
# Designing, Modeling, and Optimizing Transactional Data Structures

PhD Dissertation Defense

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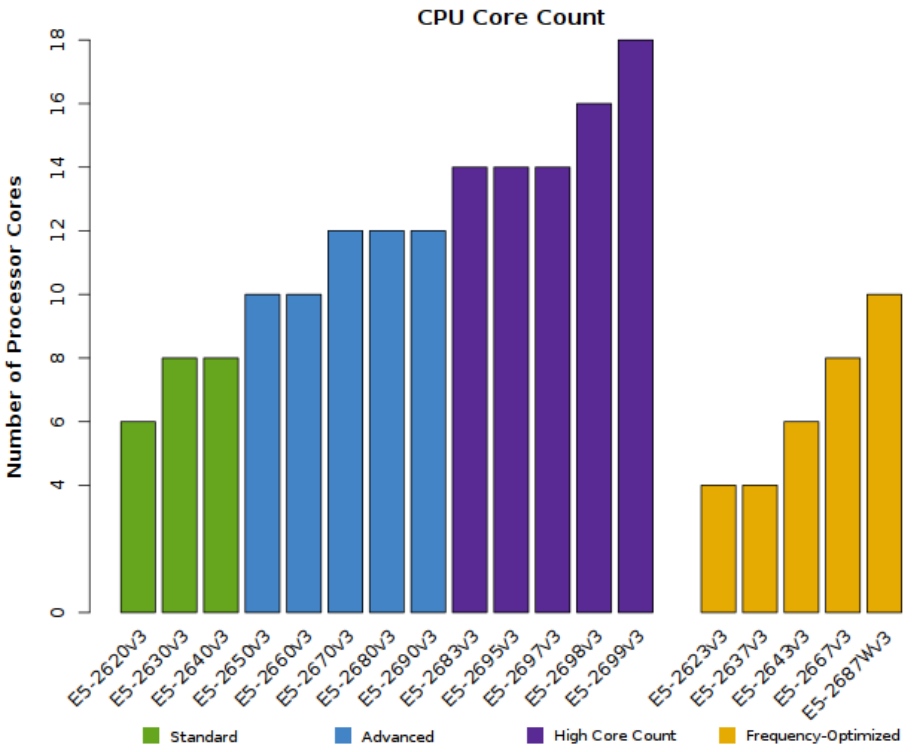
VT MENA EGYPT 2008

Virginia Tech

# State of the Art

- Multi-core architectures.
- Synchronization
  - Critical sections.
  - Using locks.
- Efficient synchronization:
  - Cache coherence protocols.
  - Atomic instructions (e.g. CAS operations).




From *www.microway.com*

# Concurrent Data Structures

# Example: Linked List

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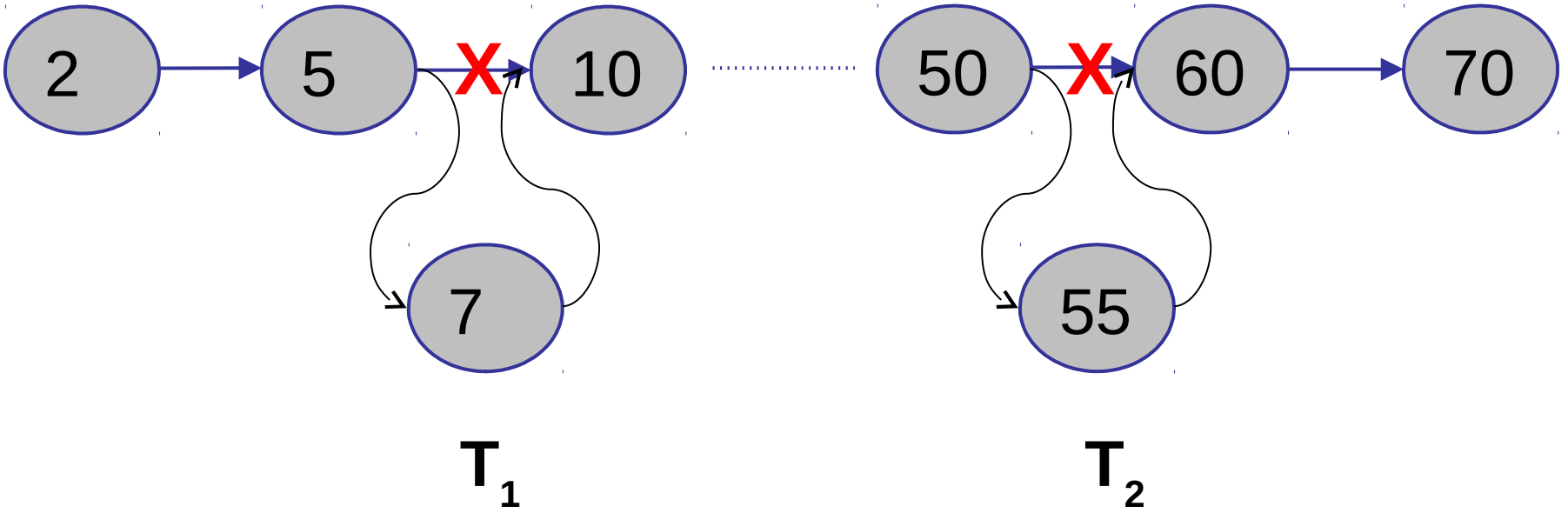


# Synchronization in Concurrent Data Structures

- Coarse-grained locking
  - Easy to implement, good for low number of small threads .
  - **But** minimizes concurrency.

- Fine-grained locking
  - Allows more concurrency.
  - **But** error prone.

- Non-Blocking Designs
  - Lock-free, obstruction-free, wait-free, …
  - Progress guarantees **But** more complex designs.

# Transactional Memory

- Use an underlying TM framework to guarantee consistency, atomicity, and isolation.

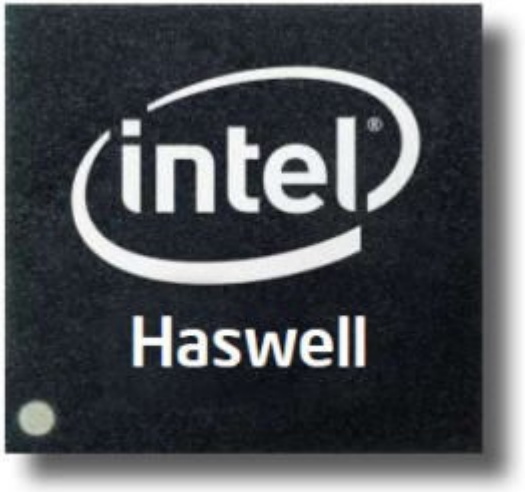
```
Thread 1

@Atomic
foo1()
{
   seqList.add(5)
}
```

- Programmable (like coarse-grained locking).
- Allows concurrency (like fine-grained locking).

# Transactional Memory Gains Traction!!

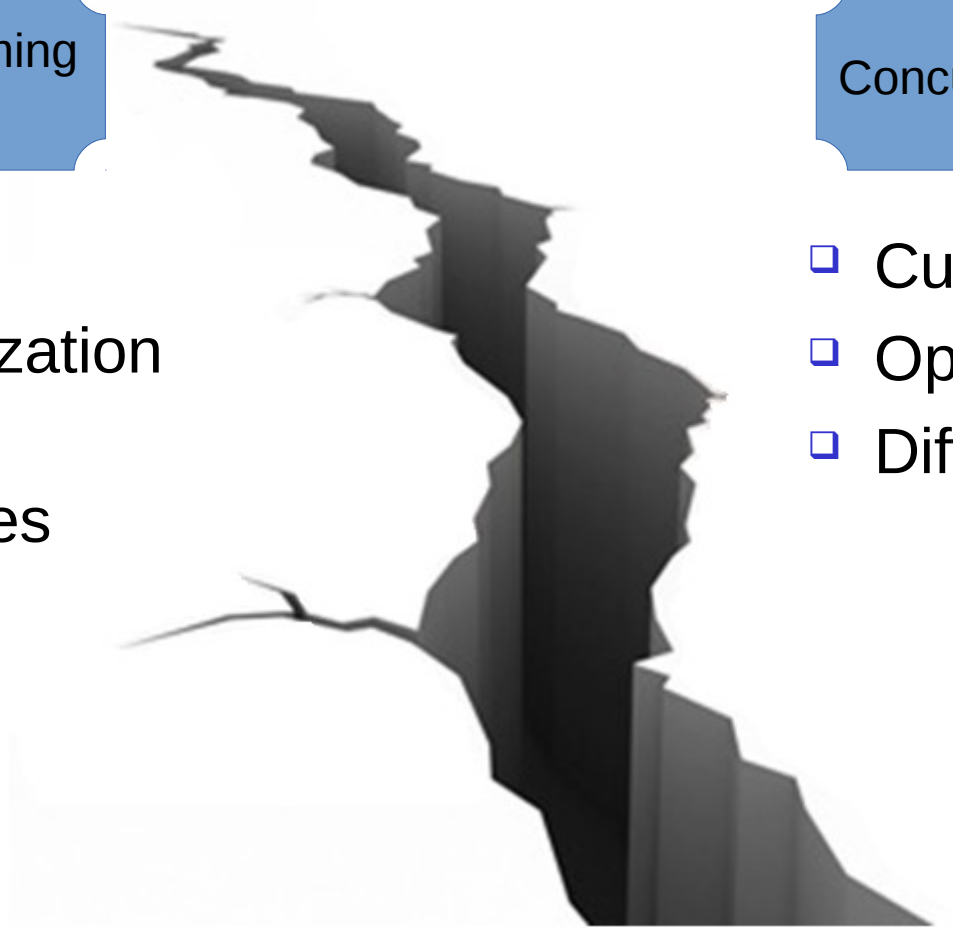
- Intel Haswell's TSX Extensions.
- IBM Power8.
- STM support in C++ and GCC.



# What about data structures?

Transactional Programming Model

- **New** APIs
- **New** synchronization primitives.
- **New** TM libraries
- **New** compiler support.
- **New** hardware components.



Concurrent Data Structures

- Customized Designs
- Optimizations
- Different Primitives

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Transactional Programming Model

Concurrent Data Structures

**Simpler interface**

**less complex designs**

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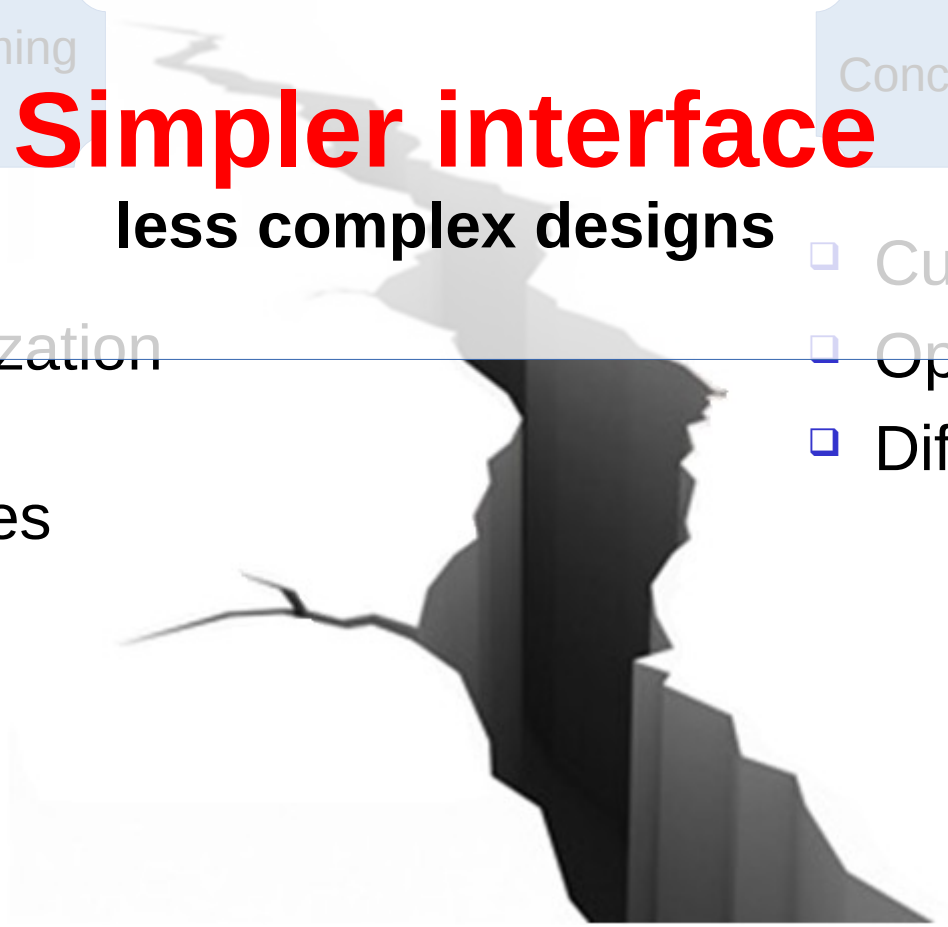

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**Atomic transaction**

**instead of atomic operations**

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- **New** hardware components.

**Simpler interface**

**less complex designs**

**Atomic transaction**

**instead of atomic operations**

**Hardware Support**

**possible performance improvement**

Concurrent Data Structures

- Customized Designs
- Optimizations
- Different Primitives

# Our Goal

## From Concurrent to Transactional Data Structures

# Challenges

- Composability.
- Integration with generic transactions.
- Modeling.

# Composability

```
Shared data: concurrentList

atomicFoo()
{
        concurrentList.add(x);
}
```

```
Shared data: concurrentList

atomicFoo()
{
        concurrentList.add(x);
        concurrentList.add(y);
}
```

```
Shared data: concurrentList1
Shared data: concurrentList2

atomicFoo()
{
        concurrentList1.remove(x);
        concurrentList2.add(x);
}
```

# Composability

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**Modify the design of concurrentList?**

**More complex designs**

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**Modify the design of concurrentList?**

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**Transactional Memory?**

**Lose optimizations of concurrentList**

```
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```
Shared data: concurrentList1
             concurrentList2

atomicFoo()
{
        concurrentList1.remove(x);
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}
```

# Integration

```
Shared data: concurrentList

atomicFoo()
{
        If(concurrentList.add(x))
                n1++;
        Else
                n2++;
}
```

## Modeling

- Different Designs and Implementations
  - Different ad-hoc approaches for proving correctness.

- Is there a unified model for concurrent data structures?
  - General enough
  - Easy to use
  - Includes composability and integration

# Our Contributions

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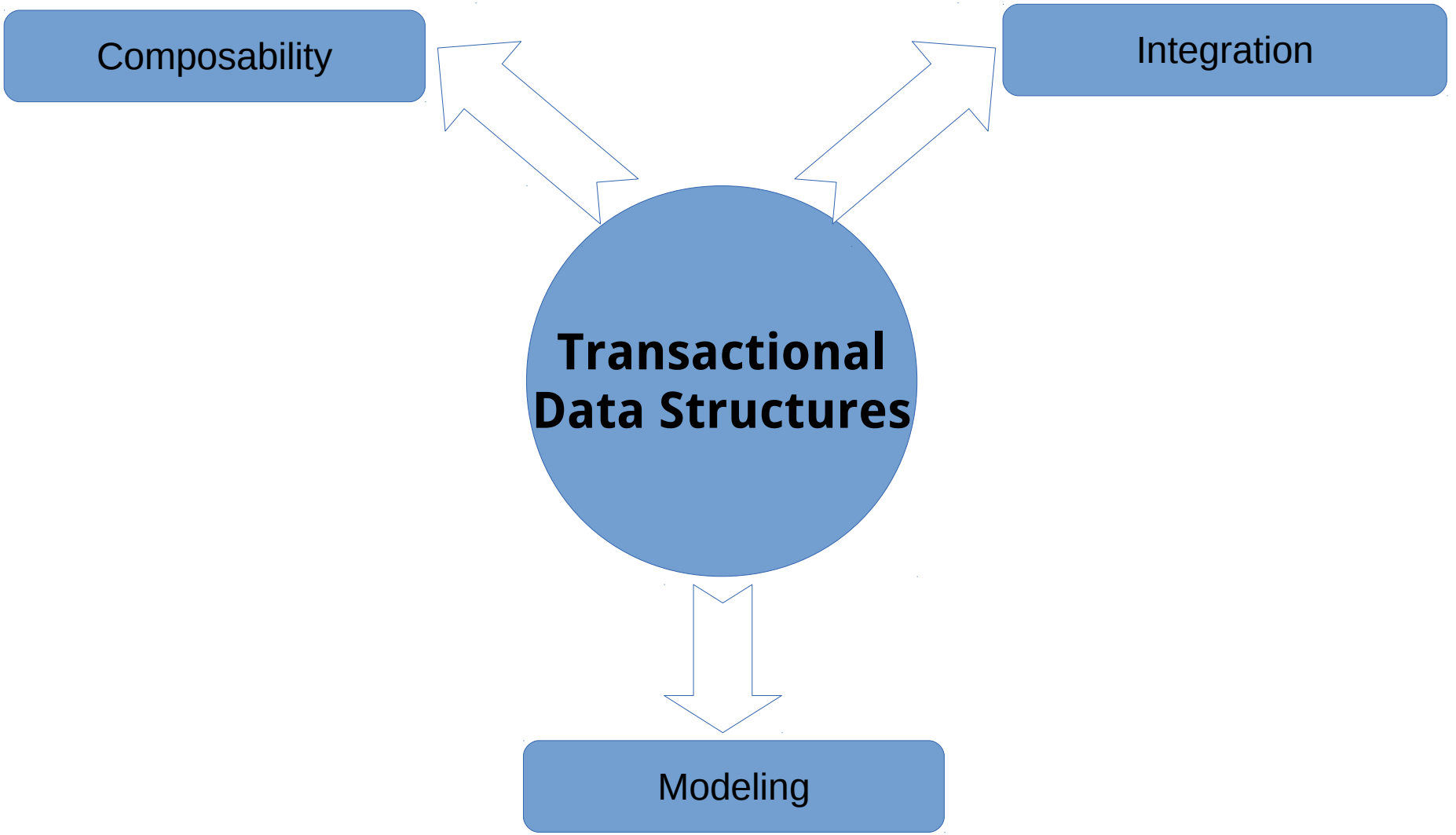

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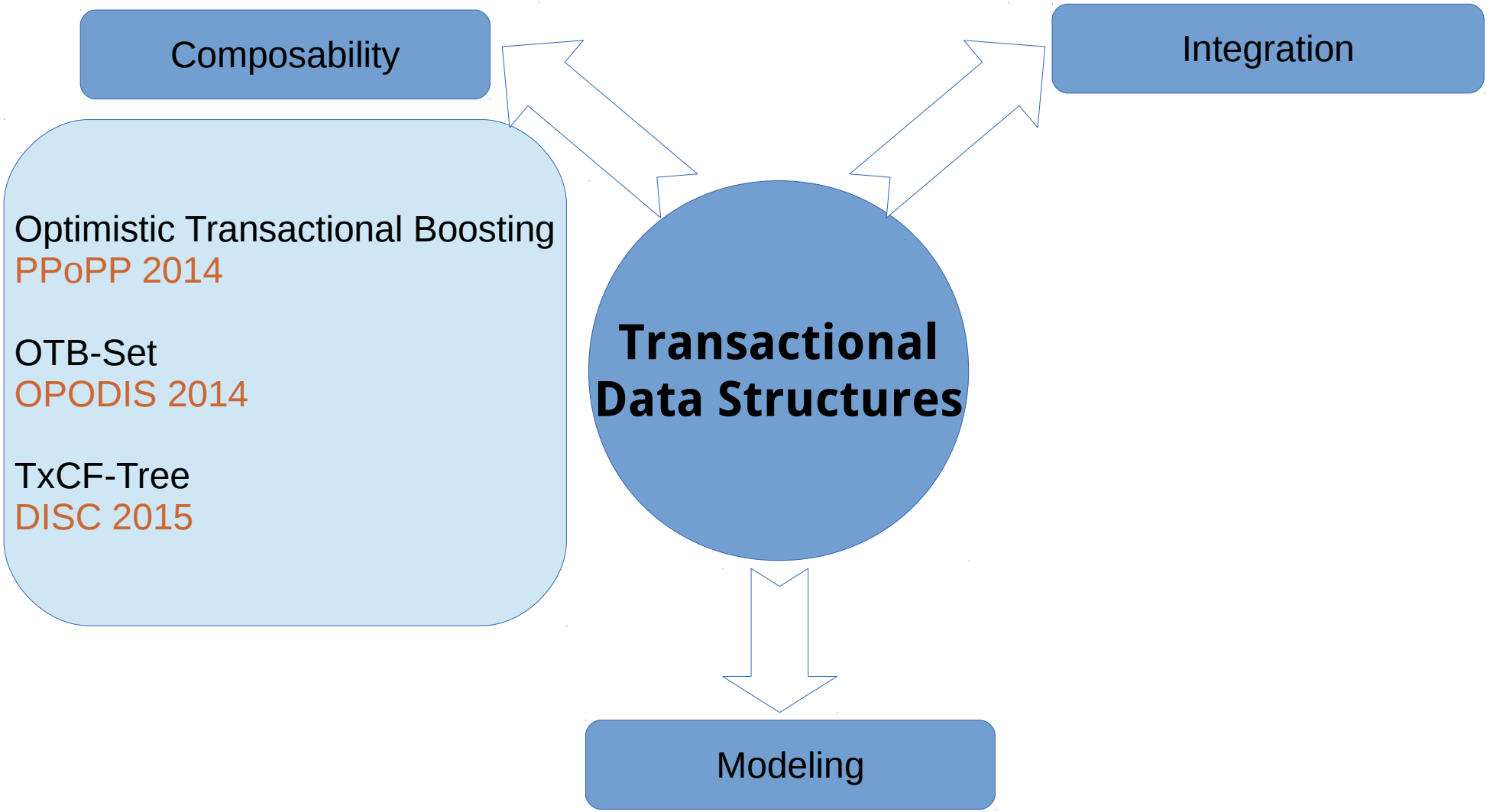

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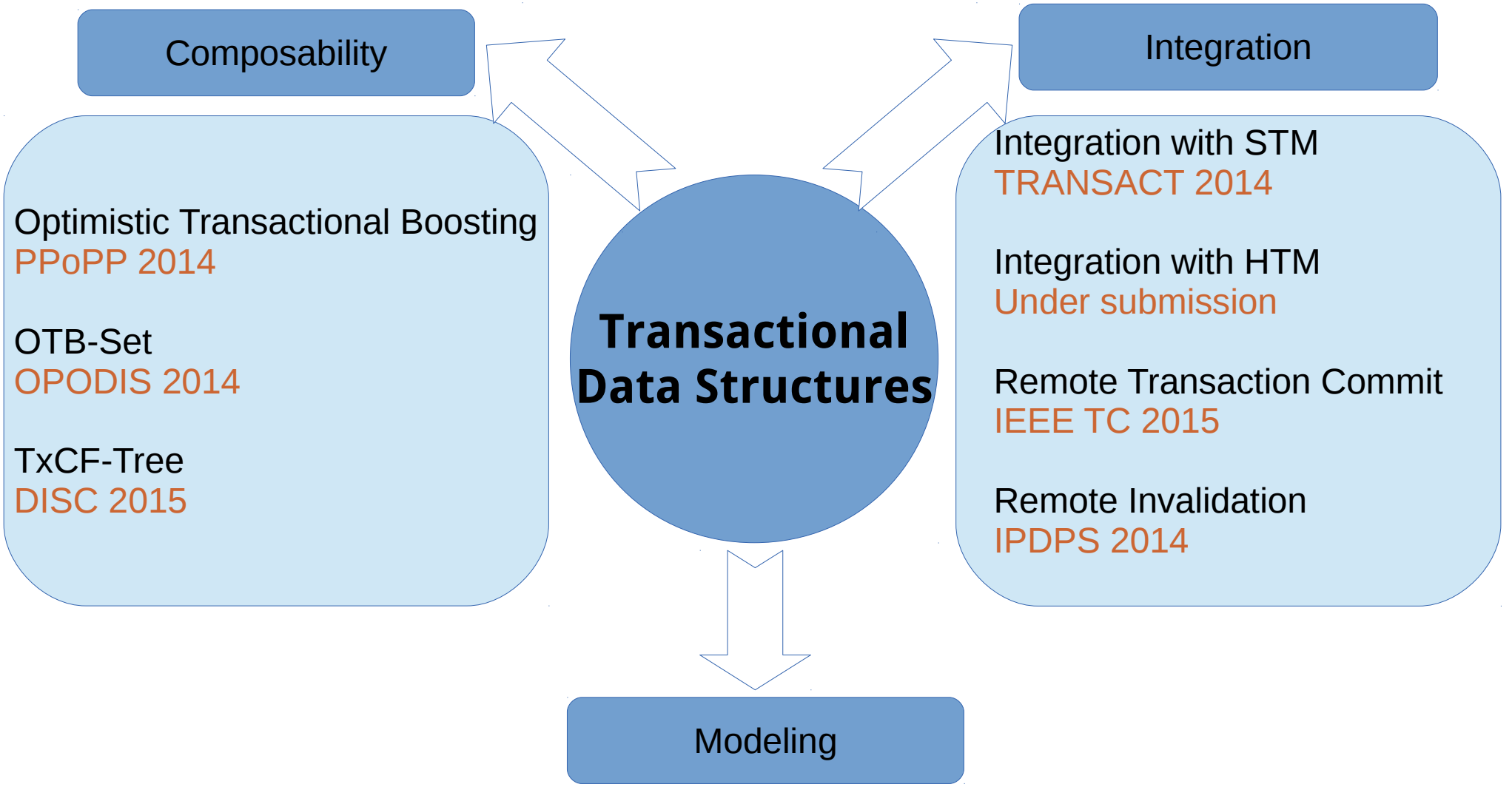

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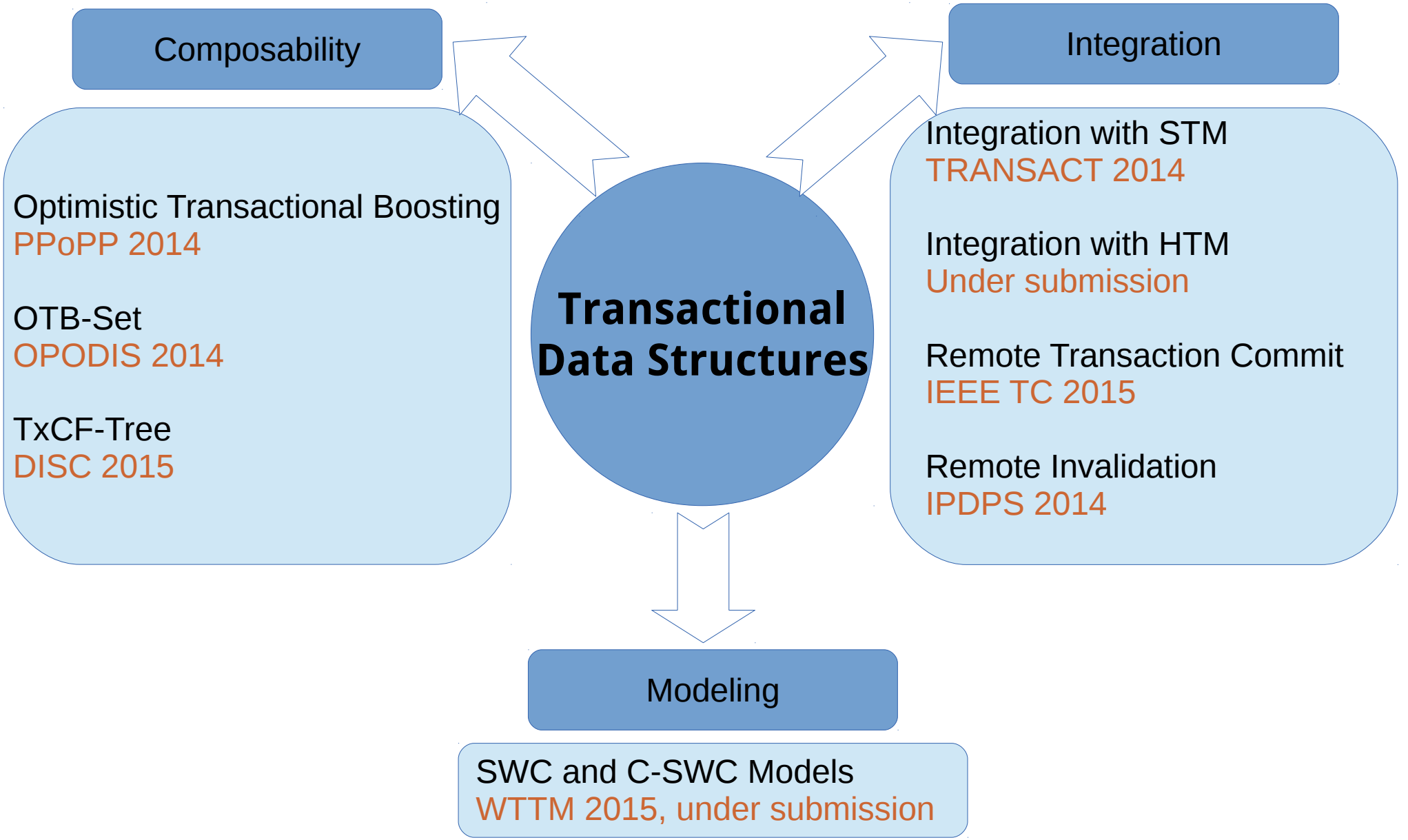

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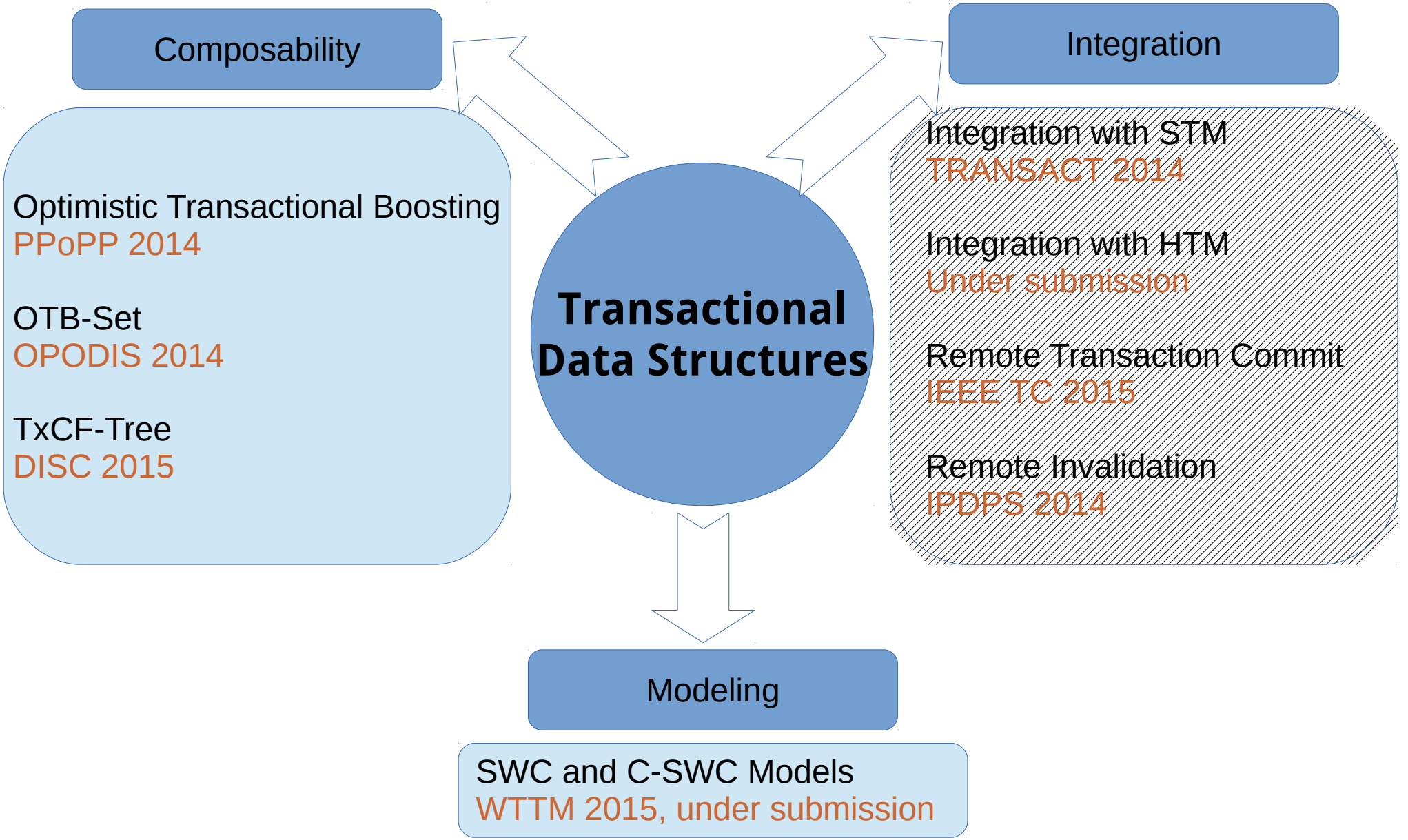

# Past and Related Work

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- Composability and Integration
  - Transactional Memory.
  - Transactional Boosting.
- Modeling
  - SWMR Model

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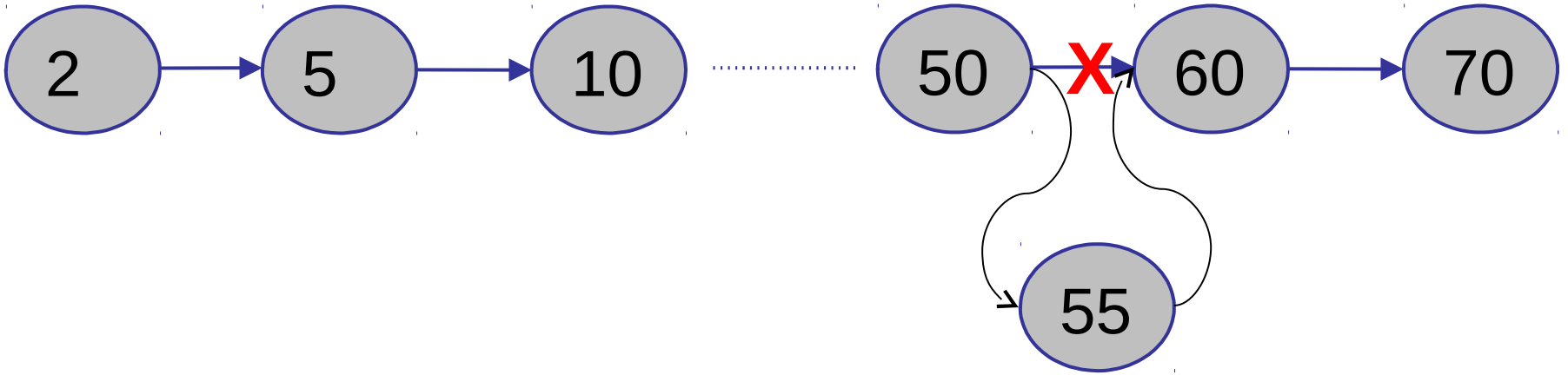
# Transactional Memory

- Software Transactional Memory (STM)
  - SW meta-data (e.g. read-sets and write-sets) on the current HW.
- Hardware Transactional Memory (HTM)
  - New HW (modify cache coherency protocols).
- Hybrid Transactional Memory (Hybrid TM)
  - HTM transactions fall-back to STM

# TM Limitation: False Conflict

Example: Linked list (Insert “55”)



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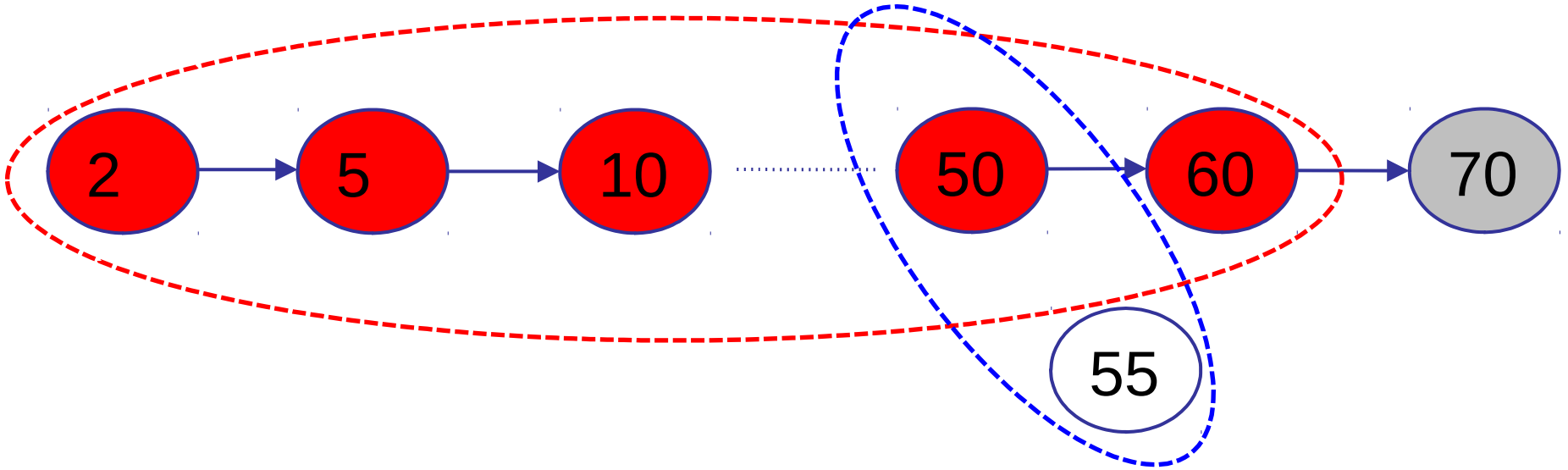


All “red” nodes are in the read-set

“50” and “55” are in the write-set

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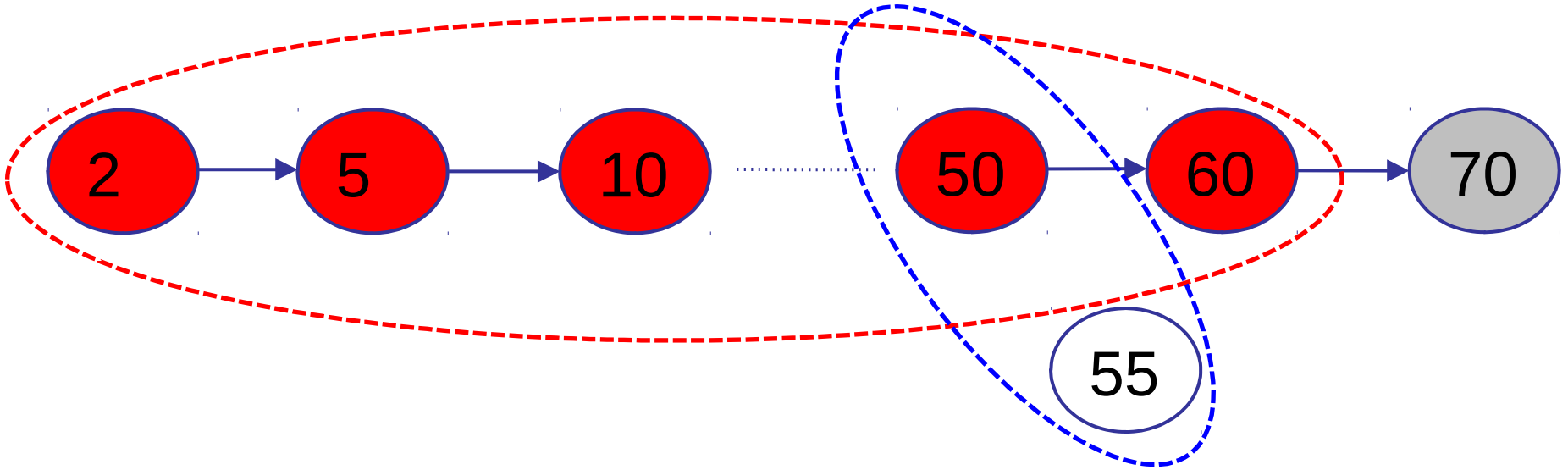


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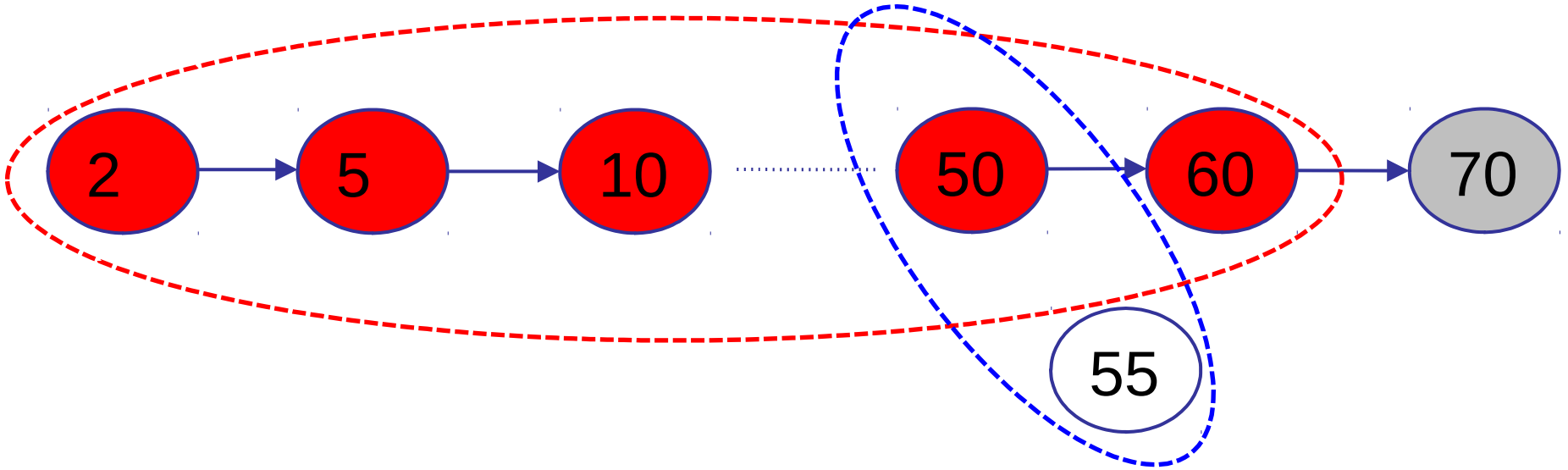


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- Convert highly concurrent data structures to be transactional.

- Composable (like STM)
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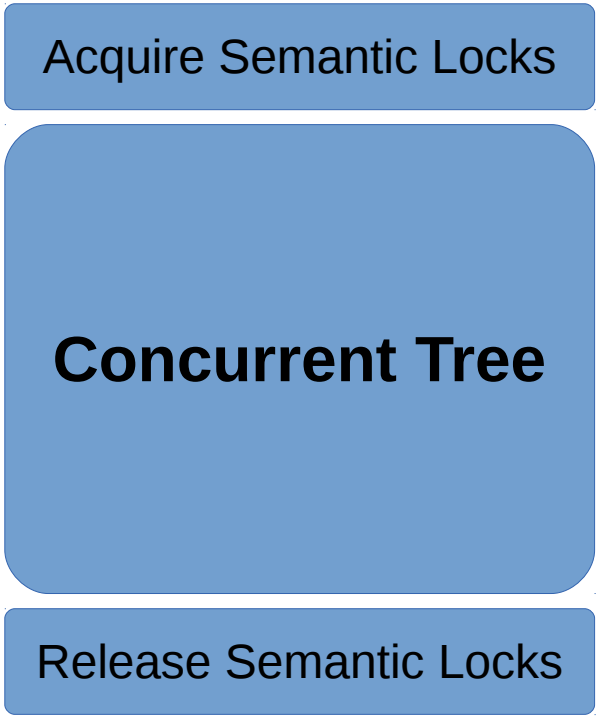
- Issues:
  - Eager locking.
  - Inverse operations.
  - Black-box concurrent data structure.
  - No Straightforward Integration

# Optimistic Transactional Boosting

# Past Solutions


Transactional Memory




Transactional Boosting

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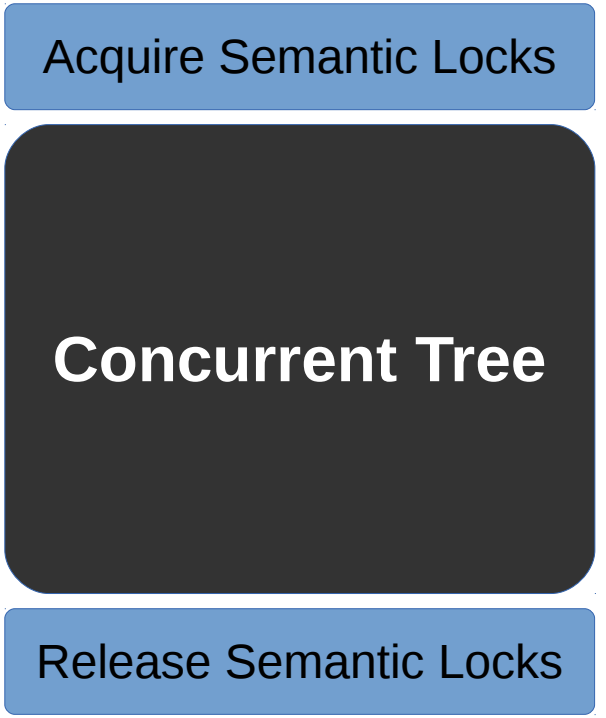


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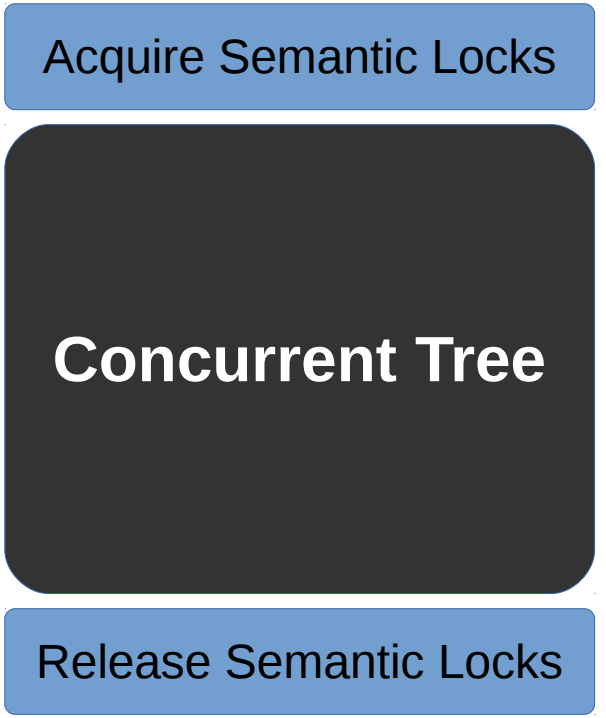


Transactional Memory

Transactional Boosting

General, BUT not optimized.

# OTB's Three Guidelines

- G1: Split operation

**Concurrent Operation (add, remove, contains, ...)**

# OTB's Three Guidelines

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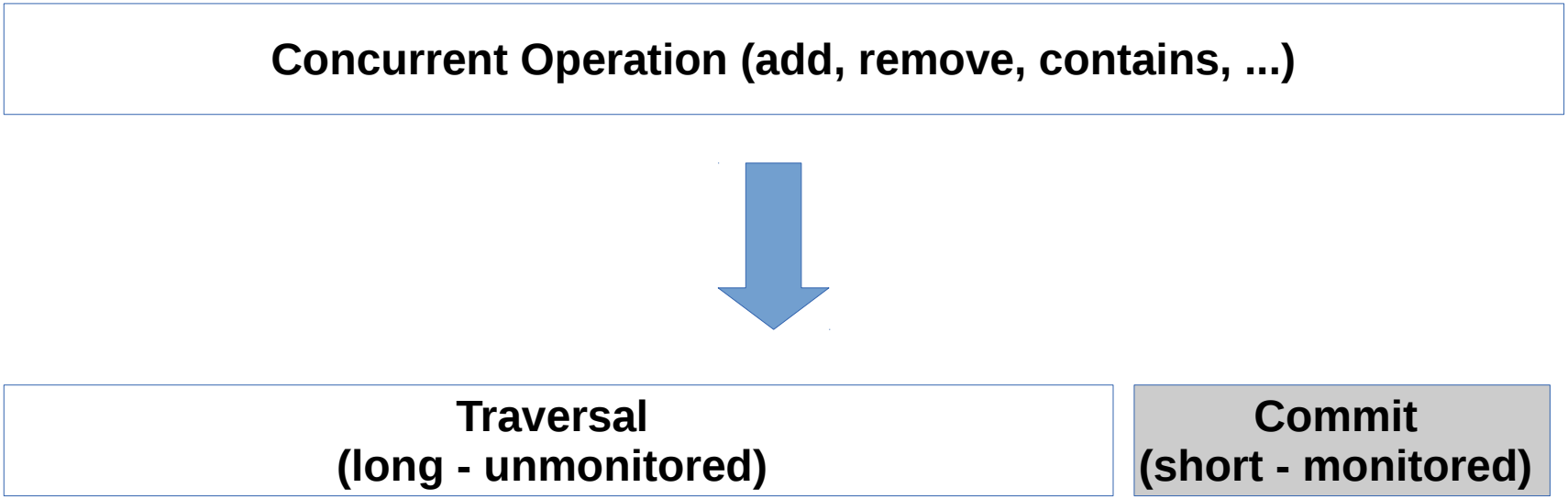

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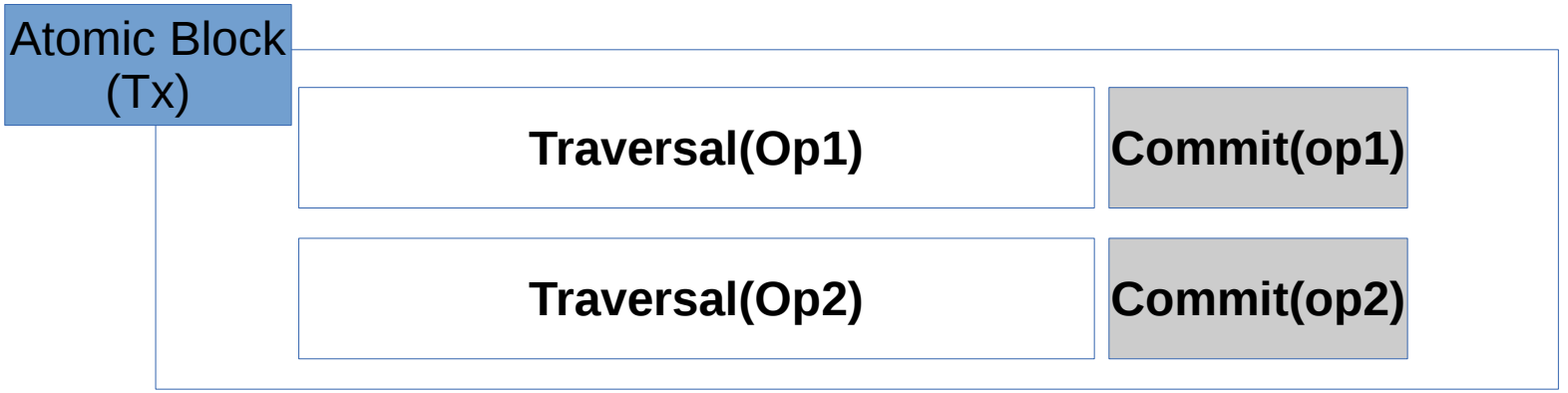

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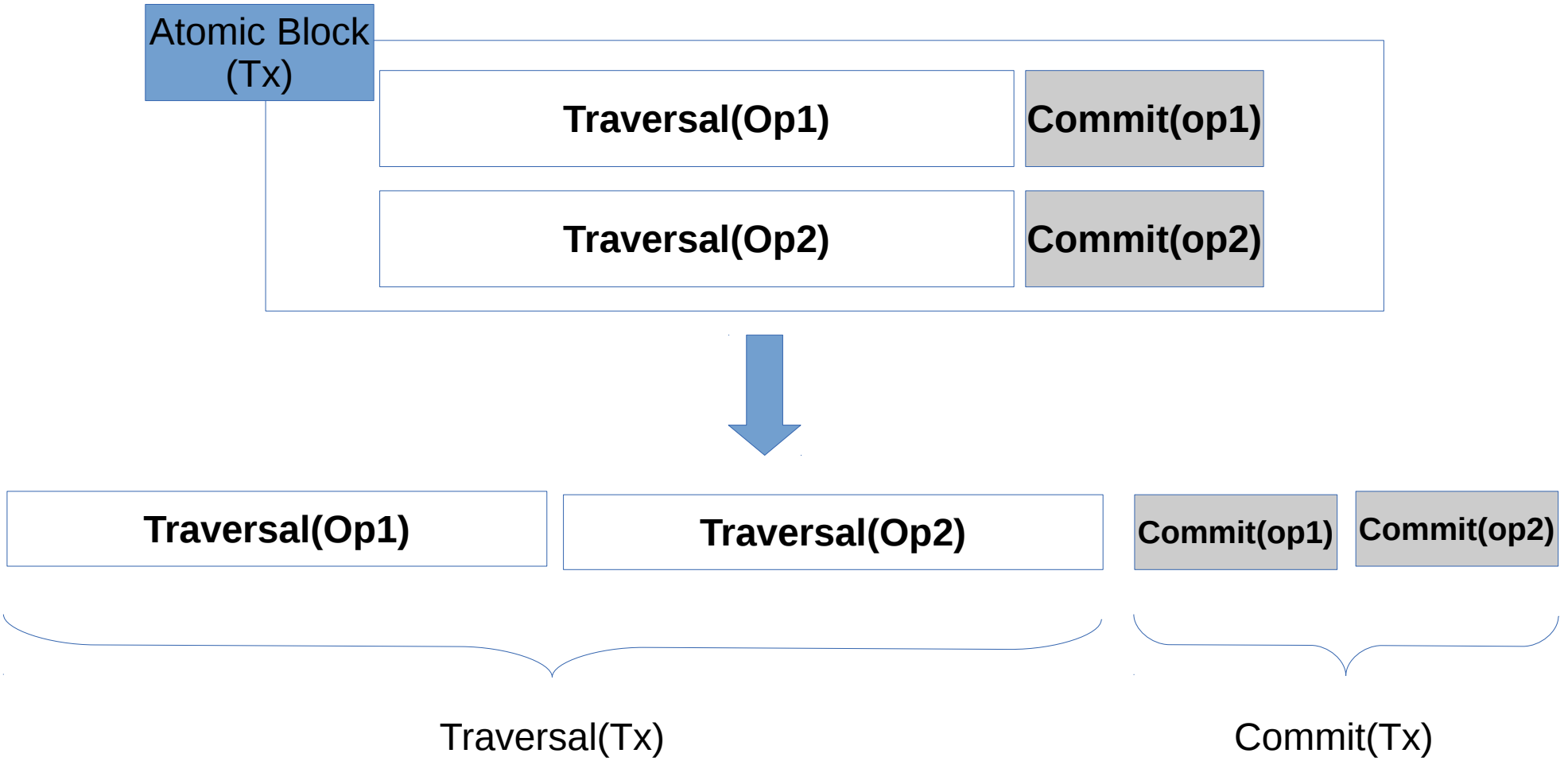

# OTB's Three Guidelines

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  - Specific to each data structure.

- Our Contribution:
  - Linked-list-based Set.
  - Skip-list-based Set.
  - Skip-list-based Priority Queue.
  - Balanced Tree

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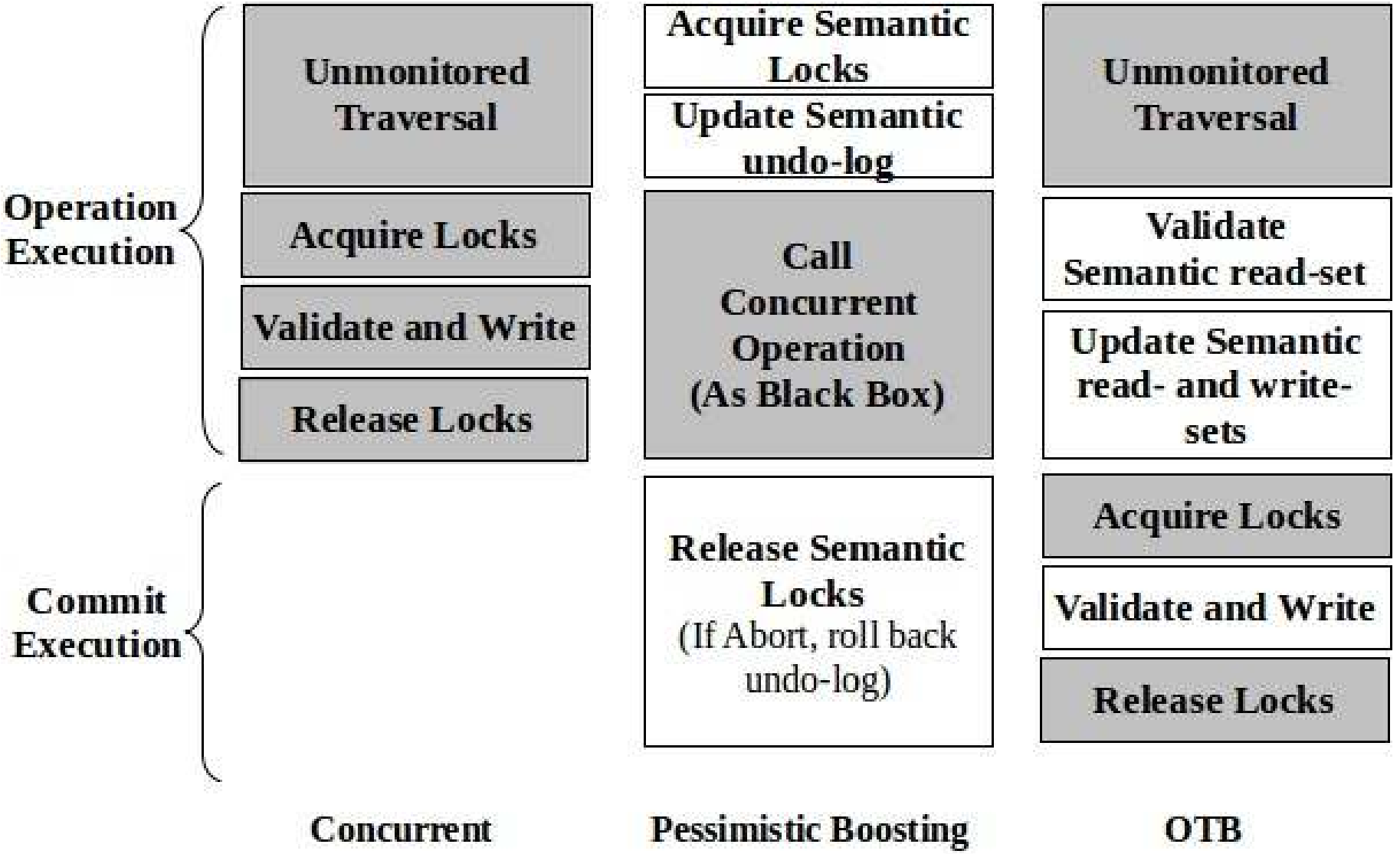
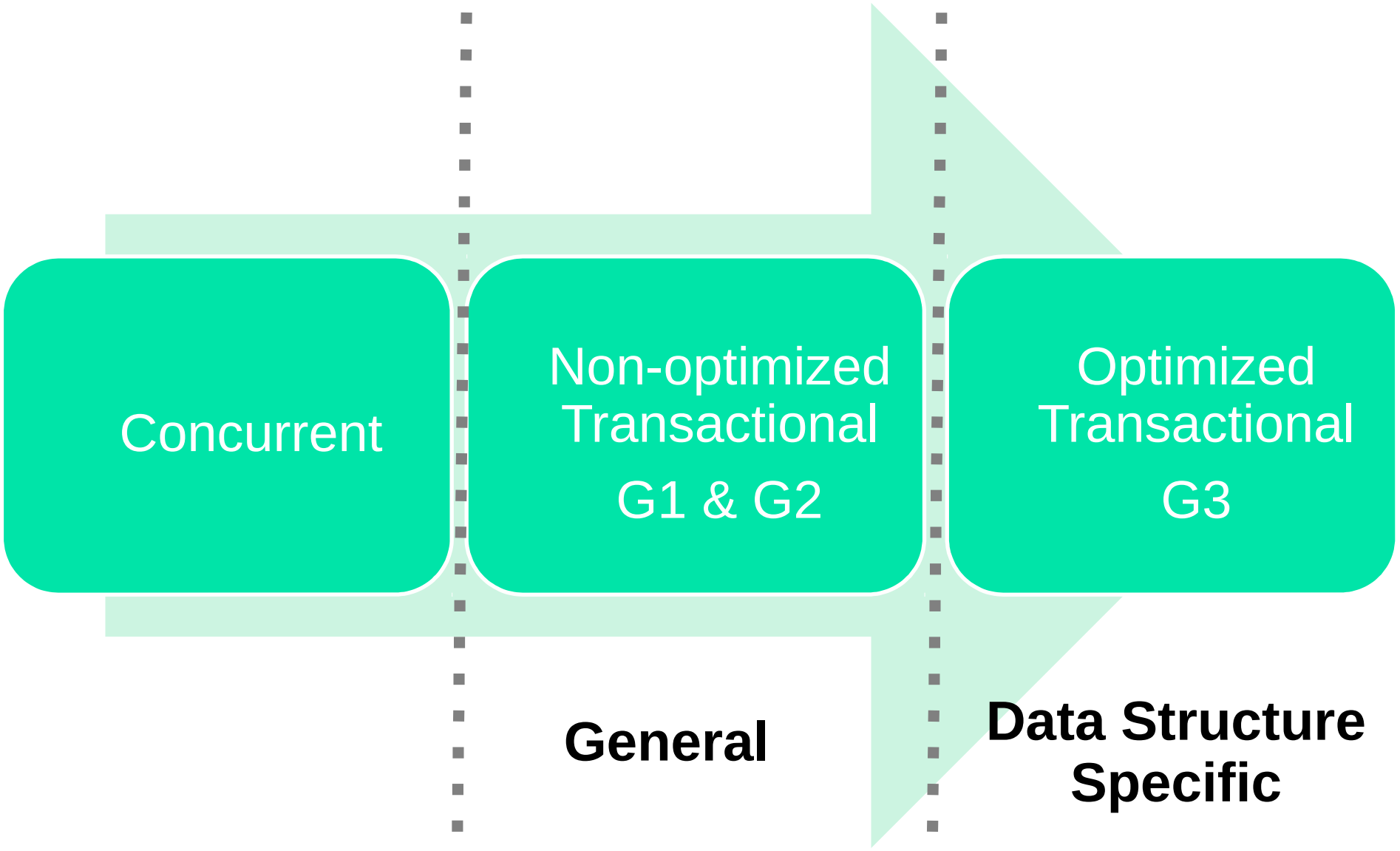
# Lazy Vs Boosting Vs Optimistic Boosting



# Example
# Bootsing “lazy” concurrent linked list

# OTB Methodology



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- Lazy Linked list (Insert “55”)

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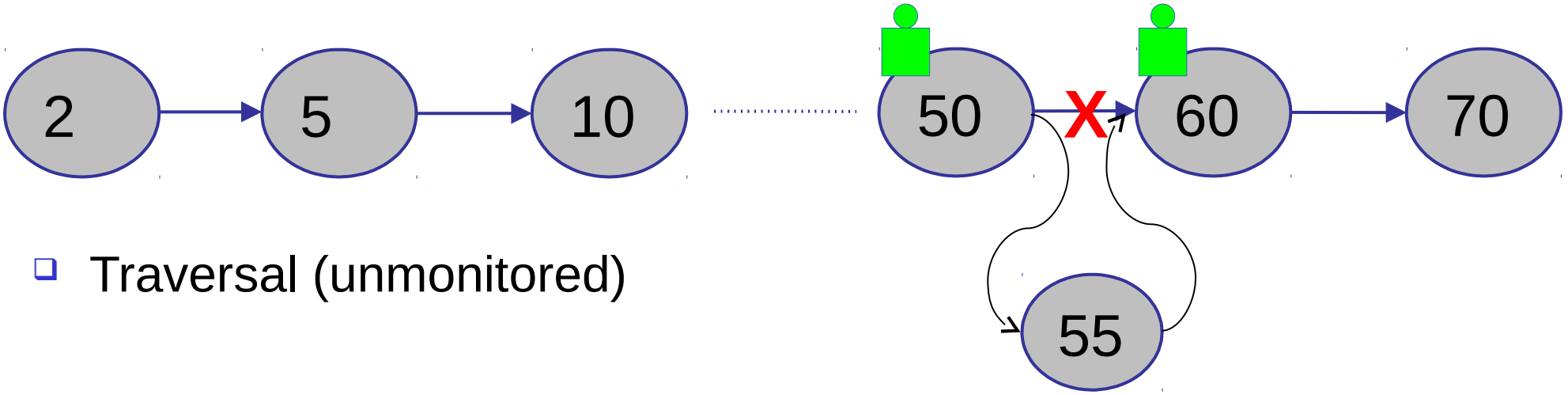


- Traversal (unmonitored)
- Validation
- Commit

# To Make it Transactional

- Results of traversal are saved in local objects:
  - Semantic read-set: to be validated.
  - Semantic write-set: to be published at commit.

# To Make it Transactional

- Example: Linked list (Insert “55”)

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read-set entry

- Pred:50, curr:60

write-set entry

- Pred:50, curr:60, new:55

# To Make it Transactional

- Example: Linked list (Insert “55”)


read-set entry
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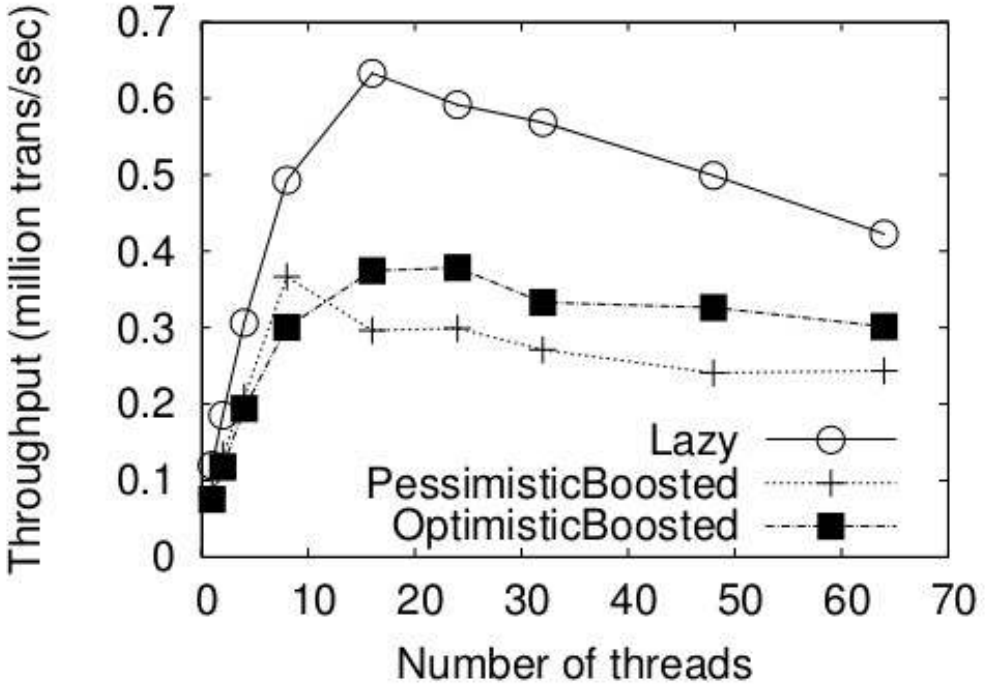
write-set entry
- Pred:50, curr:60, new:55

- Guidelines to guarantee opacity (see OPODIS'14 paper)

# Specific Optimizations

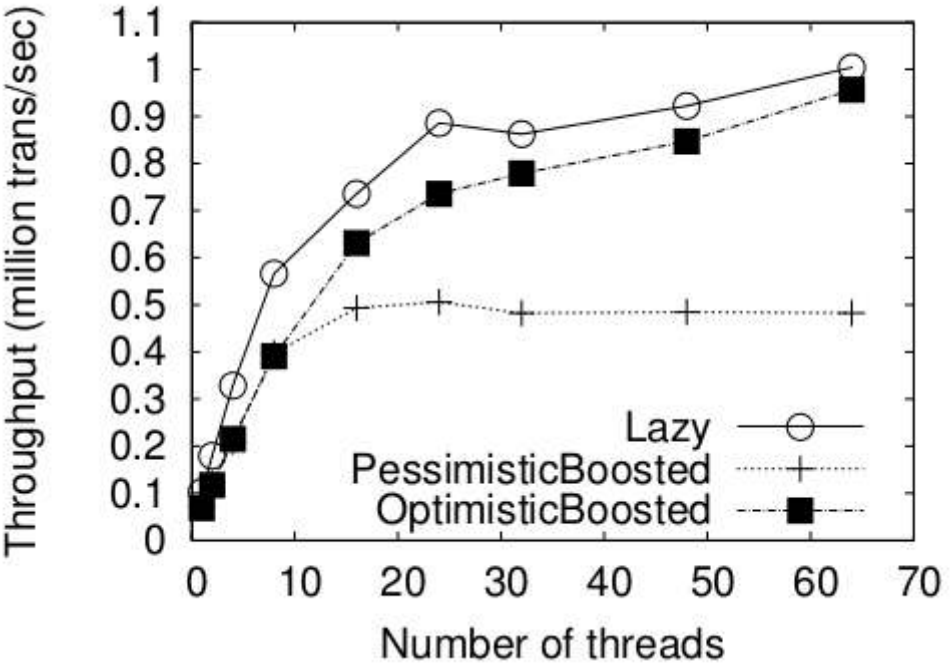
- Example optimizations on Linked-List and Skip-List
  - Local elimination:
    - Ex. Add(x) then Remove(x).
    - No need to access the shared data structure.

# Results




Skip-list 512 Nodes

5 ops/transaction




Skip-list 64K Nodes

5 ops/transaction

# Transactional Interference-less Balanced Tree

# Transactional Interference-less Balanced Trees

- Transactional: Functionality (following OTB's G1, G2).
- Interference-less: Performance (following OTB's G3).

# The Next Question

- Which concurrent balanced tree design fits OTB?

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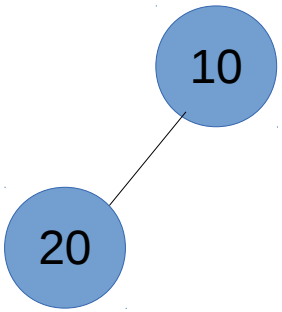
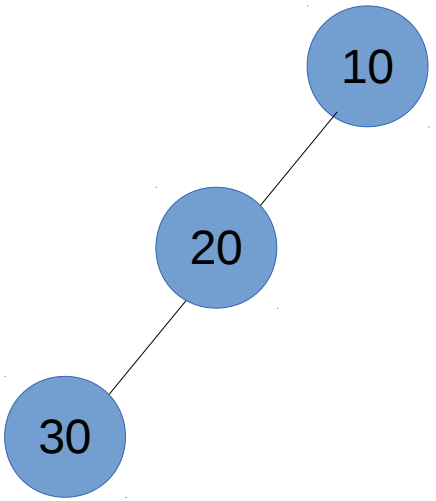
**Contention-Friendly Tree**

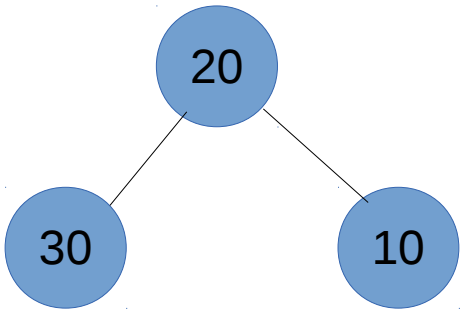
**Crain, Gramoli, & Raynal'13**

# CF-Tree

- Example: Insert 30.



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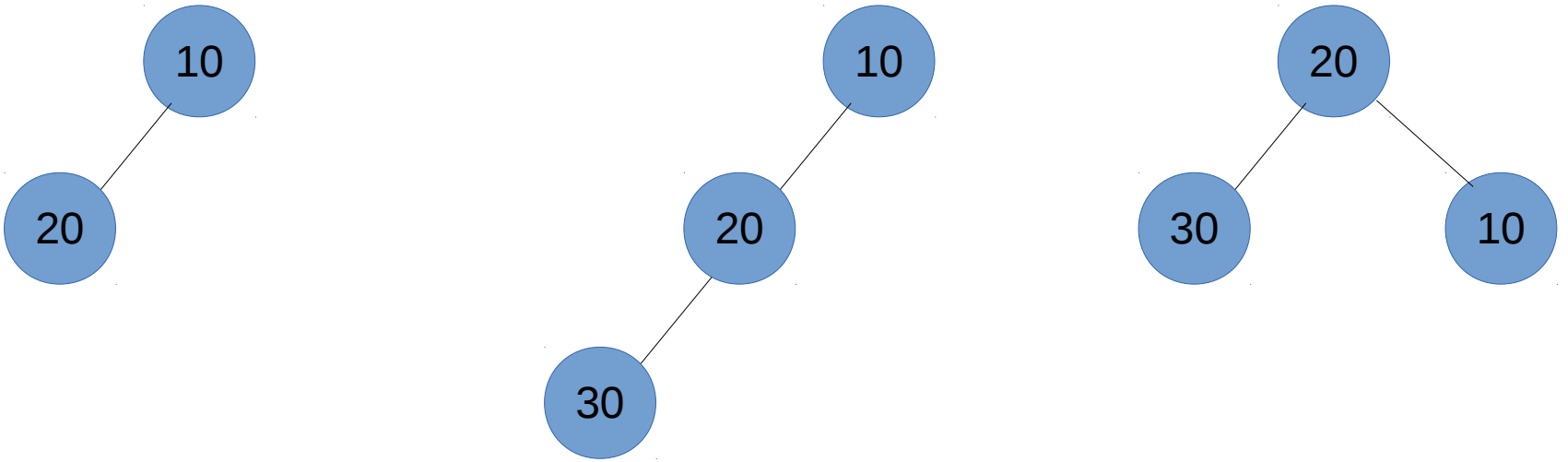


**{10, 20}** → **{10, 20, 30}** → **{10, 20, 30}**

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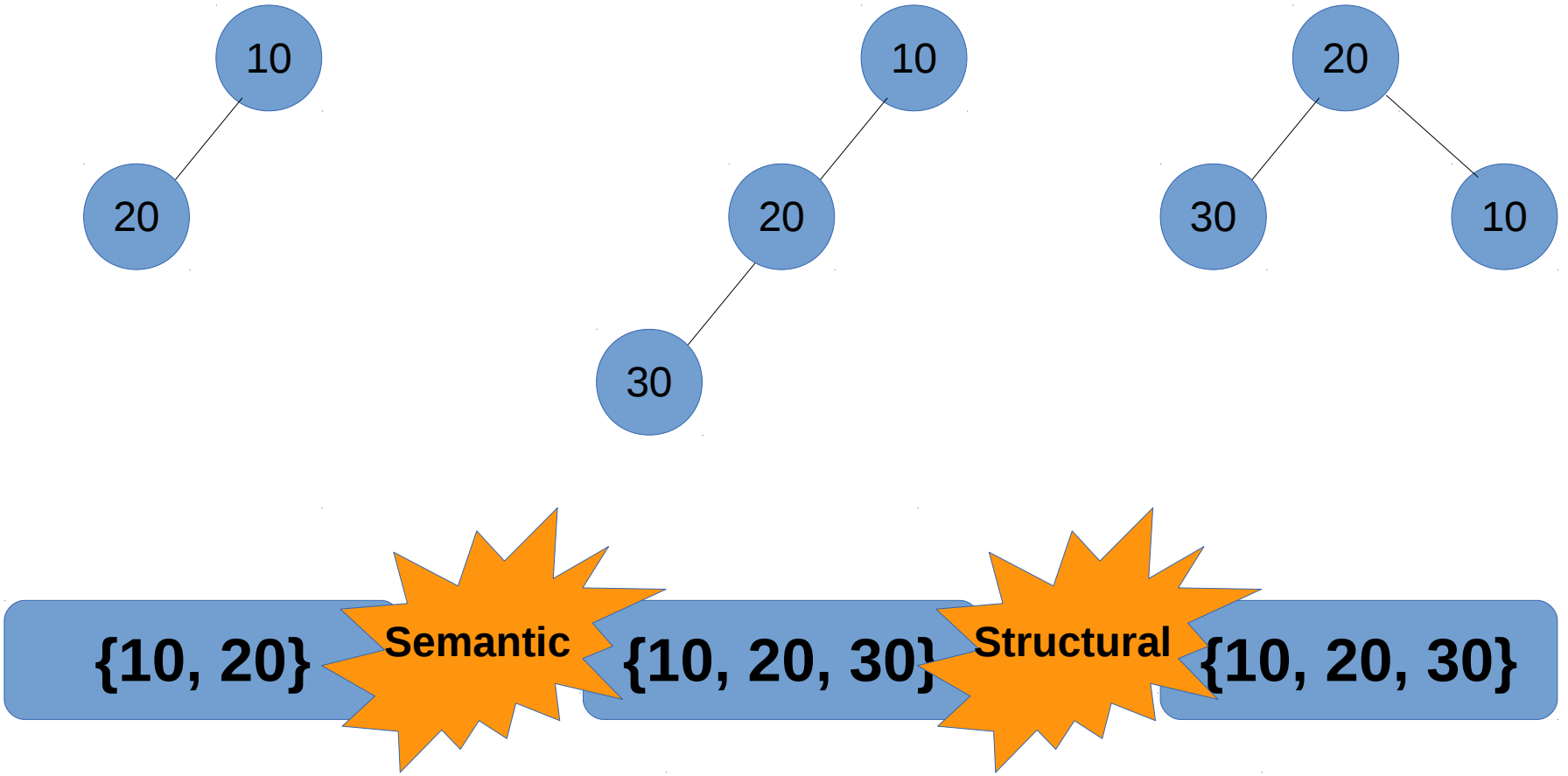

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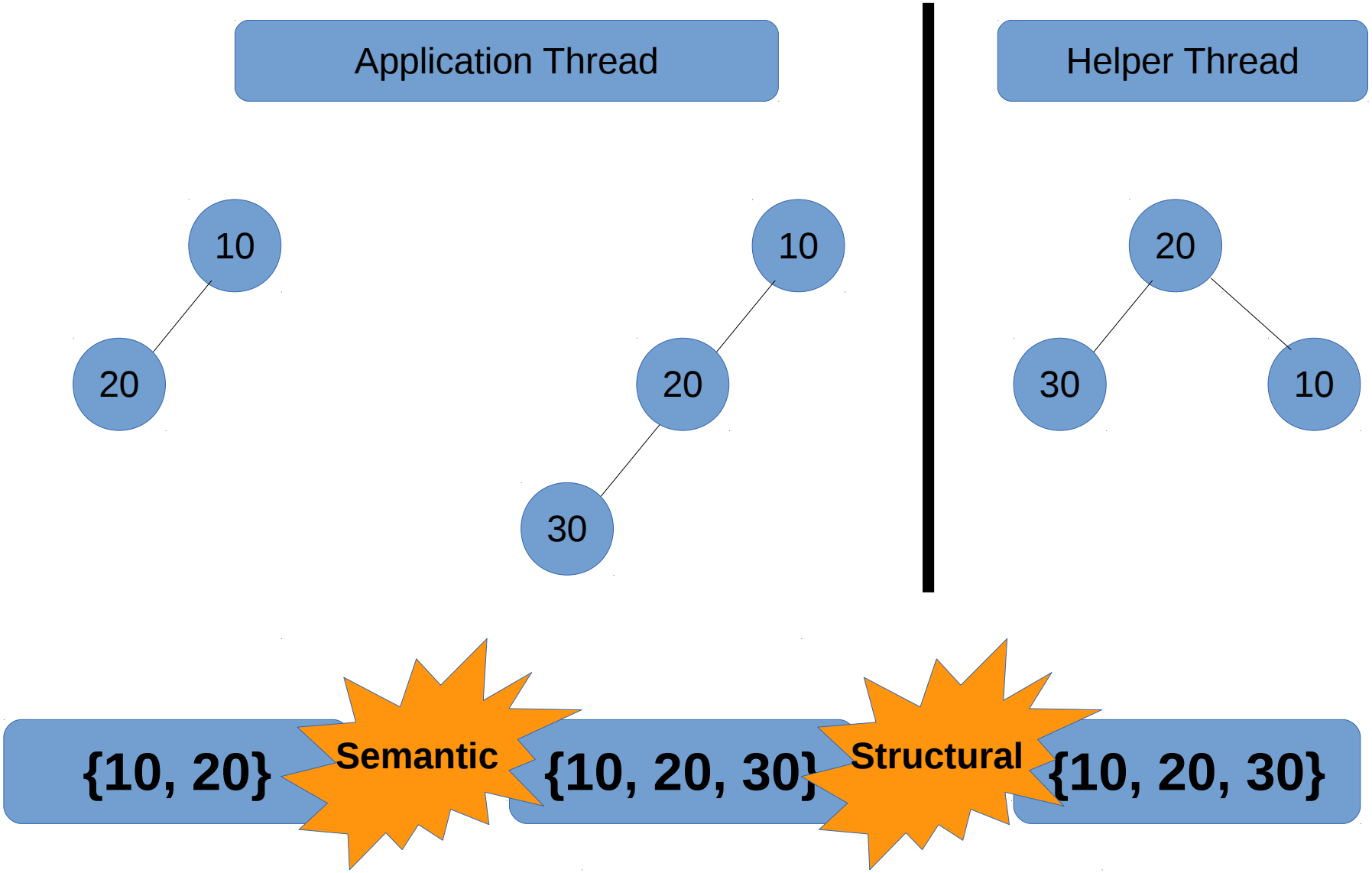
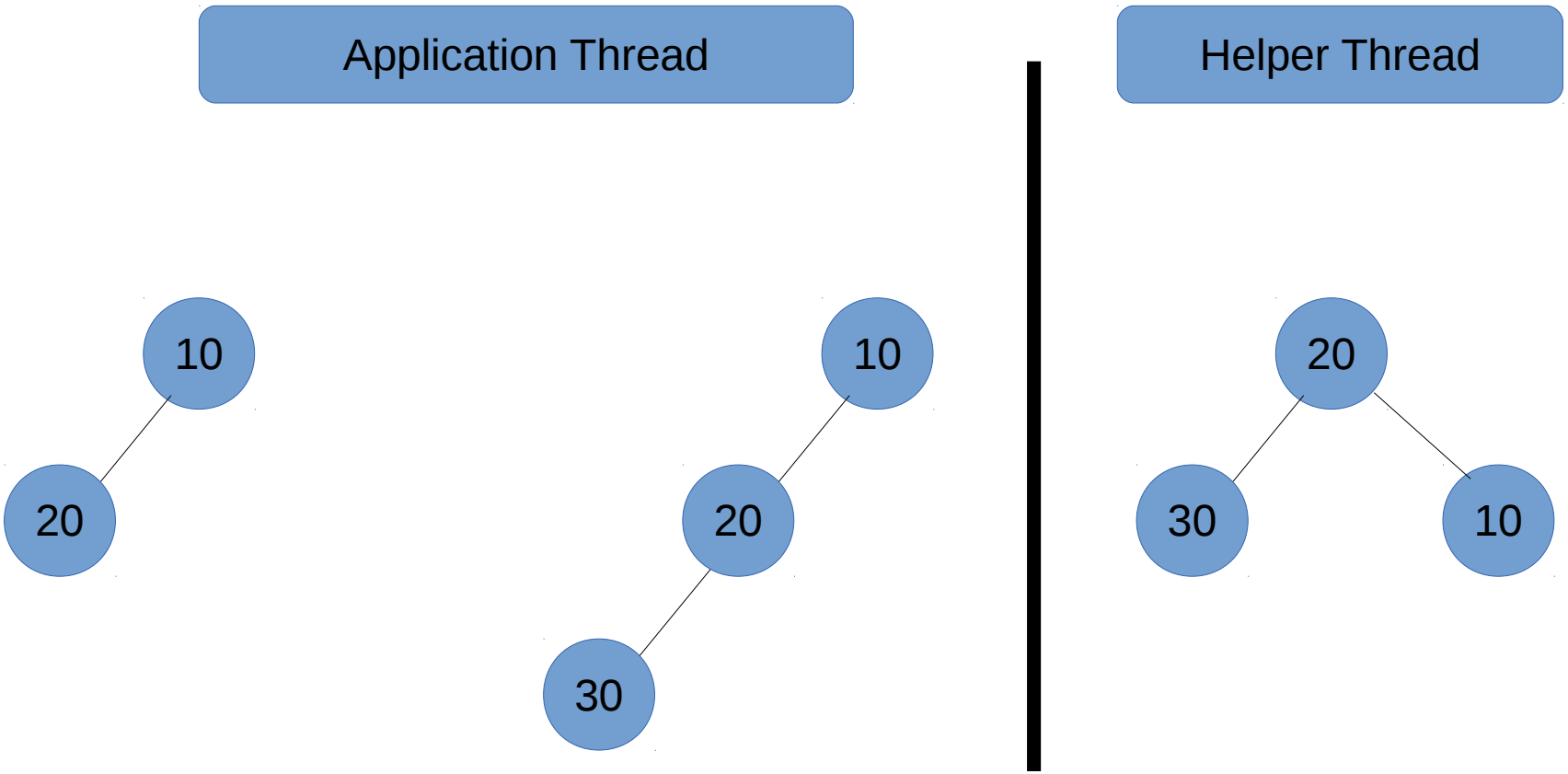


# Our Proposal

Transactionalizing CF-Tree using OTB
(TxCF-Tree)

# TxCF-Tree



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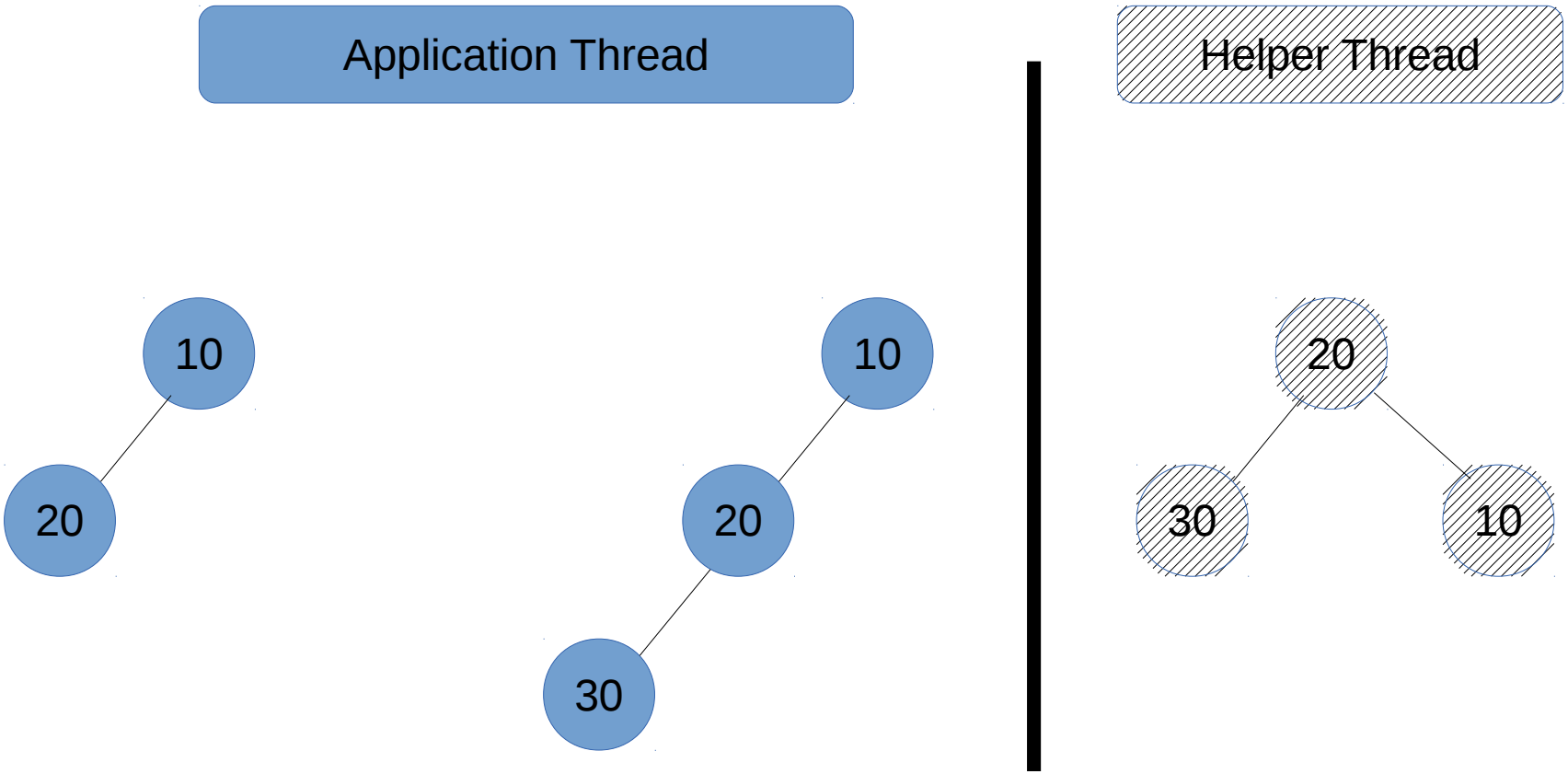

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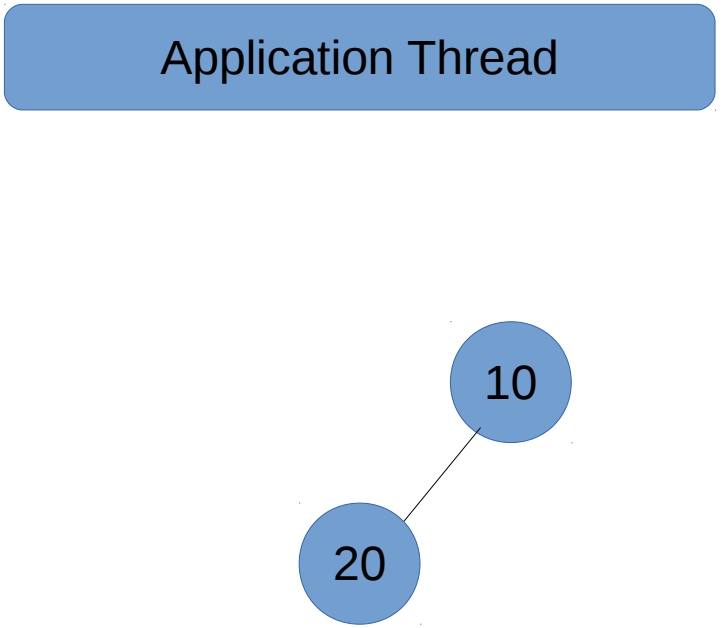

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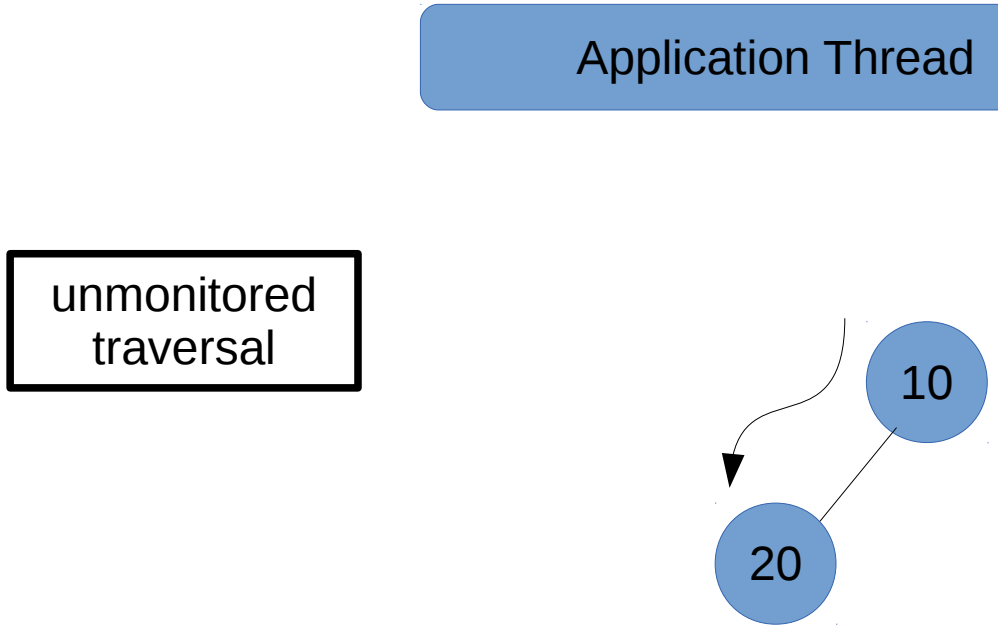
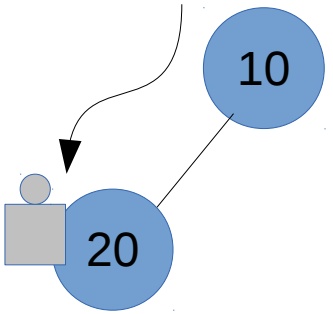


# TxCF-Tree

Application Thread

unmonitored traversal

Lock & Validate



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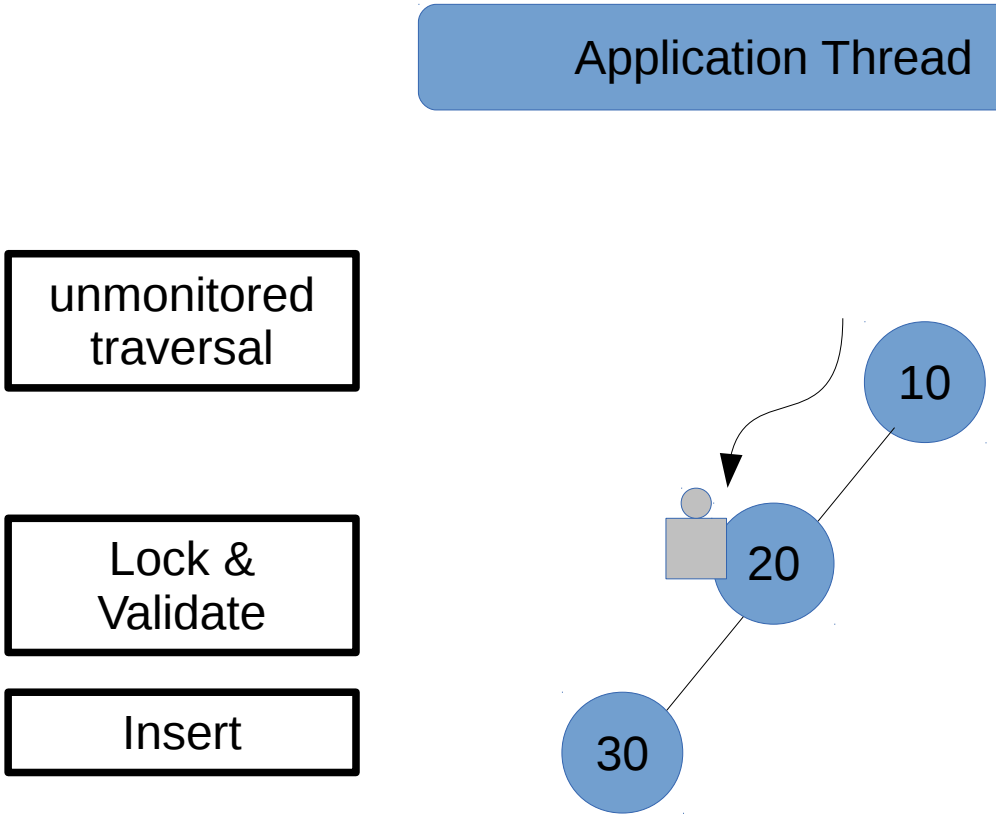
Application Thread
unmonitored traversal
Lock & Validate
Insert
10
20
30

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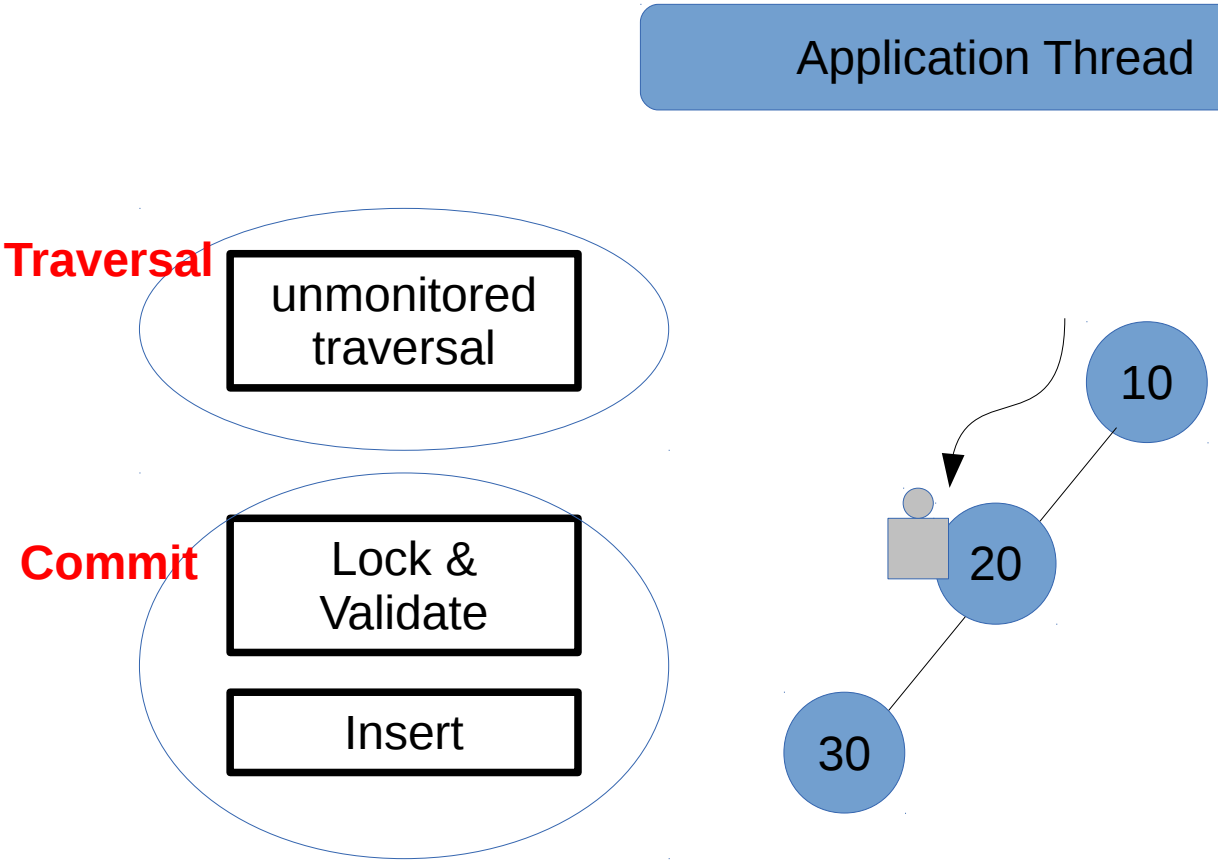
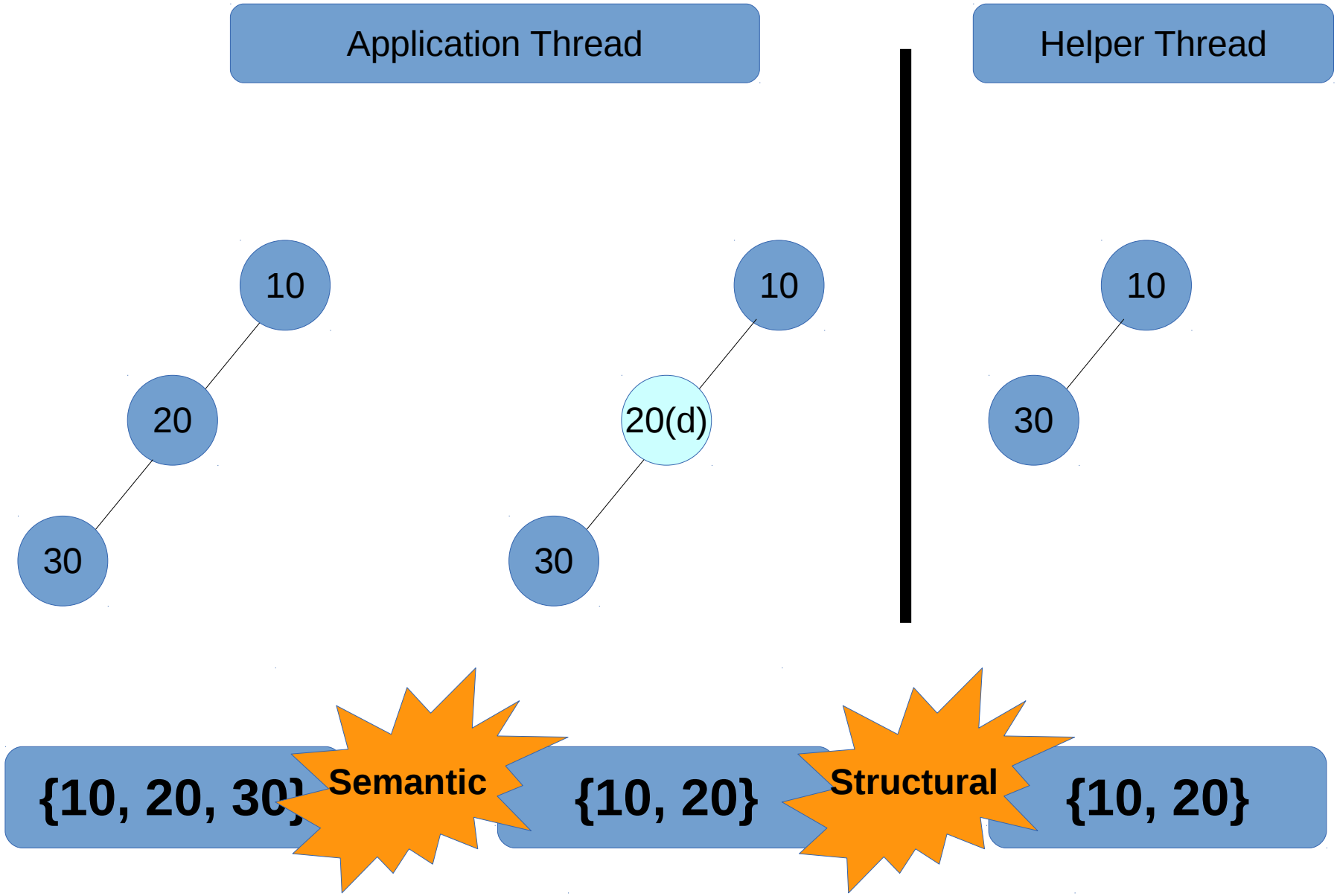
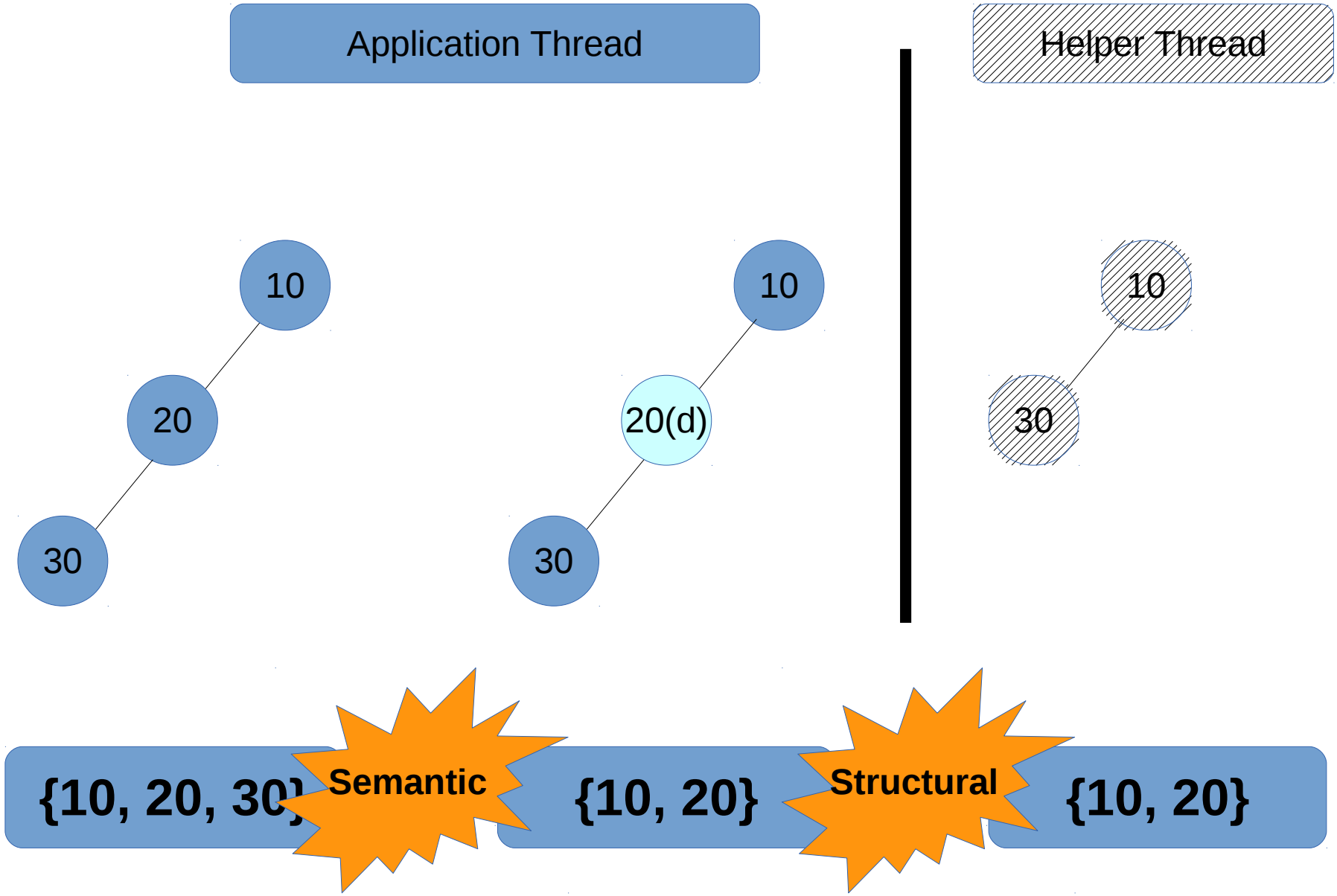

Application Thread
Traversal
unmonitored traversal
Commit
Lock & Validate
Insert
10
20
30

# Remove is similar...



# Remove is similar...



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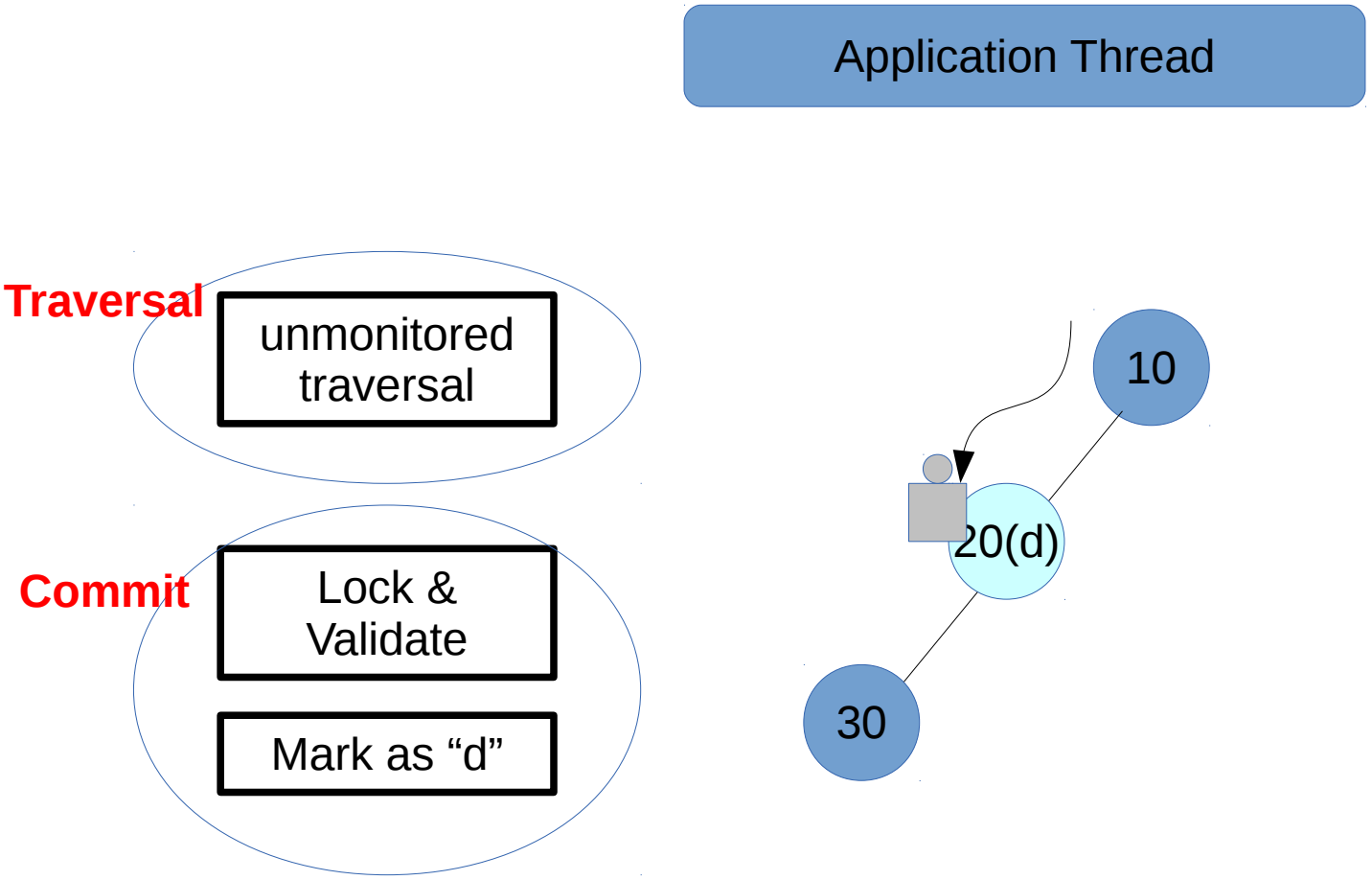

# Transactional Interference-less Tree

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- How
  - Step 1: CF-Tree!!
  - Step 2: Always give the highest priority to semantic operations over structural operations.

# Transactional Interference-less Tree

- How
  - Step 1: CF-Tree!!
  - Step 2: Always give the highest priority to semantic operations over structural operations.

- Why
  - Aborting transactions rolls back all its operations (including the non-conflicting ones).
  - Long transactions are more prone to interfere with the helper thread.

# Two building blocks

- Structural Locks.

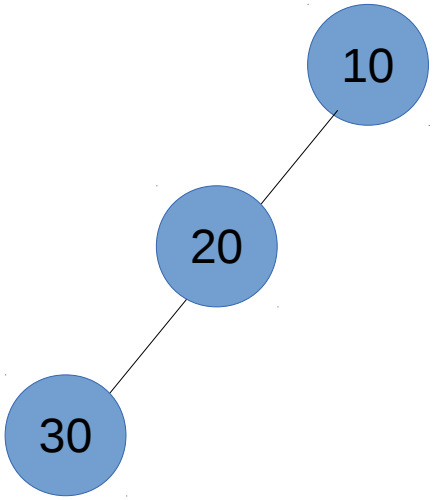
- Structural Invalidation.

# Structural Locks

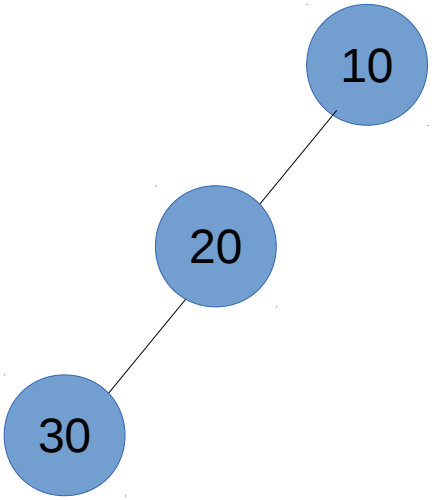
# Structural Locks

- Transaction T1 wants to delete 30.
- after traversal and before commit, assume 2 scenarios

**A concurrent rotation**




**A concurrent delete(30)**



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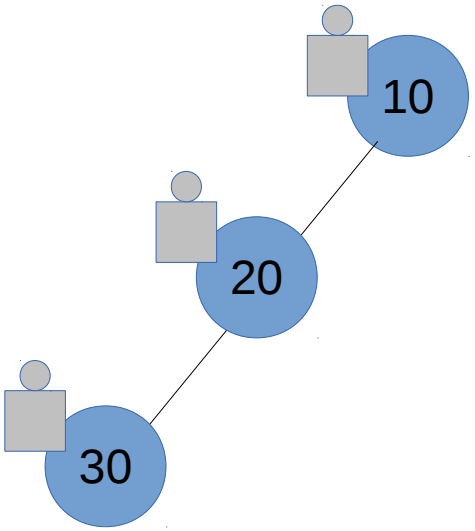


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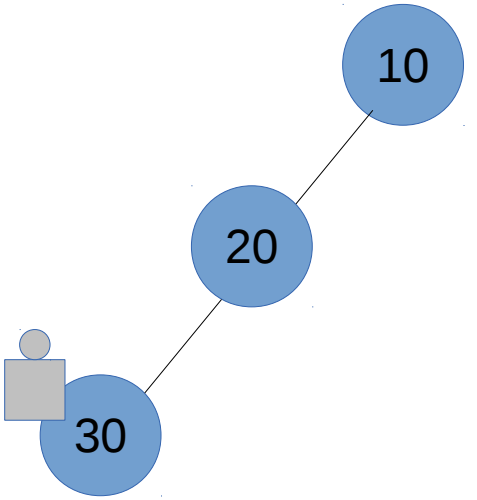

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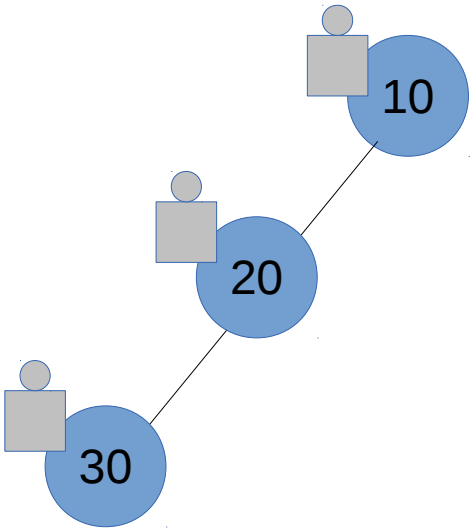


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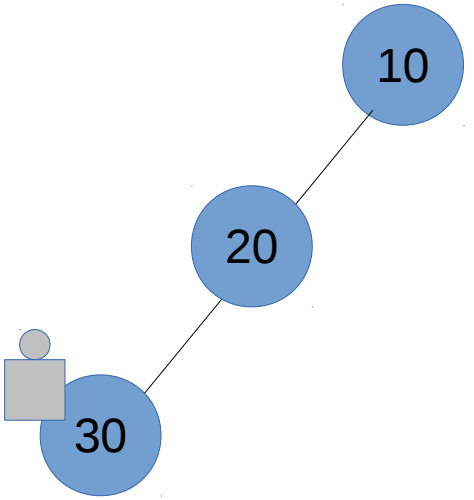


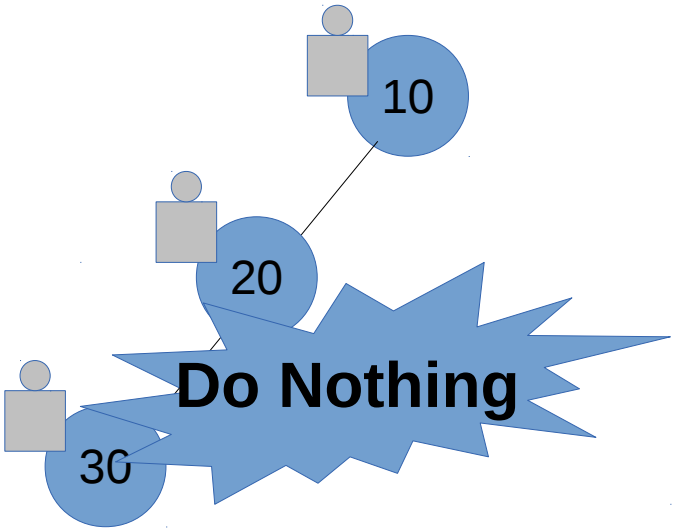

**T1 observes that “30” is locked**

**What is the best to do in both cases?**

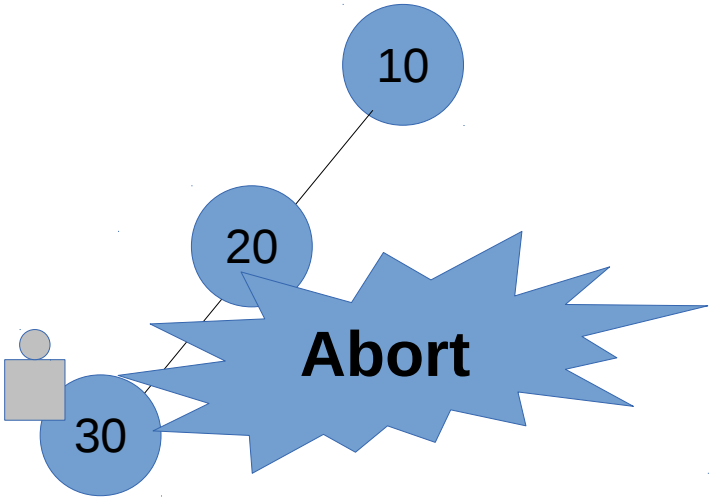
# Structural Locks

- Transaction T1 wants to delete 30.
- after traversal and before commit, assume 2 scenarios

**A concurrent rotation**




**A concurrent delete(30)**



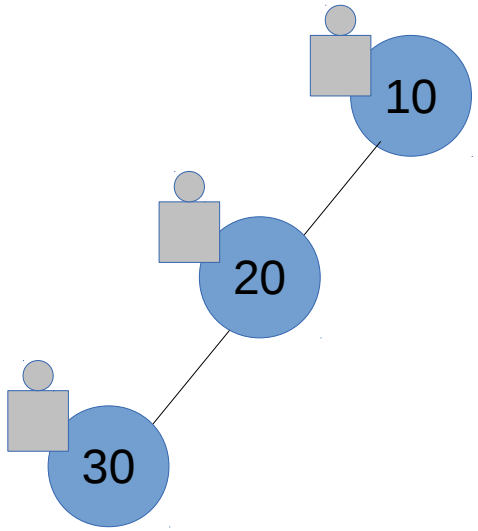

**T1 observes that “30” is locked**

**What is the best to do in both cases?**

# Structural Locks

- Transaction T1 wants to delete 30.
- after traversal and before commit, assume 2 scenarios

**A concurrent rotation**

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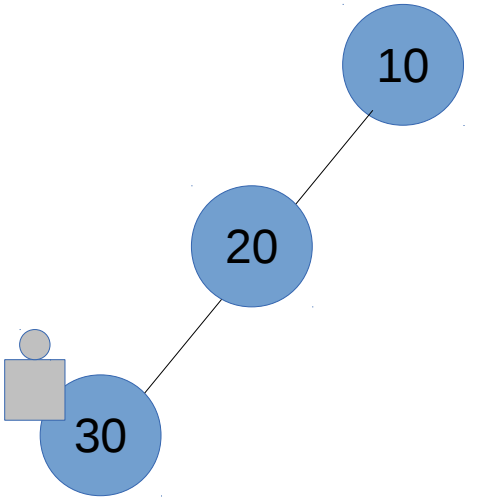


**Solution?**

# Structural Locks

- Transaction T1 wants to delete 30.
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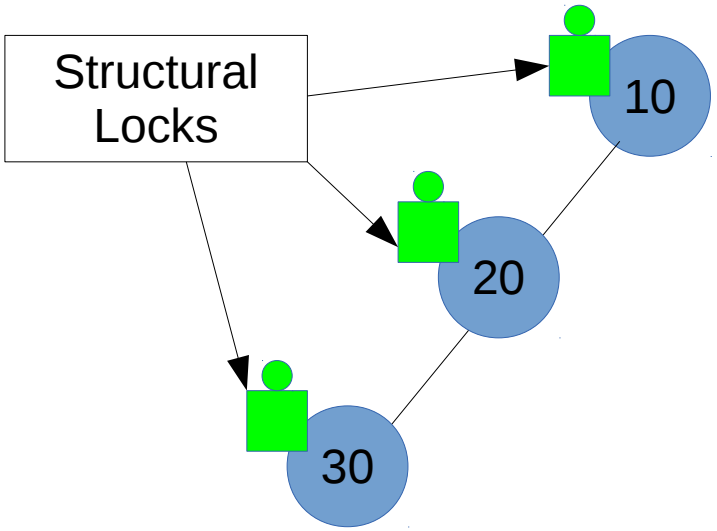


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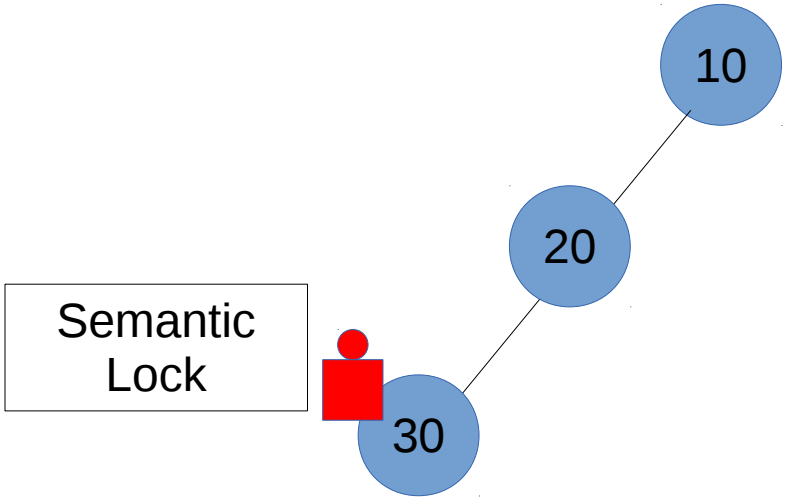



**Solution?**

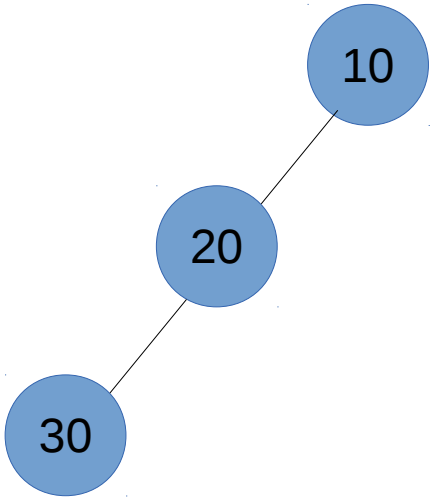
**Two types of locks**

# Structural Invalidation

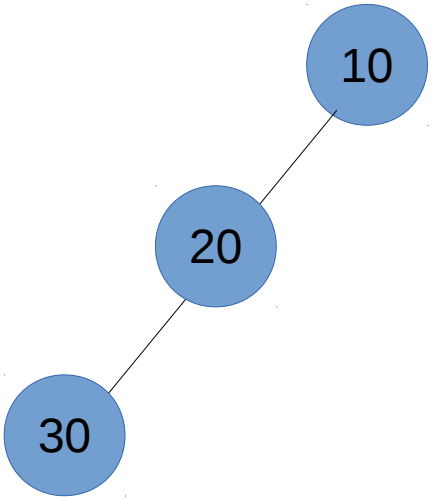
# Structural Invalidation

- Transaction T1 wants to insert 15.
- after traversal and before commit, assume 2 scenarios

**A concurrent rotation**




**A concurrent insert(15)**



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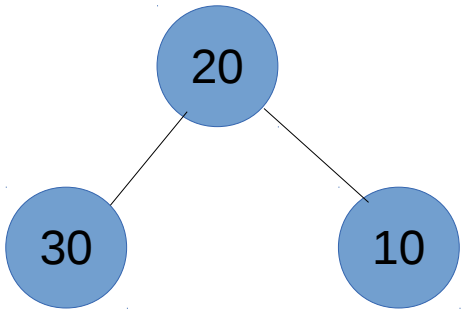
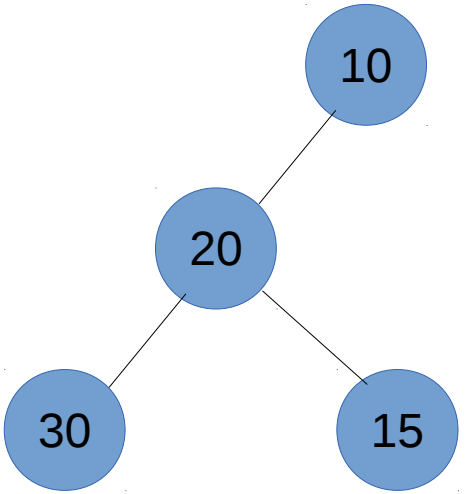



**A concurrent insert(15)**



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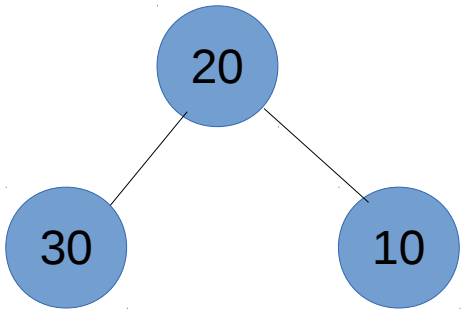


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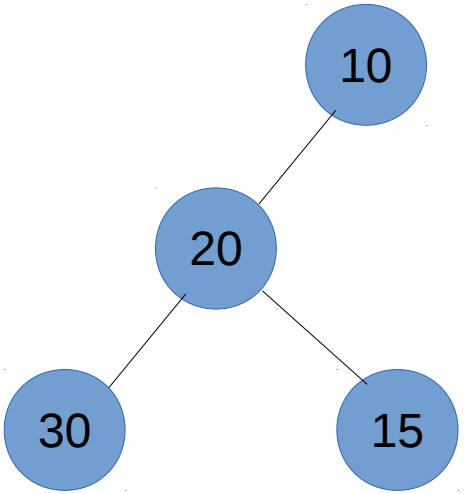


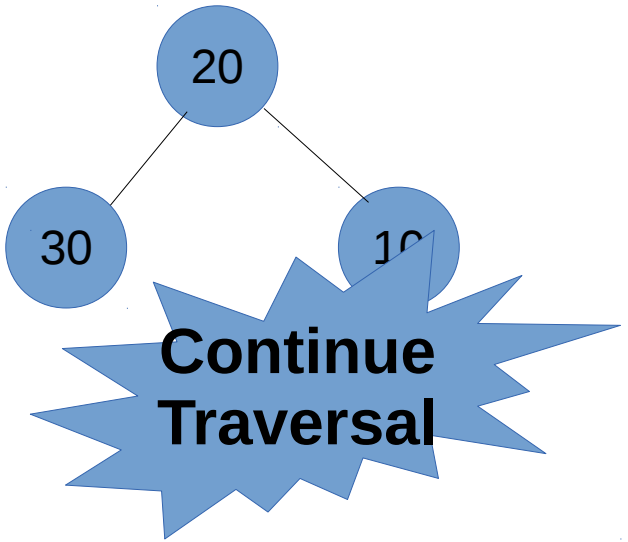

**T1 observes that the right child of “20” is not NULL**

**What is the best to do in both cases?**

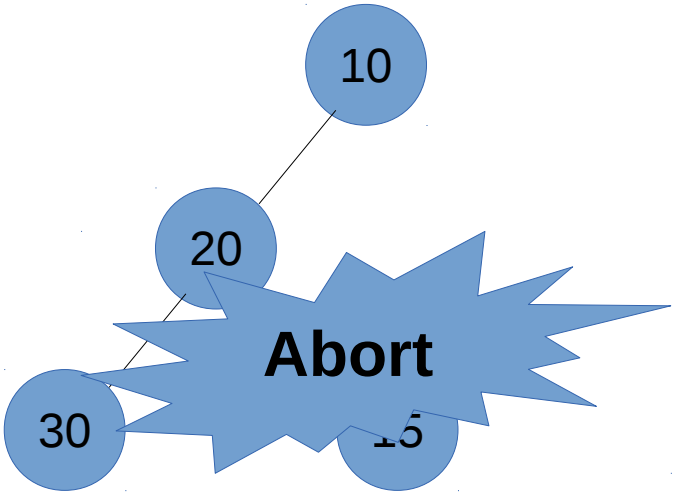
# Structural Invalidation

- Transaction T1 wants to insert 15.
- after traversal and before commit, assume 2 scenarios

**A concurrent rotation**




**A concurrent insert(15)**

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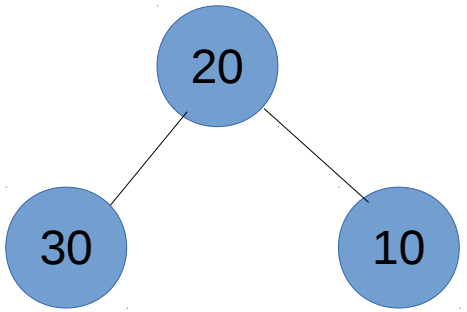


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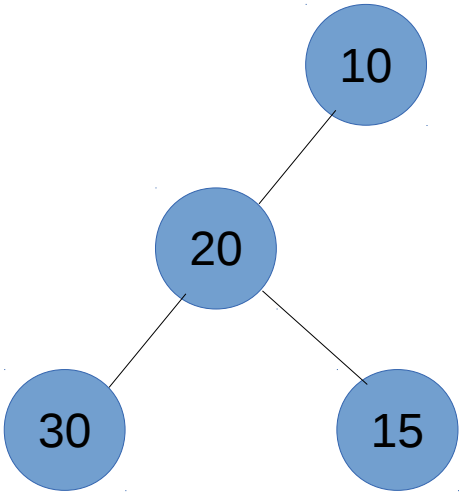


**Solution?**

# Structural Invalidation

- Transaction T1 wants to insert 15.
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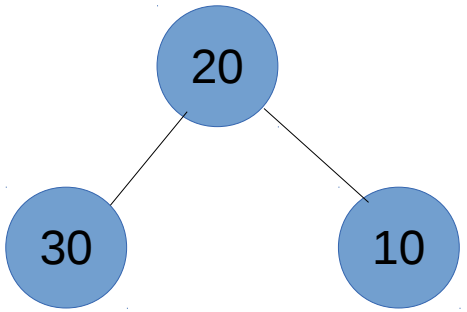


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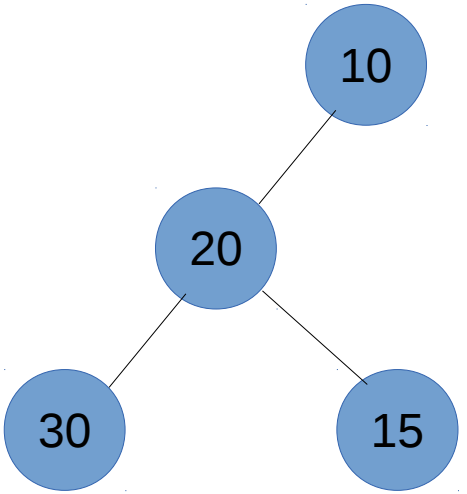
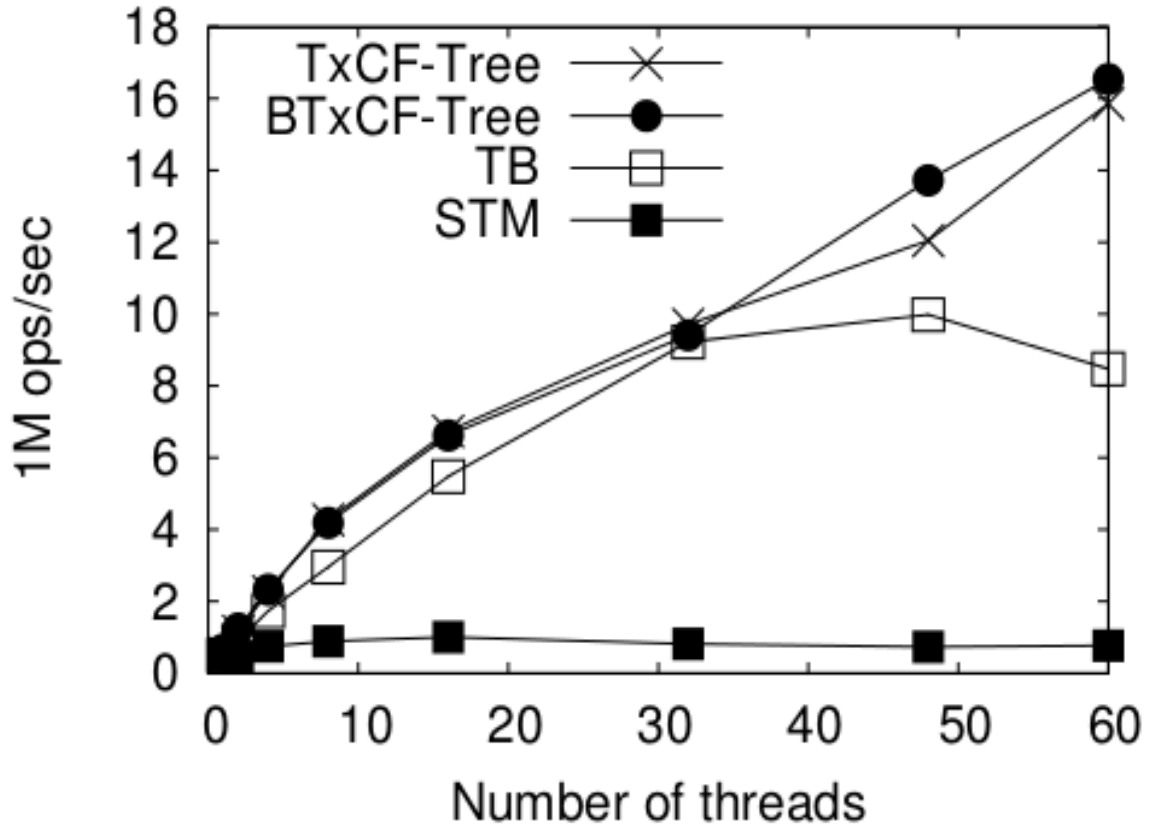



**Solution?**

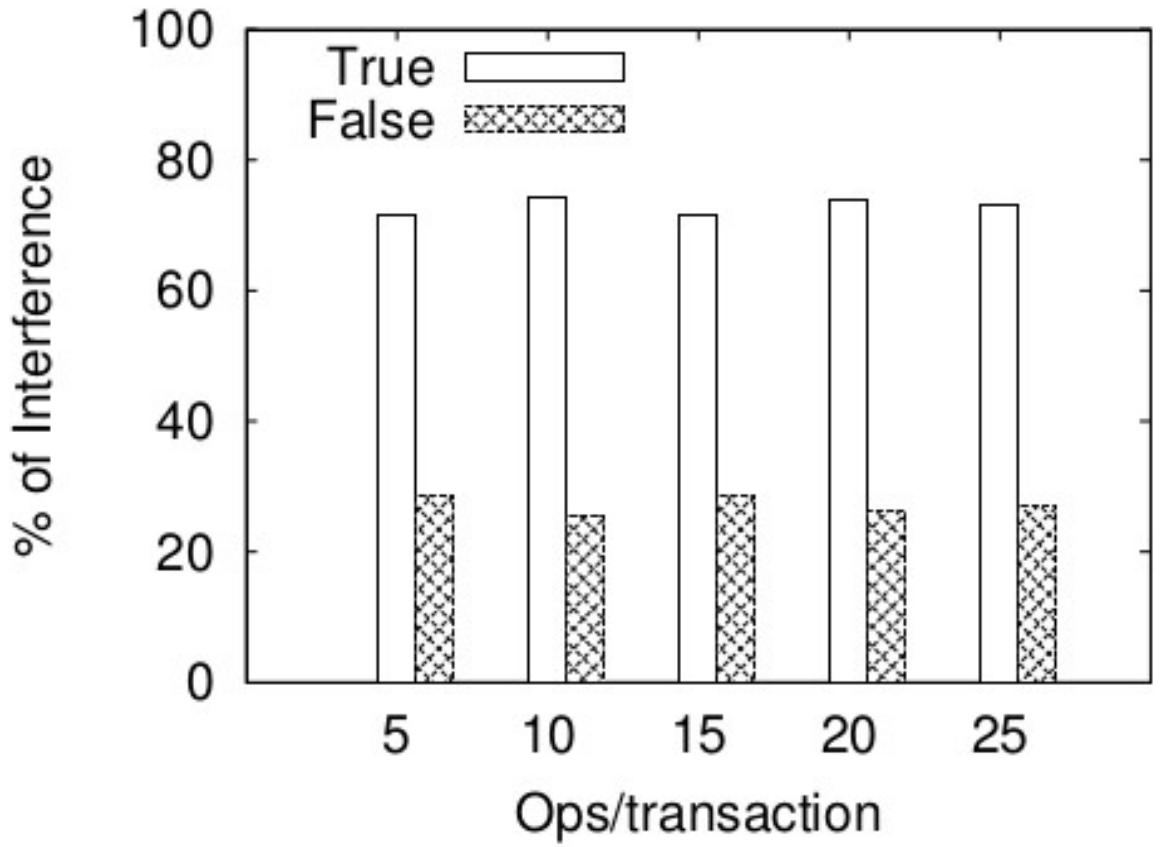
**Continue Traversal anyway**

# Evaluation




**AMD 64-cores, size 10K nodes, 50% reads, 5 ops/transaction**

# Evaluation




**AMD 64-cores, size: 10K nodes , 32 threads, 50% reads, 5 ops/transaction**

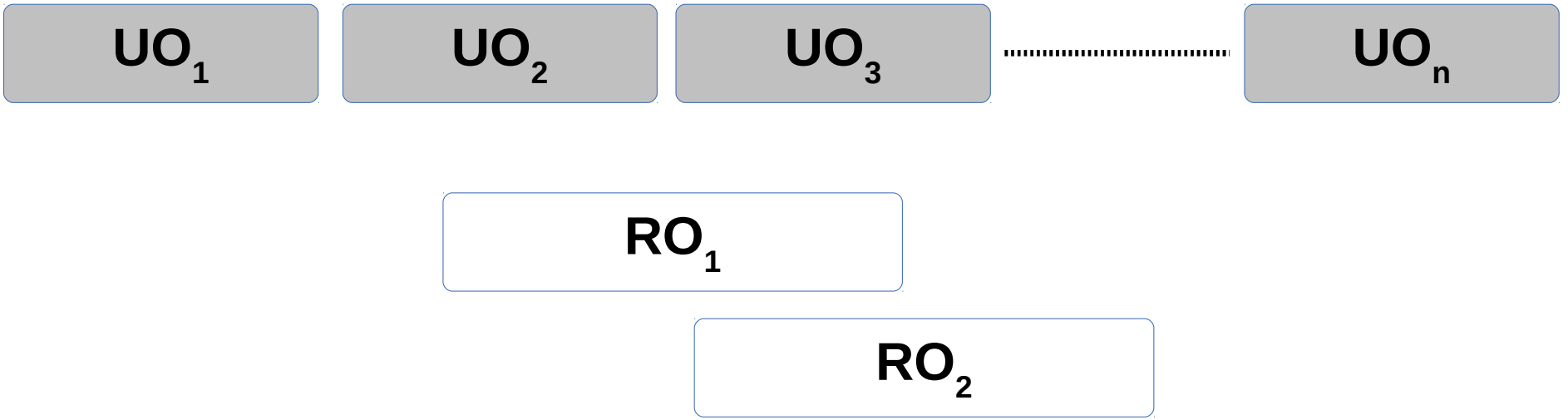
# Modeling Transactional Data Structures

## Concurrent Data Structures

- Different Designs and Implementations
  - Different ad-hoc approaches for proving correctness.

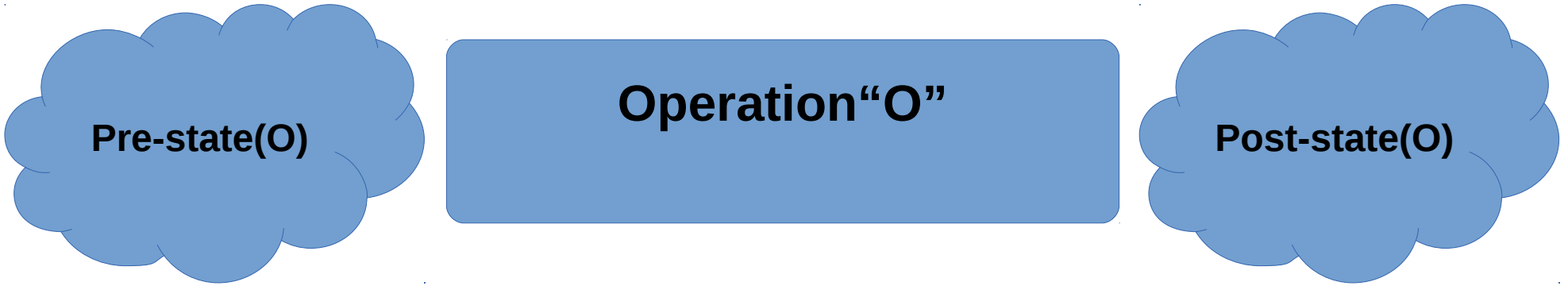
- Is there a unified model for concurrent data structures?
  - General enough.
  - Easy to use.

# SWMR Model (Lev-Ari et. Al, DISC'14)



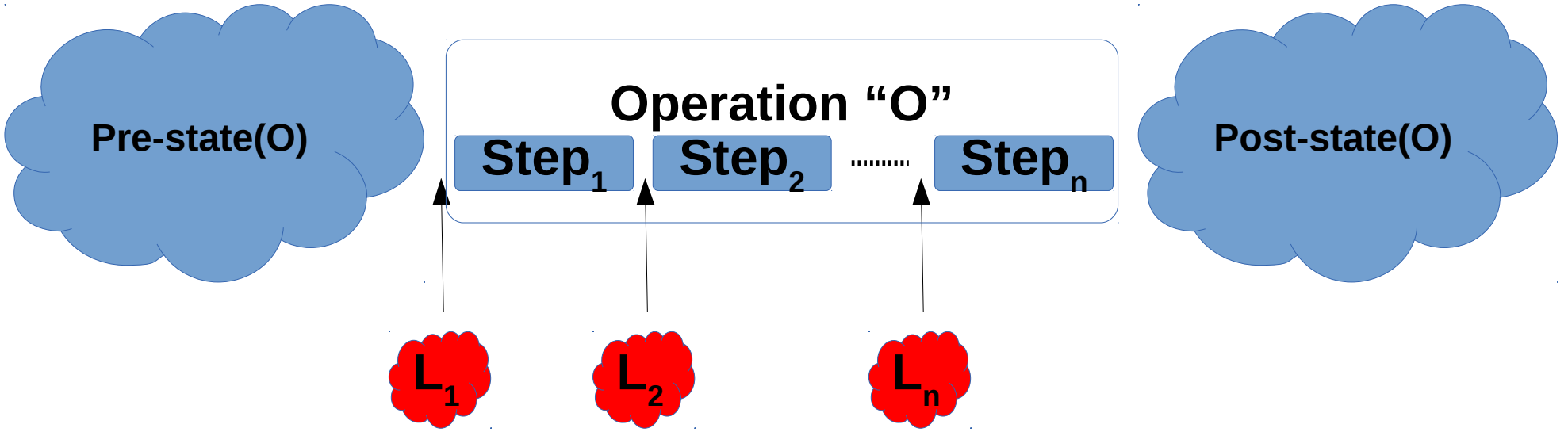
# Shared States

- Data Structure is represented as a set of shared variables.
- The values of those variables is the shared state of the data structure.

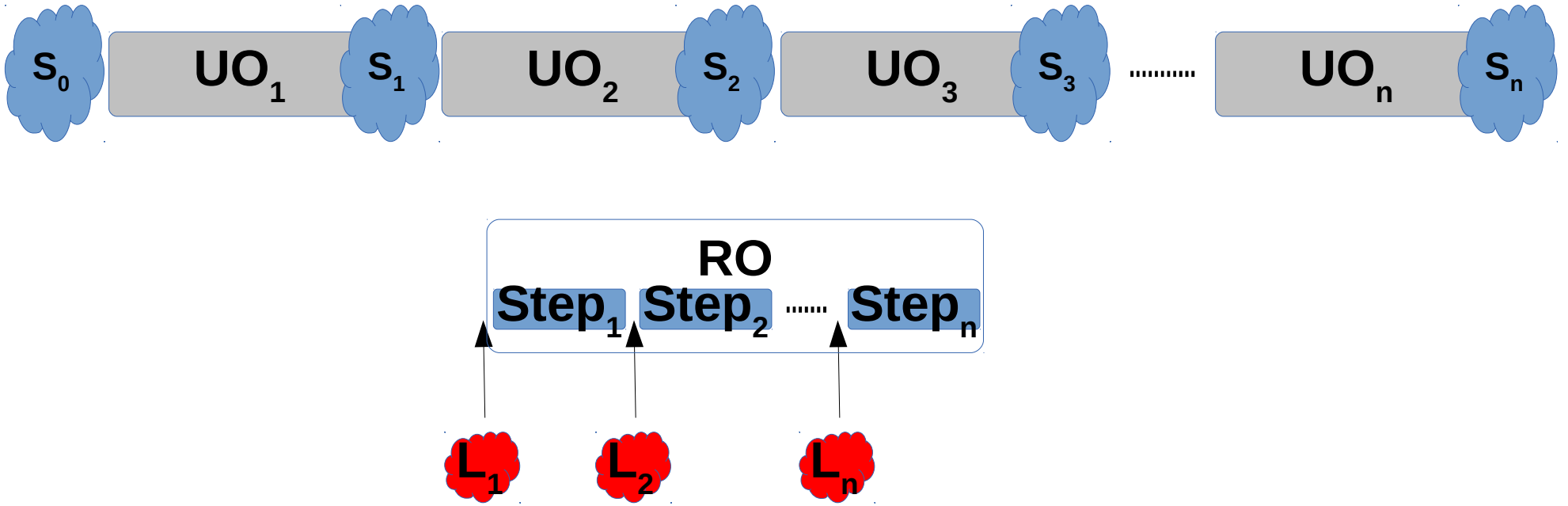


# Local States

- Operation is represented as a set of steps.
- The values of the operation's local variables before any step is the local state of the step.



# SWMR Scenario



# Validity

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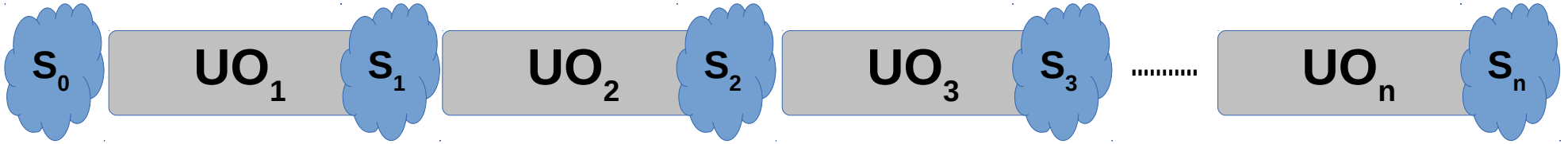


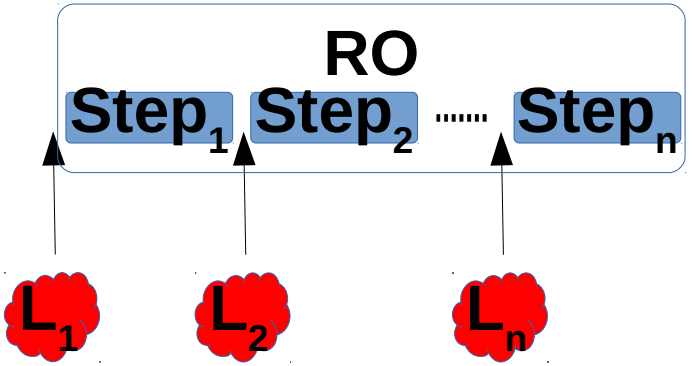

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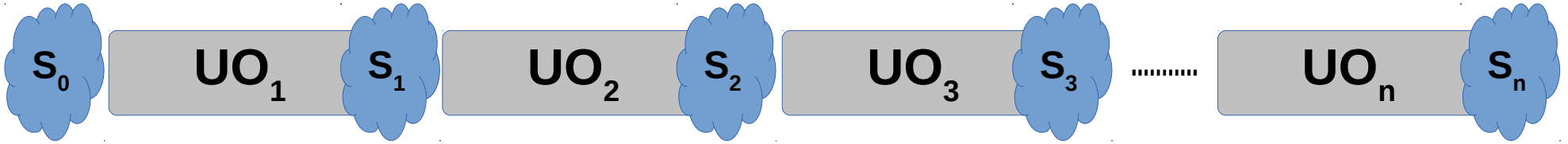
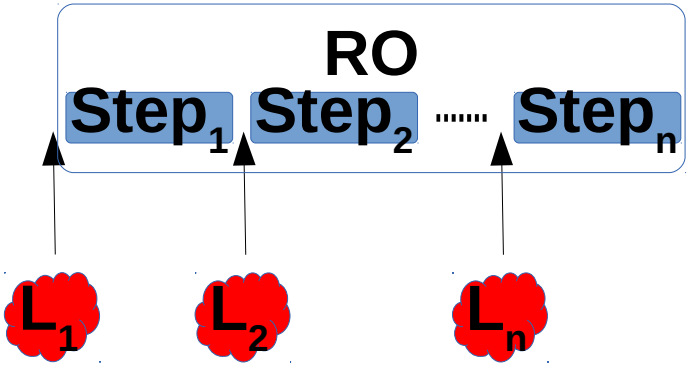
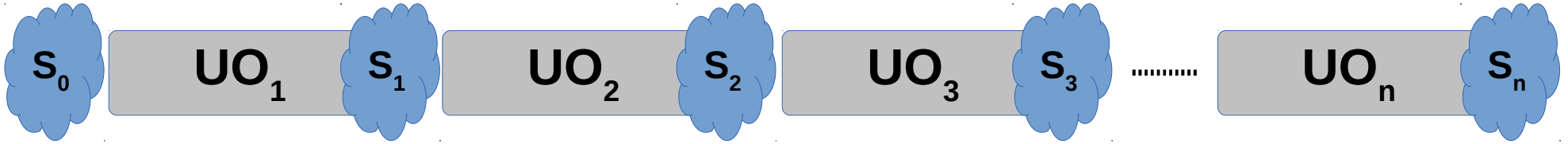
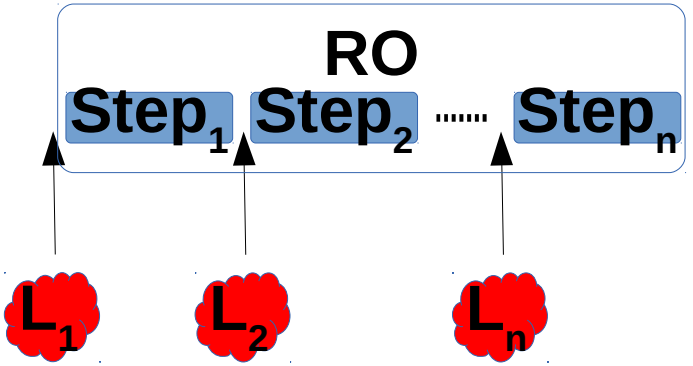



- All $S_i$ are sequentially reachable, so all $UO_i$ are valid.

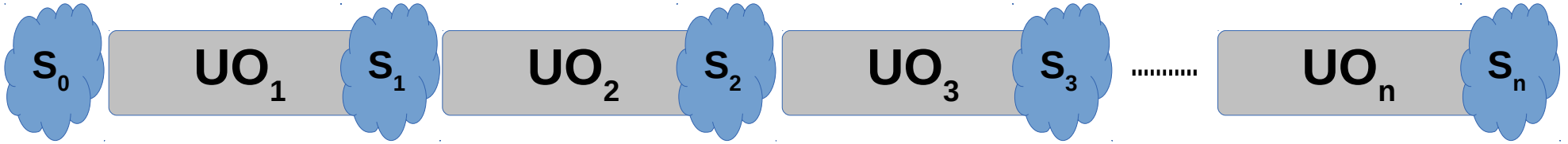
# Validity

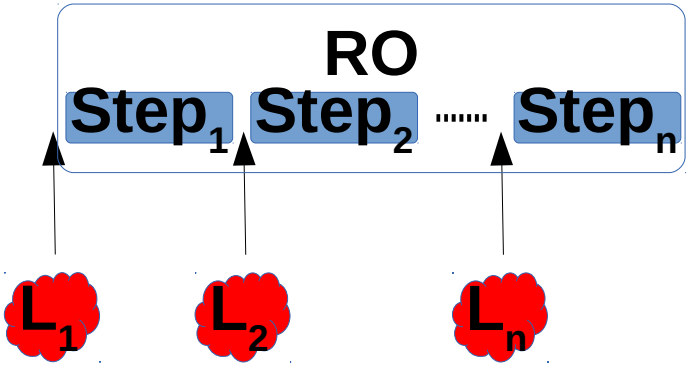



- All $S_i$ are sequentially reachable, so all $UO_i$ are valid.
- $Step_i$ in RO is valid if there is $S_j$ such that a sequential execution of RO starting from $S_j$ reaches $L_i$.

# Validity

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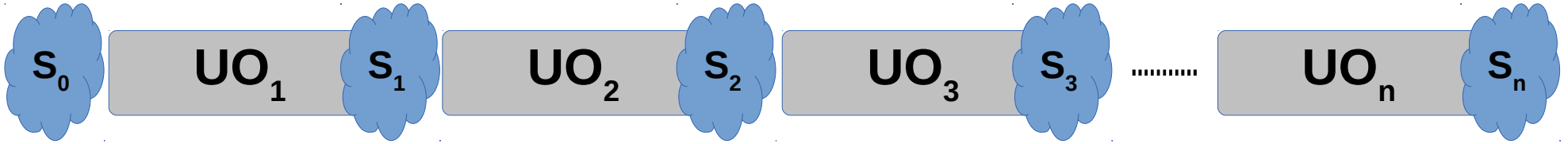


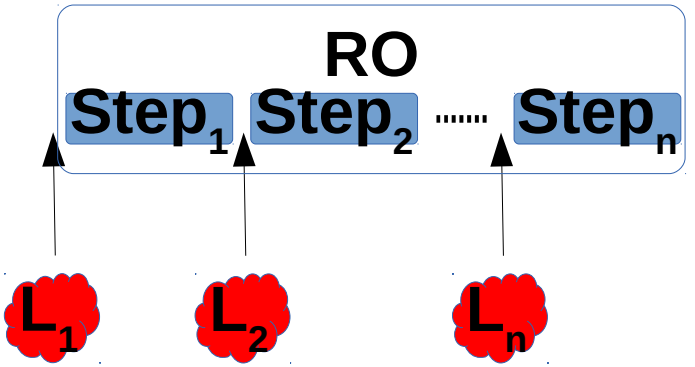


- All $S_i$ are sequentially reachable, so all $UO_i$ are valid.
- $Step_j$ in RO is valid if there is $S_i$ such that a sequential execution of RO starting from $S_i$ reaches $L_j$.

- $Step_j$ in RO is valid if there is a “base point” where the “base condition” of $step_j$ holds.

# Validity

- How to prove validity for any data structure.
  - Identify the base conditions for each step in each operation (it is sufficient to do so only for steps that access the shared memory).
  - Prove that in any concurrent execution, every step has a base point that satisfies its base condition.

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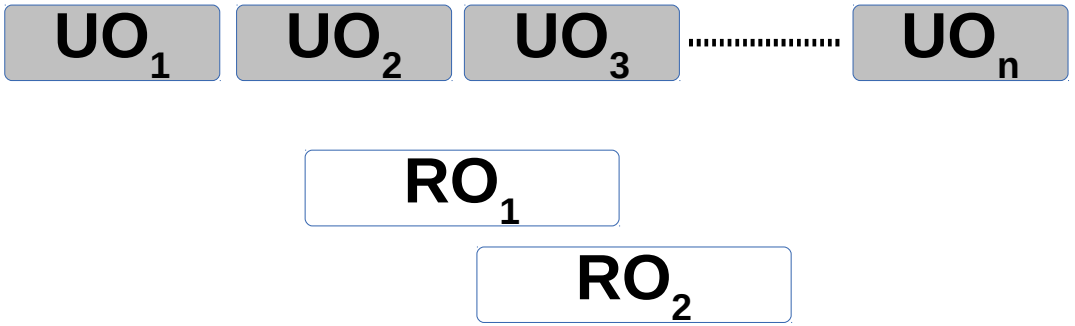
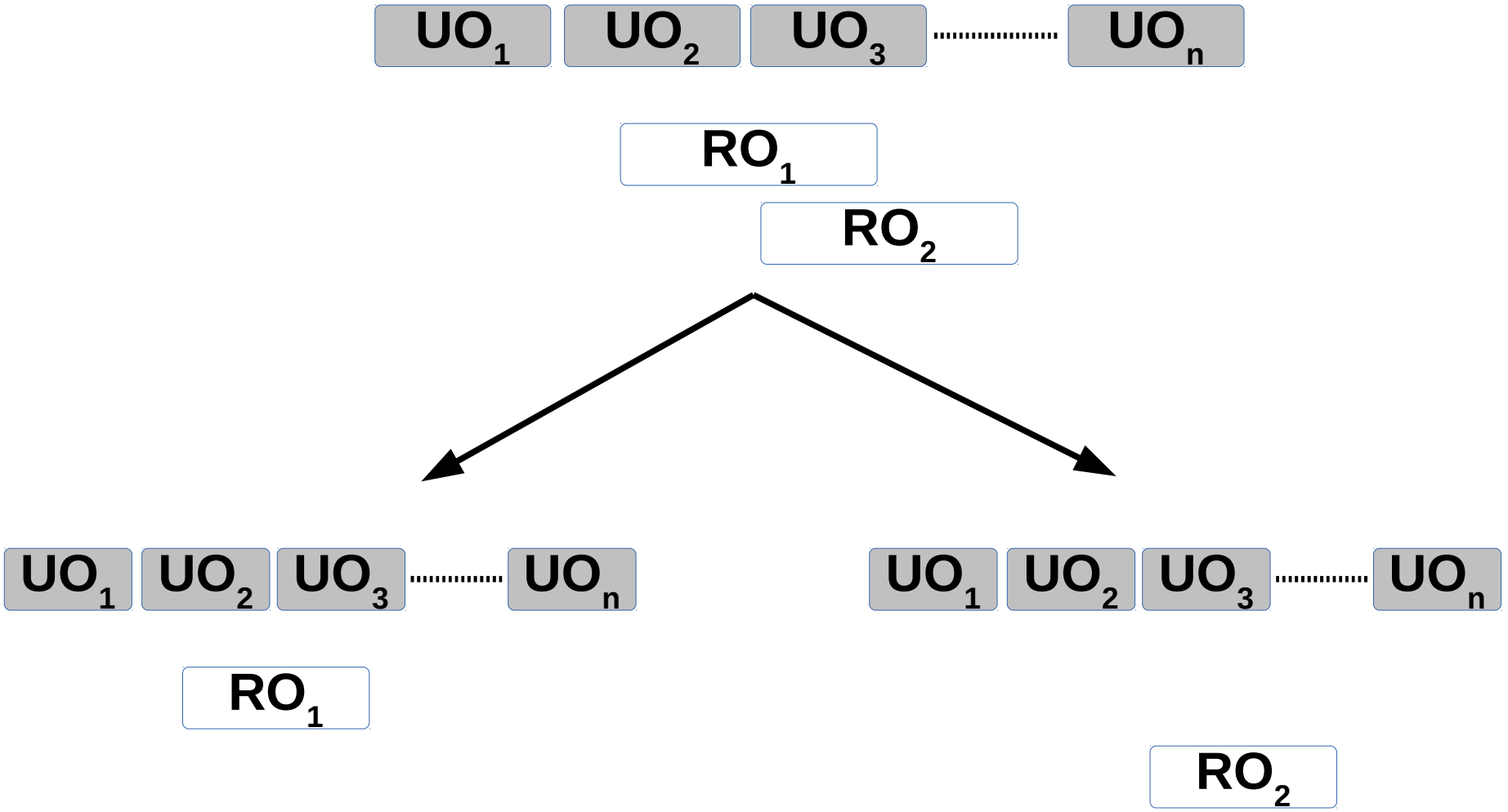
# Regularity



# Regularity



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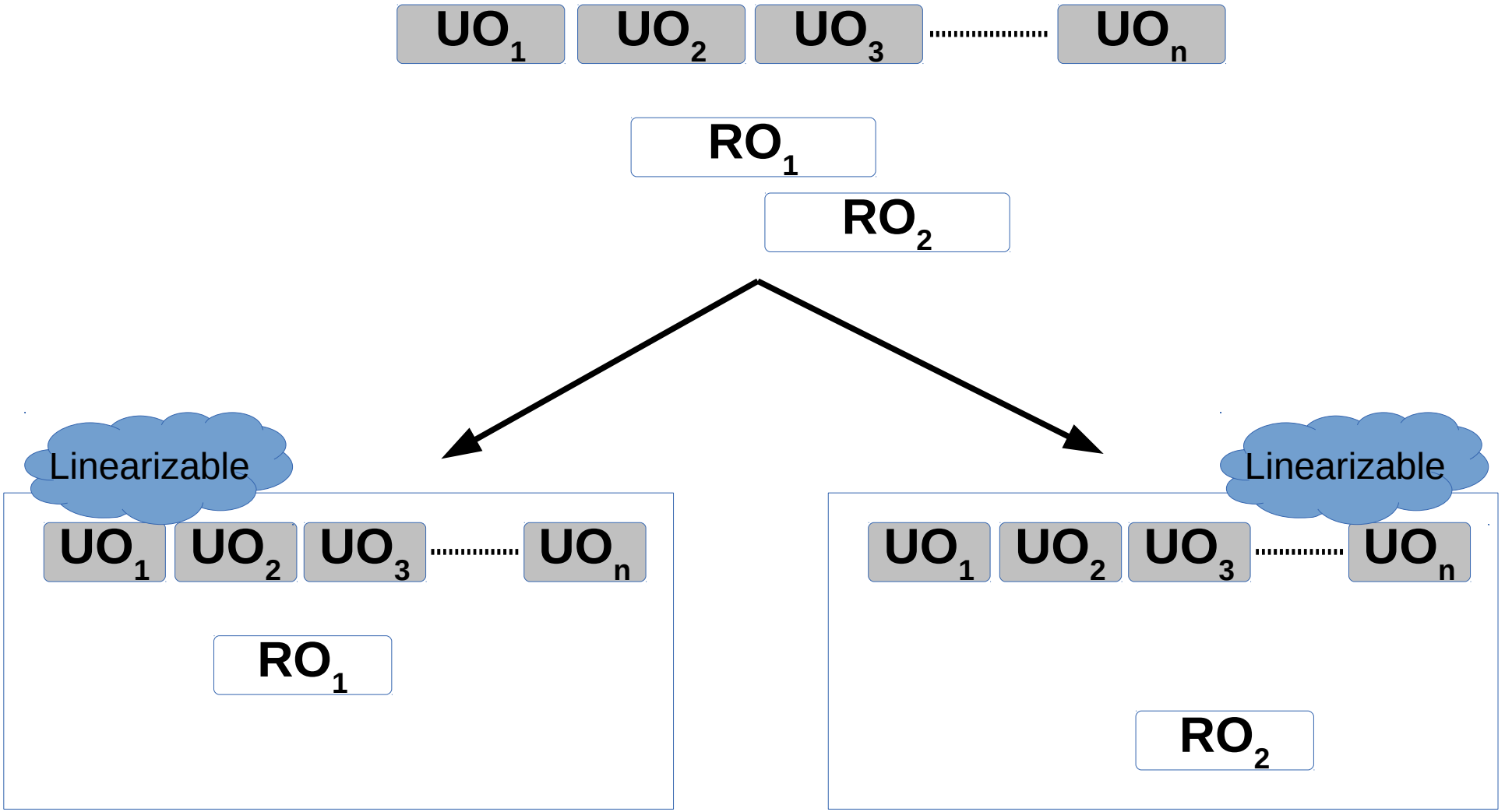
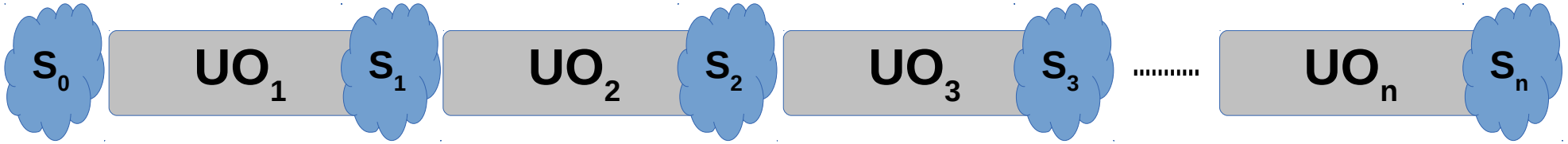
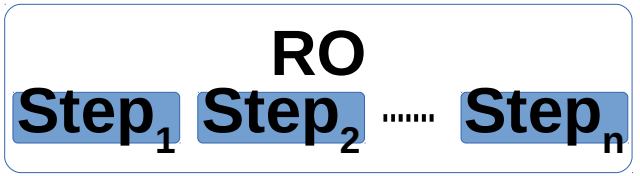

UO1 UO2 UO3 ..... UOn
RO1
RO2
Linearizable
UO1 UO2 UO3 ..... UOn
RO1
Linearizable
UO1 UO2 UO3 ..... UOn
RO2

# Regularity



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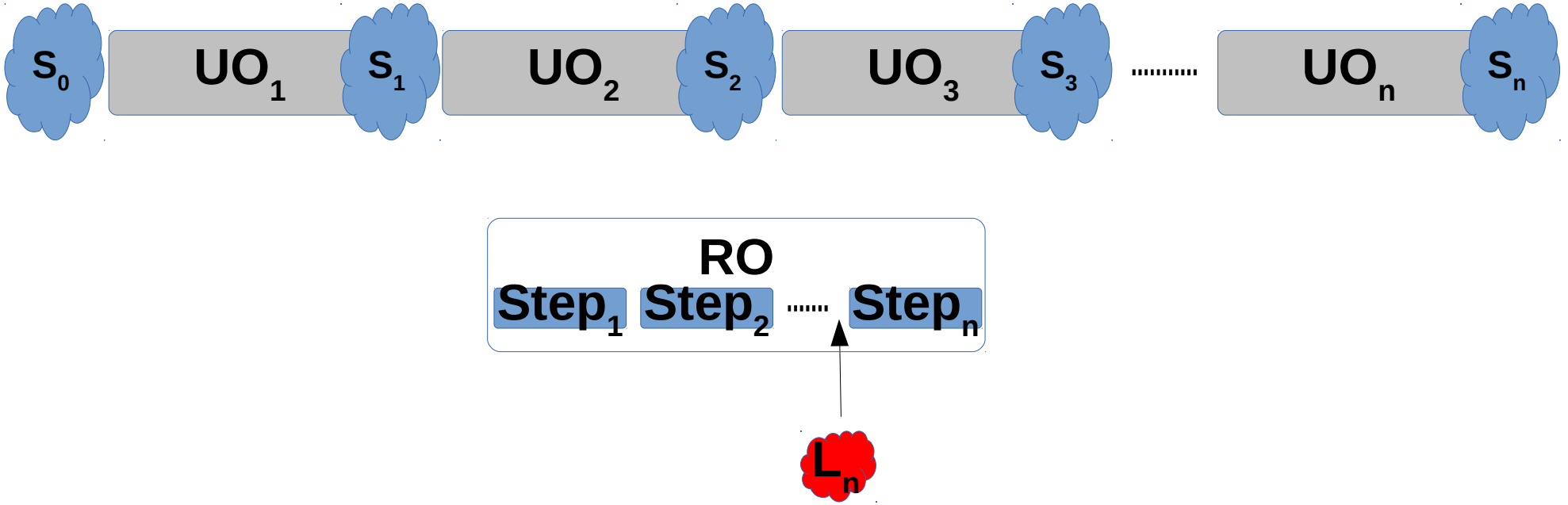


- Acceptable base points for RO's return step are only $S_1$, $S_2$, $S_3$.
  - Observes either the last update or a concurrent update.

# Example

**Function** *remove(n)*
$p \leftarrow \perp$
$next \leftarrow \textbf{\textit{read}}(head.next)$
**while** $next \neq n$
$p \leftarrow next$
$next \leftarrow \textbf{\textit{read}}(p.next)$
$\textbf{\textit{write}}(p.next,\ n.next)$

**Function** *insertLast(n)*
$last \leftarrow readLast()$
$\textbf{\textit{write}}(last.next,\ n)$

Base conditions:

$\Phi_1 : true$

$\Phi_2 : head \stackrel{*}{\Rightarrow} n$
$\Phi_3 : head \stackrel{*}{\Rightarrow} n$

**Function** *readLast()*
$n \leftarrow \perp$
$next \leftarrow \textbf{\textit{read}}(head.next)$
**while** $next \neq \perp$
$n \leftarrow next$
$next \leftarrow \textbf{\textit{read}}(n.next)$
$\textbf{\textit{return}}(n)$

# Example

**Function** *remove(n)*
$p \leftarrow \perp$
$next \leftarrow \textbf{read}(head.next)$
**while** $next \neq n$
$p \leftarrow next$
$next \leftarrow \textbf{read}(p.next)$
$\textbf{write}(p.next,\ n.next)$

**Function** *insertLast(n)*
$last \leftarrow readLast()$
$\textbf{write}(last.next,\ n)$

Base conditions:

$\Phi_1 : true$

$\Phi_2 : head \stackrel{*}{\Rightarrow} n$
$\Phi_3 : head \stackrel{*}{\Rightarrow} n$

**Function** *readLast()*
$n \leftarrow \perp$
$next \leftarrow \textbf{read}(head.next)$
**while** $next \neq \perp$
$n \leftarrow next$
$next \leftarrow \textbf{read}(n.next)$
$\textbf{return}(n)$

# Example

**Function** *remove(n)*
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$next \leftarrow \textbf{read}(head.next)$
**while** $next \neq n$
$p \leftarrow next$
$next \leftarrow \textbf{read}(p.next)$
$\textbf{write}(p.next,\ n.next)$

**Function** *insertLast(n)*
$last \leftarrow readLast()$
$\textbf{write}(last.next,\ n)$

Base conditions:

| | |
|---|---|
| $\Phi_1 : true$ | |
| $\Phi_2 : head \overset{*}{\Rightarrow} n$ | |
| $\Phi_3 : head \overset{*}{\Rightarrow} n$ | |

**Function** *readLast()*
$n \leftarrow \perp$
$next \leftarrow \textbf{read}(head.next)$
**while** $next \neq \perp$
$n \leftarrow next$
$next \leftarrow \textbf{read}(n.next)$
$\textbf{return}(n)$

Where is the Problem?

It covers only single-writer designs

It does not cover composable designs

Can we cover a wider set?

Optimistic Composable Data Structures

# Our Models

Single Writer Commit (SWC)

Composable SWC (C-SWC)

# SWC Model

$UO_1$

$UO_2$

$UO_3$

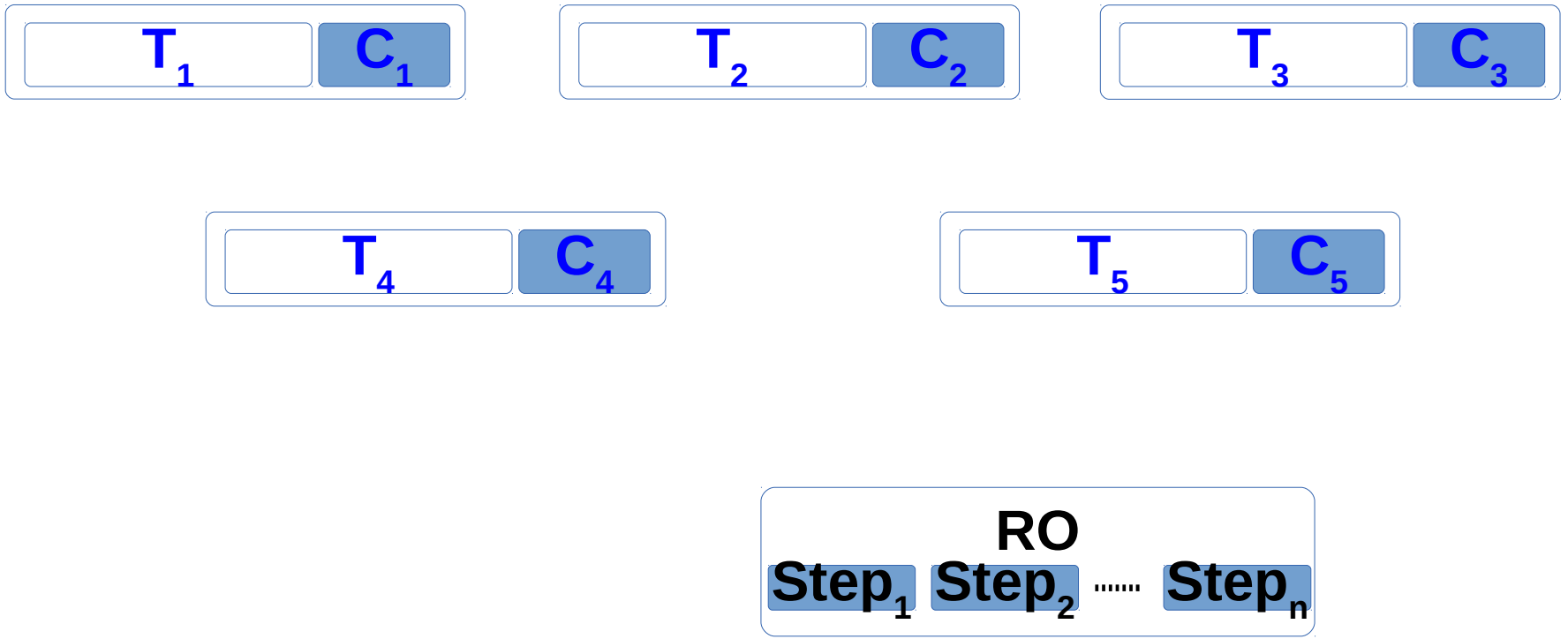
$UO_4$

$UO_5$

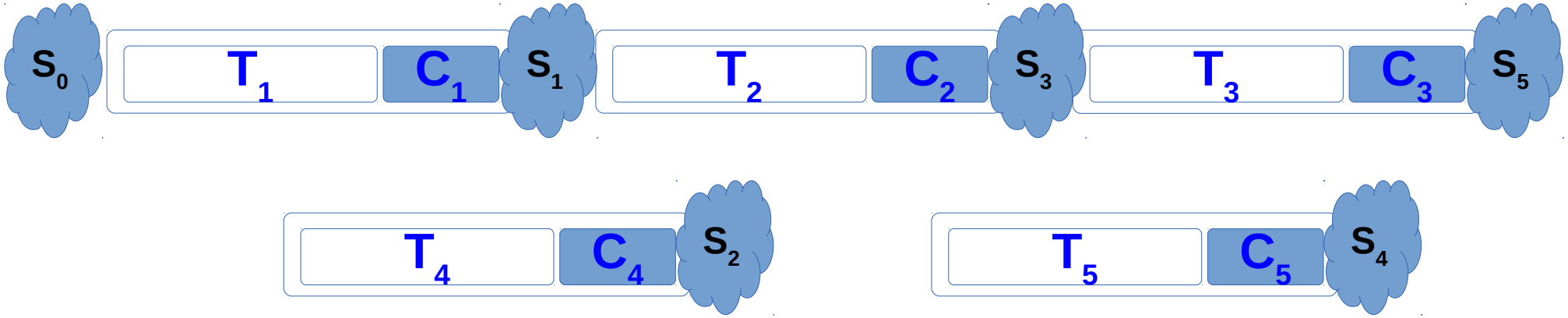
RO

$Step_1$ $Step_2$ ..... $Step_n$

# SWC Model



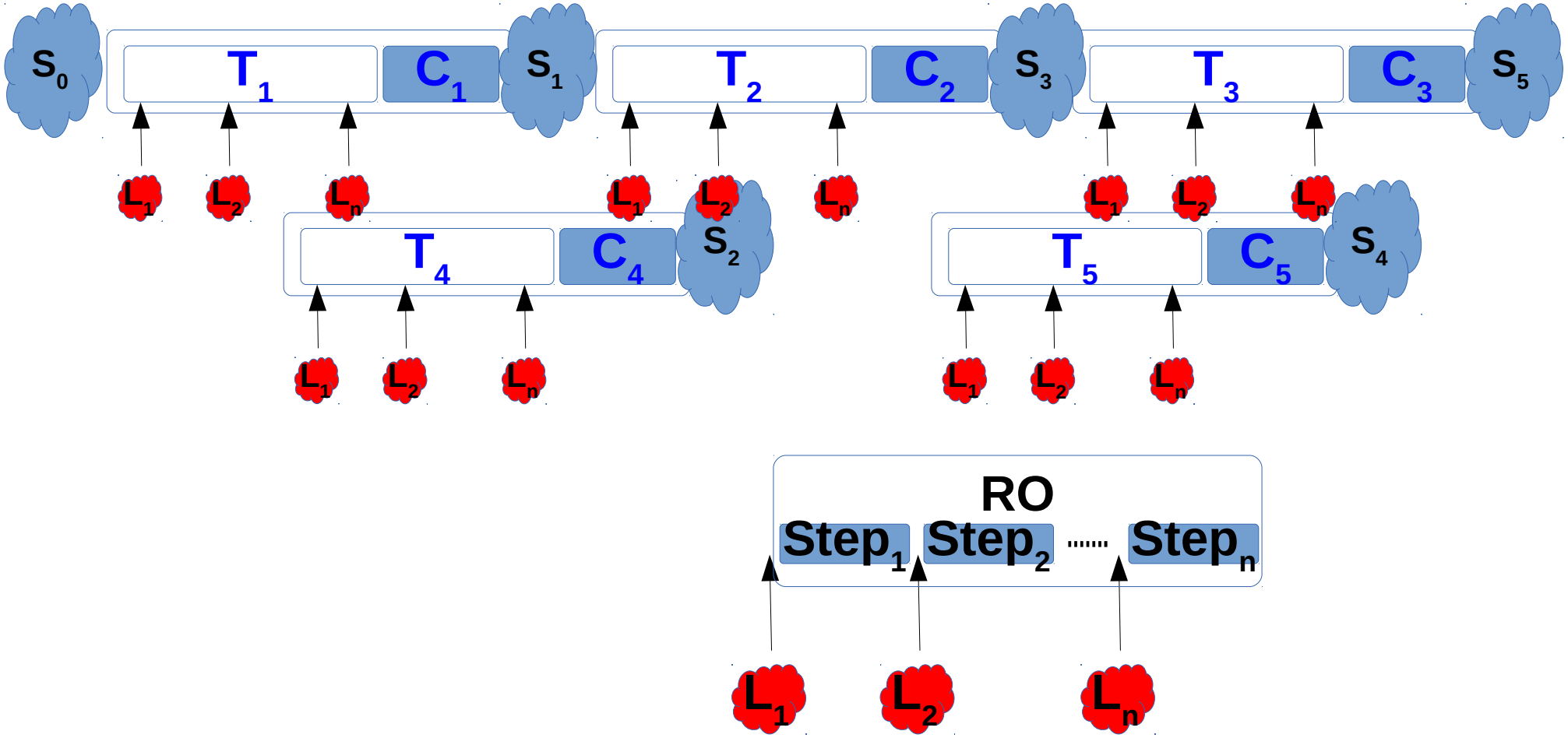
# SWC Model




RO

$Step_1$ $Step_2$ ..... $Step_n$

# SWC Model


S0
T1
C1
S1
T2
C2
S3
T3
C3
S5
T4
C4
S2
T5
C5
S4
L1
L2
Ln
RO
Step1
Step2
.....
Stepn

# Even More...

- Do we really need single commit at a time:
  - NO!!!
  - It is enough to execute commit phases atomically with *single lock atomicity (SLA)* guarantees.
  - More practical alternatives:
    - HTM (e.g. Intel TSX).
    - STM (e.g. NOrec “the SLA version”).

# Example

```
1:  procedure READLAST
2:      last ← ⊥
3:      next ← read(head.next)                          ▷ φ1 : true
4:      while next ≠ ⊥ do
5:          last ← next
6:          next ← read(last.next)                      ▷ φ2 : head ⇒* last
7:      return(last)                                    ▷ φ3 : head ⇒* last
8:  end procedure

9:  procedure INSERTLAST(n)
10:     last ← ⊥
11:     next ← read(head.next)                          ▷ φ4 : true
12:     while next ≠ ⊥ do
13:         last ← next
14:         next ← read(last.next)                      ▷ φ5 : head ⇒* last
15:     lockAcquire(gl)
                                                        ▷ φ6 : head ⇒* last
16:     if read(last.next) ≠ ⊥ then
17:         lockRelease(gl)
18:         go to 10
19:     write(last.next, n)
20:     lockRelease(gl)
21: end procedure
```

# Example

```
1:  procedure READLAST
2:      last ← ⊥
3:      next ← read(head.next)                    ▷ φ1 : true
4:      while next ≠ ⊥ do
5:          last ← next
6:          next ← read(last.next)                ▷ φ2 : head ⇒* last
7:      return(last)                              ▷ φ3 : head ⇒* last
8:  end procedure

9:  procedure INSERTLAST(n)
10:     last ← ⊥
11:     next ← read(head.next)                    ▷ φ4 : true
12:     while next ≠ ⊥ do
13:         last ← next
14:         next ← read(last.next)                ▷ φ5 : head ⇒* last
15:     lockAcquire(gl)
                                                  ▷ φ6 : head ⇒* last
16:     if read(last.next) ≠ ⊥ then
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21: end procedure
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# Example

```
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14:         next ← read(last.next)
15:     lockAcquire(gl)

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18:         go to 10
19:     write(last.next, n)
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21: end procedure
```

$\triangleright\ \phi_1 : true$

$\triangleright\ \phi_2 : head \overset{*}{\Rightarrow} last$

$\triangleright\ \phi_3 : head \overset{*}{\Rightarrow} last$

$\triangleright\ \phi_4 : true$

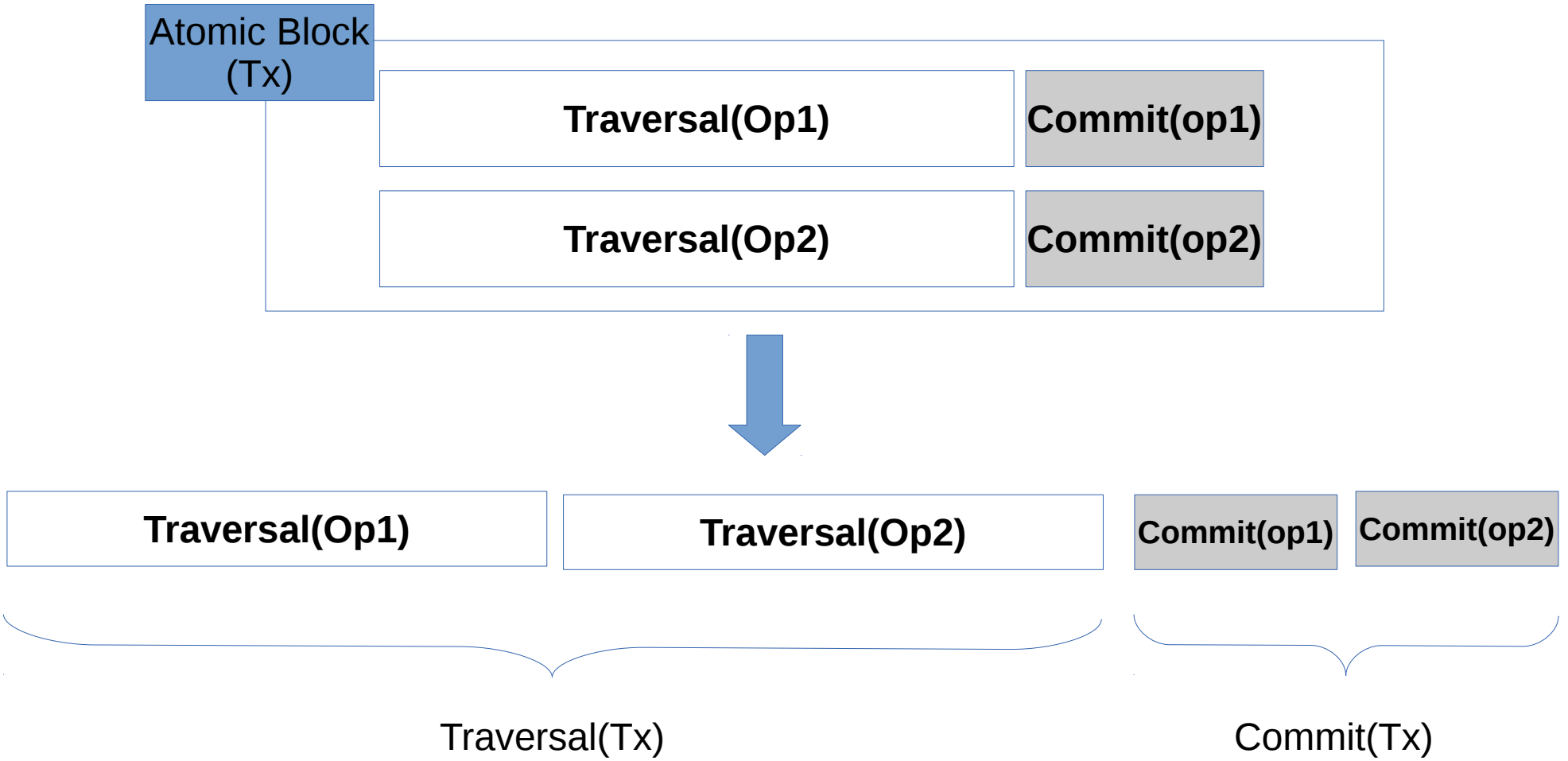
$\triangleright\ \phi_5 : head \overset{*}{\Rightarrow} last$

$\triangleright\ \phi_6 : head \overset{*}{\Rightarrow} last$

# Composable SWC Model (C-SWC)

```
1: procedure ATOMIC:T_1
2:     x = 5
3:     if readLast() ≠ x then
4:         insertLast(x)
5:     if readLast() ≠ x then
6:         ... // illegal execution
7: end procedure
```

# Composable SWC Model (C-SWC)

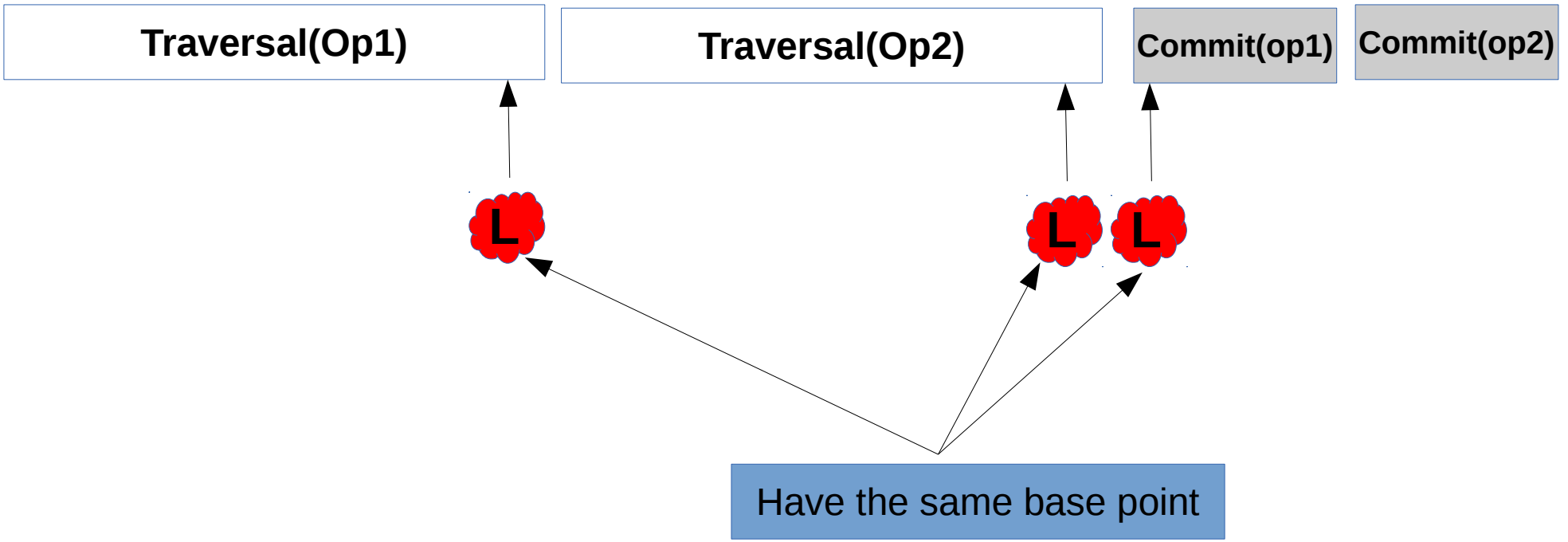


# What is remaining?

- Internal Consistency.
  - The commit phase of each operation reflects what the operation observed in its traversal.
  - The shared state of an operation is visible to subsequent operations in the same transaction.

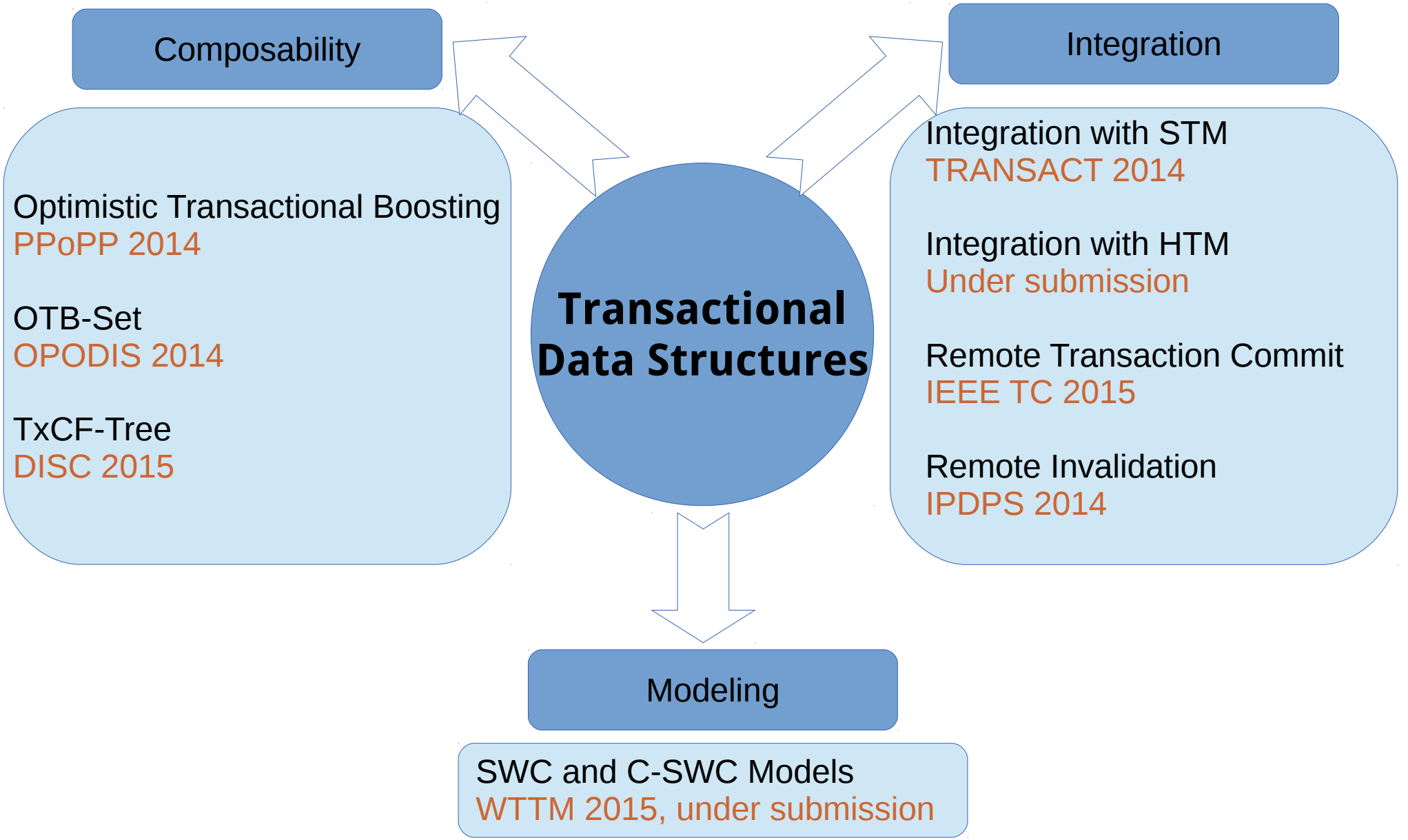
# How to prove internal consistency?

# How to prove internal consistency?



# Conclusions

# Our Contributions



## Thanks!

# Questions?