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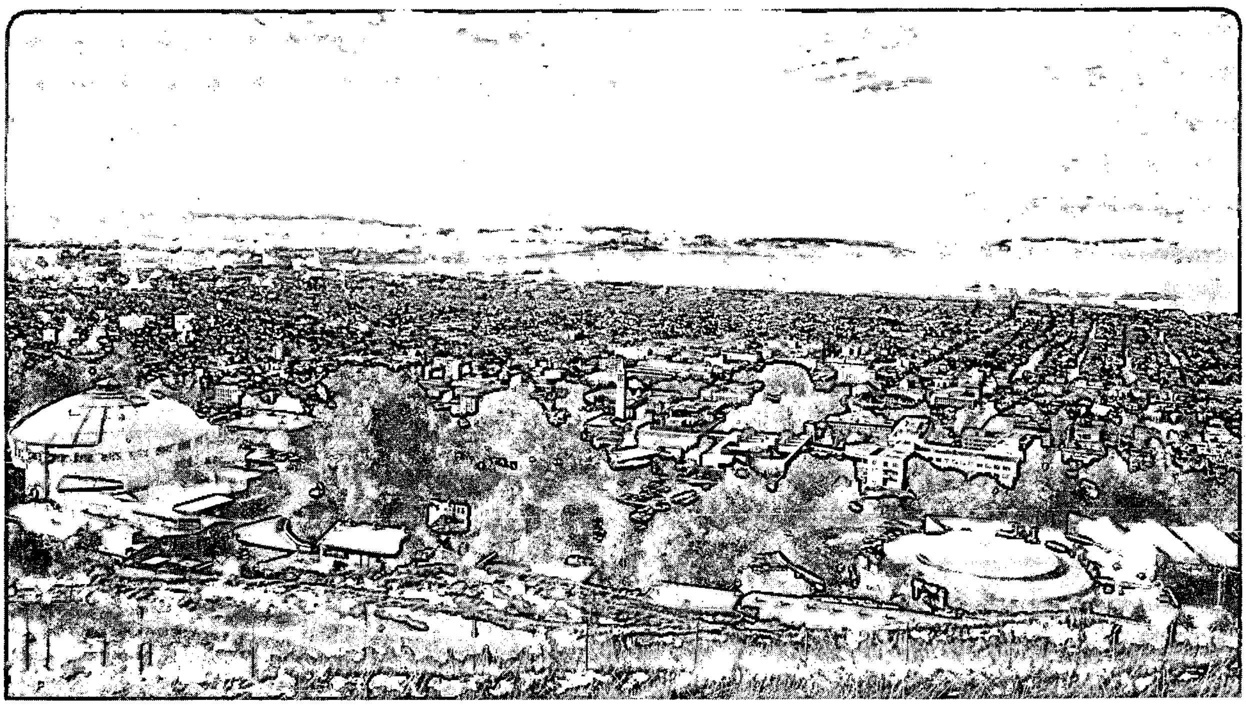
July 1994



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Detailed Characterization of a Fractured Limestone Formation Using Stochastic Inverse Approaches

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ABSTRACT

We discuss here two inverse approaches to construction of fracture flow models and their application in characterizing a fractured limestone formation. The first approach creates 'equivalent discontinuum' models that conceptualize the fracture system as a partially filled lattice of conductors which are locally connected or disconnected to reproduce the observed hydrologic behavior. An alternative approach viz. 'variable aperture lattice' models represent the tracture system as a fully filled network composed of conductors of varying apertures. The fracture apertures are sampled from a specified distribution, usually log-normal consistent with field data. The spatial arrangement of apertures is altered through inverse modeling so as to fit the available hydrologic data.

Unlike traditional fracture network approaches which rely on fracture geometry to reproduce flow and transport behavior, the inverse methods directly incorporate hydrologic data in deriving the fracture networks and thus naturally emphasize the underlying features that impact the fluid flow and transport. However, hydrologic models derived by inversion are non-unique in general. We have addressed such non-uniqueness by examining an ensemble of models that satisfy the observational data within acceptable limits. We then determine properties which are shared by the ensemble of models as well as their associated uncertainties to create a conceptual model of the fracture system.

INTRODUCTION

One recent approach to characterizing fractured rocks has been to generate statistically identical realizations of fracture networks based on data about fracture geometry and hydraulic apertures. These models represent fractures as conductive line segments or planes that are placed in space either deterministically or according to some stochastic rules.[1] Fluid flow can then be modeled through the resulting network. We call this the 'forward approach' which implicitly assumes that reproduction of fracture geometry will lead to reproduction of flow and transport behavior through the fractures. However, there is ample field evidence which indicates that often only a few fractures are hydraulically active and fluid flow may be dominated by extreme value attributes of the fractured media.[2]

The inverse approach to characterization of

References and illustrations at end of paper.

fracture zones uses fluid flow and transport information such as pressure transients from interference tests and tracer breakthrough curves directly to generate a network of conductors for subsequent analysis. Thus, in this approach, we focus on determining the hydrologic characteristics of the fracture zones conditioned by the geometry of fractures rather than letting predicted geometry predict hydrology.[3] Details of the fracture geometry play a secondary role here. However, heterogeneity at different scales can be reproduced by incorporating data from a variety of sources: cores, logs, seismic traces, well tests and tracer tests etc.

In this paper, we discuss application of two inverse approaches to characterize a fractured limestone formation (the Fort Riley formation) at the Conoco borehole test facility in Kay County, Oklahoma.[4] A series of geophysical (vertical seismic profile survey, cross-well survey) and hydrological (interference) tests were performed in a skewed five-spot pattern to identify the distribution and orientation of preferential flow paths. The pressure data collected at the producing and observation wells have been inverted to generate equiprobable patterns of fracture networks using two different approaches -- an 'equivalent discontinuum' approach and a 'variable aperture lattice' approach. The flow models derived by inversion were then verified through prediction and cross validation.

The inverse approaches described here have the advantage that they can incorporate flow as well as transport data directly in deriving the fracture networks. Thus the approach naturally emphasizes the underlying features that impact the fluid flow and transport. However, a network of flow channels, which may be rather elaborate, is likely to be poorly constrained by limited data. Thus, hydrologic models derived by inversion are likely to be non-unique. We have addressed such non-uniqueness by shifting the focus of the problem from the search for a single model that fits the data best to inferences about properties which are shared by the ensemble of acceptable models. We can then determine a most likely model and quantify the associated uncertainties.[5]

INVERSE MODELING AND THE MAXIMUM LIKELIHOOD ESTIMATE

Let us assume that we have a set of M independent observed data, $\mathbf{d_{obs}}$: $[d_1, d_2 .....d_M]$ along with associated uncertainties σ_i's. The maximum likelihood approach to parameter estimation attempts to maximize the likelihood of the data given an underlying model, $g(\mathbf{m})$. Since the measurements have been assumed to be independent, their joint probability distribution will be given by

$$P(\mathbf{d_{obs}}|\mathbf{m}) = P(d_1|\mathbf{m}).....P(d_M|\mathbf{m}) = \prod_{k=1}^{M} P(d_k|\mathbf{m}) \quad (1)$$

where $P(d_i|\mathbf{m})$ is the probability of observing the i-th data given the parameter vector $\mathbf{m}$. If we further assume that the residuals (the sum of measurement and computation errors) are Gaussian, then the likelihood function , $L(\mathbf{m}|\mathbf{d_{obs}})$ can be approximated as follows:[6]

$$L(\mathbf{m}|\mathbf{d_{obs}}) \sim P(\mathbf{d_{obs}}|\mathbf{m})$$

$$= 1/\sqrt{(2\pi)^M |\mathbf{C_D}|}\ \exp[-\rho(\mathbf{d_{obs}}, \mathbf{m})] \quad (2a)$$

where,

$$\rho(\mathbf{d_{obs}}, \mathbf{m}) = 1/2[\mathbf{g(m)} - \mathbf{d_i}]^T \mathbf{C_D^{-1}}[\mathbf{g(m)} - \mathbf{d_i}] \quad .(2b)$$

In Eq. 2b $\mathbf{C_D}$ is the covariance matrix describing the measurement errors. Typically, one attempts to maximize the log-likelihood function and this leads to the minimization of the following functional assuming that the measurement errors are uncorrelated,

$$\chi^2 = \sum_{i=1}^{m} \left\{\frac{d_i - g(\mathbf{m})}{\sigma_i}\right\}^2 . \quad (3)$$

The magnitude of this quantity is a measure of the misfit of the model. For normally distributed residuals, the sum of squares follow a chi-squared distribution. Hence, the probability that the χ^2 exceeds a given value, $P(\chi^2 > \alpha)$ may be computed.

For continuous parameters, a variety of approaches can be used for minimization of Eq. 3. These include Levenberg-Marquardt modification of the Gauss-Newton minimization or conjugate gradient algorithms together with adjoint state methods for computing the gradients.[7] However, such gradient based methods may not be suitable for fracture zone characterization since there may be isolated regions leading to discontinuities in pressure and thus, flow. We circumvent the problem by using a derivative-free minimization approach known as simulated annealing.[8,9]

In inverse modeling, most often one seeks the model which maximizes the likelihood of the data, i. e. minimizes the misfit to the data. However, given the uncertainties associated with a set of measurements, it is probable that many sets of models may fit the data within some specified tolerance. As mentioned, a network of flow channels is likely to be poorly constrained by limited data. Thus, it may not be informative to seek a single model. Instead, it may be best to generate a large collection or ensemble of models approximately satisfying the data. That is, models for which some measure of misfit, say χ^p misfit, is small. When a sufficient number of models have been accumulated, various statistical quantities may be extracted from the ensemble.

FRACTURE ZONE CHARACTERIZATION THROUGH INVERSE MODELING

We represent the fracture zone as a network of one dimensional conductors having either fixed or variable apertures. The network itself may be partially or fully connected. Depending upon the convention used, the fracture zone models are termed 'equivalent discontinuum' or 'variable aperture lattice' model. The steps involved in generating these models are discussed below.

Equivalent Discontinuum Models

These models represent the fracture zone as a network of partially connected conductors having equal apertures and hence, conductivity. Equivalent discontinuum models are derived starting from a specified lattice or template of conductors. The approach involves searching for a configuration of conductors that will satisfy observed data. We have used a derivative-free optimization scheme called simulated annealing for inversion of pressure transient data. An objective function is defined to reflect the misfit between the data and the predicted response as follows:

$$E = \sum_{j=1}^{nw} \sum_{i=1}^{nd_j} [\log\{p_{j,obs}(t^i)\} - \log\{p_{j,cal}(t^i)\}]^2 \qquad (4)$$

where nw is the total number of wells and nd_j is the number of pressure data recorded at the j-th well.

The algorithm proceeds by choosing a lattice element at random and if the element is present or 'on', it is turned off and vice versa. The change in misfit, also known as energy, is computed due to the perturbation. If the energy decreases, then the perturbation is accepted; otherwise the perturbation is accepted with a probability $P(\Delta E) = \exp(-\Delta E/T)$ where T is analogous to temperature in the Gibbs distribution. By allowing to accept changes which result in an increase in energy, the simulated annealing approach to optimization provides a mechanism of *probabilistic hill climbing* which allows the method to escape from local minima.[8] The minimization of Eq. 4 can be speeded up substantially by shifting the computed pressure response along the time axis (*X-shifting*) and/or pressure axis (*Y-Shifting*) dynamically during optimization in order to obtain a better match with the data. Such *X-Shifts* and *Y-Shifts* correspond to scaling of the transmissivities and storativities of the fracture elements and are analogous to type curve matching used in well test analysis.[10]

Once the initial template has been 'annealed' to a sufficiently low energy level, a configuration of conductors representing a conceptual model for the fracture zone is obtained. Although under the maximum likelihood hypothesis discussed before, such a model maximizes the probability of observing the given data set, it is non-unique and there are other possible configurations that will satisfy the limited data. By examining an ensemble of possible configurations, we can arrive at a conceptual model that incorporates the underlying features shared by all such models.

Variable Aperture Lattice Models

In this approach, the fracture zone is represented as a network of fully connected conductors having variable apertures and hence, conductivities. The optimization problem consists of searching for a spatial pattern of apertures that satisfy the available data. The steps involved are similar to the ones in constructing equivalent discontinuum models. However, instead of simply turning on or off, the conductors are assigned apertures sampled uniformly from a specified aperture distribution. We have chosen a log-normal distribution of apertures specified by a mean log-aperture μ_{ln} and a variance of log-apertures, σ_{ln}^2 as follows:[11]

$$p(b) = (2\pi\sigma_{\ln}^2)^{-1/2} \exp[-1/2 (\ln b - \mu_{\ln})^2/\sigma_{\ln}^2]. \quad (5)$$

The procedure consists of randomly selecting an element and assigning an aperture sampled from the specified distribution. One of the advantages of such an approach is that we can sample apertures from different distributions at different spatial locations and thus, incorporate a regionally varying aperture distribution that is more consistent with geology. The same acceptance criteria discussed for the 'equivalent discontinuum' models also apply here. The variable aperture approach generates preferential flow paths by selectively placing high apertures and thus creates a set of variable aperture channels. Such variable aperture channel models have been successfully used to interpret field tracer data by Tsang et al.[12]

WELL TESTS AND PRESSURE DATA

The inverse approaches discussed above have been applied to a set of interference test data from the Fort Riley formation underlying the Conoco bore hole test facility located in Kay county, Oklahoma. The facility consists of five shallow wells (about 45 meters in depth), GW-1 through GW-5 as shown in Fig. 1. The wells were drilled in a skewed five-spot pattern to provide maximum azimuthal coverage for both seismic and hydrologic experiments. Several well tests were conducted out of which two were chosen for detailed analysis. These tests will be denoted as Pump58 and Pump27.

During the test Pump58 water was produced from the well GW-5 and pressure responses were observed at the wells GW-1, GW-2, GW-3 and GW-4. The pumping rate was fairly constant during this test, starting at about 0.5 gpm and dropping to about 0.46 gpm by the end of the test. Background data collected before the beginning of the test indicated that the wells were recovering from rainfall when the test was started; hence, the drawdown data was corrected for rainfall before further analysis. This correction was accomplished by simply subtracting the additional drawdown due to the recovery from the rainfall. The final drawdown and recovery curves during Pump58 are shown in Fig. 2a.

The test Pump27 immediately followed test Pump58. During this test, water was produced from well GW-2 and pressure responses were observed in wells GW-1, GW-3, GW-4 and GW-5. The pumping rate during this test varied considerably, starting with about 0.40 gpm and gradually decreasing to 0.23 gpm by the end of the test. This decrease most likely was caused by progressive clogging of a water filter in the flow stream. The filter was installed to protect the flow meter. The drawdown and recovery curves during Pump27 are shown in Fig. 2b.

DEVELOPMENT OF CONCEPTUAL MODELS BY STOCHASTIC INVERSIONS

We discuss here numerical inversion of the transient pressure response at the wells during the tests Pump58 and Pump27. A channel model consisting of a network of one-dimensional conductors was used to simulate the well tests.[13] The flow field was obtained using a Galerkin finite element method. The basic template used for inversion is shown in Fig. 3. It consists of two regions -- an inner dense region to obtain greater resolution of the flow field and a coarse outer region. A sensitivity study was performed to establish the distance to the boundary to have minimal impact of the boundary conditions. A constant head outer boundary conditions was then imposed on all four sides. The inner region where the wells are located has an element spacing of 7 meters and the spacing was doubled in the outer region adjacent to the boundary. During the inversion, a criterion was imposed whereby the probability of altering an element decreased exponentially with distance beyond the inner region where the wells are located.

Discontinuum Models

As discussed before, the discontinuum models are created by randomly selecting elements and turning them on or off to minimize the difference between computed and observed values. Fig. 4 shows the matching of drawdown data from the test Pump58. The dotted lines correspond to pressure response from the initial template as shown in Fig. 3. For the producing well GW-5, early time data were excluded from matching since they were affected by wellbore storage. Overall, the inversion scheme is able to reproduce the drawdown data reasonably well. The early pressure response observed at well GW-2, the farthest well from the producing well, is indicative of a preferential flow path or fracture in that direction. The final configuration of elements obtained after inversion is shown in Fig. 5a where we have focused on the inner dense region containing the wells. The fracture patterns appear to suggest the presence of a direct pathway extending from well GW-5 to GW-2 to the north of GW-3, accounting for the early response in well GW-2. On the other hand, well GW-3, the nearest well to the pumping well GW-5 is connected to it by a long tortuous pathway resulting in a lower drawdown.

A sensitivity study was carried out to examine the impact of grid selection by choosing two alternative grids -- one oriented along the line joining GW-5 to GW-2 and the other consisting of triangular patterns. By aligning the grids along the major flow paths and allowing diagonal transport, any grid orientation effects resulting from rectangular grids should be minimized. The fracture patterns obtained from these alternative grids are shown in Figs. 5b and 5c. Again, we observe the same broad features as in Fig. 5a. A direct path between GW-5 and GW-2 to the north of well GW-3 is present in all three fracture patterns. The rest of the producing wells appear to be separated from the pumping well by long tortuous paths. Fig. 6 compares the convergence behavior for the three grid types. The triangular grid appears to work the best for this example.

For inversion of the pressure data from the test Pump27, we used an average injection rate of 0.3 gpm throughout the duration of the test. The results of the inversion are shown in Fig. 7. The fracture pattern derived from this inversion is shown in Fig. 8a. Again we observe a direct pathway between the wells GW-2 and GW-5 to the north of GW-3. On comparing Figs. 5a and 8a, we observe that both the patterns exhibit sparse fracture density in the vicinity of the central well GW-3 resulting in the lower drawdown observed during the tests. We also carried out an inversion whereby annealing was performed simultaneously on the drawdown data from both the tests discussed above. Such co-annealing is computationally intensive but should help better constrain the inverse problem. The final configuration resulting from the co-annealing is shown in Fig. 8b which shares many of the features common to the individual inversions.

In summary, the fracture patterns emerging from all the inversions exhibit a direct pathway between the wells GW-2 and GW-5. The pathway consistently appears to the north of the well GW-3 and can explain the early pressure response observed in these wells. The rest of the wells viz. GW-1, GW-3 and GW-4 appear to be connected to the producing wells through long tortuous pathways resulting in the lower drawdowns observed in these wells.

Variable Aperture Lattice Models

These models were generated by sampling apertures uniformly from a log normal distribution with a specified mean and variance. A mean aperture of 0.00065 m was chosen using cubic law based on the parameters estimated using analytical models.[14] The convergence of simulated annealing was found to be quite sensitive to the selection of log aperture variance and a value of 0.5 worked the best for both Pump58 and Pump27. The spatial pattern of apertures obtained from the inversions are shown in Figs. 9a and 9b where we have removed the mean aperture. Although the individual realizations obtained from inversions appear to be quite noisy, a clustering of high apertures forming a preferential flow path between GW-5 to GW-2 to the north of well GW-3 is apparent in these results.

ENSEMBLE STUDIES

The ensemble analysis in this section is an attempt to understand characteristics shared by the solutions of the inverse problem and provide a more rigorous and complete characterization of the inherent non-uniqueness contained within the problem. The approach is to generate and describe a collection of models that 'fit the data' in some sense and deduce inferences about properties which are shared by the ensemble of acceptable models.

A series of inversions were performed on the pressure data from the test Pump58. Specifically, several annealing runs were conducted to generate multiple discontinuum as well as variable aperture lattice models and concluded when the misfit was reduced to a specified level. When a sufficient number of models were accumulated, various statistical quantities were extracted from the ensemble. Fig. 10 shows the energy vs. iterations for twenty equivalent discontinuum models. The triangular grid was used for these inversions and Fig. 11a shows the median model derived from the ensemble. The ensemble median model from variable aperture annealing of test Pump58 is shown in Fig. 11b. Again, we have removed the mean aperture from Fig. 11b. On comparing with Fig. 9a, which is a member of this ensemble, the median model appears much less noisy and the preferential flow paths are more clearly discernible here. Figures 11a and 11b highlight characteristic features shared by several solutions to the inverse problem and appear to indicate the presence of a dominant fracture connecting GW-2 and GW-5 to the north of well GW-3. The fracture patterns here are consistent with a scaling of the pressure data from test Pump58 as shown in Fig. 12 where we have plotted drawdown vs. t/r^2 where r is the distance from the producing well. For a homogeneous medium, all the drawdown curves would have collapsed into a single curve. However, the results indicate two distinct flow regimes/heterogeneity types -- one between GW-2 and GW-5 and the other comprising of the remaining wells.

The hydrologic work discussed so far is being performed in conjunction with geophysical experiments in order to generate an integrated description of the fracture pattern at the Conoco borehole test facility. One such experiment is a reverse VSP survey in which a source was placed in well GW-3 and the resulting seismic waveforms were recorded by an arch of sensors at the surface located at 20 degree increments around the well. The sensors were placed at a distance of approximately 8 m from the well. A preliminary analysis of the reverse VSP data suggest the presence of a macrofracture to the north of well GW-3 as inferred from the hydrologic analysis.[15] Also, the orientation of the macrofracture derived from the geophysical analysis is consistent with the hydrologic model obtained from the ensemble analysis.

It is interesting to examine the characteristics of the individual models obtained by inversion in relation to the ensemble statistics. For this, we compared moving window semivariance estimates of individual models with the ensemble mean and median. The moving window semivariance describes the variance of a moving window of size h as a function of h (scale) and is defined as follows :[16]

$$\gamma_N(h) = \frac{1}{\int_0^h \xi^{d-1}\, d\xi} \int_0^h \xi^{d-1} \gamma(\xi)\, d\xi \qquad (6)$$

where d is the dimension in Euclidean space and $\gamma(h)$ is the classical semivariance estimator defined as follows:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z(x_i) - z(x_i + h)]^2. \qquad (7)$$

In Eq. 7, $z(x_i)$ is the data value at x_i and N(h) is the number of data pairs for lag distance h. The moving window semivariance estimator has been shown to be robust and superior to other such estimators and can be used to examine the 'scale effects' (variance vs. scale) of heterogeneity. Fig. 13 shows the histograms of semivariances of individual models obtained by inversion using the 'variable aperture lattice' approach. The semivariances from the ensemble median model are also superimposed in the same figure using asterisk symbols. The results indicate that whereas the individual models obtained by inversion are quite noisy, such small scale fluctuations disappear in the ensemble median model. Thus the underlying structure can be inferred more easily through the ensemble analysis.

Prediction of Build-up Data

In order to verify the conceptual models derived through inversion, an attempt was made to predict the build-up data using these models. Only the data from the test Pump58 was used for this purpose due to lack of sufficient build-up data from the other test. The data from Pump58 was affected by rainfall just before the beginning of the build-up phase. Fig. 14 shows the data versus predictions using discontinuum models. The predicted pressure recovery using these models appear to be slower than the observed data. However, the early recovery of the field data can be mainly attributed to the rainfall.

SUMMARY AND CONCLUSIONS

In this paper we have discussed two different approaches to characterization of fractured formations based on inverse methods, and their application to a fractured limestone formation using a set of interference test data. Numerical inversions have been carried out to build conceptual models for the Fort Riley formation based on both 'equivalent discontinuum' and 'variable aperture lattice' approaches. We have addressed the non-uniqueness associated with such inverse methods by focusing on the common features shared by an ensemble of models rather than trying to obtain a single model that fits the data the best. Our results suggest the following conclusions:

- Both 'equivalent discontinuum' and 'variable aperture lattice' approaches were successful in reproducing the transient pressure behavior at the pumping and observation wells and appear to be viable methods for characterizing fractured formation.

- The conceptual models for the Fort Riley formation derived based on both inverse approaches indicate the presence of a preferential pathway between wells GW-2 and GW-5 to the north of well GW-3. The results are consistent with scaling of the pressure data and a preliminary analysis of geophysical (reverse VSP) data.

- Whereas the individual models obtained by inversion were found to be noisy and non-unique, an ensemble analysis from multiple inversions resulted in a much clearer picture of the underlying heterogeneity. This was further verified through a comparison of moving window semivariance estimates of individual models with that of the ensemble median model.

- An attempt was made to verify the conceptual models by predicting the build data during the test Pump58. The field data were affected by rainfall; however, the conceptual models appeared to generate the expected trend.

NOMENCLATURE

b	Fracture Apertures
$\mathbf{C_D}$	Covariance Matrix for Measurement Errors
$\lvert\mathbf{C_D}\rvert$	Determinant of $\mathbf{C_D}$
d_i	i-th Observed Data
E	Energy or Objective Function
$g(\mathbf{m})$	Forward Model Describing Flow and Transport
$L(\mathbf{m} \lvert \mathbf{d_{obs}})$	Likelihood Function
$\mathbf{m}$	Model Vector(Apertures or Conductivities)
nd_j	Number of Data at the j-th Well
nw	Number of Wells
$p(b)$	Density Function for Apertures
$p_{j,obs}$	Observed Pressure
$p_{j,cal}$	Calculated Pressure
$P(d_i \lvert \mathbf{m})$	Conditional Probability for data d_i
T	Temperature or Control Parameter for Simulated Annealing
χ^2	Measure of Misfit
$\gamma(h)$	Semivariance for lag h
m_{ln}	Mean log-Aperture
σ_i	Measurement Error for d_i
σ_{ln}	Standard Deviation of log-Apertures

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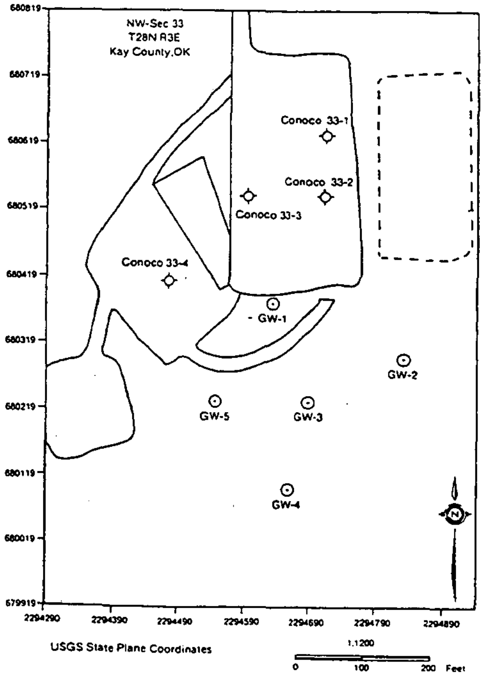

Fig. 1 Conoco Borehole Test Facility.

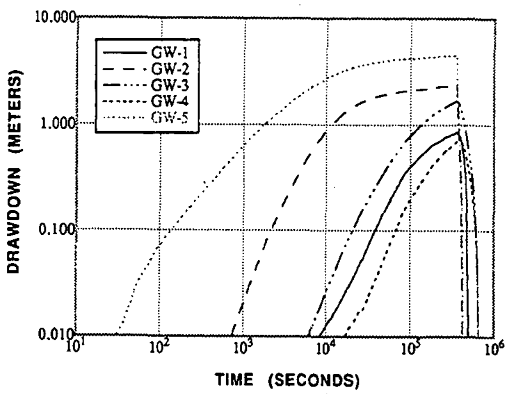

Fig. 2a Drawdown and Recovery Data During Test Pump58

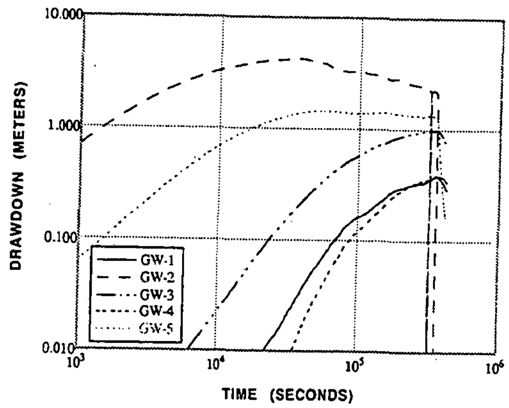

Fig. 2b Drawdown and Recovery Data During Test Pump27

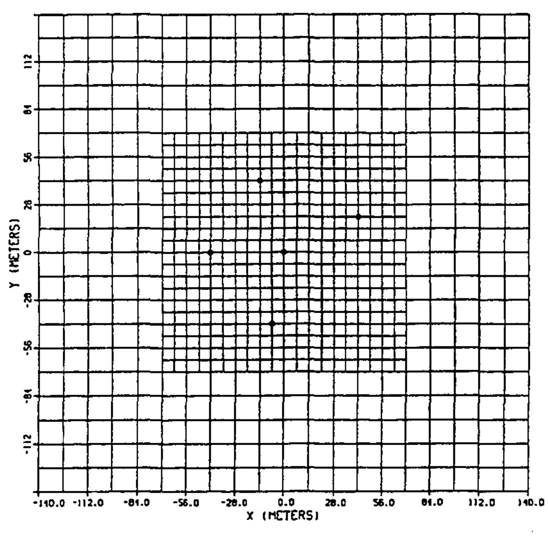

Fig. 3 Basic Template for Data Inversion.

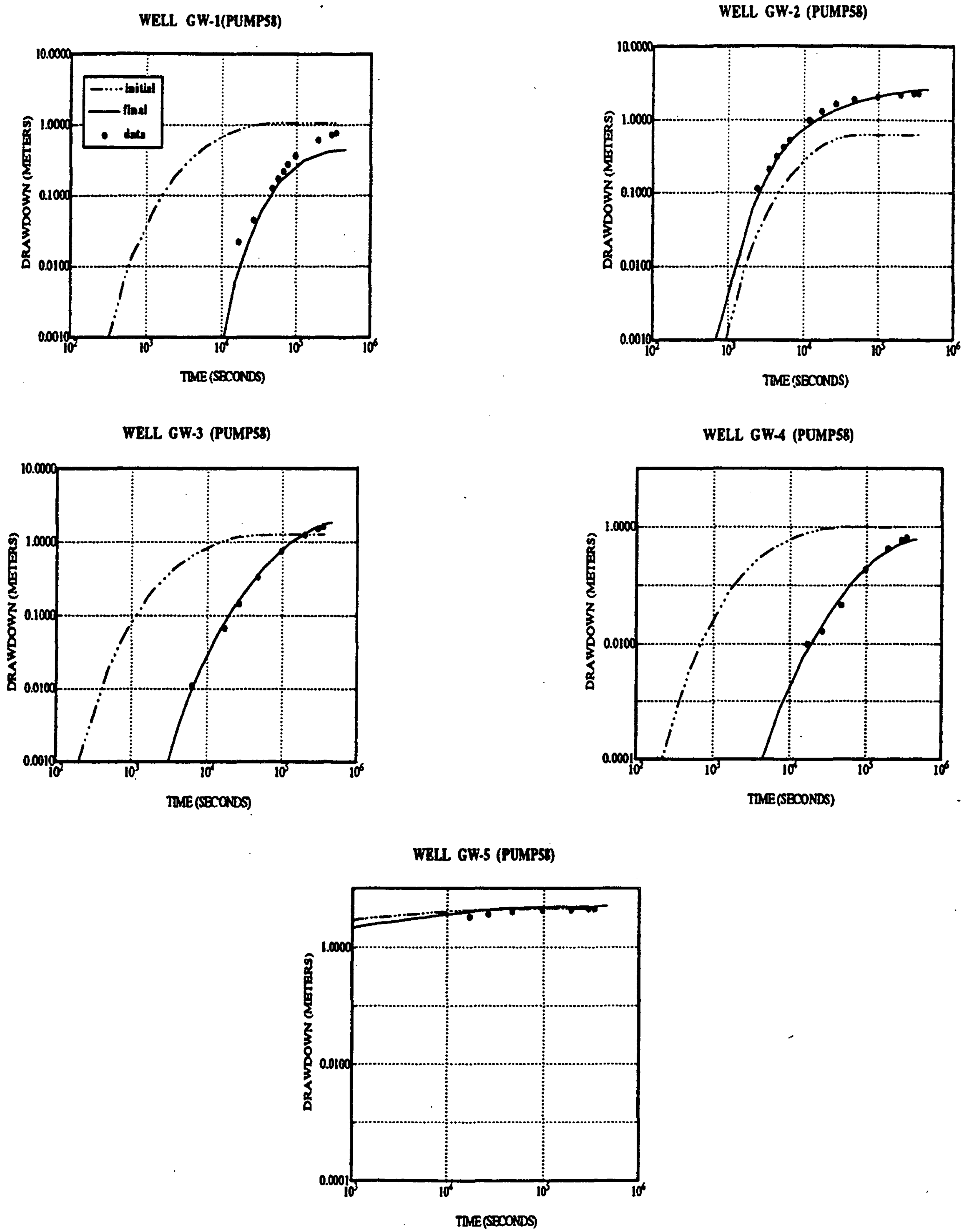

Fig. 4 Matching of Drawdown Data From the Test Pump58.

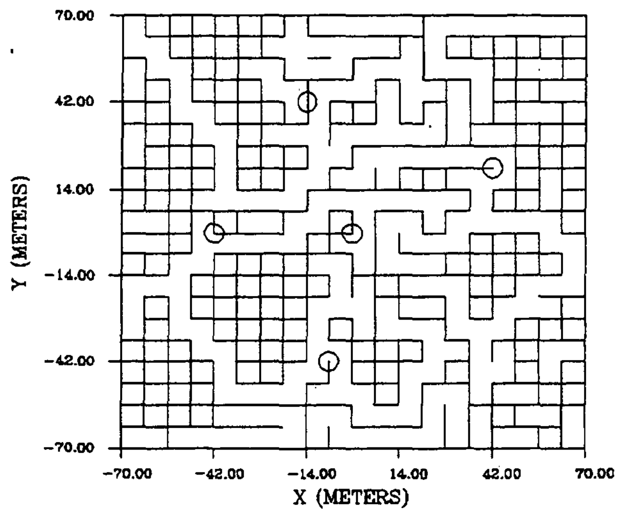

Fig.5a Dicontinuum Model From Inversion of Test Pump58.

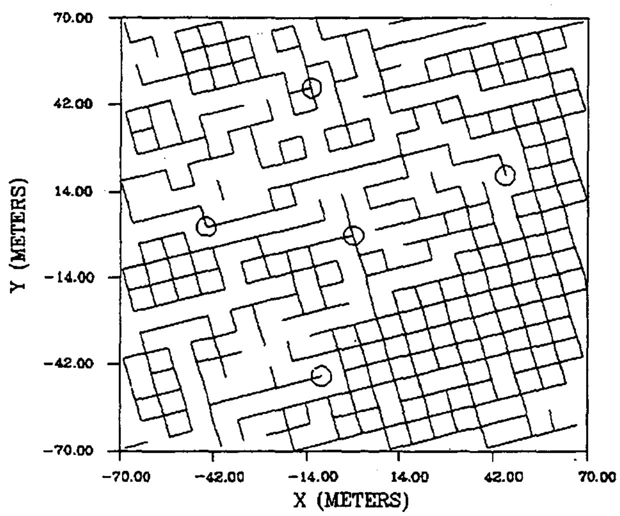

Fig. 5b Discontinuum Model Using Oriented Grids.

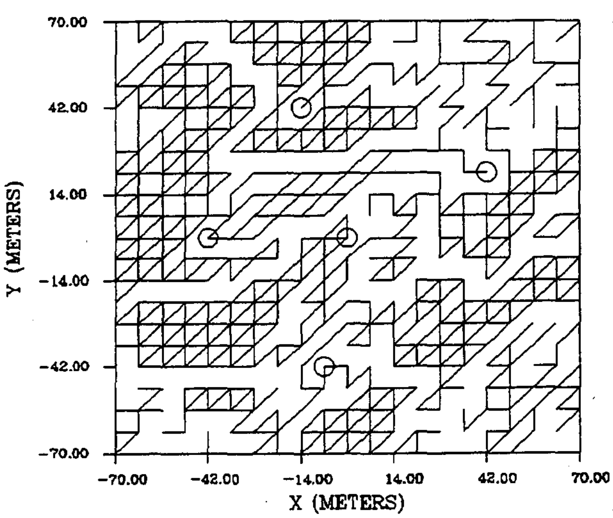

Fig. 5c Discontinuum Model Using Triangular Grids.

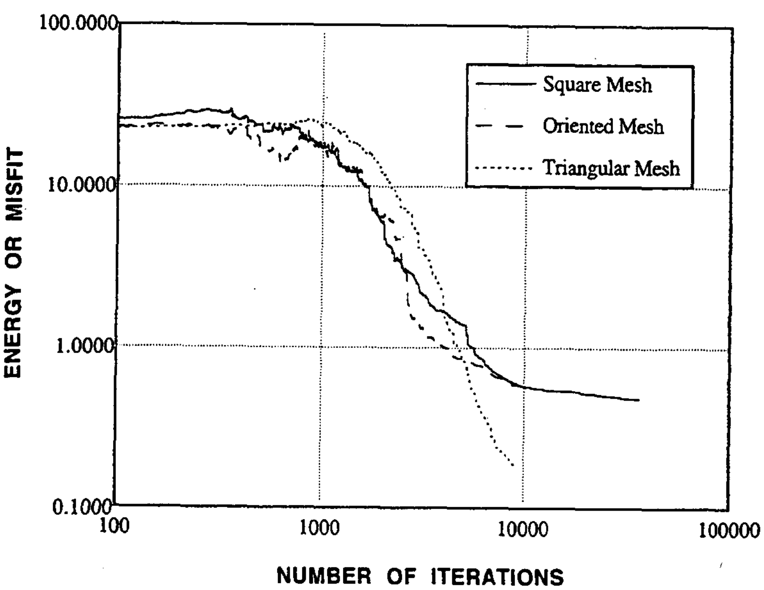

Fig. 6 Sensitivity to Grid Selection

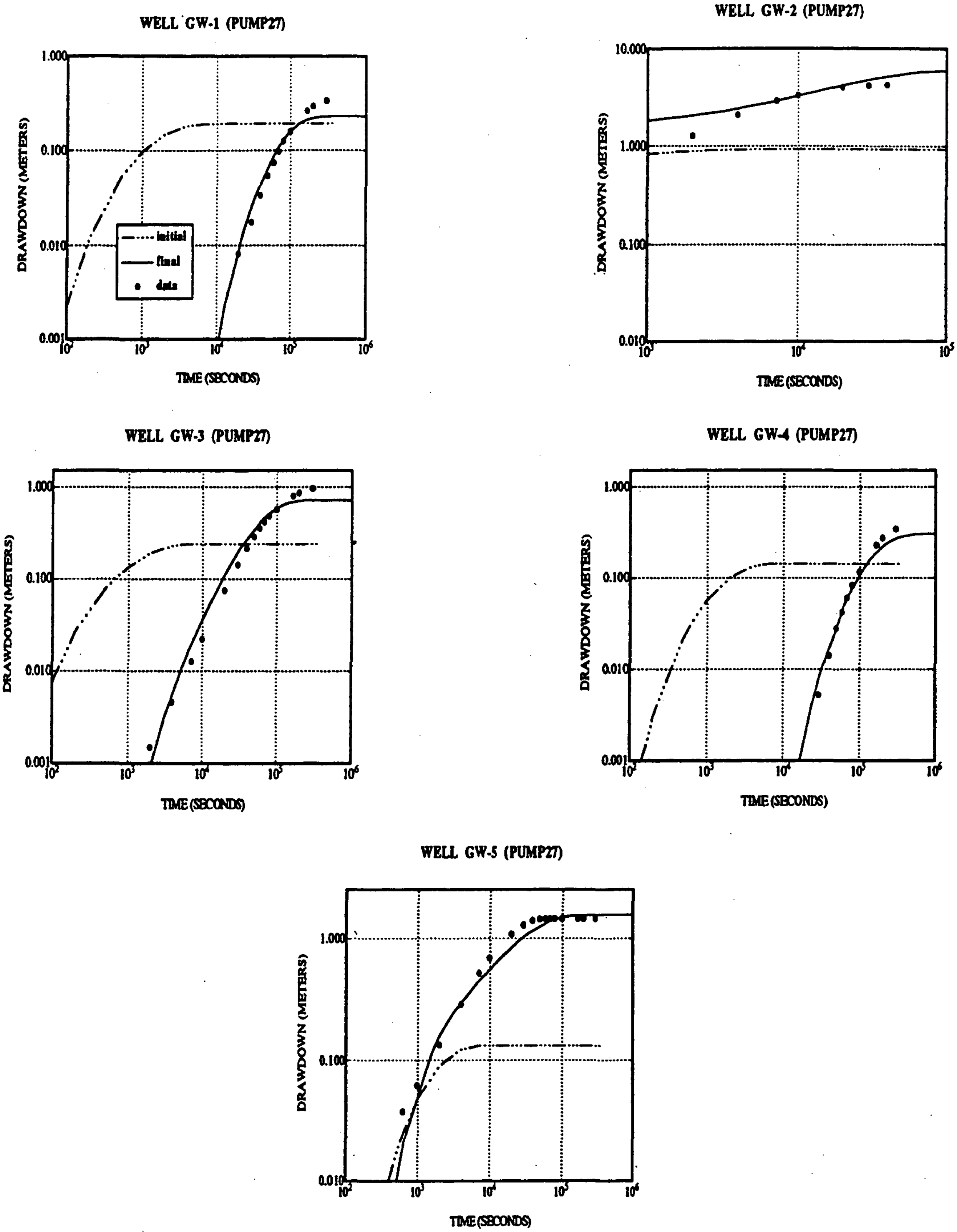

Fig. 7 Matching of Drawdown Data From the Test Pump27.

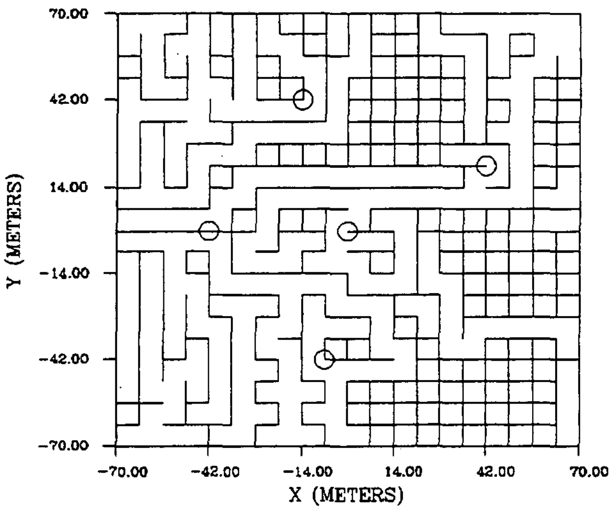

Fig. 8a Discontinuum Model From the Inversion of Test Pump27.

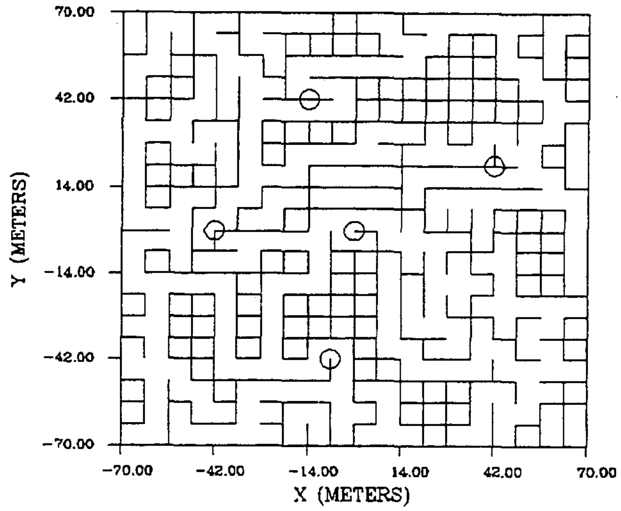

Fig. 8b Discontinuum Model From the Simultaneous Inversion of Tests Pump58 and Pump27.

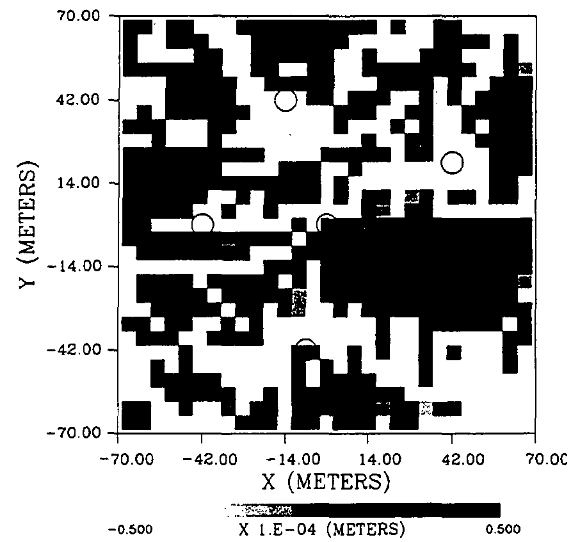

Fig. 9a Variable Aperture Model Using Test Pump58.

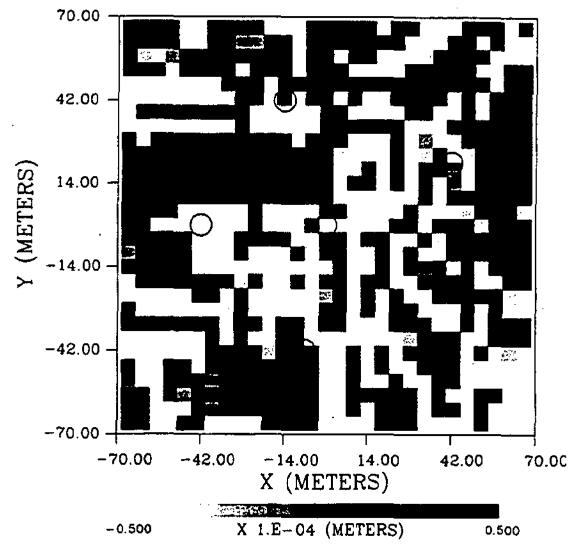

Fig. 9b Variable Aperture Model Using Test Pump27.

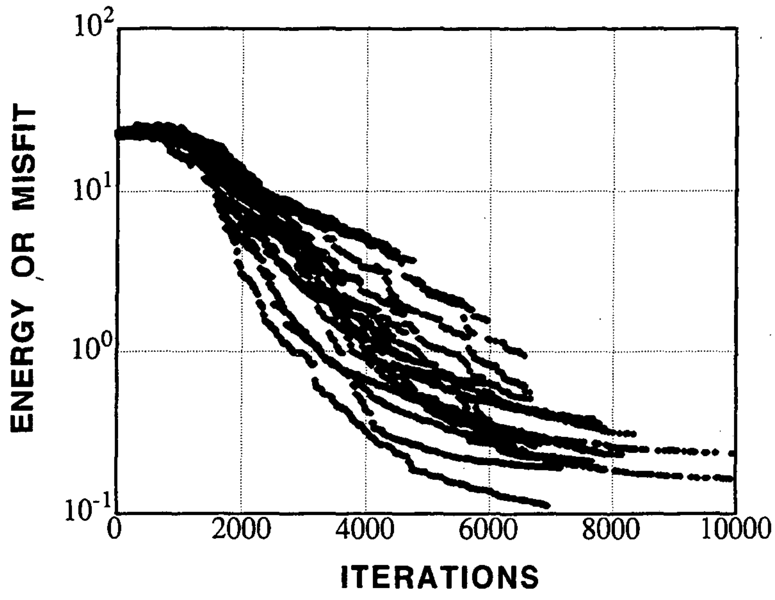

Fig. 10 Energy or Misfit vs. Iterations From an Ensemble of Inversions of Test Pump58.

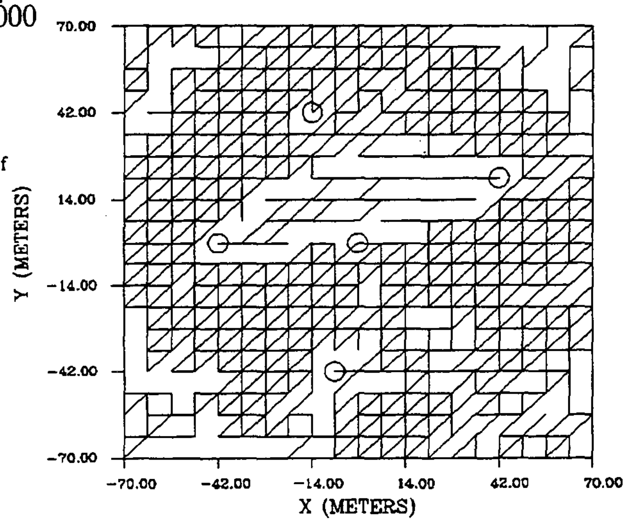

Fig. 11a Ensemble Median Model Using the Equivalent Discontinuum Approach.

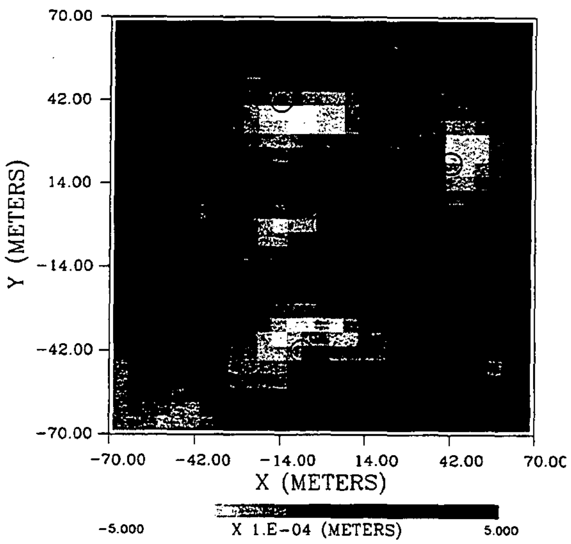

Fig. 11b Ensemble Median Model Using the Variable Aperture Lattice Model.

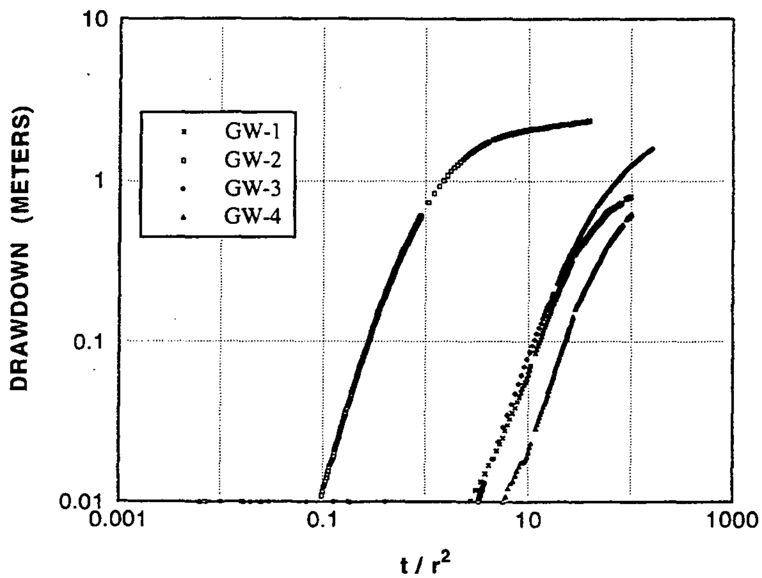

Fig. 12 Scaling of Pressure Data for Test PUMP58

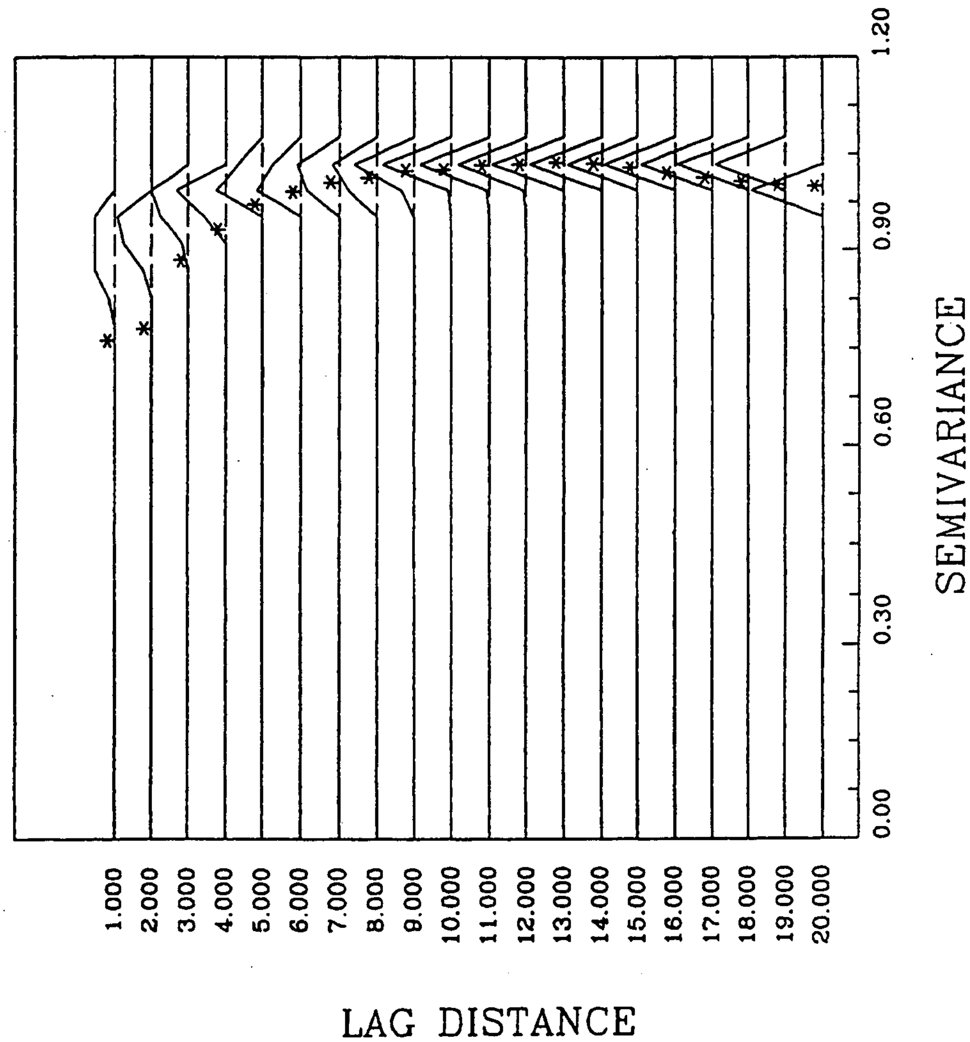

Fig. 13 Moving Window Semivariance Estimates of Individual Models and the Ensemble Median.

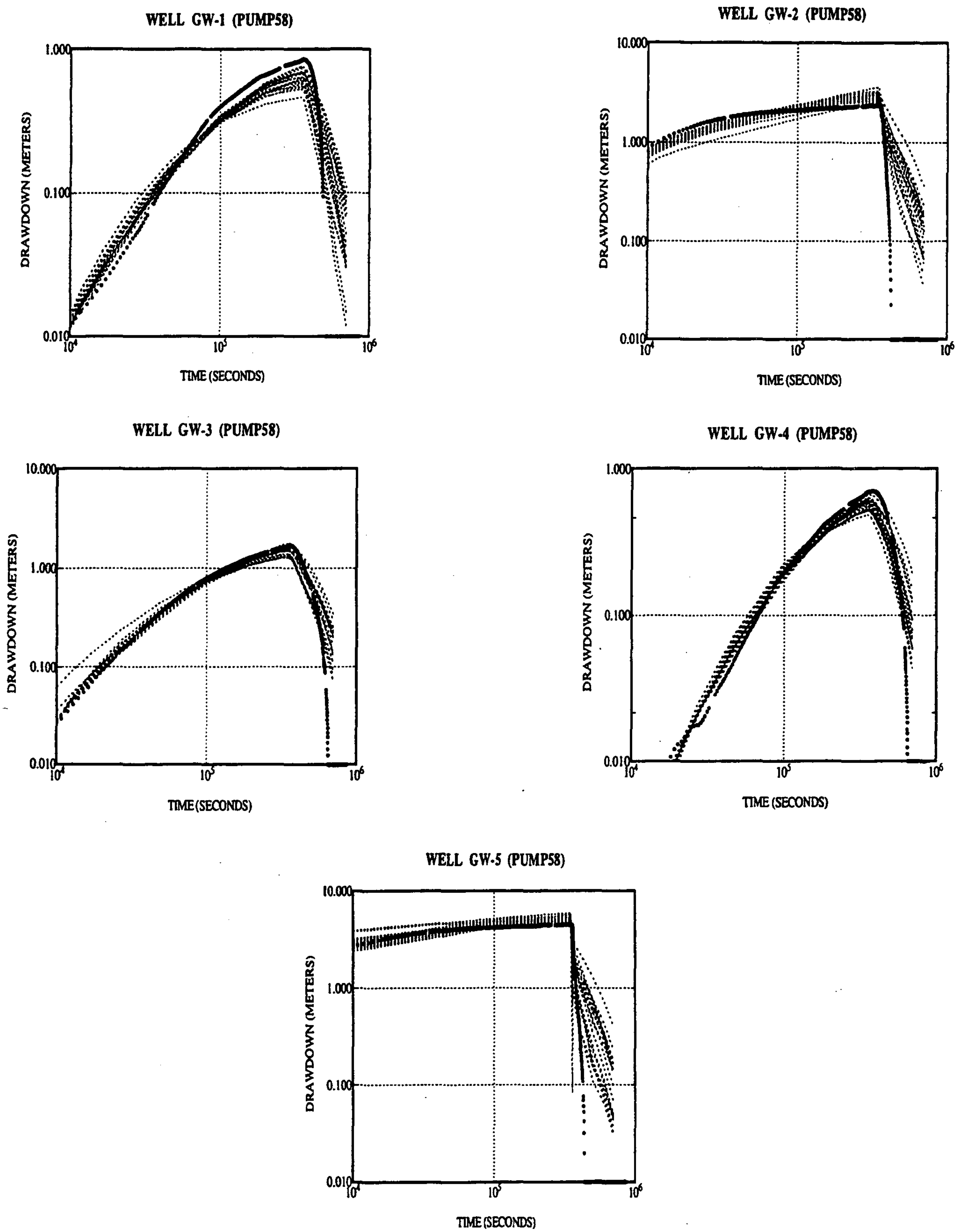

Fig. 14 Prediction of Build-up Data Using Discontinuum Models.

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