DETERMINATION OF FOUNDATION SETTLEMENTS

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ABSTRACT

This paper deals with two major problems:

·Determination of the thickness of the soil to be considered as compressed by the given load (z') using computer or grapho-analytical solution.

·Determination of foundation settlements using simple formula without determining natural and additional pressures for each elemental soil layer. When computing foundation settlement a single influence factor is used for the affected soil thickness. The influence factor is a function of foundation dimension ratio and accounts the thickness of the compressible layer (z'). The factor is tabulated for practical use. In addition, cases for a uniform and stratified soil layers are considered when determining foundation settlement.

Illustrative examples are given as comparison between the 'Summation Method' and the Proposed Formulae.

Finally, the proposed method is believed to be rapid, practical and more reliable since the cumulative errors of pressure increment by the 'Summation Method' is eliminated with

DETERMINATION OF FOUNDATION SETTLEMENTS

One of the most widely used methods for foundation settlement computations, the *'Summation Method'*, is too cumbersome for practical application as it requires determination of the compressive stress and corresponding settlement of each elemental layer as steps in the process of numerical summation or integration.

In the following proposed method, the integration process is performed with the aid of a single coefficient for the whole affected soil thickness.

In the interest of simplification of the calculation the plain stress model is adopted. The loaded material is assumed to be elastic, homogeneous, and isotropic. Furthermore, the influence of horizontal pressure (soil's lateral confinement under load) is considered through only one empirical coefficient $\beta = 0.8$, for all cases.

COMPRESSIVE STRESS AND THE AFFECTED SOIL THICKNESS

All basic assumptions of the 'Summation Method' are taken into account.

The vertical compressive stress σ_z at any point under the centroid of a uniformly loaded elastic rectangle at a depth z can be expressed by the following formula.(Love) [9].

$$\sigma_z = \frac{2p}{\pi}\left\{\frac{l/2 \cdot b/2 \cdot z}{D} \cdot \frac{(l/2)^2 + (b/2)^2 + 2z^2}{D^2z^2 + l/2(b/2)^2} + \arcsin\left(\frac{l/2 \cdot b/2}{\sqrt{(l/2)^2 + z^2}\sqrt{(b/2)^2 + z^2}}\right)\right\} \qquad (1)$$

Where l and b – dimensions of the uniformly loaded rectangle,

z – the depth of the point under consideration for σ_z (depth from footing base to elevation in soil, where increase in stress is desired),

p – external uniformly distributed load or gross pressure intensity at foundation level,

$D^2 = (l/2)^2 + (b/2)^2 + z^2$

If we denote the relative values $l/b = n$ and $2z/b = m$, then Eq. (1) becomes

$$\sigma_z = \frac{2p}{\pi}\left\{\frac{mn(1+n^2+2m^2)}{(1+m^2)(m^2+n^2)\sqrt{1+m^2+n^2}} + \arcsin\frac{n}{\sqrt{(1+m^2)(m^2+n^2)}}\right\} = \alpha p, \qquad (2)$$

Where $\alpha = f(n,m)$ = dimensionless influence factor, used when plotting graph for additional pressure $p_{oz} = \sigma_z$.

After some simplifications in Eq. (2), we obtain

$$\sigma_z = \frac{2p}{\pi}\left\{\frac{2blz(k^2 + 8z^2)}{(b^2 + 4z^2)(l^2 + 4z^2)\sqrt{k^2 + 4z^2}} + \arcsin\frac{lb}{\sqrt{(b^2 + 4z^2)(l^2 + 4z^2)}}\right\} \quad (3)$$

Where $K^2 = b^2 + l^2$

With reference to some Standard Foundation Codes such as the U.S.S.R. Standard Code of Practice for foundations [2], the affected soil thickness by foundation pressure *(z')* is determined from the following conditions:

$$\text{When}\quad \begin{aligned} E &\geq 50\ kgf/cm^2\ ;\ 0.2\ p_{nz'} = p_{oz'} \\ E &< 50\ kgf/cm^2\ ;\ 0.1\ p_{nz'} = p_{oz'} \end{aligned} \quad (4)$$

Where p_{nz}' = natural or the original overburden pressure at a depth z' below foundation level,

E = deformation modulus of the affected soil.

Using the above relations Eq. (4), and considering that $p_{nz} = (h+z)\gamma$ for a uniform soil, where γ – unit weight of soil mass or bulk density and h – foundation depth, we obtain;

$$\frac{2p}{\pi}\frac{1}{\gamma(h + z')}\left\{\frac{2blz'(k^2 + 8z'^2)}{(b^2 + 4z'^2)(l^2 + 4z'^2)\sqrt{k^2 + 4z'^2}} + \arcsin\frac{lb}{\sqrt{(b^2 + 4z'^2)(l^2 + 4z'^2)}}\right\} = 0.2\ or\ 0.1 \quad (5)$$

It is quite difficult to determine the thickness of the stress influenced zone z' analytically from Eq. (5). However, the thickness z' can be computed by using any of the two methods given below:

1. From a standard computer program for the relation:

$$G(z) = \frac{2p}{\pi\gamma(h + z)}\left\{\frac{2blz(k^2 + 8z^2)}{(b^2 + 4z^2)(l^2 + 4z^2)\sqrt{k^2 + 4z^2}} + \arcsin\frac{lb}{\sqrt{(b^2 + 4z^2)(l^2 + 4z^2)}}\right\} - 0.2\ or\ 0.1 = 0$$

2. From the graph shown in (Fig. 1): The graph of $F(z)$ can be plotted versus z, (values of p, γ, b, l and h are constant).

$$F(z) = \frac{2p}{\pi\gamma(h + z)}\left\{\frac{2blz(k^2 + 8z^2)}{(b^2 + 4z^2)(l^2 + 4z^2)\sqrt{k^2 + 4z^2}} + \arcsin\frac{lb}{\sqrt{(b^2 + 4z^2)(l^2 + 4z^2)}}\right\} = \frac{\alpha p}{\gamma(h + z)} \quad (6)$$

The dimensionless influence factor $\alpha = f(n=l/b,\ m=2z/b)$ is given in Table 1.

After plotting the graph of the function $F(z)$, it is necessary to find the point at which $F(z)$=0.2 or 0.1 as shown in Fig. 1. The value $z = z'$ corresponds to $F(z)$ = 0.2 or 0.1.

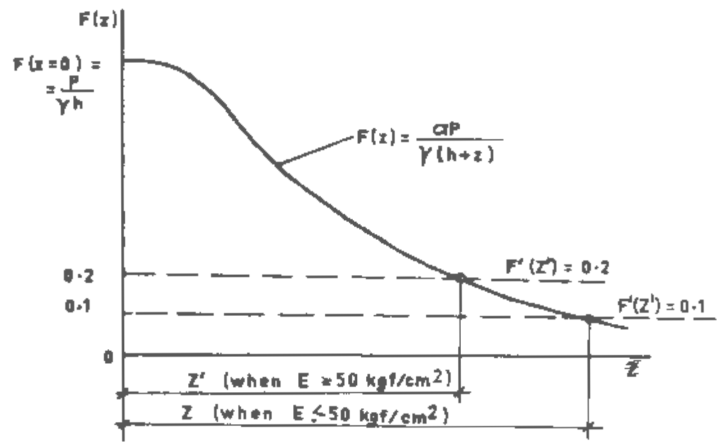

Figure 1 Typical $F(z)$ curve versus z

TOTAL FOUNDATION SETTLEMENT

According to the '*Summation Metohd*', the total foundation settlement is computed as:

$$S = \beta\sum_{i=1}^{n}\frac{\sigma_{zi}}{E_i}h_i \quad (7)$$

Where $\beta = 0.8$ – empirical coefficient for soil's lateral confinement, (function of poisson's ratio).

σ_{zi} = compressive stress at a depth z for i–th soil layer.

h_i = thickness of the i–th soil layer.

E_i = modulus of deformation for i–th soil layer.

For a uniform soil condition the above settlement expression is equivalent to: (see Fig. 2).

$$S = \frac{\beta}{E}\int_0^{z'}\sigma_z\,dz$$

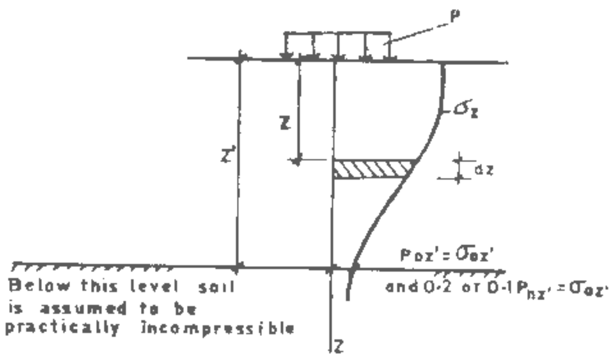

Figure 2 Typical stress curve versus z

Where σ_z is as expressed by Eq. (3).
Thus, we have

$$S = \frac{\beta}{E}\times\frac{2p}{\pi}\int_0^{z'}\left\{\frac{2blz(k^2+8z^2)}{(b^2+4z^2)(l^2+4z^2)\sqrt{k^2+4z^2}} + \arcsin\frac{lb}{\sqrt{(b^2+4z^2)(l^2+4z^2)}}\right\}dz \tag{8}$$

$$\text{Let } I_1 = \frac{2}{\pi}\int_0^{z'}\frac{2blz(k^2+8z^2)dz}{(b^2+4z^2)(l^2+4z^2)\sqrt{k^2+4z^2}} \tag{9}$$

and

$$I_2 = \frac{2}{\pi}\int_0^{z'}\left\{\arcsin\frac{lb}{\sqrt{(b^2+4z^2)(l^2+4z^2)}}\right\}dz \tag{10}$$

When solving Eqs. (9) and (10) we shall consider two possible cases;

Case 1, If $b \neq l$, (case of rectangular footing), then I_1 and I_2 are as given in Eq. (9) and (10), respectively.

Case 2, if $b = l = a$, (case of square footing), then

$$I_{1a} = \frac{4a^2}{\pi}\int_0^{z'}\frac{z(k^2+8z^2)dz}{(a^2+4z^2)^2\sqrt{k^2+4z^2}} \tag{11}$$

and

$$I_{2a} = \frac{2}{\pi}\int_0^{z'}\left\{\arcsin\frac{a^2}{a^2+4z^2}\right\}dz \tag{12}$$

After lengthy operation the final solution of Eq. (9) becomes;

$$I_1 = \frac{b}{\pi}\left\{\frac{1}{2}\ln\left(\frac{\sqrt{k^2+4z'^2}-l}{\sqrt{k^2+4z'^2}+l}\right) - \frac{1}{2}\ln\left(\frac{k-l}{k+l}\right) + \frac{l}{2b}\ln\left(\frac{\sqrt{k^2+4z'^2}-b}{\sqrt{k^2+4z'^2}+b}\right) - \frac{l}{2b}\ln\left(\frac{k-b}{k+b}\right)\right\} \tag{13}$$

If we denote the relative values $2z'/b = m'$ and $l/b = n$, and inserting these terms in Eq. (13), we obtain

$$I_1 = \frac{b}{2\pi}\left\{\ln\left(\frac{\sqrt{1+n^2+m'^2}-n}{\sqrt{1+n^2+m'^2}+n}\right) - \ln\left(\frac{\sqrt{1+n^2}-n}{\sqrt{1+n^2}+n}\right) + n\ln\left(\frac{\sqrt{1+n^2+m'^2}-1}{\sqrt{1+n^2+m'^2}+1}\right) - n\ln\left(\frac{\sqrt{1+n^2}-1}{\sqrt{1+n^2}+1}\right)\right\} \tag{14}$$

There is no closed form solution for Eq. (10). Our aim is to obtain a dimensionless coefficient as function of the ratios,

$m' = 2z'/b$ and $n = l/b$ and the coefficeent to be multiplied by foundation width b

To satisfy this, some substitutions are made and the upper limit of integration is changed to $2z'/b = m'$ (relative value of the compressed soil thickness). Thus;

$$I_2 = \frac{b}{\pi}\int_0^{2z'/b=m'}\left\{\arcsin\frac{n}{\sqrt{m^4+(1+n^2)m^2+n^2}}\right\}dm \tag{15}$$

This integral can be solved numerically for different values of m'. Finally, total settlement for case 1, ($l \neq b$) will be

$$S = \frac{\beta p}{E}(I_1+I_2) = \frac{\beta p}{E}b\left\{\frac{1}{2\pi}\left(\ln\frac{\sqrt{1+n^2+m'^2}-n}{\sqrt{1+n^2+m'^2}+n} - \ln\frac{\sqrt{1+n^2}-n}{\sqrt{1+n^2}+n} + n\ln\frac{\sqrt{1+n^2+m'^2}-1}{\sqrt{1+n^2+m'^2}+1} - n\ln\frac{\sqrt{1+n^2}-1}{\sqrt{1+n^2}+1}\right) + \frac{1}{\pi}\int_0^{m'}\left(\arcsin\frac{n}{\sqrt{m^4+(1+n^2)m^2+n^2}}\right)dm\right\} \tag{16}$$

Denoting that

$$G_{m'n} = \frac{1}{2\pi}\left(\ln\frac{\sqrt{1+n^2+m'^2}-n}{\sqrt{1+n^2+m'^2}+n} - \ln\frac{\sqrt{1+n^2}-n}{\sqrt{1+n^2}+n}\right.$$
$$\left. + n\ln\frac{\sqrt{1+n^2+m'^2}-1}{\sqrt{1+n^2+m'^2}+1} - n\ln\frac{\sqrt{1+n^2}-1}{\sqrt{1+n^2}+1}\right) \qquad (17)$$
$$+\frac{1}{\pi}\int_0^{m'}\left(\arcsin\frac{n}{\sqrt{m^4+(1+n^2)m^2+n^2}}\right)dm$$

For a rectangular footing with sides l and b, and resting on a uniform soil, strictly for the case when $n \neq l$

$$S = \frac{\beta p}{E} G_{m'n}\, b \qquad (18)$$

Performing the integration and inserting limits for Eqs (11) & (12)

$$I_{1a} = \frac{a}{\pi}\ln\left(\frac{\sqrt{2+m'^2}-1}{\sqrt{2+m'^2}+1} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) \qquad (19)$$

$$I_{2a} = \frac{a}{\pi}\int_0^{m'=2z'/a}\left(\arcsin\frac{1}{1+m^2}\right)dm \qquad (20)$$

Numerical values of Eq. (20) can be computed analogous to Eq. (15).

Similarly as for Eq. (16), for case 2 ($b = l = a$), we obtain;

$$S = \frac{\beta p}{E}(I_{1a}+I_{2a}) = \frac{\beta p}{E}a\left\{\frac{1}{\pi}\left(\ln\frac{\sqrt{2+m'^2}-1}{\sqrt{2+m'^2}+1}\right.\right. \qquad (21)$$
$$\left.\left. -\ln\frac{\sqrt{2}-1}{\sqrt{2}+1} + \int_0^{m'}(\arcsin\frac{1}{1+m^2})dm\right)\right\}$$

Denoting that

$$G^a_{m'n} = \frac{1}{\pi}\left\{\ln\frac{\sqrt{2+m'^2}-1}{\sqrt{2+m'^2}+1} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}\right. \qquad (22)$$
$$\left. + \int_0^{m'}(\arcsin\frac{1}{1+m^2})dm\right\}$$

The total settlement for a square footing; (Strictly, $b = l = a$ or $n=1$).

$$S = \frac{\beta p}{E} G^a_{m'n}\, a \qquad (23)$$

Similarly for round footings with diameter d, total settlement;

$$S = \frac{\beta p}{E} G^d_{m'n}\, d \qquad (24)$$

In this case, considering the area of a circular footing and the ratios n=1 and $m'=2z'/d = z'/r$

$$G^d_{m'n} = \frac{\pi}{4} G^a_{m'n} \qquad (25)$$

Where r = the radius of a circular footing.

$G_{m'n}$, $G^a_{m'n}$ and $G^d_{m'n}$ are dimensionless influence factors for the stress influenced zone. The influence factors are listed in Table 2 for various m' and n.

TOTAL SETTLEMENT OF FOUNDATIONS ON NON-UNIFORM SOILS

For stratified soils, the natural or the original pressure intensity at a depth z below foundation level may be expressed as,

$$p_{nz} = p_n + \sum_{i=1}^{n} \gamma_i h_i \qquad (26)$$

Where $p_n = \gamma h$ = natural or the original over-burden pressure intensity at foundation depth h, (the surcharge)

γ = soil's unit weight or bulk density within the foundation depth limit

γ_i & h_i = unit weight and the thickness of i-th soil layer within the limit of the total compressed thickness, respectively

For stratified soils we shall take an average bulk density within the limit of the compressed thickness.

$$\gamma_{av} = \frac{\sum_{i=1}^{n}\gamma_i h_i}{\sum_{i=1}^{n} h_i} \qquad (27)$$

Then Eq. (6) simplifies to

$$F(z) = \frac{\alpha p}{\gamma_{av}(h+z)} \qquad (28)$$

It can be concluded that the summation of the elemental layers settlements is the sum of the difference of influence factors $G_{m'n}$ related to i-th soil modulus of deformation.

From this fact and considering Fig. 3, the total settlement of foundations resting on stratified soil layers will be:

$$S = \beta p b \sum_{i=0}^{n} \frac{G_{m'n'(i+1)} - G_{m'n(i)}}{E_i} \qquad (29)$$

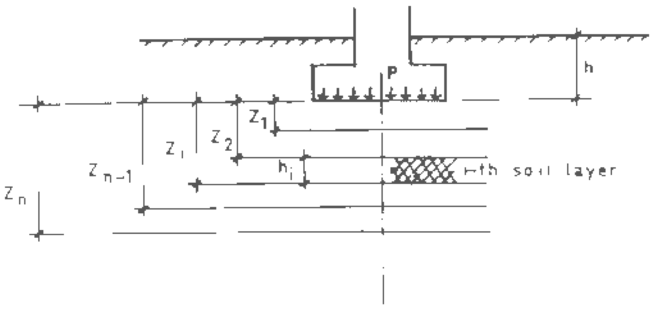

Figure 3 Settlements of foundations on non-uniform soils

Where $G_{m'n(i)} = f(n = l/b,\ m' = 2z_i/b)$

and $G_{m'n(i+1)} = f(n = l/b,\ m' = 2z_{i+1}/b)$

Obviously, if $i = 0$, then $G_{m'n(0)} = 0$,

and $S = \frac{\beta p}{E} G_{m'n} b$

SUPPLEMENTS TO SETTLEMENT COMPUTATIONS BY THE INTRODUCED FORMULA

1. When computing settlements the value of p is taken as $p_{av} - p_n = p_{av} - \gamma h$, i.e. the net pressure intensity or the difference in intensities of the average pressure after construction and the original overburden pressure.

2. As discussed before, to determine the compressed thickness z', the graph of $F(z)$ given by Eq. (6) should be plotted versus z. However, it is lengthy to calculate $F(z)$ until attaining the value $F(z) = 0.2$ or 0.1. It is, therefore, advised to determine z' within the limits of $F(a) > 0.2$ or $0.1 > F(b)$ with calculation interval 1m on the chord between $z = a$ and $z = b$ (see Fig. 4)

Then, $\frac{F(a) - F(b)}{b - a} = \frac{F(a) - 0.2 \text{ or } 0.1}{z' - a}$

If the calculation interval is 1m, obviously $b - a = 1$m.

Thus, the compressed thickness

$$z' = a + \frac{F(a) - 0.2 \text{ or } 0.1}{F(a) - F(b)} \qquad (30)$$

Strictly $F(a) > 0.2 \text{ or } 0.1 > F(b)$

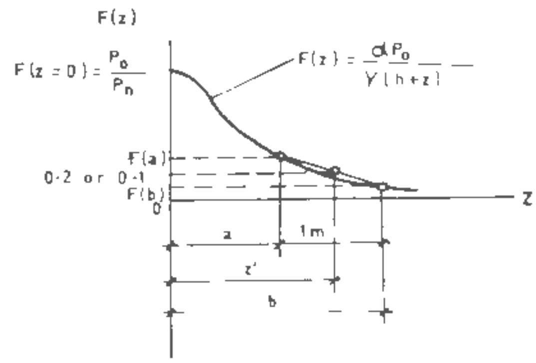

Figure 4 $F(z)$ curve versus z, the compressed thickness z'

Further, we shall compare settlement values computed using the 'Summation Method' and the proposed formulae.

Example 1

Compute total settlement of a square footing on a uniform soil given the following data

$b = l = 2$m
$p_{av} = 2$kgf/cm²
$E = 100$ kgf/cm²
$h = 1.5$m and $\gamma = 2$ ton/m³

Solution

a) 'Summation Method'

$p_n = \gamma h = 2 \times 1.5 = 3$ ton/m² $= 0.3$ kgf/cm²
$p_o = p_{av} - p_n = 2 - 0.3 = 1.7$ kgf/cm²
$h_i = 0.4a = 0.4 \times 2 = 0.8$m (thickness of the elemental layer)
$n = l/b = 1$, $m = 2z/a = 2z/2 = z$, $\alpha = f(n=1, m = z)$
$p_{oz} = \sigma_z = \alpha(p_{av} - p_n) = \alpha p_o = 1.7\alpha$, kgf/cm²
$p_{nz} = \gamma(h + z)$, kgf/cm²

Table 1: Influence Factor α

$m = 2z/b$ or $m = z/r$	Influence factor α for foundations:							
	circular	rectangular with the ratio $n = l/b$, equals						continuous footing
		1	1.4	1.8	2.4	3.2	5	$n \geq 10$
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.949	0.960	0.972	0.975	0.976	0.977	0.977	0.977
0.8	0.756	0.800	0.848	0.866	0.875	0.879	0.881	0.881
1.2	0.547	0.606	0.682	0.717	0.739	0.749	0.754	0.755
1.6	0.390	0.449	0.532	0.578	0.612	0.629	0.639	0.642
2.0	0.284	0.336	0.414	0.463	0.505	0.530	0.545	0.550
2.4	0.213	0.257	0.325	0.374	0.419	0.449	0.470	0.477
2.8	0.165	0.201	0.260	0.304	0.349	0.383	0.410	0.420
3.2	0.130	0.160	2.210	0.251	0.294	0.329	0.360	0.374
3.6	0.106	0.131	0.173	0.209	0.250	0.285	0.319	0.337
4.0	0.087	0.108	0.145	0.176	0.214	0.248	0.285	0.306
4.4	0.073	0.091	0.123	0.150	0.185	0.218	0.255	0.280
4.8	0.062	0.077	0.105	0.130	0.161	0.192	0.230	0.258
5.2	0.053	0.067	0.091	0.113	0.141	0.170	0.208	0.239
5.6	0.046	0.058	0.079	0.099	0.124	0.152	0.189	0.223
6.0	0.040	0.051	0.070	0.087	0.110	0.136	0.172	0.208
6.4	0.036	0.045	0.062	0.077	0.099	0.122	0.158	0.196
6.8	0.032	0.040	0.055	0.069	0.088	0.110	0.145	0.185
7.2	0.028	0.036	0.049	0.062	0.080	0.100	0.133	0.175
7.6	0.025	0.032	0.044	0.056	0.072	0.091	0.123	0.166
8.0	0.023	0.029	0.040	0.051	0.066	0.084	0.113	0.158
8.4	0.021	0.026	0.037	0.046	0.060	0.077	0.105	0.150
8.8	0.019	0.024	0.033	0.042	0.055	0.071	0.098	0.143
9.2	0.017	0.022	0.031	0.039	0.051	0.065	0.091	0.137
9.6	0.016	0.020	0.028	0.036	0.047	0.060	0.085	0.132
10	0.015	0.019	0.026	0.033	0.043	0.056	0.079	0.126
10.4	0.014	0.017	0.024	0.031	0.040	0.052	0.074	0.122
10.8	0.013	0.016	0.022	0.029	0.037	0.049	0.069	0.117
11.2	0.012	0.015	0.021	0.027	0.035	0.045	0.065	0.113
11.6	0.011	0.014	0.020	0.025	0.033	0.042	0.061	0.109
12.0	0.010	0.013	0.018	0.023	0.031	0.040	0.058	0.106

Note: Intermediate values of the factor can be determined by linear interpolation.

Table 2: Influence Factor *Gm'n*

$m'=2z'/b$ or $m'=z'/r$	Influence factor *Gm'n* for foundations:							
	Circular	rectangular with the ratio $n = l/b$, equals						continuous footing
		1	1.4	1.8	2.4	3.2	5	$n \geq 10$
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.4	0.155	0.198	0.199	0.199	0.199	0.199	0.199	0.199
0.8	0.295	0.375	0.382	0.384	0.385	0.385	0.386	0.386
1.2	0.405	0.516	0.535	0.542	0.547	0.548	0.549	0.549
1.6	0.488	0.621	0.656	0.672	0.681	0.686	0.688	0.689
2.0	0.548	0.698	0.750	0.775	0.793	0.802	0.806	0.807
2.4	0.595	0.757	0.823	0.859	0.885	0.899	0.907	0.910
2.8	0.630	0.802	0.881	0.926	0.961	0.982	0.995	0.999
3.2	0.658	0.838	0.928	0.981	1.025	1.053	1.072	1.078
3.6	0.680	0.866	0.965	1.026	1.079	1.113	1.139	1.148
4.0	0.698	0.889	0.997	1.064	1.125	1.166	1.199	1.211
4.4	0.713	0.908	1.023	1.097	1.164	1.212	1.253	1.269
4.8	0.727	0.925	1.045	1.124	1.199	1.253	1.301	1.322
5.2	0.738	0.939	1.064	1.148	1.229	1.289	1.345	1.371
5.6	0.748	0.952	1.081	1.169	1.255	1.321	1.384	1.416
6.0	0.757	0.964	1.096	1.188	1.279	1.350	1.421	1.458
6.4	0.766	0.975	1.110	1.204	1.300	1.377	1.454	1.498
6.8	0.774	0.985	1.122	1.220	1.319	1.401	1.485	1.535
7.2	0.782	0.995	1.134	1.234	1.337	1.423	1.514	1.570
7.6	0.789	1.004	1.145	1.247	1.353	1.443	1.541	1.604
8.0	0.796	1.014	1.155	1.259	1.369	1.462	1.566	1.635
8.4	0.803	1.022	1.165	1.270	1.383	1.480	1.589	1.666
8.8	0.810	1.031	1.174	1.281	1.396	1.496	1.611	1.694
9.2	0.816	1.039	1.184	1.291	1.408	1.511	1.632	1.722
9.6	0.822	1.047	1.193	1.301	1.420	1.526	1.652	1.748
10	0.829	1.055	1.201	1.310	1.431	1.539	1.670	1.773
10.4	0.834	1.062	1.209	1.320	1.442	1.552	1.688	1.797
10.8	0.840	1.069	1.217	1.328	1.452	1.565	1.704	1.820
11.2	0.845	1.076	1.225	1.337	1.461	1.576	1.720	1.842
11.6	0.850	1.082	1.233	1.345	1.470	1.587	1.735	1.863
12.0	0.855	1.088	1.240	1.352	1.479	1.597	1.749	1.883

Note: Intermediate values of the factor can be determined by linear interpolation.

Table 3: Accompanying Table for Settlement Computations

Points	z, m	z+h, m	$P_{nz}=\gamma(h+z)$, Kgf/cm²	0.2 p_{nz}, kgf/cm²	m = z	α	$P_{\alpha}=1.7\alpha$, kgf/cm²	P, kgf/cm²
0	0	1.5	0.300	0.060	0	1.000	1.700	
1	0.8	2.3	0.460	0.092	0.8	0.800	1.360	1.530
2	1.6	3.1	0.620	0.124	1.6	0.449	0.763	1.062
3	2.4	3.9	0.780	0.156	2.4	0.257	0.437	0.600
4	3.2	4.7	0.940	0.188	3.2	0.160	0.272	0.355
5	3.8	5.3	1.060	0.212	3.8	0.119	0.202	0.237

$$S = \frac{\beta}{E}\sum_{1}^{5} p_i h_i = \frac{0.8}{100} \times 80\ (1.530 + 1.062 + 0.600 + 0.355)$$

$$+ \frac{0.8}{100} \times 60 \times 0.237 = 2.38 \text{ cm}$$

Thus, the total settlement computed using the accompanying table (Table 3 and Fig. 5), is 2.38 cm.

b) By the Introduced Formula

In most cases, the thickness of the footing influence zone, $z \geq 1.5b$

Let $z = 3.0m,\ n = 1,\ m = \frac{2z}{b} = \frac{2}{2} \times 3 = 3$

From Table 1, $\alpha = f(n,m) = 0.1805$

Using Eq. (6),

$$F(z = 3) = \frac{0.1805 \times 17}{2(1.5 + 3)} = 0.3409 > 0.2$$

Let $z = 4.0m,\ n = 1,\ m = \frac{2z}{b} = \frac{2}{2} \times 4 = 4$

$\alpha = f(n,m) = 0.108$

$$F(z = 4) = \frac{0.108 \times 17}{2(1.5 + 4)} = 0.1669 < 0.2$$

Using Eq. (30) $z' = 3 + \frac{0.3409 - 0.2}{0.3409 - 0.1669} = 3.8\text{m}$

Then

$$m' = \frac{2z'}{b} = \frac{2 \times 3.8}{2} = 3.8,\ n = 1$$

From Table 2, $G_{m,n} = 0.8775$

The total settlement,

$$s = \frac{\beta p_o}{E} G_{m',n}\ a = \frac{0.8 \times 1.7}{100} \times 0.8775 \times 200 = 2.38 \text{ cm}$$

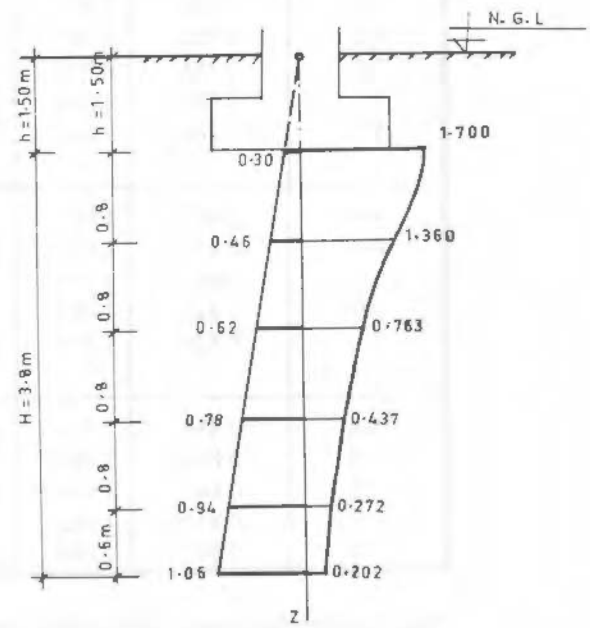

Figure 5 Intensity of pressure with depth at the center Accompany figure for settlement computations

Example 2.

Compute total settlement of a square footing on stratified soil layers given the following data :

$b = l = 2 \times 2$m
$p_{av} = 51.15$ ton/m^2
$h = 1.5$m

Soil parameters

(1) Firm clay: $I_l = 0.6$, $\gamma_1 = 2.1$ ton/m^3
$E_1 = 120$ kgf/cm^2
$h_1 = 3.0$ m

(2) Stiff Clay $I_l = 0.26$, $\gamma_2 = 2.29$ ton/m^3
$E_2 = 210$ kgf/cm^2
$h_2 = 3.0$ m

(3) Sand $\gamma_3 = 2.23$ ton/m^3,
$E_3 = 400$ kgf/cm^2

Solution

a) ' Summation Method'

$p_n = \gamma h = 2.1 \times 1.5 = 3.15$ ton/m^2
$p_o = p_{av} - p_n = 51.15 - 3.15 = 48.0$ ton/m^2
$= 4.8$ kgf/cm^2
$h_i = 0.4b = 0.4 \times 2 = 0.8$m, $n = 1$ m $= 2z/b$
$= 2/2 \times z = z$
$\alpha = f(n=1, m=z)$, $p_{oz} = \alpha p_o = 4.8\alpha$

The total settlement from the accompanying table (Table 4)

$$S = \beta \sum_{i=1}^{8} \frac{p_i h_i}{E_i} = \frac{0.8}{120}\{80(4.32 + 2.988 + 1.695) + 60 \times 1.052\}$$
$$+ \frac{0.8}{210} \times 20 \times 0.818 + \frac{0.8}{210} \times 80(0.643 + 0.444)$$
$$+ \frac{0.8}{210} \times 66 \times 0.368 = 5.68 \approx 5.7 \text{ cm}$$

Table 4 Accompanying Table for Settlement Computations

Points	z m	$z+h$, m	$p_z = \gamma(h+z)$ kgf/cm^2	$0.2\,p_z'$ kgf/cm^2	$m = z$	α	$p_{oz} = 4.8\alpha$ kgf/cm^2	p' kgf/cm^2
0	0	1.5	0.315	0.063	0	1.000	4.800	4.320
1	0.8	2.3	0.483	0.097	0.8	0.800	3.840	2.998
2	1.6	3.1	0.651	0.130	1.6	0.449	2.155	1.695
3	2.4	3.9	0.819	0.164	2.4	0.257	1.234	1.052
4	3.0	4.5	0.945	0.189	3.0	0.181	0.869	0.818
5	3.2	4.7	1.076	0.215	3.2	0.160	0.768	0.643
6	4.0	5.5	1.260	0.252	4.0	0.108	0.518	0.444
7	4.8	6.3	1.443	0.289	4.8	0.077	0.370	0.368
8	5.46	6.96	1.594	0.319	5.46	0.0608	0.292	

b) By the Formula Introduced

Let $z = 5$m

The average bulk density within the assumed depth using Eq. (27),

$$\gamma_{av} = \frac{2.1 \times 3 + 2.29 \times 2}{5} = 2.18 \text{ ton/m}^3,$$

$$\alpha = f(n = 1, m = 5) = 0.072$$

$$\beta(z) = \frac{\alpha p_o}{\gamma_{av}(h + z)} = \frac{0.072 \times 48}{2.18(1.5 + 5)} = 0.244 \approx 0.2$$

$$\text{Let } z = 6\,m, \gamma_m = \frac{2.1\times3 + 2.29\times3}{6} = 2.2\,\text{ton/m}^3$$

$$\alpha = f(n = 1,\ m = 6) = 0.051$$

$$F(z) = \frac{0.051 \times 48}{2.2(1.5 + 6)} = 0.148 < 0.2$$

The compressed thickness

$$z' = 5 + \frac{0.244 - 0.2}{0.244 - 0.148} = 5.46\,\text{m}$$

From Table 2

$$z = 0 \quad G_{m'n0} = 0$$

$$z_1 = 3m, G_{m'n1} = f\left(n = 1,\ m = \frac{2z_1}{b} = \frac{2\times3}{2} = 3\right) = 0.820$$

$$z_2 = 5.46m, G_{m'n2} = f\left(n = 1,\ m = \frac{2z_2}{2} = \frac{2\times5.46}{2} = 5.46\right)$$

$$= 0.947 \text{ (interpolated)}$$

The total settlement

$$S = \beta p_o b \sum_{i=0}^{2} \frac{G_{m'n(i+1)} - G_{m'n(i)}}{E_i}$$

$$= 0.8 \times 4.8 \times 200\left(\frac{0.820 - 0}{120} + \frac{0.947 - 0.820}{210}\right)$$

$$= 5.7 \text{ cm}$$

CONCLUSION

From the comparative analysis of total settlement no discrepancy obtained between the two methods.

It can be concluded that the proposed method is quick, accurate and easier for practical applications.

The formulas derived assume uniform contact pressure distribution at foundation level, and the values of settlement obtained are at the center of elastic footings.

Elastic settlements should not be calculated for thicknesses (z') greater than $4b$.

In the case of a rigid foundation, the settlement should be reduced by a rigidity factor. The center settlement of a rigid footing is commonly taken as 0.75 that of an elastic footing.

The calculated settelment should be corrected by an appropriate 'depth factor' to allow for the depth of foundation.

It is also to be noted that modulus of deformation (E) increases with depth for most natural soil and rock formations. Calculations based on constant modulus of deformation give exaggerated amount of settlement.

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