

# Designing for Productive Failure

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## What is Productive Failure?

Understand what students know about a novel concept that they have not been taught yet

Afford opportunities to activate and differentiate prior and intuitive knowledge....to **generate, explore, critique,** and **refine** representations and solution methods (RSMs) for solving complex problems

Invariably, such a process leads to failure (in relation to a desired goal)...

Designing for failure in the initial learning to minimize failure in the longer term...

**But,** this may precisely be the locus of deep learning... **provided** some form of structure follows subsequently

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## Two phases of the PF design

(Kapur & Bielaczyc, 2012)

### GENERATION & EXPLORATION

#### PHASE I

- Complex problems
- Collaboration
- Affective support for persistence

### CONSOLIDATION & KNOWLEDGE ASSEMBLY

#### PHASE II

- Consolidation
- Well-structured Problem solving OR Instruction OR Feedback OR Explanation, etc.

DELAY OF STRUCTURE

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## The Problem

(Grade 8/9 students)

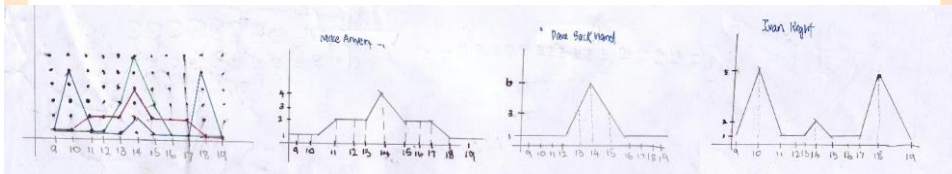
Who's the most consistent striker?

Year	Mike Arwen	Dave Backhand	Ivan Right
1988	14	13	13
1989	9	9	18
1990	14	16	15
1991	10	14	10
1992	15	10	16
1993	11	11	10
1994	15	13	17
1995	11	14	10
1996	16	15	12
1997	12	19	14
1998	16	14	19
1999	12	12	14
2000	17	15	18
2001	13	14	9
2002	17	17	10

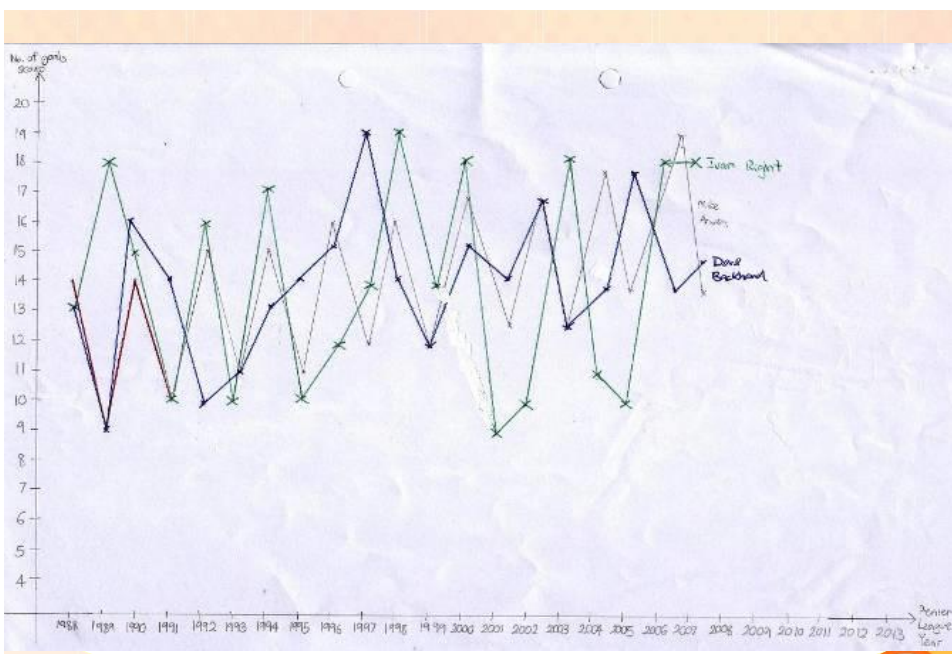
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Comparing regularity

	9	10	11	12	13	14	15	16	17	18	19
Mike Arwen : Mean = $\frac{280}{20}$ = 14 goals / year Mode = 14	1	1	2	2	2	4	2	2	2	1	1
Dave Backhand : Mean = $\frac{280}{20}$ = 14 goals / year Mode = 14	1	1	1	1	3	6	3	1	1	1	1
Ivan Right : Mean = $\frac{280}{20}$ = 14 goals / year Mode = 18 and 10	1	5	1	1	1	2	1	1	1	5	1



9 10 11 11 12 12 13 13 14 14 14 14 15 15 16 16 17 17 18 19



From Question paper: Average =  $\frac{280}{20}$

Mike has 8 years < average

4 years = average

8 years > average

Dave has 7 years < average

6 years = average

7 years > average

Ivan has 9 years < average

2 years = average

9 years > average

**Frequency of years above, below, and at average**

**Consistency = years at the mean / years away from the mean**

**Sum of year-on-year deviation**

Mike:	Dave:	Ivan:
9-14 = -5	7	5
14-9 = 5	-2	-3
10-14 = -4	-4	-5
15-10 = 5	1	1
-4	2	-6
4	1	7
-4	4	-7
5	-5	2
-4	-2	2
4	3	5
-4	-1	-5
5	3	-5
-4	-4	4
4	1	-9
-4	3	1
5	-4	8
4	4	-7
-4	4	-1
5	-4	8
-4	1	0
5	-2	-5
-4		
0		

Range  
 Mike Armen: 9 - 19 = 10  
 Dave England: 9 - 19 = 10  
 Ivan Kykt: 9 - 19 = 10

**Sum of deviations about the mean**

Year	Av	M.A	D.B	E.R	X	
1983	14	14	13	13	0	-1
1984	14	9	4	18	-5	-5
1990	14	14	16	15	0	+1
1991	14	10	14	10	-4	4
1997	16	15	10	16	+1	+2
1993	16	11	11	10	-3	-4
1998	16	15	13	17	+1	+3
1995	14	11	14	10	-3	0
1996	14	16	15	12	+2	+2
1997	14	12	14	14	-2	+5
1992	14	16	14	12	+2	0
1999	14	12	12	14	-2	-2
2000	16	17	15	18	+2	+1
2001	14	13	14	9	-1	-5
2002	14	17	17	10	+2	-4
2003	14	13	13	18	-4	-4
2004	14	18	14	11	+7	-3
2005	14	14	18	10	0	+4
2006	14	19	14	18	+5	0
2007	14	14	15	12	0	+1

**Average of year-on-year absolute deviation**

MIKE =  $\frac{5+5+4+5+4+4+4+5+4+4+4+5+4+4+4+5+4+5+4}{20} = \frac{84}{20} = 4.2$

DAVE =  $\frac{4+7+2+4+1+2+1+1+4+5+2+3+1+3+4+1+4+4+1}{19} = \frac{54}{19} = 2.84$

IVAN =  $\frac{5+3+5+1+6+7+7+2+2+5+5+4+9+1+8+7+1+8+0}{19} = \frac{84}{19} = 4.39$

DAVE is most consistent





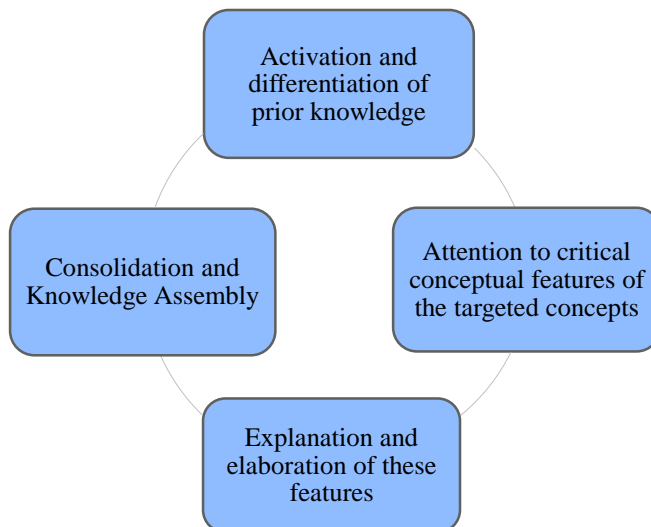
## Summary of Key Findings

- PF outperformed DI on conceptual understanding and transfer without compromising procedural fluency (Kapur, 2010, 2012; Kapur & Bielaczyc, 2012)
- Students that seem **strikingly dissimilar** on general and math ability (PSLE) appear **strikingly similar** in terms of their generative capacity (Kapur & Bielaczyc, 2012)
- RSM diversity significantly correlated with learning gains (Kapur, 2012; Kapur & Bielaczyc, 2012)
- Teachers consistently underestimate students' ability to generate RSMs
- PF teachers consistently report that they are stressed and stretched to work with students' ideas... **BUT, they themselves understood the math better...**

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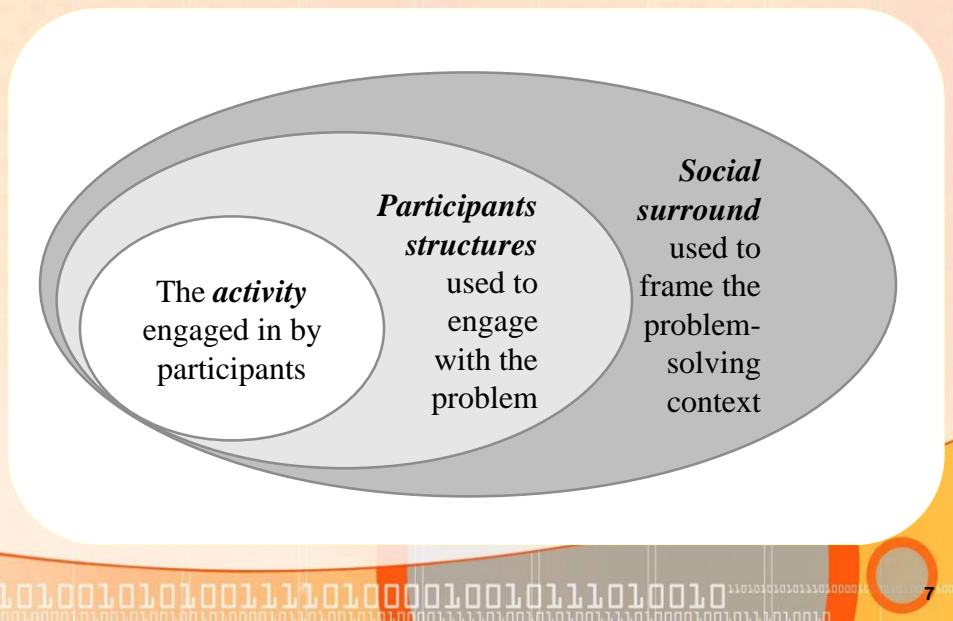
## Four mechanisms of the PF design

(Kapur & Bielaczyc, 2012)



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## Three layers of the PF design (Kapur & Bielaczyc, 2012)



## Designing for Productive Failure (Kapur & Bielaczyc, 2012)

DESIGN LAYERS AND THE EMBODIED PRINCIPLES			
DESIGN PHASES	TASK	PARTICIPATION STRUCTURES	SOCIAL SURROUND
<b>1. Generate and explore multiple representations and solutions methods (RSMs)</b>	Design tasks that are adequately complex, engaging, and draw on students' mathematical resources	Enable collaboration to allow students to elaborate, critique, explain, and evaluate shared work, thereby further enriching the shared representation and solution spaces	Create a safe space for students to explore and generate by setting appropriate socio-mathematical norms, and providing affective support for persistence
<b>2. Organization and Knowledge Assembly</b>	Compare and contrast student-generated and canonical ideas	Enable student engagement through group presentations and students' participation; teachers act as facilitators, paraphrasing student explanations, and drawing attention to critical features	Create a safe space to explore the affordances and constraints of student-generated RSMs with a view of improving upon them, and not assessing them as correct or incorrect

## Productive Failure Tasks

Targeted Concepts	Targeted Levels
Average Speed	Secondary (7 <sup>th</sup> grade)
Standard Deviation	Secondary (7 <sup>th</sup> , 8 <sup>th</sup> , 9 <sup>th</sup> grade)
Central Tendencies	Secondary (7 <sup>th</sup> grade)
Normalization	Pre-University 2 (11 <sup>th</sup> grade)
Hypothesis Testing	Pre-University 3 (12 <sup>th</sup> grade)
Fractions	Primary (3 <sup>rd</sup> grade)

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### Central Tendencies (Grade 7 students)

Which  
number  
represents  
each class?

Amount of weekly pocket money students from 1A received (\$)	Amount of weekly pocket money students from 1B received (\$)	Amount of weekly pocket money students from 1C received (\$)		Amount of weekly pocket money students from 1D received (\$)
10	10	10	15	10
11	11	11	16	11
12	12	12	16	12
13	12	13	16	13
14	12	14	16	13
14	13	14	16	13
15	14	14	17	14
15	15	14	18	15
15	16	14	19	16
16	17	15	20	17
16	18	15		18
17	18	15		19
18	18	15		19
19	19	15		20
20	20	15		30

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**Standard Deviation**  
(Grade 7/8/9 students)

Who's the most consistent striker?

Premier League Year	Mike Arwen	Dave Backhand	Ivan Right
2001	14	13	17
2002	11	11	14
2003	15	14	16
2004	12	16	11
2005	16	14	12
2006	12	12	16
2007	16	14	12
2008	13	15	16
2009	17	14	12
2010	14	17	13
2011	14	14	15

**Normalization**  
(Grade 11 students)

Who's the most outstanding performer?

Scores of Math Students between 1998 and 2012	Scores of Science students between 1998 and 2012	Scores of English Students between 1998 and 2012
72	71	72
73	72	72
74	80	75
75	80	75
75	81	78
75	81	78
81	81	81
81	81	81
81	81	84
87	81	84
87	81	87
87	82	87
88	82	90
89	<b>90 (Lisa, top science student, 2012)</b>	<b>90 (David, top English student, 2012)</b>
<b>90 (Firdaus, top math student, 2012)</b>	91	

**Hypothesis Testing**  
(Grade 12 students)

Which student's sample is least likely to come from Wala Chocolate Factory?

Net weight: 50 grams each chocolate bar

Weights of chocolate bars bought by Ahmad (g)	Weights of chocolate bars bought by Bella (g)	Weights of chocolate bars bought by Charles (g)	Weights of chocolate bars bought by Devi (g)
58	64	64	75
55	61	63	74
54	60	60	74
49	55	54	73
49	55	54	73
49	55	54	73
48	54	48	55
48	54	45	55
48	54	44	55
48	54		55
47	53		37
47	53		37
47	53		37
42	48		36
41	47		36
38	44		35