <i, j>,and constraint (22) follows. A similar argument establishes constraint (23). ■

Proof of Proposition 4 (*block-based variable upper bound constraints*)
Constraint (25) *is valid*: Suppose the [*FTD*] solution selects some starting interval $j' \le j$ for the distance range that includes interval j. Similarly, suppose weight interval $i' \le i$ is the starting interval for a range that spans weight interval i. In this case, the fee upper bound constraints (9) and the range-to-fee linkage constraints (4) and (7) ensure that P_{ijt} is no more than the lowest upper bound $u_{i'j't}$ of the block containing cell <i, j> and constraint (25) follows. A similar argument establishes constraint (26). ■

Proof of Proposition 5 (*fee difference forcing constraints*)
Constraint (27) *is valid*: Consider an [*FTD*] solution that includes both distance interval *j* and *j* + 1 in the same distance range. In this case, $\sum_{k, j_1 \le j} X^k_{j_1, j} = 0$ and constraints (27) follow from the range-to-fee linkage constraints in [*FTD*]. If not and if the solution chooses weight range $Z^h_{i_1, i_2}$ to span weight interval *i* (i.e., $i_1 \le i \le i_2$), then the RHS evaluates to $(u_{i_1, j+1, t} - l_{i_2, j, t})$ which is valid because cells (i_1, *j*+1), (i_2, *j*), (*i*, *j*), and (*i*, *j*+1) belong to the same fee block. Similarly, starting from the weight ranges and then integrating the distance range selections, we can show the validity of constraint (28). ■

Proof of Proposition 6 (*block selection forcing constraints*)
Constraint (29) *is valid*: If the intervals j_1 and j_2 belong to *different* distance ranges, the RHS of constraint (29) is zero and the constraint follows from the non-negativity of the weight range selection variables. If j_1 and j_2 belong to the same range, then the RHS of the constraint is one. Consider all of the eligible weight ranges that span weight interval *i*. The weight range selection constraints (5) and (6) ensure that the sum of all the weight ranges that span any interval *i* is one. If the solution selects $i_1 \le i \le i_2$ with $u_{i_1, j_1, t} < l_{i_2, j_2, t}$ for any *t*, then the fee value for the block containing cell <*i*, *j*> cannot simultaneously satisfy its lower and upper bounds. Therefore, the LHS of constraint (29) must be one in this case. We can establish the validity of constraint (30) following a similar argument. ■

Variable elimination rules

Proof of Proposition 2 (*variable elimination rules*)
First range. The first range must start with the first interval, therefore $j_1 = 1$. Next, the width of the first range must be the least of the range widths and as a result must be less than or equal to the average range width ($\frac{d_N}{K}$) of the subsequent $K - 1$ ranges that span a distance of $(d_N - d_{j_2})$. Finally, because each of the remaining $K - 1$ ranges must occupy at least one interval, $j_2 + (K - 1)$ must be less than or equal to *N*.
Middle range. Because $k - 1$ ranges must have been completed with each occupying at least one interval, we know that $j_1 \ge k$. Analogously, the $K - k$ ranges that follow range *k* must each cover at least one interval. Therefore, $j_2 + (K - k)$ must be less than or equal to *N* and part (b) follows. To see part (c), consider a distance range selection $X^k_{j_1, j_2}$ that implies a width of $d_{j_2} - d_{j_1 - 1}$ for range *k*. In this case, the previous ranges, each with a width no greater than that of range *k*, can have a an average width of at most $\frac{d_{j_1 - 1}}{k - 1}$, which range *k*'s width must exceed and (c) follows. Next, each following range must have a width of at least $d_{j_2} - d_{j_1 - 1}$. Therefore, the average width $\frac{d_N - d_{j_2}}{K - k}$ of the $K - k$ ranges that follow *k* must be greater than or equal to range *k*'s width and (d) follows.
Last range. The last range must include the last interval *N*, therefore $j_2 = N$. Next, the previous $K - 1$ ranges must include at least one interval each; consequently, $j_1 \ge K$. Finally, the width $(d_N - d_{j_1})$ of the last range must be at least as much as the average width $\frac{d_{j_1 - 1}}{K - 1}$ of the previous $K - 1$ ranges and (c) follows.