

# DETERMINATION OF CRITICAL TEARING ENERGY OF TIRE RUBBER

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## Abstract

This paper presents an experimental investigation to study the crack growth characteristics and determine the critical tearing energy of pure tire rubber under opening-mode and anti-plane-mode loading. Constrained tension and trousers specimens were used, respectively. In the trousers test it was observed that the crack does not propagate at a steady rate, but in a stick-slip way, that is, it arrests and reinitiates at fairly regular intervals. Thus, the force necessary to propagate the crack varies widely from a maximum value at crack initiation to a minimum value at crack arrest. In the constrained tension tests no stable crack growth was observed and crack initiation triggered catastrophic fracture. The critical tearing energies corresponding to crack initiation and arrest under anti-plane and to unstable crack growth under opening-mode loading were determined from the load-displacement record from the trousers and the constrained tension tests.

## Introduction

The Griffith criterion for growth of a crack in brittle materials was extended by Rivlin and Thomas [1] to the case of crack growth in vulcanized rubber. The energy necessary to grow the crack is supplied either from the strain energy in the deformed rubber, or as work done by the applied forces or both. Rivlin and Thomas [1] used a number of test pieces for which  $T$  can be calculated for elastic materials undergoing large deformations. From these tests they were able to determine a critical value of the tearing energy,  $T_{cr}$ , which is characteristic of the elastomer, analogous to the Griffith characteristic energy. When the tearing energy  $T$  exceeds the value  $T_{cr}$  crack growth will occur. Many investigators [2-5] have demonstrated that  $T_{cr}$  is independent of the geometry and dimensions of the test piece and can be considered as a material property. For component design the tearing energy,  $T$ , is calculated for a hypothetical crack and is compared with  $T_{cr}$  to determine if the crack will propagate.

In the present work the crack growth behavior and critical tearing energy in pure tire rubber under opening-mode and anti-plane-mode loading were investigated.

## Tearing Energy

Tearing energy is defined as the energy released per unit area of crack surface growth [6]

$$T = -\left(\frac{\partial W}{\partial A}\right)_l \quad (1)$$

where  $T$  = tearing energy (or energy release rate)  
 $W$  = elastic energy stored in specimen  
 $A$  = area of one fracture surface of the crack

The suffix  $l$  denotes differentiation with constant displacement of the boundaries over which forces are applied. If the derivative is obtained under constant applied forces, then the work done by these forces must be included into Equation. (1).

The tearing energy can be considered as the driving force for crack propagation. Experimental results indicate that the value of tearing energy at crack growth,  $T_{cr}$ , is a characteristic property of the material. When the tearing energy,  $T$ , reaches the critical value,  $T_{cr}$ , crack growth will occur. Experimental determination of tearing energy according to Equation (1) requires the measurement of force-deflection relation in specimens with different crack lengths, calculation of the energy stored and differentiation with respect to crack length.

Determination of tearing energy in terms of applied forces or deformations for an arbitrarily shaped specimen is formidable due to nonlinear behavior and large deformations of rubber. However, by suitably choosing the specimen geometry  $T$  can be determined without knowledge of the detailed strain distribution. The types of specimen that were used in this work are the trousers specimen for anti-plane-mode and the constrained tension or shear specimen for opening-mode loading.

Thomas [4] showed that the tearing energy,  $T$ , is given by

$$T = W_b d \quad (2)$$

where  $W_b$  is the work required to break a unit volume of material in simple extension in the absence of cracks and  $d$  is the diameter of the notch tip, which measures the bluntness of the notch. Since  $W_b$  is an intrinsic material constant Equation (2) indicates that  $T$  depends on the diameter  $d$  of the notch at its tip.

### **Trouser's Specimen**

The trouser's specimen has become a favorite test piece for determination anti-plane-mode critical tearing energy for elastomers. The specimen is a thin rectangular piece cut centrally along its length so that two legs are formed (Fig.1). The legs are pulled in opposite directions

out of the plane of the test piece by equal and opposite forces. The expression for tearing energy is

$$T = \frac{2\lambda P}{h} - 2bw \quad (3)$$

where:  $P$  = force on legs of specimen

$\lambda$  = extension ratio in legs (ratio of length of deformed to undeformed leg)

$h$  = specimen thickness

$b$  = width of legs

$w$  = strain energy density in the legs

The rate of crack propagation,  $\dot{a}$ , when the specimen legs are inextensible ( $\lambda = 1$ ) is

$$\dot{a} = \frac{R}{2} \quad (4)$$

where  $R$  is the crosshead speed. This means that the rate of tearing is half the crosshead speed of the testing machine.

### Constrained Tension (or Shear) Specimen

The constrained tension specimen is a wide strip of rubber material attached along its long edges to rigid grips, that constrain its lateral deformation, and contains a long edge crack (Fig.2). The specimen is subjected to a uniform displacement in a direction perpendicular to the crack. The opening-mode tearing energy [3] is

$$T = wl_o \quad (5)$$

where  $w$  is the strain energy density under conditions of constrained tension and  $l_o$  is the specimen height in the unstrained configuration. The value of strain energy density  $w$  is found from the stress-strain relation of a specimen under conditions of constrained tension.

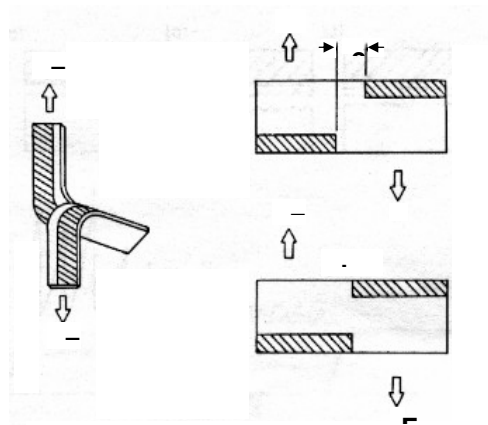


FIGURE 1. Trouser's specimen

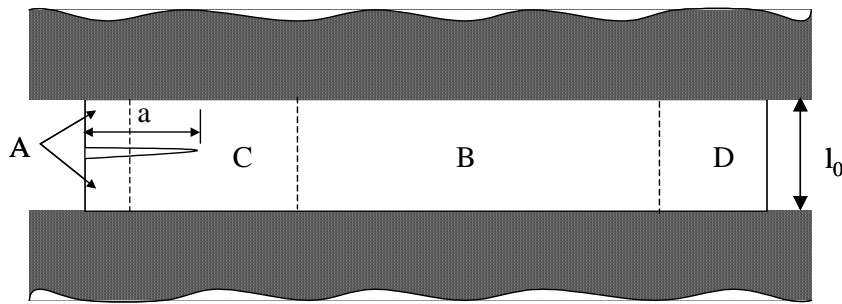


FIGURE 2. Constrained tension (shear) specimen.

## Experimental

**a. Anti-plane Loading:** The tire rubber used in this work is a blend of natural rubber (NR) and polybutadiene (BR). It is vulcanized and filled with carbon black. The anti-plane -mode critical tearing energy was determined from the trousers test (Fig. 1). The trouser's specimens had legs 12.6 mm wide and varying thickness ranging from 0.74 to 1.73 mm. Initial results revealed a stick-slip tearing mechanism during crack growth. The applied force necessary to propagate the crack varied widely from a minimum at crack arrest to a maximum at crack extension. To reduce the stick-slip tearing constrained trousers specimens were used. The specimens were reinforced locally with two thin steel shims of different widths bonded on opposite sides of the specimen along the crack. The distance between the metal shims varied between 2.54 mm and 20 mm. To ensure that the crack propagates along its initial plane direction a shallow groove was cut along the crack ligament on both sides of the specimen.

Fig. 3 shows a typical load-displacement graph during crack growth for a specimen of net thickness along the crack ligament of 1.45 mm after the depth of the two grooves has been accounted for. The distance between the steel shims is 2.54 mm and the loading rate is 5.1 mm/min. The load reaches a maximum value at crack growth and a minimum value at crack arrest. The mode of crack propagation is characterized by an increase of load with no crack growth followed by a sudden decrease of load as the crack propagates unstably and arrests. This pattern is repeated at intervals of time and jump distance that vary over a range of about 5. The tear surface of a specimen that experienced stick-slip tearing is shown in Fig. 4. Note the “shear cusps” which indicate the sites of tear initiation. At these sites the load drops to a minimum value at crack arrest.

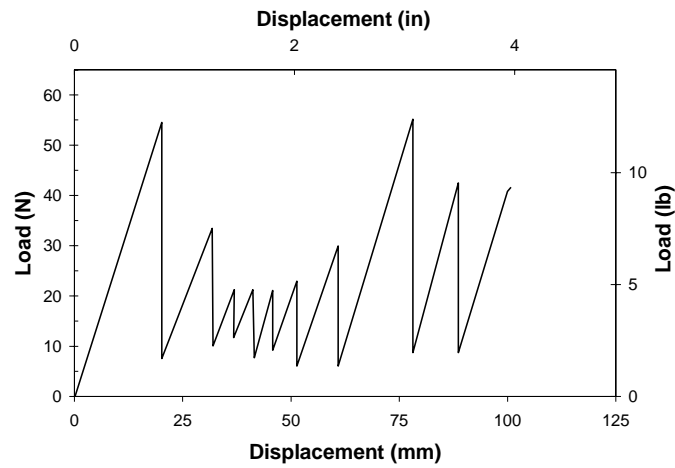


FIGURE 3 Load versus displacement curve for trousers test

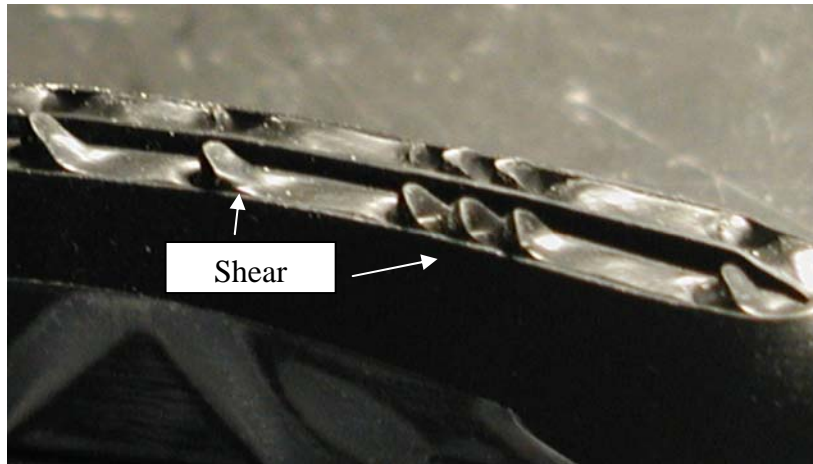


FIGURE 4. Fracture surface

The effect of crack growth rate on the critical tearing energy for applied loading rates between 5.1 and 254 mm/min was studied. Fig. 5 shows the variation of the tearing energy versus the crosshead rate.

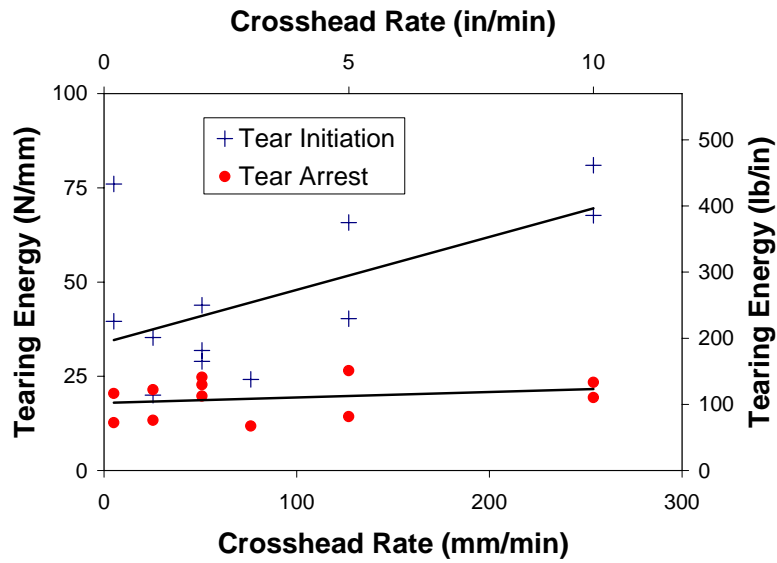


FIGURE 5 Tearing energy versus cross head rate

**Opening-mode Loading:** A series of constrained tension specimens of dimensions 101.6 x 17.8 x 1.9 mm with crack lengths of 38.1, 44.5, 50.4 and 57.1 mm were loaded in an Instron servohydraulic testing machine. Fig. 6 shows the load-displacement curves up to the point of crack initiation for cracks of various initial lengths. No stable crack growth was observed. Crack initiation coincided with rapid catastrophic failure. The load-displacement curve presents a nonlinear sigmoid behavior characteristic of rubber. The stiffness of the curve after a small linear part decreases up to a limiting strain after which it increases. Note that the curves approach each other as the crack length increases up to a limiting crack length of 50.8 mm.

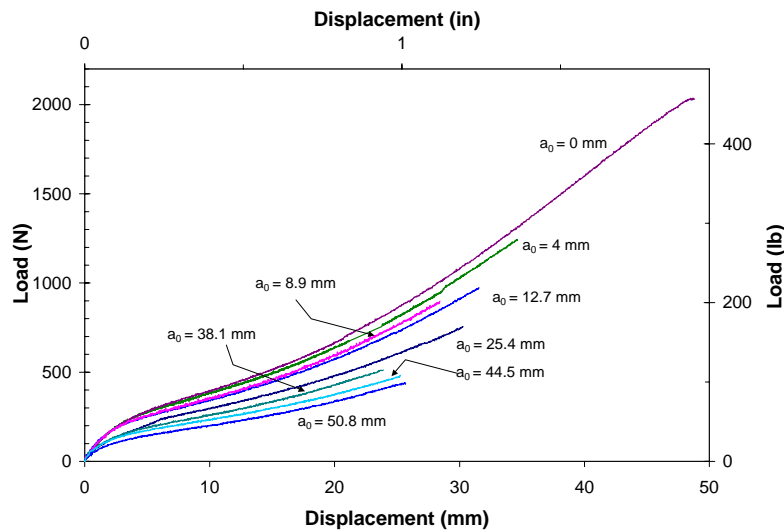


FIGURE 6. Load versus displacement curves for cracks with different initial lengths

## Results and Discussion

From the load-displacement records of Fig. 3 we observe that the force gradually increases with displacement until a maximum is reached and then it drops with increasing displacement up to a minimum value. The crack at maximum force initiates, while at minimum force arrests. This cycle of crack initiation and arrest repeats itself at intervals of time and jump distance that vary over a range of about 5. This form of crack growth is known as stick-slip tearing. The stick-slip mode of crack growth was reduced, but it was not eliminated with the bonding of the steel shims to the legs of the trousers specimen.

The observed fluctuation of the force from a maximum value at crack initiation to a minimum value at crack arrest can be justified from Equation (2). As the applied force increases the diameter,  $d$ , of the notch at its tip, which measures the bluntness of the notch, also increases up to a value at which the notch starts to propagate. At this point  $d$  reaches a maximum value and since the work  $W_b$  is a material constant, Equation (2) suggests that the tearing energy  $T$  and therefore the force  $F$  (Equation (3)) for crack growth becomes maximum. As the notch initiates at the maximum load its diameter starts to decrease to a minimum value at arrest, which leads to a minimum value of the applied force.

From the values of the applied force at crack initiation and arrest of Fig. 3 we can calculate the corresponding critical values of tearing energy. Since the legs of the trousers specimens have been reinforced with steel shims the deformation of the legs is negligible ( $\lambda = 1$ ,  $w = 0$ ). Thus, the second term in Equation (3) can be neglected. for the calculation of the critical tearing energy,  $T_{cr}$ . Results of the initiation, arrest and average tearing energy for a crosshead displacement rate of 51 mm/min and a shim separation distance  $b=38$  mm are shown in Table 1.

Specimen	(N/mm)			(lb/in)		
	Initiation	Arrest	Mean	Initiation	Arrest	Mean
1	44	20	32	250	113	181
2	32	23	27	182	130	156
3	29	25	27	165	141	153
<b>AVERAGE:</b>	<b>35</b>	<b>22</b>	<b>29</b>	<b>199</b>	<b>128</b>	<b>163</b>

The effect of the average crack growth rate on the tearing energy was studied. This study concerns the dependence of tearing energy on the average crack growth rate and does not refer to the load-rate sensitive tearing initiation and arrest resistance which, as it was previously referred to, depends on the crack growth rate from initiation to arrest. A more in-depth study of the phenomenon would concern the dependence of the crack initiation and arrest tearing energy values on the rate of crack growth from initiation to arrest. The average rate of applied load varied between 5.1 and 254 mm/min, which means that the crack propagated at a speed half of these values (Equation. (4)). It was found that as the crack propagation rate increases, the fluctuation of the load decreases at both maximum and minimum values, and therefore, the crack grows in a more stable manner at higher crack propagation rates. Furthermore, for all crack propagation rates it was found that the scatter of force values at crack arrest is in general smaller than at crack initiation. The variation of the arrest and initiation tearing energies versus the average rate of applied loading is shown in Fig.5. Note that both tearing energies increase as the loading rate increases. However, the

arrest energy increases at a much slower rate than the initiation tearing energy. This result in conjunction with the stability of tearing energies at crack arrest suggests that the critical tearing energy at crack arrest can be considered as an inherent material property. As can be observed from Fig. 5 there is a large scatter in the tearing initiation values which suggests that a statistical treatment of the problem is needed.

The load - displacement curves of Fig. 6 approach each other and tend to a limiting curve as the crack length increases up to a value of 50.8 mm. For that crack length Eq. (5) can be used for the determination of the critical tearing energy under mode-I loading. A value of 31.3 N/mm was obtained. This value is close to the critical tearing energy at initiation for anti-plane-mode loading.

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