Title	Designing the energy absorption capacity of functionally graded foam materials
Authors(s)	Cui, Liang; Kiernan, Stephen; Gilchrist, M. D.
Publication date	2009-05
Publication information	Materials Science and Engineering: A, 507 (1-2): 215-225
Publisher	Elsevier
Item record/more information	http://hdl.handle.net/10197/4619
Publisher's statement	This is the author's version of a work that was accepted for publication in Materials Science and Engineering: A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Materials Science and Engineering: A (507, 1-2, (2009)) DOI: http://dx.doi/org/10.1016/j.msea.2008.12.011
Publisher's version (DOI)	10.1016/j.msea.2008.12.011

Designing the energy absorption capacity of functionally graded foam materials

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Abstract

In this paper, a functionally graded foam model is proposed in order to improve upon the energy absorption characteristics offered by uniform foams. In this novel model, the characteristics of the foam (e.g. density) are varied through the thickness according to various gradient functions. The energy absorption ability of the novel foam is explored by performing finite element simulations of physical impact tests on flat specimens of the functionally graded foam materials. Energy absorbing capacity w.r.t. parameters including gradient functions, density difference, average density, and impact energy, is explored in detail. It is illustrated that the functionally graded foam is superior in energy absorption to the uniform foam and that convex gradients perform better than concave gradients. The performance of such foams can be improved more if the density difference is enlarged. These findings provide valuable suggestions in the design of high performance energy absorption polymeric foams.

Keywords:

Functionally graded foam, Constitutive model, Energy absorption, Impact test

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1. Introduction

Light-weight polymeric foam, composed of a large amount of microscopic polymer cellular structures, is widely used as a cushioning structure (e.g. helmet liner) to mitigate impact stresses imparted to the wearer below injurious levels. As described by Gibson and Ashby [1], the foam can absorb a great amount of energy during the plastic deformation stage when the stress reaches a roughly constant plateau value over a large percentage of total strain (typically 60-70%). Before this stage, the ability of the foam to absorb energy under impact is very inefficient and will transmit the majority of the energy in the form of stress waves to the protected object [2]. At the cellular scale, the deformation mechanisms of a single cell are cell wall bending and stretching followed by post-yield wall buckling and tearing. The bending moments provided by each cell constituent contribute to the overall load bearing capacity of the foam. By varying these micro-scale parameters (e.g. through cell wall and face thickness, and area moments of inertia), the local load bearing capacity becomes a controllable spatial variable rather than an approximately constant value [3]. Fortunately, these micro-scale parameters are heavily dependent on the foam density and can therefore be varied by correspondingly varying the bulk density of the foam.

Various constitutive models have been developed to analyse the behaviour of various types of foams under loading. Most of the foams of concern are assumed to contain approximately identical micro-scale cells and the overall characteristics are typically assumed to be isotropic and homogenous. Alternative materials used as energy absorbing structures are laminated composite materials. However, the inter-laminar stresses within laminated materials are localised at interfaces due to the strong discreteness in the material properties. This localisation of stress can lead to delamination and crack propagation [4-6]. To eliminate the stress localisation, a proper continuous gradient is required to smooth the property transition through the thickness. The current study aims to design and optimise a virtual, novel Functionally Graded Foam Material (FGFM) which contains micro-scale cells varied continuously in a predefined manner for the purpose of improving its energy absorbing characteristics under low to moderate energy impact conditions. The propagation and evolution of a stress wave generated from an impact by a projectile has been studied by Kiernan et al [7] and has shown that the stress wave profile and amplitude can be shaped by the gradient function that defines the variation in density, which consequently can improve the energy absorption capacity and reduce the severity of damage/injury induced in the object/person being protected. Foam has found use as a protective material in many diverse applications ranging from packaging, to automotive components to helmets and head protection systems [8, 9] and may come in the form of foamed polyurethane, expanded polystyrene and aluminium foam.

Functionally graded materials (FGMs) were defined as “a new generation of engineered materials wherein the microstructural details are spatially varied through a non-uniform distribution of the reinforcement phase(s), by using reinforcement with different properties, sizes and shapes, as well as by interchanging the roles of reinforcement and matrix phases in a continuous manner”[10]. FGMs originally found widespread applications as metal-ceramic composites, in which there is a gradual microstructural transition from a ceramic rich to a metal rich region [11]. More recently, interest in the mechanical response of FGMs has concentrated on optimising the load response to dynamic loading [12-15], and the energy absorbing characteristics of cellular structures [16]. Kieback et al [17] have started to develop manufacturing techniques to produce a FGFM under laboratory conditions.

Scheidler and Gazonas [18] analysed 1-D wave propagation and impact loading conditions in an elastic medium with a quadratic variation in impedance. Simulations were performed with a discretely layered model in DYNA3D and results were compared with analytical solutions. Improved solutions were obtained by increasing mesh density, but both compressive and tensile wave amplitudes were underestimated in the simulations. Bruck [19] proposed a one-dimensional model for stress wave propagation and reflection through an FGM. Bruck shows that a stress wave passing through a gradient architecture can result in a higher peak magnitude of the stress waves than through a sharp interface, while the gradient architecture introduces a time delay to the reflected wave as the stress approaches peak values. Bruck also shows that the steady-state magnitude of the stress wave reflected from the sharp interface is the same as reflected from the gradient architecture. Berezovski et al [20] extended the study of stress wave propagation from one to two dimensions. Anlas et al [21] examined the stress intensity factors for an edge cracked plate made of FGMs with various gradients, and El-Hadek and Tippur [22] analysed the crack initiation and propagation within the FGMs. Banks-Sills et al [23] simulated the dynamic loading response of five functionally graded aluminium-ceramic models, including continuous and layered models. They found that a step dynamic load applied to each model produced no significant difference in the effective stress at particular points in the time domain, while difference in the effective stress was observed at a particular time in the space domain. They concluded that a continuously changing material model was more effective than a layered model for studying the dynamic behaviour for FGMs, especially for studying crack growth problems.

Avalle et al [24] characterised compressive impact loading of polymeric foams over a range of densities using energy absorption diagrams. They showed that, for a particular density, a foam is most efficient at absorbing the kinetic energy of an impact over a limited range of stress, after which the stress rises rapidly with little corresponding increase in absorbed energy. By means of a functionally graded foam, it may be possible to combine a large range of densities to improve the energy absorbing efficiency over a wider range of stress levels.

This paper first describes the constitutive models for the polymeric foam used in the current study. Free drop weight impact tests were simulated using a crushable foam model to analyse the peak loads applied to the striker for various foam density gradients. The energy absorption abilities of various FGFMs were compared and a series of acceleration surfaces were derived which are functions of the gradient function and the incident kinetic energy of the striker. This may allow potential manufacturers to intelligently choose design parameters for these novel foams to maximise their ability to reduce peak accelerations of an impact.

2. Constitutive Models for Polymeric Foam

2.1 Constitutive model of the stress-strain relationship

The constitutive model describing the uniform foam was developed based on the model proposed by Schraad and Harlow [25] for the disordered cellular materials under uni-axial compression. In their model, the stress-strain relation of the foam was represented by a tri-linear function. With the assumption that the Poisson's ratio for low-density foam is approximately zero, the tangent stiffness E of the foam under uni-axial compression is found to be a function of its solid-volume fraction and the axial strain. That is

$$E(\varepsilon^n) = A(\varepsilon^n)E_s[\phi(\varepsilon^n)]^2 \qquad (1)$$

where ε^n is the nominal axial strain (length change per unit undeformed length) and in the range of (-1, 0), $A(\varepsilon^n)$ is a stiffness related parameter varied with the axial strain, E_s is the tangent stiffness of the parent solid material used to make the cellular material, and $\phi(\varepsilon^n)$ is the relative density, or the solid-volume fraction. As $\varepsilon^n = 0$, $A(0)=A_0$, $\phi(0)=\phi_0$, and $E(0)=E_0$. During the compression, $\phi(\varepsilon^n)$ can be expressed as (with zero Poisson's ratio)

$$\phi(\varepsilon^n) = \frac{\phi_0}{1+\varepsilon^n} = \frac{\rho_0}{\rho_s}\frac{1}{1+\varepsilon^n} \qquad (2)$$

where ρ_0 is the initial density before compression and ρ_s is the density of the parent solid material. The parameter $A(\varepsilon^n)$ defines the geometry of the stress-strain curve. For axial strains lower than the yield value, ε_1, it shows a linear elastic response with a tangent stiffness equal to E_0. The linear elasticity is controlled by buckling or stretching of walls or struts of constituent cells. As the axial strain is increased over the yield value, the plateau stress only increases slightly with the steadily increasing axial strain, yielding a smaller tangent stiffness, E_1. The plateau is associated with the collapse of cells – by elastic buckling in elastomeric foams; by brittle crushing in brittle foams; and by formation of plastic hinges in a foam which yields [1]. As the axial strain increases further to higher than a densification strain, ε_2, the cells are crushed entirely and the cell walls start to contact each other. Further strain compresses the solid itself, giving the final densification stage of sharply increasing stiffness finally approaching the stiffness of the parent solid material as the axial strain approaches 100%. The energy absorbed by a foam is proportional to the area under the stress-strain curve. It can be seen that a foam's energy absorbing efficiency, which can be defined as absorbed energy normalised by peak stress [24], is highest if the input energy is just absorbed before the strain reaches the densification strain.

As described by Schraad and Harlow [25], the transition between the three stages is over a small range of strain rather than instantaneously, due to the imperfectly homogeneous or identical cellular structure of the foam. Assuming that the imperfection of the cellular structure is distributed randomly, the transition between the linear elastic stage and the plateau stage occurs over a small range of $2\Delta\varepsilon_1$, while the transition between the plateau stage and the densification stage occurs over a small range of $2\Delta\varepsilon_2$. The geometric parameter, $A(\varepsilon^n)$, for the foam can then be expressed as

$$A(\varepsilon^n) = \begin{cases} A_0, & \varepsilon_1 + \Delta\varepsilon_1 \le \varepsilon^n \le 0, \\ \dfrac{(A_0 - A_1)\varepsilon^n - A_0(\varepsilon_1 - \Delta\varepsilon_1) + A_1(\varepsilon_1 + \Delta\varepsilon_1)}{2\Delta\varepsilon_1}, & \varepsilon_1 - \Delta\varepsilon_1 \le \varepsilon^n \le \varepsilon_1 + \Delta\varepsilon_1, \\ A_1, & \varepsilon_2 + \Delta\varepsilon_2 \le \varepsilon^n \le \varepsilon_1 - \Delta\varepsilon_1, \\ \dfrac{(A_1 - 1)\varepsilon^n - A_1(\varepsilon_2 - \Delta\varepsilon_2) + (\varepsilon_2 + \Delta\varepsilon_2)}{2\Delta\varepsilon_2}, & \varepsilon_2 - \Delta\varepsilon_2 \le \varepsilon^n \le \varepsilon_2 + \Delta\varepsilon_2, \\ 1, & -1 \le \varepsilon^n \le \varepsilon_2 - \Delta\varepsilon_2, \end{cases} \tag{3}$$

where A_0 and A_1 can be obtained from Eq. (1) and Eq. (2) as

$$A_0 = \frac{E_0}{E_s\phi_0^2} = \frac{E_0\rho_s^2}{E_s\rho_0^2} \tag{4}$$

$$A_1 = \frac{E_1}{E_s\phi(\varepsilon_1)^2} = \frac{E_1\rho_s^2}{E_s\rho_0^2}(1+\varepsilon_1)^2 \tag{5}$$

To determine the parameters in the functions of the constitutive model for a specified type of foam, a series of quasi-static uni-axial compression tests were performed on EPS foam specimens of densities ranging from 15kg/m^3 to 64kg/m^3. The assumption of a vanishing Poisson's ratio for EPS foam was validated in experiment tests under uni-axial compression up to 95% strain, which showed lateral strains to be less than 2%. A summary of the test results is shown in Fig. 1. For the solid EPS material, ρ_s=1050 kg/m^3, E_s=3.3 GPa. Based on the results of the experimental compression tests, the parameters in the model can be determined. The stress-strain curves obtained from the constitutive formula with the determined parameters are also illustrated in Fig. 1. The constitutive model was found to quantitatively match the results of the experimental tests. From this, stress-strain curves for virtual specimens of higher arbitrary densities were extrapolated and used during the numerical simulations in order to analyse a suitably wide range of densities.

Fig. 1. Comparison between the laboratory tests and the constitutive model.

Fig. 1 shows the generated stress at a given strain to be highly dependent on the foam's initial density. Stresses plotted in Fig. 1 range from 0MPa at strain of 0.0 to 3MPa at strain of 0.85. Table 1 shows the calculated yield stress for a number of densities along with the range of plateau stresses for each density. The range of plateau stresses is taken to occur between the point of initial plastic yielding and the onset of densification, which is conservatively estimated at strain of 0.65 – 0.7. Plastic yielding is defined herein on a given curve to occur at that level of strain for which the tangent modulus is equal to the average of the tangent modulus in the linear elastic region and the tangent modulus in the plateau stress region. This gives a strain defined yield point in the strain range of 0.02 – 0.04.

Table 1. Yield and plateau stresses for foams of different densities as obtained from the constitutive model.

From experiment it was observed that the EPS's deformation post yield is rate dependant. Increasing the strain rate from 0.001/s to 80/s showed that the plateau stress, up to about 50% strain, is slightly sensitive to strain rate, while deformation within the densification region was found to be influenced significantly by strain rate. In spite of this the total strain in most of the presented simulations did not reach densification strains. In this study however the presented model assumes rate independent plasticity. As strain rate effects are absent from both uniform and graded foam models (and only a single strain rate was used), the influence of introducing material gradients into cushioning structures can clearly be quantified. For a more complete material description, it is intended to include strain rate sensitivity in more advanced gradient constitutive models as part of future research.

2.2 Constitutive model for the elasticity and plasticity of the foam

Although the pseudo-plastic behaviour of a crushing foam is related to the underlying microscopic deformation mechanics and is not the same as classical metal plasticity, the macroscopic plastic behaviour can still be described by a yield surface and hardening rule. The constitutive model for the elasticity and plasticity of a cellular solid has been studied by a number of researchers, including Deshpande and Fleck [26], who established an isotropic hardening rule for metallic foams, and Zhang et al [27], who constructed a volumetric hardening rule for polymer foams. In the current numerical simulations, the ABAQUS crushable foam model with a volumetric hardening rule was adopted in conjunction with the linear elastic model. As specified in the ABAQUS user's manual [28], the crushable foam plasticity models are intended for the analysis of crushable foams that are typically used as energy absorption structures; they are intended to simulate material response under essentially monotonic loading. It is assumed that the resulting deformation is not recoverable instantaneously and can, thus, be idealized as being plastic for short duration events.

The yield surface of the crushable foam [28] is a Von Mises circle in the deviatoric stress plane and an ellipse in the meridional (p-q) stress plane, as shown in Fig. 2. The evolution of the yield surface follows either the volumetric hardening rule or the isotropic hardening rule. In the current study, only the volumetric hardening rule is used. In the volumetric hardening rule, the point on the yield ellipse in the meridional plane that represents hydrostatic tension loading is fixed and the evolution of the yield surface is driven by the volumetric compacting plastic strain, ε^{el}. The yield surface evolves in a self-similar fashion. The shape factor, α, remains constant during any plastic deformation process. It can be computed using the initial yield stress in uniaxial compression, σ^0_c, the initial yield stress in hydrostatic compression, p^0_c (the initial value of p_c), and the yield strength in hydrostatic tension, p_t as

$$\alpha = \frac{3k}{\sqrt{(3k_t + k)(3-k)}} \quad \text{with} \quad k = \frac{\sigma_c^0}{p_c^0} \quad \text{and} \quad k_t = \frac{p_t}{p_c^0} \tag{6}$$

To define the hardening behaviour, the hardening curve describing the uniaxial compression yield stress (true stress) as a function of the corresponding true plastic (logarithmic) strain should be given. The values of the yield stress in uniaxial compression as a function of the absolute values of the axial (logarithmic) plastic strain can be calculated from the compression stress-strain curve obtained from the constitutive stress-strain relationship, as described above. The magnitude of the strength of the foam in hydrostatic tension was estimated as suggested in the ABAQUS user's manual [1]: p_t is set to equal to 10% of the initial yield stress in hydrostatic compression, p_c^0. The choice of tensile strength should not have a strong effect on the numerical results unless the foam is stressed in hydrostatic tension.

Fig. 2. Crushable foam model with volumetric hardening: yield surface and flow potential in the p–q stress plane [28]

3. Impact Tests on the Functionally Graded Foam

3.1 Description of simulation parameters

To explore the energy absorbing behaviour of the functionally graded foam, a series of simulated impact tests on the foam block with various density gradient functions were performed. In the simulations, a foam block of size 150 mm × 150 mm × 50 mm, resting on a rigid anvil, was impacted by a free-drop rigid flat-end striker, as shown in Fig. 3. The horizontal cross sectional area of the flat striker is larger than that of the foam to ensure a uniform impact over the entire foam area. The evolution of the acceleration of the striker, and consequently the impact force, during the impact can then be obtained. For the functionally graded foam blocks, the direction of the striker's velocity coincides with the direction in which the density is graded.

Fig. 3. Schematic diagram of the foam specimen and the rigid striker in the impact test.

For the functionally graded foam, the density varies monotonically according to various gradient functions. The density gradients considered were logarithmic, square root, linear, quadratic, and cubic, as shown in Fig. 4. ρ_1 and ρ_2 are the densities at the incident and distal surfaces respectively. The square root, linear, quadratic, and cubic gradient functions can be described by a power-law function as

$$\rho(y) = \rho_1 + (\rho_2 - \rho_1)\left(\frac{y}{d}\right)^n, 0 \le y \le d$$

where d is the thickness and y is the position though the thickness. The logarithmic function can be described as

$$\rho(y) = \rho_1 + (\rho_2 - \rho_1)\frac{\ln(y)}{\ln(m)}, y = 1, 2, \cdots, m$$

where m is the total number of layers through the thickness, and y is the layer number.

If we define the logarithmic function to be concave and the cubic function to be convex for decreasing density, the gradient changes from concave to convex while the function changes from logarithmic to cubic, as shown in Fig. 4(a) (The normalised distance is of the foam's thickness from the incident surface to the distal surface). A similar definition can be given for increasing density. The foam model is composed of m layers (here m = 50), each assigned a unique value of E and ρ ($\nu = 0$) as defined by the gradient functions, in order to give a quasi-continuous variation in material properties from one free surface to another.

Fig. 4. Variation in density versus normalised distance (0=incident (top) surface, 1=distal (bottom) surface).

Preliminary simulations show that the energy absorbing performance of an FGFM is, *ceteris paribus*, dependent on whether the density gradient increases or decreases from the incident (impacted surface) to the distal (opposite surface). Such a dependency can be observed in Fig. 5, which shows the average internal stress in each layer through the foam's thickness, when the peak acceleration of the striker is reached. Fig. 5 illustrates that, in one single functionally graded foam block, the average internal stress in each layer of higher density is proportional to the density of that layer. If the impact is applied to the higher density surface, the stress is reduced overall as it is transmitted to the lower density surface. Conversely, if the impact is applied to the lower density surface, the stress increases as it propagates. A detailed study on the evolution of the stress wave magnitude while propagating through FGFMs has been presented elsewhere [7]. These results illustrate that, following a striker impact on the incident surface, the stress transmitted to the distal surface of the foam, which is adjacent to the object being protected, can be increased or decreased depending on the direction of the density gradient.

Fig. 5. Average internal layer stresses from top to bottom (0=incident (top) surface, 1=distal (bottom) surface) at peak acceleration (mass of striker = 1 kg, v = 3 m/s, ρ_{aver} = 20 kg/m^3)

Based on the preliminary finding of Fig. 5, the comprehensive study of the current paper only considers the simulations with decreasing density from the incident (top) surface to the distal (bottom) surface. These simulations aim to explore the sensitivity of energy absorbing performance to design parameters including: density gradient, average density (ρ_{aver}), difference between upper and lower density range limits (referred to as "density difference" or $\Delta\rho$ below), and mass of the striker. As the weight can be an important criterion when designing cushioning structures (e.g. a helmet liner), the foam blocks with five types of density gradients were designed to target the same average density and the same density difference for direct comparison with equivalent uniform foams that serve as comparison benchmarks. To keep the average density constant, the upper and lower limits of density (ρ_1 and ρ_2) decrease as the gradient varies from concave to convex. Five average densities (44, 54, 64, 84, and 104 kg/m^3) and two density differences ($\Delta\rho$=20, 40 kg/m^3) were analysed for each of the five gradients and each mass of striker. Eight striker masses (1 kg, 2 kg, 4 kg, 6 kg, 8 kg, 10 kg, 12 kg, and 14 kg) are simulated and the incident velocity is 5.425 m/s for all the impacts, giving kinetic impact energies of 14.71 J, 29.43 J, 58.86 J, 88.29 J, 117.7 J, 147.2 J, 176.6 J, and 206 J, respectively. This incident velocity is specified in certification standard EN 1384:1996 [29] for equestrian helmets. Combinations of these four parameters yield 440 simulations. The material parameters used in the simulations are listed in Table 2.

Table 2. Material gradients with density ranges used for the simulations.

3.2 Description of FE model

The rigid striker and the rigid anvil are modelled as three-dimensional 4-node rigid elements (R3D4). The rigid anvil is set to be encastred (degrees of freedom = 0) and the rigid striker is constrained to move only along the vertical direction. The contacts between all surfaces are all frictionless contacts. The foam specimen is modelled as three-dimensional 8-node linear brick elements using reduced integration with hourglass control (C3D8R). The foam specimen has 50 elements through its thickness and 10 elements along its length and width. An explicit central-difference time integration rule is used to simulate the dynamic impact behaviour. The impact velocity is 5.425m/s in all simulations. The wave propagation speed, with zero Poisson's ratio, can be predicted by

$$c_d = \sqrt{\frac{E}{\rho}}$$

Using this equation, the wave speed of the EPS foam specimen of densities ranging from 14.2 kg/m^3 to 134.4 kg/m^3 varies from 414 m/s to 437 m/s. The maximum stable time increment is given by

$$\Delta t \approx \frac{L_{\min}}{C_d}$$

where L_{min} is the smallest element dimension in the mesh.

3.3 Results and discussion

The peak acceleration transmitted to a helmeted headform during an impact certification test, e.g. E.N. standard 1384:1996 [29] is an important pass / fail criterion for determining whether the helmet can be certified to reduce the risk of head injury, e.g. a helmeted jockey falling during a racing accident. Therefore, peak accelerations of the striker for all simulations are analysed below in order to provide guidance for designing cushioning structures. A typical acceleration curve of the striker, when impacting a uniform foam block and a linearly graded foam block, is illustrated in Fig. 6 (ρ_{aver} = 64 kg/m^3, $\Delta\rho$ = 40 kg/m^3, mass of striker = 6 kg). The shapes of the acceleration curves versus time for the other functionally graded foam blocks are similar to that for the linearly graded foam block. The acceleration increases quickly until approaching the peak value for the uniform foam, while it increases gradually for the graded foams, which results in longer time being required to absorb the same input kinetic energy. The duration for which acceleration is higher than 2000 m/s^2 (approximately 200g) in the uniform foam is about 2 ms, while it is about 1.4 ms in the graded foam.

Fig. 6. Acceleration of the striker as dropped on uniform foam block and linearly graded foam block (ρ_{aver} = 64 kg/m^3, $\Delta\rho$ = 40 kg/m^3, mass of striker = 6 kg)

The peak accelerations for all the simulations are listed in Table 3(a) to Table 3(e). The comparison of accelerations between various parameters can be summarised as:

a) *Influence of density gradient.* For low kinetic energies, the graded foam performs better than the uniform foam (e.g. ρ_{aver} = 44 kg/m^3, mass = 1 kg) and the convex gradients (e.g. quadratic) perform better than the concave gradients (e.g. square root). However, for high kinetic energies, an opposite trend is observed (e.g. ρ_{aver} = 44 kg/m^3, mass = 14 kg). These trends can be explained by referring to Fig. 1 and Table 1. It is illustrated that foam with a single density is most efficient at absorbing energy when it works within the plateau strain region, up to densification, as it absorbs most energy under large plastic strains with little corresponding increase in stress. Considering a foam with an average density of 44 kg/m^3 as an example, the stress applied to the foam at the time of peak acceleration is 198 kPa for a striker mass of 1 kg and is 581 kPa for a striker mass of 14 kg. Comparing these stress values with the plateau stresses of the uniform foams shown in Fig. 1 and Table 1, it can be observed that the uniform foam block of ρ_{aver} = 44 kg/m^3, with a yield stress of 310 kPa, will absorb very little of the kinetic energy from the 1 kg striker within the elastic region, transmitting the majority of the energy as a propagating stress wave. This will result in high accelerations. However, it absorbs the kinetic energy from the 14 kg striker within the plateau stress region up to

0.6 strain. The graded foams perform better than the uniform foam when absorbing the lower energies due to their spatially varying yield surface, a direct result of the density gradient. From Table 2, for example, the density of a quadratically varying foam will vary from 54.2 kg/m^3 to 14.2 kg/m^3. At 14.2 kg/m^3, local plastic deformation will initiate at about 100kPa (Table 1), deforming to almost 0.7 strain, and approximately 20% by volume (14.2 – 28 kg/m^3) of the graded foam will yield plastically under a stress of 200 kPa. This is in stark contrast to the equivalent uniform foam, which exhibits no yielding at this stress level.

As the kinetic energy of the striker is increased the advantage gained by a varying yield surface diminishes rapidly. Low yielding regions of the FGFM are no longer effective and local deformation beyond their densification strains occurs while mitigating only a small fraction of the total energy. Results show that a uniform 44 kg/m^3 foam experiences 0.54 strain at the incident surface and 0.52 strain at the distal surface when impacted by a 14kg striker at 5.425 m/s. In contrast, the quadratically varying FGFM deforms locally to only 0.2 strain at the incident, whereas 0.98 strain at the distal face. Intuitively, and from previous work [24], it is more advantageous for a foam's entire volume to deform up to, but not beyond, its densification strain if it is to act most effectively as a cushioning structure.

b) *Influence of density difference.* Graded foams with wide density ranges (large density difference) are more effective at reducing peak accelerations in low energy impacts compared to both uniform and graded foams with a smaller density range; this benefit is negated for higher energy impacts. Similar mechanics as those described above for a) can be given to explain this influence.

c) *Influence of average density.* The stresses in uniform foams at peak accelerations for all the simulations are listed in Table 4(a), while these stresses are normalised in Table 4(b) against the stress of the 44 kg/m^3 foam for the corresponding striker mass. It can be seen that stress increases as the average density increases for all values of kinetic energies. As the impact energy increases, the stress levels observed in the denser foams tend to decrease towards the level in the lowest density foam. For the lowest energy, all the peak stresses of uniform foams occur in the elastic region; foams of higher density exhibit higher stress due to increased stiffness; therefore the efficiency of energy absorption is reduced with increasing average density. As the kinetic energy increases, the peak stresses correspondingly increase to the plateau region and the difference in peak stress between different densities reduces. It is expected that the peak stress of the lower density foam will approach or even exceed that of higher density foam when the kinetic energy increases further and the foam of lower density enters the densification region. For a given density there is an optimum stress level for which the efficiency of a foam is maximised. This efficiency is given by [24]:

$$E = \frac{\int \sigma(e)de}{\sigma}$$

It follows that a foam will increase in efficiency up to a critical stress σ_c, below which, for an additional increment of strain, the ratio of incremental energy absorbed to incremental stress is greater than one. Beyond σ_c this ratio is less than one and the efficiency will reduce as the foam densifies. It is expected then that as the density of a foam increases, its σ_c will also increase, allowing for more efficient energy management at higher energy levels.

On introduction of the functionally graded foam, it is possible for the foam of higher average density to exhibit superiority to the foam of lower average density at lower energy. Such evidence can be found in Tables 3(a) and 3(b). For example, for mass = 1 kg and $\Delta\rho$ = 40 kg/m^3, quadratic (5032.05 m/s^2) and cubic (5102.51 m/s^2) gradients with ρ_{aver} = 54 kg/m^3 exhibit lower peak accelerations than the uniform foam with ρ_{aver} = 44 kg/m^3 (5377.11 m/s^2). This indicates that the functionally graded foam, properly designed, is efficient in energy absorption over a wider range of impact energy than the uniform foam.

Table 3(a): Peak accelerations (m/s^2) in impact tests (ρ_{aver} = 44 kg/m^3).

Table 3(b): Peak accelerations in impact tests (ρ_{aver} = 54 kg/m^3).

Table 3(c): Peak accelerations (m/s^2) in impact tests (ρ_{aver} = 64 kg/m^3).

Table 3(d): Peak accelerations (m/s^2) in impact tests (ρ_{aver} = 84 kg/m^3).

Table 3(e): Peak accelerations (m/s^2) in impact tests (ρ_{aver} = 104 kg/m^3).

Table 4(a). Stress in uniform foam block of different average densities at peak acceleration for various striker masses (kPa).

Table 4(b). Stress in uniform foam block of different average densities at peak acceleration for various striker masses normalised by the stress of the 44 kg/m^3 foam at the same striker mass.

It is difficult to recognise the performance difference between various gradients from the magnitudes of the peak accelerations due to various average densities as well as various impact energies. Therefore, the peak accelerations for each gradient are normalised by the peak acceleration generated by their equivalent uniform foam (with same average density). The normalised peak accelerations for each average density and each density difference are plotted as functions of impact energy and gradient function with proper interpolations, as shown in Fig. 7 and Fig. 8 (As some of the simulations for ρ_{aver} = 44 kg/m^3 and $\Delta\rho$ = 40 kg/m^3 prematurely failed and did not run to completion, these results are not included). Although the average densities in the sub-figures of Fig. 7 or Fig. 8 are different, consistency in the magnitudes of the normalised accelerations and consistency in the shapes of the plots can be observed: for $\Delta\rho = 20$ kg/m^3, all normalised values lie within the range 0.8 to 1.05, and for $\Delta\rho = 40$ kg/m^3, within the range 0.6 to 1.1. The general trends of these plots are: acceleration decreases at lower energies as the gradient changes from concave (logarithmic) to convex (cubic), while it increases slightly at higher energies; there are some exceptions for higher average densities (84 kg/m^3 and 104 kg/m^3) at lowest energy (14.71 J), as observed in Figs. 7(d), 7(e), and 8(d). One possible reason for the inconsistency is that the fluctuations of the acceleration curves, which only occur for foams of the higher average densities at lowest energy, bring difficulties in estimating the correct value of the peak acceleration.

Fig. 7. Normalised peak acceleration as functions of impact energy and gradient function ($\Delta\rho = 20$ kg/m^3).

Fig. 8. Normalised peak acceleration as functions of impact energy and gradient function ($\Delta\rho = 40$ kg/m^3).

4. Conclusions

A functionally graded polymeric foam model was proposed and its energy absorbing ability has been analysed using the finite element method. The influence of material distribution, controlled by various explicit gradient functions, material density range, and material average density, on energy absorption under the influence of various impact energies was studied. The simulation results constitute a valuable database for designing cushioning structures using functionally graded foams. The main findings can be summarised as:

- It is shown that a functionally graded foam can exhibit superior energy absorption over equivalent uniform foams under low energy impacts, and that convex gradients perform better than concave gradients. This advantage is negated when the impact energy becomes significantly high such that low density regions of the graded foam become ineffective at bearing the higher load and they densify after absorbing only a small fraction of the total energy. What constitutes a ‘high energy impact’ is somewhat difficult to define but will depend on the average density of the foam and the density gradient.

- For a specified density range the energy absorption performance of a functionally graded foam under low energy impacts can be improved if the density range is increased. For higher energy impacts, increasing the density range can reduce the performance of the graded foams due to a higher volume fraction deforming beyond the densification strain.
- Functionally graded foams are capable of reducing the duration of the high acceleration during an impact event. This property could have wide implications in the head protection industry as many head injury criteria (HIC [30], HIP [31], GAMBIT [32]) rely on acceleration durations as indicators of the likelihood for a person suffering significant head trauma. In this respect, protective headgear, e.g. safety helmets, employing functionally graded foams as the liner constituent may be advantageous to the wearer in reducing the risk of brain injury after a fall.
- Traditionally, many helmet certification standards (e.g.[29]) require a helmet to keep the acceleration of a headform dropped from a single drop height below some certain target level - achieving this is quite simple. However, recent helmet standards (e.g.[33]) demand that helmets be effective at multiple drop heights, thus simulating both high and low energy impacts. This can be more difficult to achieve with current helmet liner technologies. Functionally graded foams have been shown to exhibit significant advantages under low energy impact conditions while still performing nearly as well as their uniform counterpart under high energy conditions. These foams, carefully manufactured, may be one possible answer to the more stringent requirements of emerging helmet standards.

Acknowledgements

This study has been funded by Enterprise Ireland (PC/2005/071), Science Foundation Ireland (08/RFP/ENM/1169), and the Turf Club of Ireland.

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List of Figures

Table 1. Yield and plateau stresses for foams of different densities as obtained from the constitutive model.

Density (kg/m^3)	Yield stress (kPa)	Plateau stress (kPa)
10	70	70~150
20	140	140~300
30	210	210~450
40	280	280~600
50	350	350~660
64	450	450~930
84	590	590~1200
104	730	730~1600

Table 2. Material gradients with density ranges used for the simulations.

Gradients	Density Range (kg/m^3) $\Delta\rho$ = 20				
Uniform	44	54	64	84	104
Logarithmic	59.2-39.2	69.2-49.2	79.2-59.2	99.2-79.2	119.2-99.2
Square Root	57.3 -37.3	67.3-47.3	77.3-57.3	97.3-77.3	117.3-97.3
Linear	54 -34	64-44	74-54	94-74	114-94
Quadratic	50.8 -30.8	60.8-40.8	70.8-50.8	90.8-70.8	110.8-90.8
Cubic	49.2 -29.2	59.2-39.2	69.2-49.2	89.2-69.2	109.2-89.2
	Density Range (kg/m^3) $\Delta\rho$ = 40				
Uniform	44	54	64	84	104
Logarithmic	74.4-34.4	84.4-44.4	94.4-54.4	114.4-74.4	134.4-94.4
Square Root	70.6-30.6	80.6-40.6	90.6-50.6	110.6-70.6	130.6-90.6
Linear	64-24	74-34	84-44	104-64	124-84
Quadratic	57.5-17.5	67.5-27.5	77.5-37.5	97.5-57.5	117.5-77.5
Cubic	54.2-14.2	64.2-24.2	74.2-34.2	94.2-54.2	114.2-74.2

Table 3(a): Peak accelerations (m/s^2) in impact tests (ρ_{aver} = 44 kg/m^3).

Gradients	Density Range (kg/m^3)	Striker Mass (Kinetic Energy):							
		1kg (14.71J)	2kg (29.43J)	4kg (58.86J)	6kg (88.29J)	8kg (117.7J)	10kg (147.2)	12kg (176.6J)	14kg (206J)
Uniform	44	5377.11	2955.83	1715.78	1308.86	1120.49	1018.72	959.436	925.677
Logarithmic	59.2-39.2	4915.53	2836.18	1713.83	1332.05	1144.08	1039.39	977.322	942.245
Square Root	57.3 -37.3	4906.47	2834.22	1735.46	1345.34	1148.28	1042.1	980.883	945.262
Linear	54 -34	4632.68	2800.41	1750.68	1351.63	1157.11	1052.6	989.517	954.253
Quadratic	50.8 -30.8	4467.77	2891.08	1776.58	1359.64	1164.69	1055.67	993.785	956.965
Cubic	49.2 -29.2	4679.565	2951.16	1780.35	1363.61	1163.04	1052.74	989.792	954.47
Logarithmic	74.4-34.4	4404.87	2620.48	1674.49	1334.49	1170.93	1077.71	1023.41	989.503
Square Root	70.6-30.6	4222.66	2595.7	1694.55	1369.33	1202.72	1103.44	1042.07	1005.27
Linear	64-24	3715.96	2533.82	1749.07	1444.81	1270.61	1155.23	1076.29	1026.67
Quadratic	57.5-17.5	3648.88	2652.37	1875.76	Fail	Fail	Fail	Fail	Fail
Cubic	54.2-14.2	4067.82	2815.26	1934.67	Fail	Fail	Fail	Fail	Fail

Table 3(b): Peak accelerations in impact tests (ρ_{aver} = 54 kg/m^3).

Gradients	Density Range (kg/m^3)	Striker Mass (Kinetic Energy):							
		1kg (14.71J)	2kg (29.43J)	4kg (58.86J)	6kg (88.29J)	8kg (117.7J)	10kg (147.2)	12kg (176.6J)	14kg (206J)
Uniform	54	7889.1	4193.34	2319.22	1708.42	1406.16	1233.99	1118.37	1043.35
Logarithmic	69.2-49.2	7050.25	3987.25	2296.97	1717.54	1426.53	1250.73	1133.54	1058.14
Square Root	67.3-47.3	7017.75	4079.19	2319.38	1725.29	1428.82	1251.61	1138.13	1061.1
Linear	64-44	6568.13	3836.75	2313.85	1741.04	1432.59	1253.93	1144.69	1067.83
Quadratic	60.8-40.8	6596.08	3933.22	2366.6	1753.09	1443.92	1265.26	1148.14	1070.4
Cubic	59.2-39.2	6429.82	4025.79	2379.49	1746.41	1441.06	1262.92	1147.27	1068.32
Logarithmic	84.4-44.4	6622.27	3666.46	2199.82	1684.81	1421.72	1260.64	1156.42	1087.35
Square Root	80.6-40.6	6121.14	3587.88	2206.89	1703.41	1447.9	1288.12	1180.72	1104.34
Linear	74-34	5451.34	3395.65	2197.41	1742.42	1495.92	1336.25	1219.52	1135.84
Quadratic	67.5-27.5	5032.05	3363.19	2302.91	1843.95	1546.84	1350.27	1227.18	1142.47
Cubic	64.2-24.2	5102.51	3559.17	2420.36	1867.69	1529.14	1341.76	1212.31	1102.75

Table 3(c): Peak accelerations (m/s^2) in impact tests (ρ_{aver} = 64 kg/m^3).

Gradients	Density Range (kg/m^3)	Striker Mass (Kinetic Energy):							
		1kg (14.71J)	2kg (29.43J)	4kg (58.86J)	6kg (88.29J)	8kg (117.7J)	10kg (147.2)	12kg (176.6J)	14kg (206J)
Uniform	64	10436.45	5669.82	3078.63	2201.19	1769.09	1512.99	1346.32	1227.03
Logarithmic	79.2-59.2	9631.38	5422.33	3016.72	2189.09	1774.9	1520.11	1358.58	1239.91
Square Root	77.3-57.3	9482.29	5366.36	3028.55	2199.79	1782.89	1526.36	1363.97	1243.39
Linear	74-54	8838.41	4919.7	3003.34	2218.85	1792.09	1533.78	1365.74	1249.33
Quadratic	70.8-50.8	8476.05	5088.2	3032.91	2235.94	1798.72	1539.22	1368.97	1249.37
Cubic	69.2-49.2	8366.23	4849.32	3088.82	2234.63	1796.88	1540.8	1368.7	1250.37
Logarithmic	94.4-54.4	8768.07	4836.54	2853.32	2125.59	1748.46	1517.24	1362.51	1254.17
Square Root	90.6-50.6	8013.9	4886.41	2832.07	2134.1	1763.32	1538.76	1384.57	1275.67
Linear	84-44	7340.35	4244.31	2754.3	2123.69	1791.16	1573.38	1421.23	1303.37
Quadratic	77.5-37.5	6509	4272.96	2813.44	2222.3	1868.28	1619.33	1438.81	1315.35
Cubic	74.2-34.2	6455.33	4395.27	2926.61	2295.31	1890.3	1610.21	1430.58	1296.72

Table 3(d): Peak accelerations (m/s^2) in impact tests (ρ_{aver} = 84 kg/m^3).

Gradients	Density Range (kg/m^3)	Striker Mass (Kinetic Energy):							
		1kg (14.71J)	2kg (29.43J)	4kg (58.86J)	6kg (88.29J)	8kg (117.7J)	10kg (147.2)	12kg (176.6J)	14kg (206J)
Uniform	84	12471.3	8775.75	4933.71	3417.96	2691.09	2242.48	1942.94	1736.05
Logarithmic	99.2-79.2	12247.9	8318.51	4788.01	3373.05	2673.06	2232.49	1949.08	1738.93
Square Root	97.3-77.3	12117.7	8143.04	4648.88	3366.93	2676.86	2245.54	1950.72	1746.1
Linear	94-74	11871.2	7809.48	4683.26	3356.63	2637.8	2254.55	1957	1746.05
Quadratic	90.8-70.8	11762.4	7601.85	4602.395	3412.26	2699.97	2260.76	1962.65	1756
Cubic	89.2-69.2	11638.6	7611.33	4680.5	3455.38	2708.14	2266.06	1961.73	1756.32
Logarithmic	114.4-74.4	11807.4	7738.15	4498.36	3256.23	2591.22	2188.43	1927.82	1730.18
Square Root	110.6-70.6	11554.9	7296.24	4386.35	3198.94	2600.07	2197.68	1929.6	1744.24
Linear	104-64	10782.9	6746.8	4326.53	3142.25	2559.07	2200.29	1945.12	1758.72
Quadratic	97.5-57.5	9849.65	6418.04	4090.25	3164.9	2609.38	2267.83	1999.18	1801.96
Cubic	94.2-54.2	9464.09	6375.17	4224.41	3234.53	2698.2	2308.21	2019.43	1800.06

Table 3(e): Peak accelerations (m/s^2) in impact tests (ρ_{aver} = 104 kg/m^3).

Gradients	Density Range (kg/m^3)	Striker Mass (Kinetic Energy):							
		1kg (14.71J)	2kg (29.43J)	4kg (58.86J)	6kg (88.29J)	8kg (117.7J)	10kg (147.2)	12kg (176.6J)	14kg (206J)
Uniform	104	13554.8	10317.6	7176.02	4992.59	3865.91	3167.17	2717.28	2392.32
Logarithmic	119.2-99.2	13447.1	10265.8	6979.05	4889.69	3792.15	3146.05	2696.43	2390.08
Square Root	117.3-97.3	13465.1	10253.7	6789.36	4830.86	3785.39	3154.01	2708.69	2395.45
Linear	114-94	13488.9	10136.8	6539.65	4741.1	3791.7	3138.5	2715.35	2397.28
Quadratic	110.8-90.8	13543.1	10026.6	6526.63	4806.57	3772.53	3169.6	2734.85	2406.73
Cubic	109.2-89.2	13577.8	9983.88	6527.97	4771.74	3826.91	3178.01	2736.8	2411.35
Logarithmic	134.4-94.4	13371.3	10108.6	6412.12	4661.73	3663.32	3059.07	2655.42	2346.97
Square Root	130.6-90.6	13406.5	9905.71	6346.71	4531.5	3652.15	3044.56	2640.65	2355.29
Linear	124-84	13374.8	9389.75	5993.37	4395.56	3538.06	2998.3	2620.4	2349.04
Quadratic	117.5-77.5	13000.4	8863.48	5814.44	4357.46	3529.07	3045.91	2659.69	2398.39
Cubic	114.2-74.2	12910.3	8620.05	5684.41	4366.32	3598.76	3084.02	2720.77	2433.3

Table 4(a). Stress in uniform foam block of different average densities at peak acceleration for various striker masses (kPa).

ρ_{aver} (kg/m^3)	1kg (14.71J)	2kg (29.43J)	4kg (58.86J)	6kg (88.29J)	8kg (117.7J)	10kg (147.2)	12kg (176.6J)	14kg (206J)
44	238.98	262.74	305.03	349.03	398.40	452.76	511.70	575.98
54	350.63	372.74	412.31	455.58	499.97	548.44	596.46	649.20
64	463.84	503.98	547.31	586.98	629.01	672.44	718.04	763.49
84	617.15	788.21	877.10	911.46	956.83	996.66	1036.23	1080.21
104	684.45	990.89	1275.74	1331.36	1374.55	1407.63	1449.22	1488.55

Table 4(b). Stress in uniform foam block of different average densities at peak acceleration for various striker masses normalised by the stress of the 44 kg/m^3 foam at the same striker mass.

ρ_{aver} (kg/m^3)	1kg (14.71J)	2kg (29.43J)	4kg (58.86J)	6kg (88.29J)	8kg (117.7J)	10kg (147.2)	12kg (176.6J)	14kg (206J)
44	1	1	1	1	1	1	1	1
54	1.47	1.42	1.35	1.31	1.25	1.21	1.17	1.13
64	1.94	1.92	1.79	1.68	1.58	1.49	1.40	1.33
84	2.58	3.00	2.88	2.61	2.40	2.20	2.03	1.88
104	2.86	3.77	4.18	3.81	3.45	3.11	2.83	2.58