Designing Optimal Gain Sharing Plans to Promote Energy Conservation

by

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Abstract

We analyze the optimal design of gain sharing plans to promote energy conservation. We show how the optimal plan varies as industry conditions and the regulator's information change. We demonstrate the importance of allowing the energy supplier a choice among plans, some of which offer the prospect of both pronounced financial gains for superior performance and substantial losses for inferior performance.

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1 Introduction

Energy conservation has taken center stage as a top priority in energy sectors around the world, and *gain sharing* has emerged as a popular means to secure conservation.[1] Gain sharing attempts to motivate a supplier of energy conservation services to promote conservation while securing for energy consumers a substantial portion of the resulting benefits. Gain sharing pursues this goal by specifying in advance how the realized benefits of an energy conservation program will be divided between the supplier and energy consumers.[2]

In designing gain sharing plans, regulators have benefited from the guidance of thoughtful discussion about desirable properties of these plans. However, with few exceptions, this guidance has not been derived from formal economic models that permit a specification of optimal gain sharing plans. The purpose of this research is to develop a streamlined formal model of energy conservation and employ the model to characterize the properties of optimal gain sharing plans. Our model provides some conclusions that are consistent with the advice that has been offered to regulators. The model also provides distinct conclusions that can promote more effective and more pronounced energy conservation.

In our model, a firm can exert effort to promote energy conservation. This firm might be either the same firm that supplies energy to consumers or an independent energy services company (an ESCO) that focuses solely on delivering energy conservation services. The firm's effort in promoting energy conservation is not readily measured because it reflects, for example, the care with which the firm designs an energy conservation program and the diligence with which it implements the program.[3] A regulator implements a gain sharing plan to motivate the firm to deliver energy conservation effort.[4] The plan specifies the fraction of

[1] Dixon et al. (2010) review the history of energy conservation and efficiency policies in the United States. Tanaka (2011) summarizes the corresponding policies employed in other countries.

[2] See Stoft and Gilbert (1994), Stoft et al. (1995), Munns (2008), and the Institute for Electric Efficiency (2010), for example, for descriptions of gain sharing plans that have been implemented in practice.

[3] If this effort were readily observed and if the firm's cost of delivering the effort were known, no sharing of realized gains would be required. The firm could simply be compensated for its effort and all of the gains resulting from the effort could be delivered to consumers.

[4] We use the terms "supplier" and "firm" interchangeably throughout the ensuing analysis.

the realized benefit from the energy conservation plan (i.e., the "gain" from the plan) that is awarded to the firm. Consumers receive the remaining fraction of the realized gain. The gain from the plan might reflect the realized reduction in consumers' expenditures on energy, for example.[5]

The regulator faces a fundamental trade-off in this setting. She can motivate the supplier to deliver more energy conservation effort by promising the supplier a larger share of the realized gain. However, the larger is the share of the realized gain that is awarded to the supplier, the smaller is the share that can be delivered to consumers. The optimal gain sharing plan balances these considerations.

Our analysis provides five primary conclusions. First, the ideal gain sharing plan varies with the information that is available to the regulator and the supplier. For example, the supplier often is afforded a smaller fraction of the realized gain when the supplier is better informed than the regulator about how difficult it is to achieve substantial energy conservation.

Second, a gain sharing program should subject the supplier to substantial downside risk. If the supplier can be penalized severely when the conservation plan does not achieve specified goals, then the supplier also can be rewarded handsomely when the plan exceeds the goals without affording the firm excessive profit (i.e., rent). The prospect of substantial rewards can motivate the firm to pursue large gains aggressively, which is beneficial for consumers.[6]

Third, regulators should not restrict themselves to a single gain sharing plan. A regulator can secure a substantial increase in consumer welfare by allowing the supplier to choose one plan from a carefully structured menu of plans. The increase in consumer welfare that such

[5] Alternatively, when the energy supplier also provides energy conservation services, the gain might be the reduction in the supplier's cost of serving its customers as their energy consumption declines. This gain can also be viewed as the sum of (i) the reduction in customer expenditures on energy; and (ii) the increase in the energy supplier's measured profit, i.e., its revenues less its measured costs. These measured costs can include any observed expenditures that promote energy conservation.

[6] This finding is consistent with Eto et al. (1998, p. 47)'s recommendation to "use higher marginal incentive rates than are currently found in practice, but limit total incentive payments by adding a fixed charge." Similarly, Stoft and Gilbert (1994, p.4) advise that a "shared-savings incentive scheme with a very high marginal incentive rate should be the principle form of incentive scheme" (p. 4).

choice generates is particularly pronounced when the plans can impose considerable downside risk on the supplier.

Fourth, the ideal extent of gain sharing varies systematically with the environment in which the gain sharing program is implemented. We identify how the optimal levels of gain sharing vary with the regulator's objective, the political pressures she faces, her *ex ante* beliefs about industry conditions, realized industry conditions, and the maximum loss the firm can be forced to bear under the gain sharing program.

Fifth, it may be optimal to intentionally cede rent to the firm, even when the regulator does not value this rent and could readily prevent the firm from securing any rent. When the firm cannot be compelled to suffer large financial losses, the regulator can only motivate the firm by promising it substantial returns when large gains are realized. Although consumers only secure modest gains in such circumstances, modest gains are preferable to the even smaller gains that would arise if the firm's incentive to generate gains were reduced.

We develop these findings as follows. Section 2 describes the key elements of our formal model. Section 3 considers the hypothetical setting where the regulator can observe the firm's effort in promoting energy conservation. Section 4 considers another benchmark setting in which the regulator shares the firm's knowledge of the industry, but cannot monitor the firm's effort. Section 5 identifies the properties of the best single gain sharing plan in the setting of primary interest where the firm is better informed than the regulator about the prospects for substantial gains from energy conservation. Section 6 explains the merits of offering the firm a menu of optional gain sharing plans and characterizes the optimal such menu. Section 7 illustrates the magnitude of the welfare gains that can be secured through careful design of a menu of optional gain sharing plans. Section 8 offers concluding observations and discusses directions for future research. The Appendix provides the proofs of all formal conclusions.

Our research complements earlier, more informal work on the design of gain sharing plans. Moskovitz (1989), Stoft and Gilbert (1994), Stoft et al. (1995), and Eto et al. (1998) review the different types of energy efficiency programs that are employed in practice, explain their relative strengths and weaknesses, and discuss the properties of effective programs. Although these studies offer thoughtful discussions of the appropriate design of gain sharing plans, they do not analyze formal models that admit explicit derivation of the properties of optimal plans.

Eom and Sweeney (2009) employ a formal model to characterize gain sharing policies that maximize social net benefits while achieving an exogenously specified energy conservation target. The authors consider a setting where (as in our benchmark analysis in section 4) the regulator shares the firm's substantial knowledge of industry conditions, including the potential for energy conservation. Eom (2009) extends this analysis to allow the firm to be privately informed about the prevailing potential for energy conservation. However, Eom restricts attention to settings in which a single gain sharing plan is implemented (as in our analysis in section 5). We identify the conditions under which this restriction is limiting and illustrate the welfare gains that can arise more generally.

In describing the design of gain sharing (or shared savings) programs in practice, Blank and Gegax (2011, p. 5) observe that "Although previous authors have addressed the optimal design of incentive mechanisms, the actual shared savings mechanisms adopted by regulatory commissions seem to have arbitrarily selected the level of shared savings."[7] Our goal is to derive from formal economic models practical guidance that can assist regulators in the future as they design new and improved gain sharing plans. The guidance we offer largely reflects standard conclusions in the theoretical literature on the design of reward structures in the presence of limited information.[8] Our contribution is primarily to develop these findings in the context of an energy conservation program in order to help synthesize and refine the messages that have been offered in the (often informal) literature on the design of gain

[7] The authors' footnote (#3) has been omitted in this quote. The footnote identifies Stoft and Gilbert (1994), Eto et al. (1998), and Eom and Sweeney (2009) as the studies that analyze the optimal design of incentive mechanisms.

[8] See, for example, Laffont and Tirole (1993), Laffont and Martimort (2002), and Bolton and Dewatripont (2005). We contribute to this theoretical literature by providing a detailed analysis of how optimal reward structures change as industry conditions (including minimum payment constraints) change.

sharing plans to motivate energy conservation.

2 Elements of the Model

We consider a setting in which a firm (an energy supplier or an ESCO) can devote effort to promoting energy conservation. This effort can take the form of designing and implementing programs that help customers achieve energy conservation most economically, for instance.[9] These programs might encompass: (i) inspecting customers' premises and repairing problems that are found to be reducing energy efficiency; (ii) encouraging customers to install more energy efficient appliances; or (iii) educating customers on how to reduce their energy consumption without sacrificing comfort, for example. The firm's effort produces a gain that reflects the benefit generated by the energy conservation plan. As noted above, this benefit might reflect the reduction in customers' expenditures on energy, for example.

We assume the realized gain, g, can be measured accurately.[10] However, the regulator cannot measure the effort and the associated cost the firm incurs to secure the realized gain. We will denote by $K(G, k)$ the unmeasured effort cost the firm incurs to achieve expected gain G when the prevailing cost parameter is k. This cost parameter indexes the firm's unmeasured effort cost of securing energy conservation gains.[11] For simplicity, we assume this cost can be either relatively high or relatively low. Formally, k can take on one of two possible values, k_1 or k_2. It is costless for the firm to secure no expected gain for both realizations of k, so $K(0, k_1) = K(0, k_2) = 0$. The cost of achieving a positive expected gain increases at an increasing rate for both realizations of k. Formally, $K_G(G, k_i) > 0$ and $K_{GG}(G, k_i) > 0$ for all $G > 0$ for $i = 1, 2$, where the subscripts G and GG denote

[9] We use the terms "customer" and "consumer" interchangeably throughout the ensuing analysis.

[10] As we discuss further in the concluding section, components of the realized gain can sometimes be difficult to monitor accurately in practice. To illustrate, when the gain includes the change in profit an energy supplier experiences due to reduced investment in new capacity, the return the supplier would have secured from the investment must be calculated. As Blank and Gegax (2011) observe, this calculation entails important subtleties.

[11] Some of the costs the firm incurs to promote energy conservation (e.g., rebates given to consumers who purchase new energy efficient appliances) may be readily measured. These costs can be viewed as a component of the net benefit (i.e., the gain) generated by the energy conservation program.

the first and second partial derivatives with respect to G, respectively. The firm's marginal cost of achieving a positive expected gain is higher when the firm operates in the high cost environment (i.e., when $k = k_2$) than when it operates in the low cost environment (i.e., when $k = k_1$). Formally, $K_G(G, k_2) > K_G(G, k_1)$ for all $G > 0$.[12] In practice, the high cost environment could prevail when energy customers are reluctant to change their energy consumption behavior, when the customers have been unusually diligent in ensuring the energy efficiency of their dwellings, or when the supplier of energy conservation services is not highly skilled in designing and implementing energy conservation programs, for example.

The supplier of energy conservation services is assumed to have better information than the regulator about its unmeasured cost of achieving energy conservation gains.[13] This information advantage might reflect, for example, the firm's extensive industry experience and its privileged knowledge of its personnel and internal operations. Formally, the information advantage is captured by assuming the firm knows the realization of the cost parameter $k \in \{k_1, k_2\}$ from the outset of its interaction with the regulator. The regulator does not know the realization of k. She believes the low cost environment (k_1) prevails with probability $\phi_1 \in (0, 1)$ and the high cost environment (k_2) prevails with probability ϕ_2 $(= 1 - \phi_1)$.

To motivate the firm to develop and implement a successful energy conservation program, the regulator promises the firm: (i) a fixed payment, F, that does not vary with the program performance; and (ii) a share, $s \in [0, 1]$, of the realized gain from the program.[14] A negative fixed payment ($F < 0$) implies that the firm must achieve a positive gain in order to avoid a financial loss from the energy conservation program.[15]

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[12] The higher marginal cost when $k = k_2$ implies a higher total cost of achieving any positive expected gain, i.e., $K(G, k_2) > K(G, k_1)$ for all $G > 0$. To avoid uninteresting cases in which either no expected gain or an infinite expected gain are pursued, we assume $K_G(G, k_i)|_{G=0} = 0$ and $\lim_{G \to \infty} K_G(G, k_i) = \infty$ for $k_i \in \{k_1, k_2\}$.

[13] Our formal analysis focuses on the case in which the firm has superior knowledge of its cost of achieving a specified expected gain. The analysis also pertains to settings in which the firm has superior knowledge of the expected gain that can be secured from its energy conservation effort.

[14] This linear structure is not without loss of generality, although the structure is common in practice. The concluding section discusses alternative structures.

[15] The gain, g_0, that ensures zero profit for the supplier (so $F + s\,g_0 = 0$) might be viewed as the "goal" of the energy conservation plan. Under this view, the supplier secures positive profit if it surpasses the goal

In practice, there may be limits on the loss a firm can be required to suffer under an energy conservation program. Too large a loss could compel an ESCO to declare bankruptcy. A large loss also could threaten the financial integrity of an energy supplier and thereby limit its ability to deliver uninterrupted, high quality service to its customers. To reflect such considerations, we assume the fixed payment to the firm cannot be less than a specified minimum value, $-\underline{F}$. To illustrate, if $\underline{F} = 0$, then the firm cannot be required to deliver any payment to consumers in return for the opportunity to retain a share of the realized gain.[16]

The firm seeks to maximize its expected profit from the energy conservation program, which is the sum of the fixed payment (F) and the share of the expected gain ($s\,G$) it receives, less its unmeasured cost ($K(\cdot)$). The firm will agree to undertake the energy conservation program as long as it anticipates non-negative profit from doing so. To facilitate the interpretation of our analysis as pertaining both to ESCOs and to energy suppliers, we do not explicitly analyze the profit earned by the industry's energy supplier. Note, though, that if the same firm supplies both energy and energy conservation services, then any reduction in its measured profit from supplying energy that results from its energy conservation effort can be viewed as a measured cost of the conservation plan. The firm can be directly reimbursed for some or all of these costs if the realized gain that is shared does not explicitly include the measured costs of the energy conservation program.[17]

The regulator designs the gain sharing program to maximize expected consumer welfare and the fraction $\alpha \in [0, 1)$ of the firm's profit from the energy conservation program. Expected consumer welfare is the fraction of the expected gain awarded to consumers ($[1 - s]\,G$) less the payment to the firm (F), which is financed by revenues collected from consumers.

(i.e., if $g > g_0$), whereas the supplier incurs negative profit if it fails to achieve the goal (i.e., if $g < g_0$).

[16] Munns (2008) explains how the gain sharing plans employed in California limit the losses that the energy supplier incurs from failure to achieve specified energy conservation targets.

[17] The concluding section provides additional discussion of the appropriate reimbursement of observed program costs. The concluding section also discusses some of the new considerations that arise when the same firm supplies energy and energy conservation services and when this firm's profit from energy supply is considered explicitly.

The parameter α reflects the regulator's valuation of the firm's profit relative to her valuation of consumer welfare. Because $\alpha < 1$, the regulator's primary concern is consumer welfare. If $\alpha = 0$, consumer welfare is the regulator's exclusive concern.

The regulator is aware that the firm – and only the firm – knows the prevailing environment. Consequently, the best the regulator can do is offer the firm a choice between gain sharing plans and design the plans to ensure the firm chooses one plan, (F_1, s_1), when the low cost environment prevails (i.e., when $k = k_1$) and the other plan, (F_2, s_2), when the high cost environment prevails (i.e., when $k = k_2$). Formally, the regulator's problem, [P], is:

$$\underset{s_i,\, F_i \geq -\underline{F}}{\text{Maximize}} \quad \sum_{i=1}^{2} \phi_i \{ [1 - s_i] \; G_i - F_i + \alpha \; \pi_i (F_i, s_i) \} \tag{1}$$

subject to, for $j \neq i, \;\; i, j \in \{1, 2\}$:

$$\pi_i (F_i, s_i) \;\; \equiv \;\; F_i + s_i \, G_i - K(G_i, k_i) \;\; \geq \;\; 0 \, ; \;\; \text{and} \tag{2}$$

$$\pi_i (F_i, s_i) \;\; \geq \;\; F_j + s_j \, G_{ji} - K(G_{ji}, k_i) \, , \tag{3}$$

where

$$G_{ji} \;\; = \;\; \arg\max_{G} \;\; \{ F_j + s_j \, G - K(G, k_i) \} \quad \text{and} \quad G_i \;\; = \;\; G_{ii}. \tag{4}$$

Expression (1) reflects the regulator's objective to maximize expected consumer welfare and the fraction α of the firm's profit. The participation constraints in expression (2) reflect the fact that the firm will only agree to undertake the energy conservation program if it anticipates nonnegative profit from doing so.[18] The incentive compatibility constraints in expression (3) identify (F_i, s_i) as the gain sharing plan the firm will select when $k = k_i$. Expression (4) identifies G_i as the expected gain the firm will pursue when $k = k_i$ and it selects the (F_i, s_i) plan. It also identifies G_{ji} as the expected gain the firm would implement if it selected the (F_j, s_j) plan when $k = k_i$.

[18] Profit should be interpreted as extranormal profit, which is the minimum profit required to induce the firm to undertake the activity in question. Here and throughout the ensuing analysis, we assume that when the firm is indifferent among actions, it will undertake the action most preferred by the regulator. This simplifying assumption is without essential loss of generality.

3 The Full Information Benchmark

Discussions of the design of energy conservation programs often presume the regulator is fully informed about all elements of the energy conservation process and can observe all relevant actions undertaken by the supplier. If this were the case in practice, the regulator's task would be straightforward. She could calculate the expected gain that guarantees the greatest combined net benefit for consumers and the firm, direct the firm to deliver the effort that ensures this expected gain, and award the firm a payment that just covers its cost of implementing the identified expected gain if and only if it delivers the requisite effort.[19]

Formally, consider the *full information setting* in which the regulator, like the firm, knows the prevailing cost environment (k_i) and so knows the firm's cost ($K(G, k_i)$) of achieving a specified expected gain, G. In this setting, the regulator will instruct the firm to deliver the effort required to ensure the expected gain that maximizes total expected surplus (i.e., the sum of the expected net benefits for the supplier and energy consumers), $G_i^* \equiv \arg\max_G \{G - K(G, k_i)\}$. Furthermore, because she values consumer welfare more highly than she values the firm's profit, the regulator will limit the firm's expected profit to zero. She can do so, for example, by promising the firm the share $s_i^* = K(G_i^*, k_i)/G_i^*$ of the realized gain (with no accompanying fixed payment) if and only if the firm delivers the effort required to ensure expected gain G_i^*. The firm's expected profit if it delivers the specified effort under this reward structure is:

$$s_i^* \, G_i^* - K(G_i^*, k_i) \;=\; \left[\frac{K(G_i^*, k_i)}{G_i^*}\right] G_i^* - K(G_i^*, k_i) \;=\; 0\,.$$

Consequently, this reward structure ensures that the surplus-maximizing expected gain is pursued and the firm receives only the minimum compensation it requires to pursue energy conservation.[20]

[19] As Stoft and Gilbert (1994, p. 18) note, "If the regulator's information is perfect, the incentive problem can be solved easily through a forcing contract."

[20] Other reward structures also can ensure the regulator's preferred outcome in the full information setting. To illustrate, the regulator can secure this outcome by promising the firm a fixed payment $F = K(G_i^*, k_i)$ (and no share of the realized gain) if and only if the firm delivers the effort that ensures expected gain G_i^*.

4 The Symmetric Information Benchmark

For the reasons identified above, it often is unreasonable to assume the regulator can observe perfectly all of the firm's energy conservation activities.[21] Consequently, the regulator typically is unable to link the firm's share of the realized gain to its conservation efforts or, equivalently, to the expected gain it implements. Instead, the regulator can only link the firm's payments to the realized gain, g.

Even when she cannot monitor the firm's activities, the regulator may still be able to secure the payoff she achieves in the full information setting if she knows the prevailing cost environment (k_i). She can do so if she can impose substantial penalties on the firm when meager gains are realized (i.e., if $\underline{F}$ is sufficiently large). This conclusion is recorded formally in Observation 1.

Observation 1. *Suppose* $\underline{F} \geq G_i^* - K(G_i^*, k_i)$ *and the regulator knows* $k = k_i$*. Then she can secure the same expected payoff she achieves in the full information setting by awarding the firm the entire realized gain (so* $s = 1$*) and setting the fixed payment to ensure exactly zero expected profit for the firm (i.e.,* $F = -\{G_i^* - K(G_i^*, k_i)\}$*).*

Observation 1 reflects the well known conclusion that a regulator's inability to observe a firm's actions is not constraining when the firm is not averse to risk and can be compelled to bear the full consequences of its actions.[22] In such a setting, the regulator simply calculates the maximum expected surplus ($G_i^* - K(G_i^*, k_i)$) that is attainable in the prevailing environment. She then requires the firm to deliver this expected surplus as a lump sum payment to consumers. In return, the firm retains the entire gain it generates. When it anticipates receiving the entire gain it secures, the firm will choose its effort to maximize the difference between the expected gain and the cost of securing the expected gain. Consequently, the firm is motivated to deliver precisely the effort the regulator seeks, and so the regulator is

[21] As Stoft and Gilbert (1994, p. 18) note, "Frequently in actual regulation scenarios, the regulator faces a serious informational gap. The regulator may not be able to observe a critical component of the utility's cost."

[22] This conclusion dates back at least to Loeb and Magat (1979) and Shavell (1979).

not harmed by her inability to monitor the firm's energy conservation activities.

Observation 1 reflects the advice offered in earlier analyses of the design of gain sharing plans. In settings where the regulator and the firm share the same information about the prevailing environment, the regulator should promise the firm a large share of the realized gain (i.e., set $s = 1$) and protect consumers by requiring the firm to pay a substantial sum for the opportunity to retain the large share of the realized gain.[23]

Such a reward structure will not be feasible if the potential surplus from an energy conservation program is large and the regulator cannot credibly threaten to impose a substantial financial loss on the firm (so $\underline{F}$ is relatively small). When she is unable to extract the full expected surplus from the firm in the form of a lump sum payment (or a substantial penalty for failing to achieve a specified target gain), the regulator optimally affords the firm less than the full gain it realizes. Doing so reduces the firm's incentive to deliver energy conservation effort. However, the associated loss is outweighed by the increased surplus that consumers secure from the larger share of the realized gain they receive. This conclusion is stated formally as Observation 2.

Observation 2. *Suppose* $\underline{F} < G_i^* - K(G_i^*, k_i)$ *and the regulator knows the prevailing cost environment* (k_i)*. Then the regulator optimally sets* $F = \underline{F}$ *and* $s < 1$*. The share of the realized gain delivered to the firm* (s) *declines as the maximum loss the firm can be compelled to bear* ($\underline{F}$) *declines.*[24]

5 The Optimal Single Gain Sharing Plan

Now return to the setting of primary interest where the energy supplier is privately informed about the prevailing cost environment. If the regulator were to offer only a single gain sharing plan that the firm will agree to regardless of the prevailing cost environment, the regulator would offer the (F, s) structure that solves the following problem, [P-1]:

[23] See Stoft and Gilbert (1994), Stoft et al. (1995), and Eto et al. (1998), for example. As noted above, the lump sum payment by the firm can take the form of a penalty for failing to achieve a target realized gain.

[24] Once $\underline{F}$ is sufficiently small that the firm's participation constraint does not bind, the optimal s does not change as $\underline{F}$ declines. The rationale for this conclusion is discussed below.

$$\underset{s,\ F \geq -\underline{F}}{\text{Maximize}} \quad \sum_{i=1}^{2} \phi_i \{ [1-s]\ G_i - F + \alpha\,[F + s\,G_i - K(G_i, k_i)] \} \tag{5}$$

subject to, for $i = 1, 2$:

$$F + s\,G_i - K(G_i, k_i) \ \geq \ 0\,, \tag{6}$$

where

$$G_i \ = \ \arg\max_{G} \{ F + s\,G - K(G, k_i) \}\ . \tag{7}$$

Problem [P-1] is analogous to problem [P] except that the regulator offers a single plan (F, s) that (as expression (6) ensures) must generate nonnegative expected profit for the firm in both of the cost environments that might prevail (i.e., both when $k = k_1$ and $k = k_2$).

Lemma 1 is helpful in understanding the solution to problem [P-1]. The lemma refers to $\Delta\pi(F, s)$, which is the difference between the firm's expected profit under the gain sharing plan (F, s) in the low cost environment (k_1) and in the high cost environment (k_2).

Lemma 1. $\Delta\pi(F, s)$ *is strictly increasing in* s.

To understand the implications of Lemma 1, suppose the regulator implements a gain sharing plan that eliminates the firm's rent in the high cost environment. Lemma 1 implies that the rent the firm secures under this plan in the low cost environment increases as the share of the realized gain the plan promises to the firm increases. The firm's rent stems from the lower costs it enjoys in the low cost environment. This cost advantage delivers more rent to the firm the larger is the expected gain it implements, which increases with the share of the realized gain it is promised.

Conclusion 1 identifies the properties of the optimal gain sharing plan when the regulator offers only a single plan. The Conclusion refers to π_2, which is the firm's expected profit at the solution to [P-1] in the high cost environment (i.e., when $k = k_2$).

Conclusion 1. *There exist two distinct values of* $\underline{F}$, *namely* $\underline{F}_L < \underline{F}_H$, *such that at the solution to [P-1], the optimal single gain sharing plan has the following features:*

(i) *If* $\underline{F} \geq \underline{F}_H$, *then* $s = \overline{s} < 1$, $\frac{d\overline{s}}{d\underline{F}} = 0$, *and* $\pi_2 = 0$.

(ii) If $\underline{F} \in (\underline{F}_L, \underline{F}_H)$*, then* $s \in (\underline{s}, \overline{s})$, $\frac{ds}{d\underline{F}} > 0$, $F = -\underline{F}$*, and* $\pi_2 = 0$.

(iii) If $\underline{F} \leq \underline{F}_L$*, then* $s = \underline{s} < \overline{s}$, $\frac{ds}{d\underline{F}} = 0$, $F = -\underline{F}$*, and* $\pi_2 \geq 0$*, with strict inequality if and only if* $\underline{F} < \underline{F}_L$.

Conclusion 1(i) reports an important change in the optimal gain sharing plan that arises when the firm has better information than the regulator about the prevailing cost environment. Even when large penalties can be imposed on the firm for poor performance (so $\underline{F} \geq \underline{F}_H$), it is no longer optimal to promise the firm the entire realized gain. Such a reward structure would harm consumers by delivering excessive rent to the firm if the low cost environment prevails. Recall from Lemma 1 that the rent the firm secures in the low cost environment under any (F, s) gain sharing plan increases as s increases. Therefore, to limit the firm's rent (and thereby secure greater expected surplus for consumers), the regulator optimally sets $s < 1$ and selects F to eliminate the firm's expected profit in the high cost environment. This plan (unavoidably) allows the firm to secure some rent in the low cost environment, but less rent than would arise under a corresponding plan that awarded the firm a larger share of the realized gain.

Conclusion 1(ii) reports that when bounds on the minimum loss the firm can incur prevent the regulator from limiting the firm's rent by reducing the fixed payment to the firm (F), the regulator instead reduces the share (s) of the realized gain awarded to the firm. The reduction in s reduces the firm's incentive to devote effort to the energy conservation program. However, the reduction in s also reduces the firm's rent in the low cost environment (recall Lemma 1). As long as s is not too small (i.e., when $s > \underline{s}$), the rent reduction secured by reducing s outweighs the corresponding reduction in the total expected surplus due to the firm's reduced effort.

Conclusion 1(iii) reports that the regulator does not continually reduce s as $\underline{F}$ declines. At some point (i.e., once s declines to $\underline{s}$), further reductions in s would harm consumers by reducing unduly the effort the firm devotes to the conservation program. Consequently, once the maximum penalty that can be imposed on the firm for poor performance becomes sufficiently limited (i.e., when $\underline{F} \leq \underline{F}_L$), the regulator does not reduce s as the maximum feasible penalty declines further (i.e., $\frac{ds}{d\underline{F}} = 0$). Instead, the regulator intentionally cedes rent to the firm even in the high cost environment (so $\pi_2 > 0$) so as not to diminish unduly the firm's effort supply.[25]

6 The Optimal Pair of Gain Sharing Plans

Although it may be simpler to design and implement a single gain sharing plan, the regulator often can secure a higher level of welfare by allowing the supplier to select one of two carefully structured gain sharing plans. The optimal such pair of plans is described in Conclusion 2. The Conclusion refers to $\widehat{\pi}_2$, which is the supplier's expected profit in the high cost environment (i.e., when $k = k_2$) at the solution to [P].

Conclusion 2. *There exist two values of $\underline{F}$, namely $\underline{F}_L < \widehat{\underline{F}}_H$, such that, at the solution to [P], the optimal pair of gain sharing plans $\{(F_1, s_1), (F_2, s_2)\}$ has the following properties:*

(i) If $\underline{F} \geq \widehat{\underline{F}}_H$, then $s_1 = 1$, $s_2 = \overline{s}_2 < 1$, $F_1 < F_2$, $\frac{d\,\overline{s}_2}{d\underline{F}} = 0$, and $\widehat{\pi}_2 = 0$.

(ii) If $\underline{F} \in [\underline{F}_L, \widehat{\underline{F}}_H)$, then $s_2 \leq s_1 < 1$, $F_2 \geq F_1 = -\underline{F}$, and $\widehat{\pi}_2 = 0$. In addition, if $K_{GGG}(G, k_i) \geq 0$ and $K_{GG}(G, k_2) \geq K_{GG}(G, k_1)$ for all G and for $k_i \in \{k_1, k_2\}$, then there exists an $\widehat{\underline{F}}_L \in [\underline{F}_L, \widehat{\underline{F}}_H)$, such that $s_1 = s_2$ for $\underline{F} \in [\underline{F}_L, \widehat{\underline{F}}_L]$, whereas $s_2 < s_1$ for $\underline{F} \in (\widehat{\underline{F}}_L, \widehat{\underline{F}}_H)$. Furthermore, $\frac{ds_1}{d\underline{F}} = \frac{ds_2}{d\underline{F}} > 0$ for $\underline{F} \in (\underline{F}_L, \widehat{\underline{F}}_L)$, whereas $\frac{ds_1}{d\underline{F}} > 0$, $\frac{ds_2}{d\underline{F}} < 0$, $\frac{dF_1}{d\underline{F}} < 0$, and $\frac{dF_2}{d\underline{F}} > 0$ for $\underline{F} \in (\widehat{\underline{F}}_L, \widehat{\underline{F}}_H)$.

(iii) If $\underline{F} < \underline{F}_L$, then $s_1 = s_2 = \underline{s}$, $F_1 = F_2 = -\underline{F}$, $\frac{d\,\underline{s}}{d\underline{F}} = 0$, and $\widehat{\pi}_2 > 0$.

Furthermore, the two plans provide the same expected profit for the supplier in the low cost environment for all values of $\underline{F}$.

Conclusion 2(i) demonstrates how the regulator can avoid a fundamental conflict when she allows the firm to select one of two possible gain sharing plans rather than implementing a single plan. The regulator would like to both limit the firm's rent and motivate the firm to

[25] Eom (2009) and Eom and Stoft (2009) also observe that the regulator may intentionally cede rent to the firm so as not to unduly reduce its energy conservation activities.

devote substantial effort to the energy conservation program when the low cost environment prevails. These two objectives conflict when the regulator implements only one gain sharing plan. A single plan that the firm will accept in the high cost environment necessarily affords the firm rent in the low cost environment. Furthermore, this rent increases as the share of the realized gain the firm receives (s) increases. (Recall Lemma 1.) Therefore, a small value of s best limits the firm's rent in the low cost environment. However, a small value of s diminishes the firm's incentive to devote effort to the energy conservation program. Thus, the regulator cannot achieve both objectives with a single gain sharing plan.

The regulator can enhance her ability to pursue both objectives if she allows the firm to choose between two carefully designed gain sharing plans. One plan – the (F_2, s_2) plan that eliminates the firm's rent in the high cost environment – can award the firm a relatively small share of the realized gain and impose a relatively small penalty on the firm when meager gains are realized. Because this plan entails a relatively small s_2, it provides relatively little rent to the firm in the low cost environment. The low value of s_2 would limit the firm's energy conservation effort in the low cost environment if the firm chose the (F_2, s_2) plan in this environment. However, the firm can be induced to choose the (F_1, s_1) plan instead by setting $s_1 = 1$, thereby affording the entire realized gain to the firm. Because it awards the firm the entire realized gain, the (F_1, s_1) plan induces the firm to pursue the surplus-maximizing expected gain (G_1^*) in the low cost environment. To limit the firm's rent in this environment, the regulator can (when $\underline{F} \geq \underline{\widehat{F}}_H$) reduce F_1 to the level that ensures the firm anticipates the same profit under the (F_1, s_1) plan and the (F_2, s_2) plan in the low cost environment.[26] Thus, by introducing two distinct gain sharing plans rather than a single plan, the regulator can both limit the firm's rent and induce it to pursue the surplus-maximizing expected gain in the low cost environment.[27]

[26] As shown in the proof of Conclusion 2, this structuring of the (F_1, s_1) gain sharing plan renders it unprofitable for the firm in the high cost environment. Consequently, the firm prefers the (F_2, s_2) plan to the (F_1, s_1) plan in the high cost environment, even though the firm secures no rent under the (F_2, s_2) plan.

[27] The relatively small value of s_2 diminishes the effort the firm supplies in the high cost environment. However, a reduction in s_2 diminishes the rent that must be ceded to the firm when the low cost environment prevails. The factors that influence the optimal value of s_2 are analyzed below.

The regulator's ability to pursue these two objectives simultaneously is restricted when she is unable to impose large penalties on the firm. As Conclusion 2(ii) reports, when the regulator cannot reduce F_1 to the level that equates the firm's rent in the low cost environment under the (F_1, s_1) gain sharing plan and the regulator's preferred (F_2, s_2) plan, the regulator pursues alternative means to limit the firm's rent. In particular, she reduces the share of the realized gain awarded to the firm in the low cost environment. Under the specified conditions, this share (s_1) is reduced further below 1 as the maximum feasible penalty ($\underline{F}$) declines.

The regulator also increases s_2 as $\underline{F}$ declines. The increase in s_2 benefits consumers by inducing the firm to devote more effort to the energy conservation program in the high cost environment. The increase in s_2 (and the corresponding reduction in F_2 that leaves $\widehat{\pi}_2 = 0$) increases the rent the (F_2, s_2) plan affords the firm in the low cost environment, and so increases the rent the (F_1, s_1) plan must afford the firm to ensure the firm selects this plan in the low cost environment. However, this increased rent simply reflects the increase dictated by the binding lower bound on F_1, which limits the regulator's ability to extract rent from the firm by reducing F_1.[28]

As Conclusion 2(ii) reports, the two gain sharing plans the regulator offers become more similar as $\underline{F}$ decreases below $\widehat{\underline{F}}_H$. In particular, as $\underline{F}$ decreases, s_1 decreases toward s_2 while s_2 increases toward s_1, and F_1 increases toward F_2 as F_2 decreases toward F_1. Eventually, when the regulator's ability to impose financial penalties on the firm is sufficiently constrained (i.e., when $\underline{F} \leq \widehat{\underline{F}}_L$), the regulator optimally implements only a single gain sharing plan, even when she has the ability to offer multiple plans. Hence, implementing a single gain sharing plan when the firm is privately informed about the unmeasured cost of achieving gains from energy conservation programs can be optimal, but only when the regulator's

[28] In principle, the regulator could respond to a reduction in $\underline{F} \in (\widehat{\underline{F}}_L, \widehat{\underline{F}}_H)$ by reducing s_2 further below 1 and thereby reducing the rent the (F_2, s_2) plan affords the firm in the low cost environment. Under the conditions specified in Conclusion 2(ii), though, such a reduction in s_2 would sacrifice total surplus unduly, and so is not optimal. Notice that these conditions hold if, for example, $K(G, k) = \frac{k}{2} G^2$. This case is considered in section 7.

ability to impose financial losses on the firm is sufficiently limited.[29]

Conclusion 3 provides additional information about how the share (s_2) of the realized gain that is optimally awarded the firm in the high cost environment changes as industry parameters change.[30] The Conclusion refers to the following conditions on the firm's unmeasured cost of achieving expected gain G, given cost parameter k.[31]

Condition 1. $K_{GGk}(G,k) \geq K_{GGG}(G,k)\left[\frac{K_{Gk}(G,k)}{K_{GG}(G,k)}\right]$ for all G and k.

Condition 2. $K_{GGk}(G,k) \leq K_{GGG}(G,k)\left[\frac{K_{Gk}(G,k)}{K_{GG}(G,k)}\right]$ for all G and k.

Conclusion 3. *Suppose the regulator's objective function is a concave function of s_2. Then at the solution to [P]:*

(i) s_2 *increases as* ϕ_2 *increases or as* α *increases;*

(ii) s_2 *decreases as* k_2 *increases if Condition 1 holds; and*

(iii) s_2 *increases as* k_1 *increases if* $\underline{F} > \widehat{\underline{F}}_H$ *or if* $\underline{F} \leq \widehat{\underline{F}}_L$ *and Condition 2 holds.*

Conclusion 3(i) indicates that as the regulator becomes more certain the high cost environment prevails, she will increase s_2 in order to encourage the firm to devote more effort to the energy conservation program in this environment. The regulator also will increase s_2 when she values the firm's profit more highly, and so is less concerned about the increased rent that must be afforded the firm in the low cost environment (and in the high cost environment when $\underline{F} \leq \widehat{\underline{F}}_L$) as s_2 increases.

[29] Conclusion 2(iii) simply reflects the finding reported in Conclusion 1(iii). Once the sharing rate in the single plan the regulator offers is sufficiently small ($s = \underline{s}$), the regulator will not reduce the sharing rate further as $\underline{F}$ declines. Although such a reduction would reduce the supplier's rent, it would reduce the supplier's conservation effort unduly and thereby sacrifice excessive total expected surplus.

[30] Once the relevant changes in s_2 are identified, the corresponding changes in s_1, F_1, and F_2 are readily derived from Conclusion 2. In particular, s_1 remains at 1 as industry parameters change when $\underline{F} \geq \widehat{\underline{F}}_H$. If $\underline{F} \in (\widehat{\underline{F}}_L, \widehat{\underline{F}}_H)$, then as s_2 increases, F_2 adjusts to ensure $\widehat{\pi}_2 = 0$, F_1 remains at $-\underline{F}$, and s_1 increases to ensure the two gain sharing plans provide the same expected profit to the firm in the low cost environment. Of course, when $\underline{F} \leq \widehat{\underline{F}}_L$, $s_1 = s_2$ (and $F_1 = F_2 = -\underline{F}$) and so the changes in s_2 identified in Conclusion 3 are precisely the relevant changes in s_1.

[31] For analytic ease, the following conditions treat $K(G,k)$ a differentiable function of k. Subscripts denote the relevant partial derivatives in the conditions. Observe that Conditions 1 and 2 both hold if, for example, $K(G,k) = \frac{k}{2}G^2$. This case is considered in section 7.

An increase in k_2 has two effects. First, it increases the firm's cost advantage – and thus the rent it enjoys – in the low cost environment. Second, an increase in k_2 increases the firm's costs and thereby reduces the magnitude of the expected gain it will pursue in the high cost environment. The first effect encourages the regulator to reduce s_2 in order to limit the firm's rent in the low cost environment. The second effect could induce the regulator to increase s_2 in order to avoid an unduly large reduction in the expected gain in the high cost environment. However, the reduction is relatively limited when Condition 1 holds. Therefore, as Conclusion 3(ii) reports, the first effect outweighs the second in this case, and so the regulator reduces s_2.

An increase in k_1 reduces the firm's cost advantage in the low cost environment. As Conclusion 3(iii) indicates, the reduced cost advantage and the associated reduction in rent the firm secures under the (F_2, s_2) gain sharing plan ensure the regulator will increase s_2 when her ability to penalize the firm is not constrained (i.e., when $\underline{F} > \underline{\widehat{F}}_H$). The regulator will also increase s_2 as k_1 increases when Condition 2 holds and her ability to penalize the firm is so constrained that she optimally implements a single gain sharing plan (i.e., when $\underline{F} \leq \underline{\widehat{F}}_L$). When Condition 2 holds, the firm reduces the expected gain it pursues in the low cost environment relatively rapidly as k_1 increases. An increase in $s_1 = s_2$ helps to counteract this tendency.[32]

7 The Quadratic Setting

Conclusions 2 and 3 identify the key general features of optimal gain sharing programs and explain how these features change as industry conditions change. Additional details of optimal gain sharing programs can be derived if more information about the supplier's unmeasured costs, $K(\cdot)$, is available. To derive this additional detail in a case of potential interest, consider the *quadratic setting* in which $K(G, k_i) = \frac{1}{2} k_i G^2$.

[32] When $\underline{F} \in (\underline{\widehat{F}}_L, \underline{\widehat{F}}_H)$, the binding $F_1 \geq -\underline{F}$ constraint at the solution to [P] forces the regulator to set s_1 lower and s_2 higher than she would in the absence of the constraint. Consequently, an increase in k_1 that relaxes constraint (3) in [P] can encourage the regulator to increase s_1 and reduce s_2. This additional consideration renders the impact of an increase in k_1 on s_2 ambiguous when $\underline{F} \in (\underline{\widehat{F}}_L, \underline{\widehat{F}}_H)$.

Table 1 characterizes the optimal gain sharing plans and the regulator's expected payoff in the quadratic setting when the two possible cost environments are equally likely (so $\phi_1 = \phi_2 = \frac{1}{2}$), the regulator only values consumer welfare (so $\alpha = 0$), and lower bounds on payments to the supplier are not constraining (so $\underline{F} \geq \max\{\underline{F}_H, \widehat{\underline{F}}_H\}$). The cost parameter k_2 is set equal to 1 and k_1 varies between 0.1 and 0.9 in the setting of Table 1.

k_1	s	s_1	s_2	$W_{[P-1]}$	$W_{[P]}$	$\frac{W_{[P]}}{W_{[P-1]}}$
0.1	0.55	1.0	0.1	1.513	2.525	1.669
0.2	0.60	1.0	0.2	0.900	1.300	1.444
0.3	0.65	1.0	0.3	0.704	0.908	1.290
0.4	0.70	1.0	0.4	0.613	0.725	1.184
0.5	0.75	1.0	0.5	0.563	0.625	1.111
0.6	0.80	1.0	0.6	0.533	0.567	1.063
0.7	0.85	1.0	0.7	0.516	0.532	1.031
0.8	0.90	1.0	0.8	0.506	0.513	1.012
0.9	0.95	1.0	0.9	0.501	0.503	1.003

Table 1. Optimal Sharing Rates and Expected Welfare in the Quadratic Setting ($k_2 = 1$, $\phi_1 = \phi_2 = \frac{1}{2}$, $\alpha = 0$, and $\underline{F} \geq \max\{\underline{F}_H, \widehat{\underline{F}}_H\}$).

The second column in Table 1 presents the optimal sharing rate, s, at the solution to [P-1], i.e., when the regulator offers at most one gain sharing plan. The fifth column lists the regulator's expected welfare ($W_{[P-1]}$) in this case.[33] The third and fourth columns in Table 1 provide the sharing rates, s_1 and s_2, that are optimally implemented in the low cost environment and the high cost environment, respectively, at the solution to [P], i.e., when the regulator can offer the supplier a choice between gain sharing plans. The sixth column in

[33] From (5), $W_{[P-1]} = \sum_{i=1}^{2} \phi_i \{ [1-s]\, G_i - F \}$ at the solution to [P-1].

the table presents the regulator's expected welfare ($W_{[P]}$) in this case.[34] The last column in the table provides the ratio of $W_{[P]}$ to $W_{[P-1]}$. This ratio reflects the proportionate increase in expected welfare the regulator can achieve by allowing the supplier to choose between gain sharing plans.

Table 1 illustrates the more general conclusion that the regulator often can secure a substantial increase in expected welfare if she affords the supplier a choice between gain sharing plans. For example, when $k_2 = 1$ and $k_1 = 0.1$, so the supplier's cost of securing energy conservation gains is substantially higher in the high cost environment than in the low cost environment, the regulator can increase her expected welfare by more than two-thirds (66.9%) if she allows the supplier to choose between optimally designed gain sharing plans. This choice allows the regulator to maximize the total expected surplus in the low cost environment without affording excessive rent to the supplier in this environment.[35]

The increase in expected welfare the regulator secures by offering two distinct gain sharing plans declines as her ability to impose penalties on the supplier declines. Recall from Conclusion 2 that as $\underline{F}$ declines between $\widehat{\underline{F}}_H$ and $\widehat{\underline{F}}_L$, the gain sharing plans the regulator optimally presents to the supplier become more similar. Figure 1 illustrates this finding in the quadratic setting when $k_2 = 1$, $k_1 = 0.5$, $\phi_1 = \phi_2 = \frac{1}{2}$, and $\alpha = 0$. Figure 1(a) depicts how the supplier's profit ($F_i + s_i\, g$) varies with the realized gain (g) under the optimal pair of gain sharing plans when $\underline{F} \geq \widehat{\underline{F}}_H = 0.875$, so the bound on feasible payments to the supplier is not constraining. The slopes of the linear segments in Figure 1 reflect the relevant sharing rates (s_i). Figure 1(b) illustrates how the (F_1, s_1) and (F_2, s_2) plans become more similar when $\underline{F}$ declines to 0.5, so the supplier cannot be penalized as heavily as the regulator would like when the supplier fails to achieve the target gain in the low cost environment. Figure 1(c) reflects the finding in Conclusion 2 that when $\underline{F} \leq \widehat{\underline{F}}_L = 0.125$, the regulator

[34] From (1), $W_{[P]} = \sum_{i=1}^{2} \phi_i \{ [1 - s_i]\, G_i - F_i \}$ at the solution to [P].

[35] As Table 1 reveals, the increase in expected welfare the regulator secures by offering the supplier a choice between gain sharing plans is more pronounced the more disinct are the cost environments in which the supplier might be operating (i.e., the larger is the difference between k_2 and k_1).

optimally offers the supplier only a single gain sharing plan, so the opportunity to offer multiple plans is of no value to the regulator.

Figure 2 illustrates the impact of finer variations in $\underline{F}$ on the optimal sharing rates in this same setting. When $\underline{F}$ exceeds $\widehat{\underline{F}}_H = 0.875$, the regulator optimally induces the supplier to choose a gain sharing plan with sharing rate $s_1 = 1$ in the low cost environment and a plan with sharing rate $s_2 = 0.50$ in the high cost environment. As $\underline{F}$ declines below 0.875, the regulator's more limited ability to penalize the supplier for poor performance causes her to offer plans with more similar sharing rates (i.e., s_1 declines below 1 and s_2 increases above .50). When $\underline{F}$ is sufficiently small (i.e., when $\underline{F} \leq \widehat{\underline{F}}_L = 0.222$), the regulator optimally implements only one gain sharing plan. The sharing rate in this plan declines from 0.66 to 0.50 as $\underline{F}$ declines from $\widehat{\underline{F}}_L = 0.222$ to $\underline{F}_L = 0.125$. The optimal sharing rate does not decline below 0.50 as $\underline{F}$ declines below $\underline{F}_L = 0.125$. Although a lower sharing rate would reduce the firm's rent in the low cost environment, it would reduce unduly the supplier's energy conservation activity and thus total expected surplus. (Recall Conclusion 2(iii).)

8 Conclusions

We have developed a streamlined economic model of energy conservation activity and employed the model to identify the properties of an optimal gain sharing program. We have demonstrated the importance of: (i) allowing the supplier to choose a plan from a carefully structured set of plans; (ii) ensuring that the supplier faces the prospect of substantial financial reward for exceptional performance and considerable financial penalty for substandard performance; and (iii) tailoring gain sharing plans to the prevailing industry conditions and to the information available to the regulator.

Our research represents a first step in deriving from formal economic models the properties of optimal gain sharing plans. Additional research is required to provide comprehensive, practical guidance to regulators as they continue to design and implement gain sharing plans. In closing, we mention several extensions of our analysis that await further research.

First, alternative representations of the supplier's private information should be considered. In practice, this private information may not be reasonably characterized as the realization of a binary random variable. When the supplier's private information is not binary, it typically will be optimal to allow the supplier to choose one gain sharing plan from a richer set of plans. However, the basic considerations and trade-offs identified above will persist more generally.

Second, multi-dimensional information asymmetry merits explicit consideration. We have analyzed a setting in which the supplier is privately informed about its cost of delivering energy conservation effort, but the net benefit the effort produces is readily measured. In practice, key components of the net benefit of energy conservation may not be easily measured. To illustrate, Blank and Gegax (2011) note the difficulties of measuring an energy supplier's opportunity cost of reducing consumer demand for energy.[36] If the net benefit of an energy conservation program is systematically under-estimated or over-estimated, then the policies derived above will induce too little or too much energy conservation activity. The optimal manner in which to modify these policies to reflect relevant information asymmetries about program benefits or costs merits further study.[37]

Third, the interaction between measured and unmeasured inputs in the energy conservation process merits careful study. For simplicity, our formal analysis considered a setting in which the supplier's observed costs of promoting energy conservation (e.g., rebates delivered to customers who purchase new energy efficient appliances) did not affect the supplier's unobserved cost of securing energy conservation (e.g., the effort cost associated with diligent oversight and management of the conservation program). In some settings, observed energy conservation expenditures (e.g., wages paid to home energy auditors) may reduce

[36] The difficulty in measuring the impact of an energy conservation program on consumer welfare should also be noted in this regard. Welfare gains are not necessarily proportional to reductions in energy consumption or energy expenditures, for example. As Brennan (2011) observes, energy conservation activities will cause consumer demand for energy to increase at certain price levels.

[37] Stoft and Gilbert (1994, p. 10) observe that the net benefits derived from "energy-management service programs that are informational in nature" can be difficult to measure. Consequently, rather than link financial rewards to measured net benefits, regulators sometimes set rewards to reflect the supplier's observed expenditures on the program.

the unobserved effort the supplier must deliver to achieve energy conservation gains. Less than full reimbursement of observed expenditures may be optimal in order to limit excessive expenditures in such cases. Alternatively, or in addition, the supplier's incremental reward for realized gains may optimally be reduced as reimbursement of its observed expenditures increases.

Fourth, the optimal use of additional policy instruments should be considered. To illustrate, consider the setting in which the energy supplier also delivers energy conservation services. In this setting, the regulator might adjust the marginal price of energy to influence the supplier's energy conservation activity.[38] The potential benefit of limiting the extent to which the firm's revenue from energy sales declines as realized energy consumption declines has been noted in this regard (Brennan, 2010). It would be useful to determine the optimal extent of such "decoupling" of revenues and energy consumption and to analyze how such energy pricing policies are optimally combined with gain sharing programs.[39]

Fifth, more general gain sharing programs merit analysis. The supplier's financial reward need not vary linearly with realized net benefits in a gain sharing plan. Nonlinear reward structures can be particularly valuable when the regulator and/or the energy supplier are averse to risk. When the supplier is averse to risk or when large financial rewards to the supplier would create political difficulties for the regulator, the supplier's incremental reward for improved performance typically will decline as the realized performance improves under an optimal gain sharing plan.[40]

We note in closing that different regulatory objectives also merit consideration. For example, a regulator might value particularly highly gains that accrue to low-income customers. To encourage such gains, the energy supplier might be directed to undertake conservation

[38] In settings where consumers can undertake unobserved energy conservation activities, energy prices might also be structured to influence these activities.

[39] Lewis and Sappington (1992) examine how energy prices are optimally employed to motivate energy conservation in a setting where the firm has superior knowledge of consumer demand for energy but the regulator is fully informed about the firm's operating costs and there are no limits on feasible payments to the firm.

[40] Stoft et al. (1995, pp. 37-41) discuss how risk aversion affects the design of gain sharing plans.

activities that are specifically designed to produce gains for low-income customers (e.g., complementary home energy audits for these customers). Alternatively, or in addition, the supplier might be promised a relatively large reward for securing gains (e.g., bill reductions) for low-income customers.

Appendix

Proof of Observation 1.

It is apparent from (4) that the firm will implement expected gain G_i^* when $s = 1$. The firm's expected profit will be $F + G_i^* - K(G_i^*, k_i) = 0$ when $F = -\{G_i^* - K(G_i^*, k_i)\}$. This gain sharing plan is feasible under the maintained assumptions. Because the plan maximizes the total expected surplus $(G - K(G, k_i))$ and eliminates the firm's rent, the plan secures for the regulator the same expected payoff she achieves in the full information setting. ■

Proof of Observation 2.

Let [P-k] denote the regulator's problem when she knows the prevailing cost parameter is k. This problem is:

$$\underset{s,\, F \geq -\underline{F}}{\text{Maximize}} \quad -F + [1-s]\, G + \alpha \{ F + s\, G - K(G,k) \}$$

$$\text{subject to:} \quad F + s\, G - K(G,k) \geq 0\,, \tag{8}$$

$$\text{where} \quad K_G(G,k) = s\,. \tag{9}$$

Let λ denote the Lagrange multiplier associated with constraint (8), and let $\underline{\lambda}$ denote the Lagrange multiplier associated with the $F \geq -\underline{F}$ constraint. Then the necessary conditions for a solution to [P-k] are:

$$s: \quad G\,[-1 + \alpha + \lambda] + [1-s]\,\frac{dG}{ds} = 0\,; \quad \text{and} \tag{10}$$

$$F: \quad -1 + \alpha + \lambda + \underline{\lambda} = 0\,. \tag{11}$$

From (9):

$$\frac{dG}{ds} = \frac{1}{K_{GG}(G,k)} > 0. \tag{12}$$

It is readily verified that the $F \geq -\underline{F}$ constraint binds at the solution to [P-k] when $\underline{F} < G_i^* - K(G_i^*, k_i)$. Consequently, $\underline{\lambda} > 0$. Therefore, from (10), (11), and (12):

$$[1-s]\,\frac{dG}{ds} = G\,\underline{\lambda} > 0 \quad \Rightarrow \quad s < 1\,.$$

If $\underline{F}$ is sufficiently small that constraint (8) does not bind at the solution to [P-k], then $\lambda = 0$, and so $\underline{\lambda} = 1 - \alpha > 0$, from (11). Consequently, $F = \underline{F}$. Furthermore, from (10) and (12):

$$\frac{1-s}{K_{GG}(G,k)} = [1-\alpha]\,G \quad \Rightarrow \quad 1 - s = [1-\alpha]\,G\,K_{GG}(G,k)$$

$$\Rightarrow \quad s = 1 - [1-\alpha]\,G\,K_{GG}(G,k) \equiv \widetilde{s}\,.$$

Let $\underline{F}^o$ denote the largest value of $\underline{F}$ for which constraint (8) does not bind at the solution to [P-k]. Then as $\underline{F}$ increases from $\underline{F}^o$ to $\underline{F}^* \equiv G_i^* - K(G_i^*, k_i)$, s increases monotonically from $\widetilde{s}$ to 1. This is the case because the $F \geq -\underline{F}$ constraint and constraint (8) both bind at the solution to [P-k] for all $\underline{F} \in (\underline{F}^o, \underline{F}^*)$. Therefore:

$$-\underline{F} + s\,G - K(G,k) \;=\; 0\,. \tag{13}$$

Differentiating (13) and using (9) provides:

$$-d\,\underline{F} + \left\{ G + [\, s - K_G(G,k)\,] \frac{dG}{ds} \right\} ds \;=\; 0$$

$$\Rightarrow \quad -\,d\,\underline{F} + G\; ds \;=\; 0 \quad \Rightarrow \quad \frac{ds}{d\,\underline{F}} \;=\; \frac{1}{G} \;>\; 0\,. \quad \blacksquare$$

Proof of Lemma 1.

$$\Delta\pi(F,s) \;=\; \max_G \{\, F + s\,G - K(G,k_1)\,\} \;-\; \max_G \{\, F + s\,G - K(G,k_2)\,\}\,. \tag{14}$$

(14) and the envelope theorem imply:

$$\frac{d\Delta\pi(F,s)}{ds} \;=\; G_1(s) - G_2(s) \;>\; 0\,, \quad \text{where} \;\; G_i(s) \;=\; \max_G \{\, s\,G - K(G,k_i)\,\}\,. \tag{15}$$

The inequality in (15) holds because $K_G(G_1(s),k_1) \;=\; s \;=\; K_G(G_2(s),k_2)$, $K_{GG}(G,k_i) > 0$ for $i = 1,2$, and $K_G(G,k_2) > K_G(G,k_1)$ for all $G > 0$. $\blacksquare$

Proof of Conclusion 1.

Let λ_i denote the Lagrange multiplier associated with constraint (6), and let $\underline{\lambda}$ denote the Lagrange multiplier associated with the $F \geq -\underline{F}$ constraint. Then the necessary conditions for a solution to [P-1] include:

$$s: \qquad \sum_{i=1}^{2} G_i\,[-\phi_i\,(1-\alpha) + \lambda_i] + \sum_{i=1}^{2} \phi_i\,[1-s]\,\frac{dG_i}{ds} \;=\; 0\,; \quad \text{and} \tag{16}$$

$$F: \qquad -\,1 + \alpha + \lambda_1 + \lambda_2 + \underline{\lambda} \;=\; 0\,. \tag{17}$$

From (7):

$$s \;=\; K_G(G_i,k_i) \quad \Rightarrow \quad \frac{dG_i}{ds} \;=\; \frac{1}{K_{GG}(G_i,k_i)} \;>\; 0\,. \tag{18}$$

Since $K(G,k_2) \;>\; K(G,k_1)$ for all $G \;>\; 0$, constraint (6) does not bind for $i \;=\; 1$. Therefore, $\lambda_1 \;= 0$ at the solution to [P-1]. Consequently, (17) provides:

$$\lambda_2 \;=\; 1 - \alpha - \underline{\lambda}\,. \tag{19}$$

Define problem [P-1]$'$ to be problem [P-1] without the participation constraints (6) imposed. (19) implies that $\underline{\lambda} \;=\; 1-\alpha \;>\; 0$ at the solution to [P-1]$'$, and so $F \;=\; -\,\underline{F}$. Furthermore, from (16):

$$[1-s]\sum_{i=1}^{2}\phi_i\,\frac{dG_i}{ds} \;=\; \phi_1\,[1-\alpha]\,G_1+\phi_2\,[1-\alpha]\,G_2 \tag{20}$$

Let $\underline{s}$ denote the value of s that solves (20). Then $(-\underline{F},\underline{s})$ is the solution to $[\text{P-1}]'$.

Define $\underline{F}_L$ to be the largest value of $\underline{F}$ for which no participation constraint binds at the solution to [P-1] (so $\underline{F}_L \;=\; \max\limits_G\,\{\,\underline{s}\,G-K(G,k_2)\}\,)$. Observe that if $\underline{F}\;\leq\;\underline{F}_L$, then $(-\underline{F},\underline{s})$, the solution to $[\text{P-1}]'$, is a feasible solution to [P-1], and so is the solution to [P-1]. Note from (20) that $\frac{d\underline{s}}{d\underline{F}}\;=\;0$ when $\underline{F}\;<\;\underline{F}_L$.

Now define problem $[\text{P-1}]''$ to be problem [P-1] without the $F\;\geq-\underline{F}$ constraint imposed. (19) implies that $\lambda_2\;=\;1-\alpha\;>\;0$ at the solution to $[\text{P-1}]''$, and so $\pi_2=0$. Furthermore, from (16):

$$-\,\phi_1\,[1-\alpha]\,G_1+[1-\alpha]\,G_2\,[1-\phi_2]+[1-s]\sum_{i=1}^{2}\phi_i\,\frac{dG_i}{ds}\;=\;0$$

$$\Leftrightarrow\quad [1-s]\sum_{i=1}^{2}\phi_i\,\frac{dG_i}{ds}\;=\;\phi_1\,[1-\alpha]\,[G_1-G_2]\;>\;0\,. \tag{21}$$

The inequality in (21) holds because $G_1\;>\;G_2$ from (7), since $K_G(G,k_2)\;>\;K_G(G,k_1)$. Since $\frac{dG_i}{ds}>0$ for $i=1,2$ from (18), (21) implies that $s\;<\;1$. Let $\overline{s}$ denote the value of s that solves the equality in (21).

Define $\underline{F}_H$ to be the smallest value of $\underline{F}$ for which the solution to $[\text{P-1}]''$ is a feasible solution (and thus the solution) to [P-1].

It remains to show that $\underline{s}<\overline{s}$, and so $\underline{F}_L<\underline{F}_H$, since:

$$-\,\underline{F}_L+\max_G\,\{\underline{s}\,G-K(G,k_2)\}\;=\;0\;=\;-\,\underline{F}_H+\max_G\,\{\,\overline{s}\,G-K(G,k_2)\}$$

$$\Rightarrow\quad \underline{F}_L\;=\;\underline{F}_H+\max_G\,\{\underline{s}\,G-K(G,k_2)\}-\max_G\,\{\overline{s}\,G-K(G,k_2)\}<\underline{F}_H\quad\text{when }\underline{s}\;<\;\overline{s}\,.$$

First observe from (20) and (21) that $\overline{s}\neq\underline{s}$. Now suppose that $\underline{s}>\overline{s}$, and so $\underline{F}_L>\underline{F}_H$. Consider two values of $\underline{F}$, namely $\underline{F}_1$ and $\underline{F}_2$, such that $\underline{F}_1\neq\underline{F}_2$ and $\underline{F}_1,\,\underline{F}_2\in(\underline{F}_H,\underline{F}_L)$. If $\underline{F}=\underline{F}_i$ for $i=1$ or $i=2$, then $(-\underline{F}_H,\overline{s})$, the solution to $[\text{P-1}]'$, remains a feasible solution to [P-1] since $\underline{F}_i>\underline{F}_H$. Hence, $(-\underline{F}_H,\overline{s})$ is a solution to [P-1].

Furthermore, $(-\underline{F}_i,\underline{s})$, the solution to $[\text{P-1}]''$ when $\underline{F}=\underline{F}_i$, remains a feasible solution to [P-1] since $\underline{F}_i<\underline{F}_L$. Hence, $(-\underline{F}_i,\underline{s})$ is a solution to [P-1]. Therefore, the regulator is indifferent between the $(-\underline{F}_H,\overline{s})$ and the $(-\underline{F}_i,\underline{s})$ plans for $i=1$ and $i=2$. Consequently, the regulator must be indifferent between the $(-\underline{F}_1,\underline{s})$ plan and the $(-\underline{F}_2,\underline{s})$ plan. However, the regulator strictly prefers the $(-\underline{F}_2,\underline{s})$ plan to the $(-\underline{F}_1,\underline{s})$ plan because the former provides systematically less compensation for the firm and the two plans generate the same total expected surplus. Therefore, by contradiction, it must be the case that $\underline{s}<\overline{s}$, and so $\underline{F}_L<\underline{F}_H$.

Three possibilities arise at the solution to [P-1]: (i) the participation constraint (6) when $k=k_2$ is the unique binding constraint; (ii) the $F\geq-\underline{F}$ constraint is the unique binding constraint; or (iii) both constraints bind. We have shown that possibility (i) arises if and only if $\underline{F} \geq \underline{F}_H$. We have also shown that possibility (ii) arises if and only if $\underline{F} \leq \underline{F}_L$. Therefore, possibility (iii) arises if and only if $\underline{F} \in (\underline{F}_L, \underline{F}_H)$. In this case, $F = -\underline{F}$ and:

$$-\underline{F} + s\,G_2 - K(G_2, k_2) \;=\; 0 \quad\Rightarrow\quad -\,d\underline{F} + G_2\,ds \;=\; 0 \quad\Rightarrow\quad \frac{ds}{d\underline{F}} \;=\; \frac{1}{G_2} \;>\; 0\,. \quad \blacksquare$$

Proof of Conclusion 2.

Let λ_i and λ_{ij} denote the Lagrange multipliers associated with constraints (2) and (3), respectively. Also let $\underline{\lambda}_i$ denote the Lagrange multiplier associated with the $F_i \geq -\underline{F}$ constraint. Then the necessary conditions for a solution to [P] include:

$$s_i: \quad G_i\left[-\phi_i\,(1-\alpha) + \lambda_i + \lambda_{ij}\right] - \lambda_{ji}\,G_{ij} + \phi_i\,[1-s_i]\,\frac{dG_i}{ds_i} \;=\; 0\,; \text{ and} \tag{22}$$

$$F_i: \quad -\,\phi_i\,[1-\alpha] + \lambda_i + \lambda_{ij} - \lambda_{ji} + \underline{\lambda}_i \;=\; 0\,. \tag{23}$$

(22) and (23) provide:

$$\phi_i\,[1-s_i]\,\frac{dG_i}{ds_i} \;=\; \lambda_{ji}\,[G_{ij} - G_i] + \underline{\lambda}_i\,G_i \quad \text{for } j \neq i, \quad i,j \in \{1,2\}\,. \tag{24}$$

From (4):

$$K_G(G_i, k_i) \;=\; s_i \text{ and } K_G(G_{ij}, k_j) \;=\; s_i \quad\Rightarrow\quad G_{21} \geq G_2 \text{ and } G_1 \geq G_{12}\,. \tag{25}$$

The inequalities in (25) hold because $K_G(G, k_1) < K_G(G, k_2)$ and $K(\cdot)$ is an increasing, convex function of G. The inequalities in (25) hold as strict inequalities if a positive expected gain is induced when $k = k_1$.

The following lemmas constitute the remainder of the proof of the Conclusion.

Lemma 2. *The participation constraint (2) when $k = k_1$ does not bind at the solution to [P].*

Proof. The conclusion holds because the firm's expected profit is strictly higher when $k = k_1$ than when $k = k_2$ under any non-trivial gain sharing plan.[41] $\square$

Lemma 3. $G_1 > G_2$, $F_2 \geq F_1$, *and* $s_2 \leq s_1$ *under any feasible solution to [P] that entails a non-trivial gain sharing plan.*

Proof. To show that $G_1 > G_2$, observe that the incentive compatibility constraints (3) ensure:

$$\pi_1(s_1, F_1) - \pi_1(s_2, F_2) \;\geq\; 0 \;\geq\; \pi_2(s_1, F_1) - \pi_2(s_2, F_2)$$

[41] A non-trivial gain sharing plan (F, s) is one: (i) that the firm selects either when $k = k_1$ or when $k = k_2$; and (ii) in which the firm implements a strictly positive expected gain ($G > 0$) when it operates under the plan.

$$\Rightarrow \quad \pi_1(s_1, F_1) + \pi_2(s_2, F_2) \;\geq\; \pi_2(s_1, F_1) + \pi_1(s_2, F_2)\,. \tag{26}$$

Further observe that:

$$\pi_1(s_1, F_1) + \pi_2(s_2, F_2) \;=\; F_1 + s_1\, G_1 - K(G_1, k_1) \;+\; F_2 + s_2\, G_2 - K(G_2, k_2)\,; \text{ and} \tag{27}$$

$$\pi_2(s_1, F_1) + \pi_1(s_2, F_2) \;\geq\; F_1 + s_1\, G_1 - K(G_1, k_2) \;+\; F_2 + s_2\, G_2 - K(G_2, k_1)\,. \tag{28}$$

The inequality in (28) holds because G_i is not necessarily the profit-maximizing expected gain under the (s_i, F_i) gain sharing plan when $k = k_j$ for $j \neq i$. (26), (27), and (28) provide:

$$\begin{aligned} 0 \;&\leq\; \pi_1(s_1, F_1) + \pi_2(s_2, F_2) - [\pi_2(s_1, F_1) + \pi_1(s_2, F_2)] \\ &\leq\; K(G_1, k_2) - K(G_2, k_2) - [K(G_1, k_1) - K(G_2, k_1)] \qquad (29) \\ &=\; \int_{G_2}^{G_1} \left[\frac{\partial}{\partial G} K(G, k_2) - \frac{\partial}{\partial G} K(G, k_1) \right] dG \quad \Rightarrow \quad G_1 \;>\; G_2\,. \end{aligned}$$

To show that $s_1 \;\geq\; s_2$, observe that:

$$\pi_2(s_1, F_1) + \pi_1(s_2, F_2) \;\geq\; F_1 + s_1\, G_2 - K(G_2, k_2) + F_2 + s_2\, G_1 - K(G_1, k_2)\,. \tag{30}$$

The inequality in (30) holds because G_j is not necessarily the profit-maximizing expected gain under the (s_i, F_i) gain sharing plan when $k = k_j$ for $j \neq i$. (26), (27), and (30) provide:

$$0 \;\leq\; \pi_1(s_1, F_1) + \pi_2(s_2, F_2) - [\pi_2(s_1, F_1) + \pi_1(s_2, F_2)] \;\leq\; [G_1 - G_2][s_1 - s_2]. \tag{31}$$

(31) implies that $s_1 \geq s_2$, since $G_1 > G_2$. Therefore, because incentive compatibility ensures it cannot be the case that $F_1 > F_2$ and $s_1 > s_2$, it must be the case that $F_2 \;\geq\; F_1$. □

Lemma 4. *The $F_2 \geq -\underline{F}$ limited liability constraint does not bind at the solution to [P].*

Proof. From Lemma 3, $F_2 \;\geq\; F_1$ under any feasible nontrivial gain sharing plan. Consequently, the $F_2 \geq -\underline{F}$ limited liability constraint will be satisfied at the solution to [P] as long as the $F_1 \geq -\underline{F}$ constraint is imposed. Therefore, the $F_2 \geq -\underline{F}$ limited liability constraint does not bind at the solution to [P]. □

Lemmas 2 and 4 imply that $\lambda_1 = 0$ and $\underline{\lambda}_2 = 0$ at the solution to [P].

Lemma 5. *When the regulator offers two distinct, non-trivial gain sharing plans to the firm, the firm cannot be indifferent between the two plans both when $k = k_1$ and when $k = k_2$.*

Proof.

$$\frac{\partial}{\partial s} \left\{ \max_{G_i} \; [\, s\, G_i - K(G_i, k_1)] \;-\; \max_{G_i} \; [\, s\, G_i - K(G_i, k_2)] \right\} \;=\; G_{i1} - G_{i2} \;\geq\; 0\,. \tag{32}$$

The inequality in (32), which follows from (25), implies that:

$$\max_G \{ s_1 G - K(G, k_1) \} \;-\; \max_G \{ s_1 G - K(G, k_2) \}$$
$$\geq \max_G \{ s_2 G - K(G, k_1) \} \;-\; \max_G \{ s_2 G - K(G, k_2) \} . \qquad (33)$$

When the firm is indifferent between the two plans both when $k = k_1$ and when $k = k_2$, the weak inequality in (33) will hold as an equality. Consequently, it must be the case that a zero expected gain ($G = 0$) is induced under both plans. But then the plans are not distinct, non-trivial plans. Therefore, when the regulator offers two distinct, non-trivial gain sharing plans to the firm, only one of the incentive compatibility constraints will bind. □

Lemma 6. *If neither participation constraint (2) binds at the solution to [P], then the regulator optimally offers only a single gain sharing plan.*

Proof. If neither participation constraint binds at the solution to [P], then $\lambda_1 = \lambda_2 = 0$. Consequently, from (23):

$$\underline{\lambda}_1 \;=\; [1-\alpha]\,\phi_1 + \lambda_{21} - \lambda_{12} \quad \text{and} \quad \underline{\lambda}_2 \;=\; [1-\alpha]\,\phi_2 + \lambda_{12} - \lambda_{21} \,. \qquad (34)$$

Since $\underline{\lambda}_2 = 0$ from Lemma 4, (34) implies that $\lambda_{21} > 0$. (34) also implies that $\underline{\lambda}_1 = \underline{\lambda}_1 + \underline{\lambda}_2 = 1 - \alpha > 0$. Therefore, $F_1 = -\underline{F}$.

(24) and (25) imply:

$$\phi_2\,[1 - s_2]\,\frac{dG_2}{ds_2} \;=\; \lambda_{12}\,[G_{21} - G_2] + \underline{\lambda}_2\,G_2$$
$$\Rightarrow \quad s_2 \;<\; 1 \text{ if and only if } \underline{\lambda}_2 > 0 \text{ or } \lambda_{12} > 0 \,. \qquad (35)$$

Lemma 5 implies that $\lambda_{12} = 0$, since $\lambda_{21} > 0$. Consequently, $s_2 = 1$, from (35). But then it cannot be optimal for the regulator to offer two distinct gain sharing plans because the single (F_2, s_2) plan would deliver no more rent to the firm and would generate a higher level of expected total surplus. □

Lemma 7. *Suppose $\underline{F}$ is sufficiently large that the $F_i \geq -\underline{F}$ constraints do not bind at the solution to [P]. Then $s_2 < s_1 = 1$, $\lambda_2 > 0$, and $\lambda_{12} > 0$ at the solution to [P].*

Proof. Since $\underline{\lambda}_1 = \underline{\lambda}_2 = 0$ in this case, (23) and Lemma 2 imply that $\lambda_2 = \lambda_1 + \lambda_2 = 1 - \alpha > 0$. (23) also implies that $\lambda_{12} = \lambda_{21} + [1-\alpha]\,\phi_1 > 0$. Therefore, $\lambda_{21} = 0$, from Lemma 5. Consequently, from (24):

$$\phi_1\,[1 - s_1]\,\frac{dG_1}{ds_1} \;=\; 0 \quad \Rightarrow \quad s_1 \;=\; 1 \,.$$

(24) implies that when $\underline{\lambda}_2 = 0$:

$$\phi_2\,[1 - s_2]\,\frac{dG_2}{ds_2} \;=\; \lambda_{12}\,[G_{21} - G_2] \;>\; 0 \quad \Rightarrow \quad s_2 \;<\; 1 \,. \qquad (36)$$

The first inequality in (36) reflects (25). □

Lemma 8. *Suppose the participation constraint (2) when $k = k_2$ and the $F_1 \geq -\underline{F}$ limited liability constraint both bind at the solution to [P]. Then $s_2 \leq s_1 < 1$.*

Proof. Since $\underline{\lambda}_1 > 0$ in this case, (24) implies:

$$\phi_1[1 - s_1] \frac{dG_1}{ds_1} > 0 \quad \Rightarrow \quad s_1 < 1 .$$

Furthermore, $s_2 \leq s_1$ from Lemma 3. Therefore, since $\underline{\lambda}_2 > 0$ from Lemma 4, (24) implies that $\lambda_{12} > 0$. □

Define $\widehat{\underline{F}}_H$ to be the smallest value of $\underline{F}$ for which the $F_1 \geq -\underline{F}$ constraint does not bind at the solution to [P]. Then Lemma 7 implies that when $\underline{F} \geq \widehat{\underline{F}}_H$, $s_2 < s_1 = 1$, $\widehat{\pi}_2 = 0$, $F_2 > F_1 = -\widehat{\underline{F}}_H$, and the firm secures the same expected profit under the two gain sharing plans in the low cost environment at the solution to [P].

Recall that $\underline{F}_L = \max_G \{ \underline{s}\, G - K(G, k_2)\}$ is the largest value of $\underline{F}$ for which no participation constraint binds at the solution to [P-1]. Lemma 6 implies that the solution to [P] is the solution to [P-1] when $\underline{F} \leq \underline{F}_L$. Therefore, from the proof of Conclusion 1, $s_1 = s_2 = \underline{s}$, $F_1 = F_2 = -\underline{F}$, $\frac{d\underline{s}}{d\underline{F}} = 0$, and $\widehat{\pi}_2 > 0$ at the solution to [P] when $\underline{F} \leq \underline{F}_L$.

The definition of $\underline{F}_L$ and Lemma 2 imply that $\widehat{\pi}_2 = 0$ at the solution to [P] when $\underline{F} > \underline{F}_L$. Furthermore, if the $F_1 \geq -\underline{F}$ constraint binds and $s_1 = s_2 = \widehat{\underline{s}}$ at the solution to [P], it must be the case that $\frac{d\widehat{\underline{s}}}{d\underline{F}} > 0$ (to ensure $\widehat{\pi}_2 = 0$) when $\underline{F} > \underline{F}_L$.

Lemma 9. $\widehat{\underline{F}}_L < \widehat{\underline{F}}_H$.

Proof. We first show that $\widehat{\underline{F}}_L \neq \widehat{\underline{F}}_H$. To do so, suppose $\widehat{\underline{F}}_L = \widehat{\underline{F}}_H$. Lemma 6 and (34) imply that $(-\widehat{\underline{F}}_L, \underline{s})$ is the optimal plan when $\underline{F} = \widehat{\underline{F}}_L = \widehat{\underline{F}}_H$. Furthermore, $\widehat{\underline{s}} < 1$ and $\widehat{\pi}_2 = 0$ under this plan. Lemma 7 implies that the $\{(F_2, s_2), (-\widehat{\underline{F}}_H, 1)\}$ gain sharing program is also optimal and $\widehat{\pi}_2 = 0$ under this program. Notice that the firm strictly prefers the $(-\widehat{\underline{F}}_H, 1)$ plan to the $(-\widehat{\underline{F}}_L, \underline{s})$ plan because $\widehat{\underline{F}}_L = \widehat{\underline{F}}_H$ and $\widehat{\underline{s}} < 1$. Therefore, it cannot be the case that $\widehat{\pi}_2 = 0$ under both plans. Hence, by contradiction, $\widehat{\underline{F}}_L \neq \widehat{\underline{F}}_H$.

Now suppose $\widehat{\underline{F}}_L > \widehat{\underline{F}}_H$, and consider a value of $\underline{F} \in (\widehat{\underline{F}}_H, \widehat{\underline{F}}_L)$. Since $\underline{F} > \widehat{\underline{F}}_H$, the $\{(F_2, s_2), (-\widehat{\underline{F}}_H, 1)\}$ gain sharing program identified in Lemma 7 is a solution to [P]. Since $\underline{F} < \widehat{\underline{F}}_L$, the $(-\underline{F}, \widehat{\underline{s}})$ gain sharing plan identified in Lemma 6 is also a solution to [P]. As $\underline{F}$ increases in this range, the regulator's expected payoff increases under the $(-\underline{F}, \widehat{\underline{s}})$ plan because the payment to the firm $(-\underline{F})$ declines. In contrast, the regulator's expected payoff does not change under the $\{(F_2, s_2), (-\widehat{\underline{F}}_H, 1)\}$ program because this program does not change as $\underline{F}$ increases. Therefore, both of the identified solutions cannot be optimal and so, by contradiction, $\widehat{\underline{F}}_L \leq \widehat{\underline{F}}_H$.

Since $\widehat{\underline{F}}_L \leq \widehat{\underline{F}}_H$ and $\widehat{\underline{F}}_L \neq \widehat{\underline{F}}_H$, it must be the case that $\widehat{\underline{F}}_L < \widehat{\underline{F}}_H$. □

Lemma 10. *Suppose* $\underline{F} \in [\underline{F}_L, \widehat{\underline{F}}_H)$. *Then* $s_2 \leq s_1 < 1$, $F_2 \geq F_1 = -\underline{F}$, *and* $\widehat{\pi}_2 = 0$. *In addition, if* $K_{GGG}(G, k_i) \geq 0$ *and* $K_{GG}(G, k_2) \geq K_{GG}(G, k_1)$ *for all* G *and for* $k_i \in \{k_1, k_2\}$, *then there exists an* $\widehat{\underline{F}}_L \in [\underline{F}_L, \widehat{\underline{F}}_H)$, *such that* $s_1 = s_2$ *for* $\underline{F} \in [\underline{F}_L, \widehat{\underline{F}}_L]$, *whereas* $s_2 < s_1$ *for* $\underline{F} \in (\widehat{\underline{F}}_L, \widehat{\underline{F}}_H)$. *Furthermore,* $\frac{ds_1}{d\underline{F}} = \frac{ds_2}{d\underline{F}} > 0$ *for* $\underline{F} \in (\underline{F}_L, \widehat{\underline{F}}_L)$, *whereas* $\frac{ds_1}{d\underline{F}} > 0$, $\frac{ds_2}{d\underline{F}} < 0$, $\frac{dF_1}{d\underline{F}} < 0$, *and* $\frac{dF_2}{d\underline{F}} > 0$ *for* $\underline{F} \in (\widehat{\underline{F}}_L, \widehat{\underline{F}}_H)$.

Proof. If $\underline{F} \in [\underline{F}_L, \widehat{\underline{F}}_H)$, then the participation constraint (2) when $k = k_2$ and the $F_1 \geq -\underline{F}$ constraint both bind at the solution to [P]. Consequently, $\widehat{\pi}_2 = 0$ and $F_1 = -\underline{F}$. Furthermore: (i) $F_2 \geq F_1$ from Lemma 3; (ii) $s_2 \leq s_1 < 1$ from Lemma 8; and (iii) $\lambda_{12} > 0$ from the proof of Lemma 8.

From (1), the regulator maximizes:

$$\sum_{i=1}^{2} \phi_i \{ [1 - s_i]\, G_i - F_i + \alpha\, \pi_i(F_i, s_i) \}$$

$$= \sum_{i=1}^{2} \phi_i\, \{ G_i - K(G_i, k_i) - [1 - \alpha]\, \pi_i(F_i, s_i) \}\,. \tag{37}$$

When $s_2 < s_1$, the regulator can be viewed as choosing the optimal value of s_2. The corresponding optimal values of F_2 and s_1 are then readily determined because $\widehat{\pi}_2 = 0$ and $\lambda_{12} > 0$. Differentiating (37), recognizing that $\frac{d\pi_2(\cdot)}{ds_2} = 0$, provides:

$$\sum_{i=1}^{2} \phi_i \left\{ [1 - K_G(G_i, k_i)] \frac{dG_i}{ds_i} \right\} ds_i - \phi_1 [1 - \alpha]\, G_1\, ds_1$$

$$= \sum_{i=1}^{2} \phi_i \left[\frac{1 - K_G(G_i, k_i)}{K_{GG}(G_i, k_i)} \right] ds_i - \phi_1 [1 - \alpha]\, G_1\, ds_1 = 0\,. \tag{38}$$

The first equality in (38) holds because $\frac{dG_i}{ds_i} = \frac{1}{K_{GG}(G_i, k_i)}$, since $K_G(G_i, k_i) = s_i$ from (25).

Since $\widehat{\pi}_2 = 0$:

$$F_2 + s_2\, G_2 - K(G_2, k_2) = 0 \quad \Rightarrow \quad dF_2 + G_2\, ds_2 = 0\,. \tag{39}$$

Since $\lambda_{12} > 0$:

$$-\underline{F} + s_1\, G_1 - K(G_1, k_1) = F_2 + s_2\, G_{21} - K(G_{21}, k_1)\,. \tag{40}$$

Differentiating (40), using (39), provides:

$$G_1\, ds_1 = dF_2 + G_{21}\, ds_2 = [G_{21} - G_2]\, ds_2\,. \tag{41}$$

(38) and (41) imply that when $s_2 < s_1$ at the solution to [P]:

$$\phi_1 \left[\frac{1 - K_G(G_1, k_1)}{K_{GG}(G_1, k_1)} \right] G_1\, ds_1 + \phi_2 \left[\frac{1 - K_G(G_2, k_2)}{K_{GG}(G_2, k_2)} \right] \left[\frac{G_1}{G_{21} - G_2} \right] ds_1$$

$$- \phi_1 [1 - \alpha]\, G_1\, ds_1 = 0$$

$$\Rightarrow \quad \phi_1 \left[\frac{1-K_G(G_1,k_1)}{K_{GG}(G_1,k_1)}\right] \frac{1}{G_1} + \phi_2 \left[\frac{1-K_G(G_2,k_2)}{K_{GG}(G_2,k_2)}\right] \left[\frac{1}{G_{21}-G_2}\right] - \phi_1 \left[1-\ \alpha\right] \ = \ 0\,. \quad (42)$$

G_2 and G_{21} are readily calculated for any given s_2. Given G_2 and G_{21}, G_1 can be derived from (42). We now show that G_1 (and therefore s_1) is uniquely determined by s_2 and that s_1 is a monotone decreasing function of s_2.

Differentiating (42) provides:

$$\begin{aligned}
&\phi_1 \left\{ \left[\frac{1-K_G(G_1,k_1)}{K_{GG}(G_1,k_1)}\right] \left[-\frac{1}{G_1^2}\right] \right. \\
&\qquad \left. + \frac{-K_{GG}^2(G_1,k_1) - \left[1-K_G(G_1,k_1)\right] K_{GGG}(G_1,k_1)}{K_{GG}^2(G_1,k_1)} \left[\frac{1}{G_1}\right] \right\} \left[\frac{dG_1}{ds_1}\right] ds_1 \\
&+ \phi_2 \left\{ \frac{1-K_G(G_2,k_2)}{K_{GG}(G_2,k_2)} \left[\frac{1}{(G_{21}-G_2)^2}\right] \right. \\
&\qquad \left. + \frac{-K_{GG}^2(G_2,k_2) - \left[1-K_G(G_2,k_2)\right] K_{GGG}(G_2,k_2)}{K_{GG}^2(G_2,k_2)} \left[\frac{1}{G_{21}-G_2}\right] \right\} \frac{dG_2}{ds_2}\, ds_2 \\
&+ \phi_2 \left[\frac{1-K_G(G_2,k_2)}{K_{GG}(G_2,k_2)}\right] \left[-\frac{1}{(G_{21}-G_2)^2}\right] \left[\frac{dG_{21}}{ds_2}\right] ds_2 \ = \ 0\,. \qquad (43)
\end{aligned}$$

Since $\frac{dG_i}{ds_i} = \frac{1}{K_{GG}(G_i,k_i)}$, the terms that multiply ds_1 in (43) can be written as:

$$\begin{aligned}
&\frac{\phi_1}{K_{GG}^2(G_1,k_1)\,G_1^2} \left\{ -\left[1-K_G(G_1,k_1)\right] K_{GG}(G_1,k_1) - G_1\, K_{GG}^2(G_1,k_1) \right. \\
&\qquad \left. - G_1 \left[1-K_G(G_1,k_1)\right] K_{GGG}(G_1,k_1) \right\} \frac{1}{K_{GG}(G_1,k_1)}\, ds_1 \ < \ 0\,. \qquad (44)
\end{aligned}$$

The inequality in (44) holds when $K_{GGG}(\cdot) \geq 0$ because $K_G(G_1,k_1) = s_1 < 1$.

Similarly, the terms that multiply ds_2 in (43) can be written as:

$$\begin{aligned}
&\frac{\phi_2}{K_{GG}^2(G_2,k_2)\left[G_{21}-G_2\right]^2} \left\{ \left[1-K_G(G_2,k_2)\right] K_{GG}(G_2,k_2) - \left[G_{21}-G_2\right] K_{GG}^2(G_2,k_2) \right. \\
&\qquad \left. - \left[G_{21}-G_2\right]\left[1-K_G(G_2,k_2)\right] K_{GGG}(G_2,k_2) \right\} \frac{1}{K_{GG}(G_2,k_2)} \\
&\qquad + \phi_2 \left[\frac{1-K_G(G_2,k_2)}{K_{GG}(G_2,k_2)}\right] \left[-\frac{1}{(G_{21}-G_2)^2}\right] \frac{1}{K_{GG}(G_{21},k_1)} \\
&< \ \frac{\phi_2\left[1-K_G(G_2,k_2)\right]}{K_{GG}(G_2,k_2)\left[G_{21}-G_2\right]^2} \left[\frac{1}{K_{GG}(G_2,k_2)} - \frac{1}{K_{GG}(G_{21},k_1)}\right] \ \leq \ 0\,. \qquad (45)
\end{aligned}$$

The first inequality in (45) holds when $K_{GGG}(G,k) \geq 0$ since $K_G(G_2,k_2) = s_2 < 1$ and $G_{21} > G_2$. The last inequality in (45) holds because $K_{GG}(G_{21},k_1) \leq K_{GG}(G_2,k_2)$ when $K_{GGG}(G,k_i) \geq 0$ and $K_{GG}(G,k_2) \geq K_{GG}(G,k_1)$ for all G and for $k_i \in \{k_1,k_2\}$.

(43), (44), and (45) imply that for each s_2, there is a unique s_1 that decreases as s_2 increases (so $\frac{ds_1}{ds_2} < 0$) at the solution to [P]. Lemma 1 implies that the firm's profit in the low cost environment at the solution to [P] increases as s_2 increases and s_1 decreases. Therefore, since $\lambda_{12} > 0$, there is a unique F_1 that increases as s_2 increases.

Let $\overline{s}_2$ denote the value of s_2 at the solution to [P] when $\underline{F} = \widehat{\underline{F}}_H$. Also let $\widehat{s}$ denote the largest share of the realized gain awarded the supplier when $s_1 = s_2$ at the solution to [P]. In addition, let $\widehat{\underline{F}}_L \geq \underline{F}_L$ denote the value $\underline{F}_L$ at which $s_1 = s_2 = \widehat{s}$ at the solution to [P]. Since $F_1 = -\underline{F}$ when $\underline{F} \in [\underline{F}_L, \widehat{\underline{F}}_H)$, it follows that s_2 increases from $\overline{s}_2$ to $\widehat{s}$ as $\underline{F}$ declines from $\widehat{\underline{F}}_H$ to $\underline{F}_L$. Therefore, $s_2 < s_1$ and $\frac{ds_1}{d\underline{F}} > 0$, $\frac{ds_2}{d\underline{F}} < 0$, $\frac{dF_1}{d\underline{F}} < 0$, and $\frac{dF_2}{d\underline{F}} > 0$ when $\underline{F} \in (\widehat{\underline{F}}_L, \widehat{\underline{F}}_H)$. □ ■

<u>Proof of Conclusion 3</u>.

Let (F_i, s_i) denote the gain sharing plan the firm chooses when $k = k_i$. Then consumer surplus when $k = k_i$ is:

$$CS_i \equiv -F_i + [1-s_i]\,G_i\,. \tag{46}$$

Total surplus when $k = k_i$ is:

$$T_i \equiv G_i - K(G_i,k_i). \tag{47}$$

The firm's rent when $k = k_i$ is:

$$R_i \equiv F_i + s_i\,G_i - K(G_i,k_i)\,. \tag{48}$$

The regulator's objective is to maximize:

$$W \equiv \sum_{i=1}^{2} \phi_i\,[CS_i + \alpha\,R_i] = \sum_{i=1}^{2} \phi_i\,[T_i - (1-\alpha)R_i]\,. \tag{49}$$

<u>Case I</u>. $\underline{F} \geq \widehat{\underline{F}}_H$.

The regulator can be viewed as determining the optimal s_2. Conclusion 2 implies that once s_2 is determined, F_2 is set to ensure the firm earns no rent when $k = k_2$. Furthermore, $s_1 = 1$ and F_1 is chosen so that the firm is indifferent between the (F_1, s_1) plan and the (F_2, s_2) plan when $k = k_1$. This indifference implies:

$$R_1 = F_2 + s_2\,G_{21} - K(G_{21},k_1)\,, \tag{50}$$

where G_{21} is the success probability the firm would implement under the (F_2, s_2) plan in the low cost environment.

Because $R_2 = 0$:

$$0 = \frac{dR_2}{ds_2} = \frac{\partial R_2}{\partial s_2} + \frac{\partial R_2}{\partial G_2}\left[\frac{dG_2}{ds_2}\right] + \frac{\partial R_2}{\partial F_2}\left[\frac{dF_2}{ds_2}\right] = G_2 + \frac{dF_2}{ds_2} \;\Rightarrow\; \frac{dF_2}{ds_2} = -\,G_2\,. \tag{51}$$

The third equality in (51) reflects the envelope theorem and the fact that $\frac{\partial R_2}{\partial s_2} = G_2$ and $\frac{\partial R_2}{\partial F_2} = 1$, from (48).

$\frac{dW}{ds_2} = 0$ at the solution to [P]. We will determine how changes in parameter values affect $\frac{dW}{ds_2}$. If $\frac{dW}{ds_2}$ becomes positive (negative) as a parameter increases, then the optimal s_2 will increase (decrease), given the presumed concavity of W.

From (48):

$$K_G(G_i, k_i) \;=\; s_i \quad \Rightarrow \quad \frac{dG_i}{ds_i} \;=\; \frac{1}{K_{GG}(G_i, k_i)} \quad \text{for } i = 1, 2\,. \tag{52}$$

Because $s_1 = 1$, T_1 is not affected by changes in s_2, i.e., $\frac{dT_1}{ds_2} = 0$.

From (47), using (52):

$$\begin{aligned} \frac{dT_2}{ds_2} &= \frac{\partial T_2}{\partial s_2} + \frac{\partial T_2}{\partial G_2}\left[\frac{dG_2}{ds_2}\right] + \frac{\partial T_2}{\partial F_2}\left[\frac{dF_2}{ds_2}\right] \;=\; \frac{\partial T_2}{\partial G_2}\left[\frac{dG_2}{ds_2}\right] \\ &= [1 - K_G(G_2, k_2)]\,\frac{dG_2}{ds_2} \;=\; [1 - s_2]\,\frac{dG_2}{ds_2} \;=\; \frac{1 - s_2}{K_{GG}(G_2, k_2)}\,. \end{aligned} \tag{53}$$

The second equality in (53) holds because $\frac{\partial T_2}{\partial s_2} = \frac{\partial T_2}{\partial F_2} = 0$, from (47). The last two equalities in (53) reflect (52). From (48):

$$\begin{aligned} \frac{dR_1}{ds_2} &= \frac{\partial R_1}{\partial s_2} + \frac{\partial R_1}{\partial G_{21}}\left[\frac{dG_{21}}{ds_2}\right] + \frac{\partial R_1}{\partial F_2}\left[\frac{dF_2}{ds_2}\right] \\ &= \frac{\partial R_1}{\partial s_2} + \frac{\partial R_1}{\partial F_2}\left[\frac{dF_2}{ds_2}\right] \;=\; G_{21} + \frac{dF_2}{ds_2} \;=\; G_{21} - G_2\,. \end{aligned} \tag{54}$$

The second equality in (54) reflects the envelope theorem. The third equality in (54) follows from (50). The last equality in (54) reflects (51).

(49), (53), and (54) imply:

$$\frac{dW}{ds_2} \;=\; \phi_2\left[\frac{1 - s_2}{K_{GG}(G_2, k_2)}\right] - \phi_1\,[1 - \alpha]\,[G_{21} - G_2]\;. \tag{55}$$

Differentiating (55) with respect to α provides:

$$\frac{d}{d\alpha}\left(\frac{dW}{ds_2}\right) \;=\; \phi_1\,[G_{21} - G_2] \;>\; 0. \tag{56}$$

The inequality in (56) implies that the optimal s_2 increases as α increases.

Differentiating (55) with respect to ϕ_1 provides:

$$\frac{d}{d\phi_1}\left(\frac{dW}{ds_2}\right) \;=\; -\,[1 - s_2]\,\frac{1}{K_{GG}(G_2, k_2)} - [1 - \alpha]\,[G_{21} - G_2] \;<\; 0.$$

This inequality implies that the optimal s_2 decreases as ϕ_1 increases.

Differentiating (55) with respect to k_2 provides:

$$\frac{d}{dk_2}\left(\frac{dW}{ds_2}\right) \;=\; \phi_1\,[1-\alpha]\,\frac{dG_2}{dk_2} - \phi_2\,[1-s_2]\,\frac{K_{GGG}(G_2,k_2)\frac{dG_2}{dk_2} + K_{GGk}(G_2,k_2)}{[K_{GG}(G_2,k_2)]^2} \;<\; 0. \quad (57)$$

The inequality in (57) holds when Condition 1 holds because $\frac{dG_2}{dk_2} = -\frac{K_{Gk}(G_2,k_2)}{K_{GG}(G_2,k_2)} < 0$, since $s_2 = K_G(G_2,k_2)$. The inequality in (57) implies that the optimal s_2 decreases as k_2 increases.

Differentiating (55) with respect to k_1 provides:

$$\frac{d}{dk_1}\left(\frac{dW}{ds_2}\right) \;=\; -\,\phi_1\,[1-\alpha]\,\frac{dG_{21}}{dk_1} \;>\; 0. \qquad (58)$$

The inequality in (58) holds because $\frac{dG_{21}}{dk_1} = -\frac{K_{Gk}(G_{21},k_1)}{K_{GG}(G_{21},k_1)} < 0$, since $K_G(G_{21},k_1) \;=\; s_2$. The inequality in (58) implies that the optimal s_2 increases as k_1 increases.

Case II. $\underline{F} \in (\widehat{\underline{F}}_L, \widehat{\underline{F}}_H)$.

The regulator can again be viewed as determining the optimal s_2. Conclusion 2 implies that once s_2 is determined, F_2 is set to ensure the firm earns no rent when $k = k_2$. Furthermore, $F_1 = -\underline{F}$ and s_1 is chosen so that the firm is indifferent between the (F_1, s_1) and (F_2, s_2) plans when $k = k_1$.

$\frac{dT_2}{ds_2}$ in this case is as specified in (53). Furthermore, from (47), using (52):

$$\begin{aligned}\frac{dT_1}{ds_2} \;&=\; \frac{\partial T_1}{\partial G_1}\left[\frac{dG_1}{ds_2}\right] \;=\; [1-K_G(G_1,k_1)]\,\frac{dG_1}{ds_2}\\ &=\; [1-s_1]\,\frac{dG_1}{ds_1}\left[\frac{ds_1}{ds_2}\right] \;=\; \left[\frac{1-s_1}{K_{GG}(G_1,k_1)}\right]\frac{ds_1}{ds_2}\,. \qquad (59)\end{aligned}$$

From (50), (51), and the envelope theorem:

$$-\,\underline{F} + s_1\,G_1 - K(G_1,k_1) \;=\; F_2 + s_2\,G_{21} - K(G_{21},k_1)$$

$$\Rightarrow \quad G_1\,\frac{ds_1}{ds_2} \;=\; \frac{dF_2}{ds_2} + G_{21} \quad \Rightarrow \quad \frac{ds_1}{ds_2} \;=\; \frac{G_{21}-G_2}{G_1} \;>\; 0\,. \qquad (60)$$

In addition, from (50):

$$\begin{aligned}\frac{dR_1}{ds_2} \;&=\; \frac{\partial R_1}{\partial s_2} + \frac{\partial R_1}{\partial G_{21}}\left[\frac{dG_{21}}{ds_2}\right] + \frac{\partial R_1}{\partial F_2}\left[\frac{dF_2}{ds_2}\right]\\ &=\; \frac{\partial R_1}{\partial s_2} + \frac{\partial R_1}{\partial F_2}\left[\frac{dF_2}{ds_2}\right] \;=\; G_{21} + \frac{dF_2}{ds_2} \;=\; G_{21} - G_2\,. \qquad (61)\end{aligned}$$

The second equality in (61) reflects the envelope theorem. The third equality in (61) holds because $\frac{\partial R_1}{\partial s_2} = G_{21}$ and $\frac{\partial R_1}{\partial F_2} = 1$, from (50). The last equality in (61) reflects (51).

(49), (53), (59), and (61) imply:

$$\frac{dW}{ds_2} \;=\; \phi_1 \left[\frac{1-s_1}{K_{GG}(G_1,k_1)} \right] \frac{ds_1}{ds_2} + \phi_2 \left[\frac{1-s_2}{K_{GG}(G_2,k_2)} \right] - \phi_1 \, [1-\alpha] \, [G_{21} - G_2] \; . \tag{62}$$

Differentiating (62) with respect to α provides:

$$\frac{d}{d\alpha} \left(\frac{dW}{ds_2} \right) \;=\; \phi_1 \, [G_{21} - G_2] \;>\; 0 \, .$$

This inequality implies that the optimal s_2 increases as α increases.

Differentiating (62) with respect to ϕ_1 provides:

$$\begin{aligned} \frac{d}{d\phi_1} \left(\frac{dW}{ds_2} \right) \;&=\; - \, [1-\alpha] \, [G_{21} - G_2] - \frac{1-s_2}{K_{GG}(G_2,k_2)} + \frac{1-s_1}{K_{GG}(G_1,k_1)} \left[\frac{ds_1}{ds_2} \right] \\ &=\; - \, \frac{1-s_2}{K_{GG}(G_2,k_2)} - \frac{\phi_2}{\phi_1} \left[\frac{1-s_2}{K_{GG}(G_2,k_2)} \right] \;<\; 0 \, . \end{aligned} \tag{63}$$

The last equality in (63) follows from (62), since $\frac{dW}{ds_2} = 0$ at the optimal value of s_2. The inequality in (63) implies that the optimal s_2 decreases as ϕ_1 increases.

Differentiating (62) with respect to k_2 provides:

$$\frac{d}{dk_2} \left(\frac{dW}{ds_2} \right) \;=\; \phi_1 \, [1-\alpha] \, \frac{dG_2}{dk_2} - \phi_2 \, [1-s_2] \, \frac{K_{GGG}(G_2,k_2) \frac{dG_2}{dk_2} + K_{GGk}(G_2,k_2)}{\left[K_{GG}(G_2,k_2) \right]^2} \;<\; 0 \, .$$

This inequality holds when Condition 1 holds because $\frac{dG_2}{dk_2} = - \frac{K_{Gk}(G_2,k_2)}{K_{GG}(G_2,k_2)} < 0$, since $s_2 = K_G(G_2,k_2)$. The inequality implies that the optimal s_2 decreases as k_2 increases.

The proofs for the settings in which $\underline{F} \leq \underline{\widehat{F}}_L$ are analogous, and so are omitted. ■

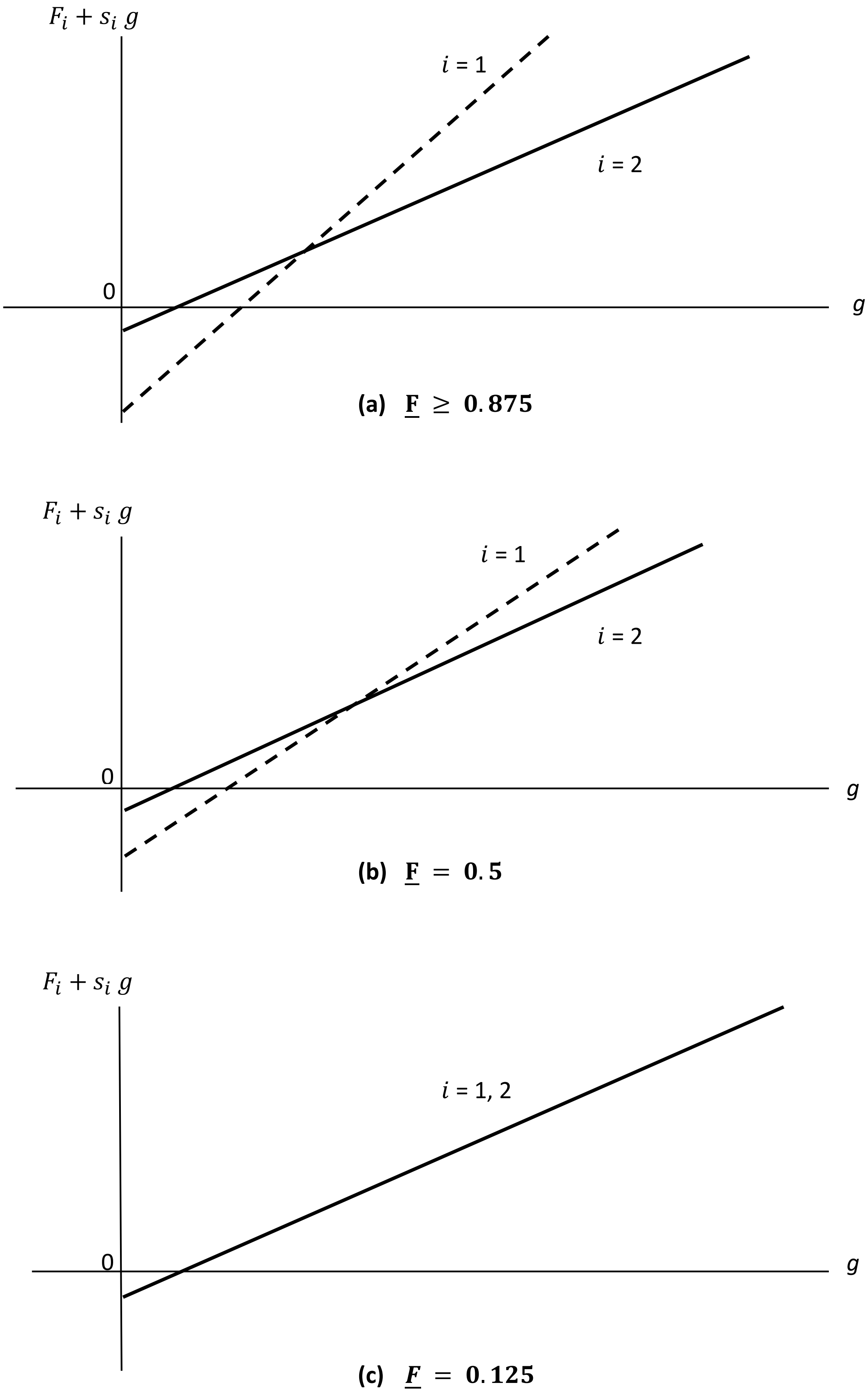

Figure 1. Optimal Gain Sharing Plans $\{(F_1, s_1), (F_2, s_2)\}$ as $\underline{F}$ Changes ($k_1 = 0.5,\ \ k_2 = 1.0,\ \ \phi_1 = \phi_2 = 0.5,\ \ \alpha = 0$).

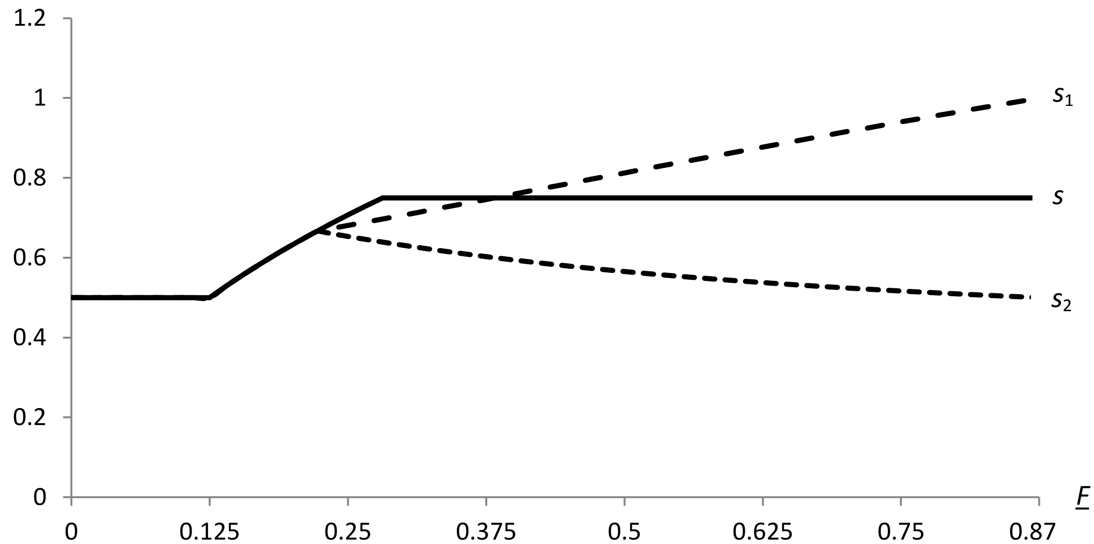

Figure 2. Optimal Sharing Rates Under a Single Gain Sharing Plan (s) and Under Two Optional Plans (s_1, s_2) ($k_1 = 0.5$, $k_2 = 1.0$, $\phi_1 = \phi_2 = 0.5$, $\alpha = 0$).

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