DETERMINATION OF ERROR IN INDIVIDUAL DISCHARGE MEASUREMENTS

By V. B. Sauer and R. W. Meyer

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ABSTRACT

The uncertainty, or standard error, for individual measurements of stream discharge is computed based on a root-mean-square error analysis of the individual component errors. The component errors include errors in the measurement of width, depth, and velocity, and in computation procedures. This analysis can be used to evaluate the uncertainty for most discharge measurements made with the vertical axis, cup-type current meter. The procedures do not apply to other types of current meters or other methods such as dilution gaging or ultrasonic methods. The study indicates that standard errors for individual discharge measurements can range from about 2 percent under ideal conditions to about 20 percent when conditions are poor and shortcut methods are used. Most measurements will have standard errors ranging from about 3 percent to 6 percent. Some conditions, such as wind, ice, boundary effects, flow obstructions, improper equipment, as well as incorrect measuring procedures and carelessness, can result in larger errors than indicated by the error analysis. A computer program is available for making the error computations.

INTRODUCTION

The U.S. Geological Survey (USGS), Water Resources Division, has the responsibility for determining streamflow in the Nation's streams, rivers, and estuaries. In the course of conducting this work, many thousands of individual measurements of discharge are made each year. In fact, a reasonable estimate would be about 100,000 measurements per year. These measurements are used in many ways, but the primary use is to define stage-discharge relations so that continuous records of discharge can be computed from a continuous record of stream stage. The accuracy of the discharge records depends to a large extent on the accuracy of the individual discharge measurements. It is important, therefore, to be able to quantitatively assess the accuracy of the discharge measurements.

Many studies have been made to define uncertainties in measuring the various components of a discharge measurement, and then to combine these uncertainties into an overall estimate of the discharge error. In the United States, the principal error studies have been conducted by Carter and Anderson (1963), Smoot and Carter (1968), and Schneider and Smoot (1976). In England, Herschy (1971 and 1985) has defined measurement error sources and magnitudes, and has published a number of papers on the subject. The International Organization for Standardization (ISO) (1979) has published standards for error determination, and these standards are based largely on the work of Herschy and Carter, but include information from other countries in Europe, the U.S.S.R, and Canada. The World Meteorological Organization (WMO) (1980), also has published guidelines for estimating errors in discharge measurements.

An extensive review of procedures for computing discharge measurement errors was published by Dickinson (1967). This review was extended and updated by Pelletier (1988). His report is an excellent summary and presentation of the many studies that have been performed, and it contains an extensive list of references. No attempt will be made herein to do a literature review. The interested reader should refer to the reports of Dickinson (1967) and Pelletier (1988).

The error studies of discharge measurements generally are not used by the USGS to compute the error of individual measurements. Rather, these studies have been used to confirm a quasi-quantitative method that has been used for many years to evaluate the accuracy of each measurement. This method is based on a qualitative evaluation of several factors, such as cross-section uniformity, velocity uniformity, stream bed conditions, and other factors that might, in the opinion of the streamgager, affect the accuracy of the measurement. The streamgager then assigns one of the following accuracy ratings to the measurement:

Excellent (within 2 percent of the actual "true" discharge)
Good (within 5 percent)
Fair (within 8 percent)
Poor (measured discharge 8 percent greater or less than the true discharge)

The error studies have shown that this is a reasonable rating system. However, with increased need for streamflow records and better quantification of the accuracy of the records, it is desirable to provide a more precise estimate of the accuracy of the individual discharge measurements.

PURPOSE AND SCOPE

The purpose of this report is to identify error sources in the measurement of stream discharge, and to quantify these errors to the extent possible on the basis of previous studies and practical experience. A procedure for combining the individual errors into an overall discharge measurement error will be defined.

The procedures defined in this report apply only to discharge measurements made by the velocity-area method using vertical axis cup-type current meters. The procedures do not apply to dilution methods, measurement methods utilizing structures (such as weirs and flumes), electromagnetic meter methods, or ultrasonic meter methods. Likewise, the methods do not apply to measurements made under ice cover, where measurements of width, depth, and velocity can be subject to large, undefined errors, or to measurements made outside the defined range of measuring conditions for the equipment used.

The standard errors given in this report are at the 68 percent level of significance (one standard deviation from the true value). That is, 68 percent of measurements made under similar conditions are expected to differ from the true value by less than one standard error. It should be noted that the errors given in much of the literature from other countries use the 95 percent level of significance (two standard deviations from the true value). The term uncertainty is used in much of this literature to refer to error at 95 percent or 99 percent significance levels. In this paper, however, the term uncertainty is used in its everyday, nontechnical sense.

MEASUREMENT ERROR SOURCES

The discharge of a stream usually is calculated from a series of measurements of width, depth, and velocity along a cross section of the stream. Theoretically, the true discharge would be an integration of the velocity and area throughout the cross section. In actual practice, however, the discharge is approximated by a series of finite summations,

$$Q = \sum_{i=1}^{N} (b_i \times d_i \times v_i) \tag{1}$$

where Q is the total calculated discharge, N is the number of segments in the cross section, b_i is the width of segment i, d_i is the depth of segment i, and v_i is the mean velocity in segment i. Measurement of segment widths, depths, and velocities are described in the following paragraphs.

Most discharge measurements are made by first laying out a cross section of the stream that is as near as possible perpendicular to the direction of flow. For shallow streams, measurements are most commonly made by wading and depths are determined using a calibrated rod. Width measurements are made with a tape measure or calibrated line (tagline) that is stretched across the stream and along the cross section. For streams too deep to wade, the measurements generally are made from a boat, a cableway, a bridge, or other structure. For these types of measurements, depths usually are determined using a weight suspended by a cable from a reel arrangement, calibrated so that depth can be read from a dial. Widths are determined by measuring along the cableway, bridge, or other structure with a tape or tagline. In the case of boat measurements, widths are measured with a tagline, or with surveying techniques. Velocities are measured in a series of verticals across the width of the stream with a Price AA or a Pygmy current meter. A vertical is defined as the vertical line in which depth and velocity measurements are made for the purpose of estimating the mean depth and mean velocity for a segment of the stream cross section. The segment extends, on each side, halfway to the adjacent vertical, if one exists, or all the way to the edge of the water. In each vertical, velocity is measured at one or two points to determine the average velocity in the vertical. In some instances, velocities may be measured at more than two points in each vertical.

Verticals are chosen so that flow in each segment of the streamflow measurement is approximately 5 percent or less of the total flow. This usually requires 25 to 30 verticals for each measurement.

Discharge measurements are computed using the mid-section method. This method assumes that the depth and mean velocity for a vertical apply throughout a segment extending half the distance to the verticals on either side of the vertical being measured.

Error sources consist of the following:

o Errors in cross sectional area, which relate to errors in measurement of width and depth, and errors in the assumption that the measured depth in a vertical represents the mean depth of a segment.

- o Errors in mean stream velocity, which relate to current meter errors, vertical and horizontal velocity distributions, velocity pulsation, oblique flow, stream turbulence, and other factors.
- o Errors associated with the computation method.
- o Errors caused by change in stage during the measurement, boundary effects, ice, obstructions, wind, incorrect equipment, incorrect measuring technique, poor distribution of the measurement verticals, carelessness, and other factors.

The following sections of this report address and quantify these error sources. These error sources are then combined in an equation that defines the overall discharge measurement error.

Uncertainties in Cross Section Area

Two primary factors, width and depth, enter into the determination of the cross-section area. Very little research has been done to quantify the errors that may occur in measuring these two quantities.

Width Errors

Width measurement errors are considered by most investigators to be small (less than 1 percent) or negligible, especially where width is determined by use of a measuring tape or tagline that spans the stream. For purposes of this report, width errors are considered insignificant, and are neglected in the overall computation of discharge measurement error.

Depth Errors

The uncertainty of making individual measurements of depth is considered significant; however, there is little or no information to quantify these errors. From practical experience, it is evident that depth errors may sometimes be quite large. For instance, if the streambed consists of “soft” sediments such as silt, mud, and muck, and a heavy sounding weight is used, it is often difficult for the streamgager to sense the streambed when sounding. In some cases, there may be high velocities and deep depths, causing drag on the sounding weight and line. Depths must then be corrected by applying wetline and dryline corrections. Uncertainties in measuring the vertical angle of the sounding line and uncertainties in the forces acting on the weight, meter, and sounding line can cause significant errors in depth determination. Mobile streambeds (sand) may be changing throughout the discharge measurement as a result of dunes and antidunes moving through the streamflow reach. A streamgager wading in the channel or a sounding weight resting on or near the streambed can cause scour under some conditions. Measurement of depth under these conditions can be subject to considerable uncertainty. Uneven, rough streambeds (cobbles, rocks, boulders,) also can cause depth measurement errors. Depth measurements made with a rod in high velocities will produce “pile-up” of water on the rod at the water surface, and if this is not properly accounted for, depth measurement errors will result. The type of sounding equipment (rod, cable and weight, and acoustic) used depends on the depth being measured, which relates to the percentage of error that may occur. Rod measurements are usually used for wading measurements when depths are less than about 3 to 4 feet. Cable and weight sounding is usually used when depths are greater than about 3 to 4 feet. Acoustic sounding methods are sometimes used for depths greater than about 5 feet.

Table 1 presents approximate standard errors, or methods for computing approximate average standard errors, in percent, attributable to individual depth measurement errors (S_d) for various measuring and streambed conditions. The depth measurement errors shown in table 1 are highly subjective and arbitrary, as there is little or no experimental data upon which to base the errors. They do conform as much as possible to information noted by some investigators. For instance, Dickinson (1967) reports that for a "well selected gaging site," the standard deviation would be less than 1 percent of the mean depth. Herschy (1971) states that depth measurements have a "tolerance" of 1 to 3 percent, depending on the magnitude of the depth. Both of these investigators are reporting approximate standard deviations for individual depth measurements. The 2 percent standard error shown for condition (A) in table 1 is partly based on these reports.

Table 1. Standard errors attributable to individual depth measurement errors in discharge measurements
[*D*, Depth, in feet; ≤, equal to or less than; ≥, equal to or greater than; --, not applicable]

Depth measuring conditions	Standard error, or method for computing average standard error, in percent, for indicated type of measurement		
	Rod suspension $D \leq 4$ ft	Cable suspension $D \geq 3$ ft	Acoustic $D \geq 5$ ft
(A) Stable streambed (even, firm, smooth)	2	2	2
(B) Soft streambed (silt, mud, muck)	$2\sqrt{1+\left(\frac{5}{2D}\right)^2}$	$2\sqrt{1+\left(\frac{30}{2D}\right)^2}$	$2\sqrt{1+\left(\frac{30}{2D}\right)^2}$
(C) Stable streambed (Uneven, gravel, cobbles, for rod suspension. Uneven, cobbles, boulders, for cable suspension.)	$2\sqrt{1+\left(\frac{10}{2D}\right)^2}$	$2\sqrt{1+\left(\frac{30}{2D}\right)^2}$	$2\sqrt{1+\left(\frac{30}{2D}\right)^2}$
(D) Mobile streambed (shifting sand, dunes)	10	10	10
(E) Stable streambed (high velocity and some vertical angles)	--	5	--
(F) Unstable streambed (high velocity and some vertical angles)	--	15	--

For conditions (B) and (C), the methods shown in table 1 for computing average standard errors are based on a root mean square of the errors defined for condition (A) and an absolute error accounting for the soft or uneven streambeds. The absolute error used for rod measurements in a soft streambed is +/- 0.05 ft, and for rod measurements on an uneven stable streambed, +/- 0.1 ft.

For cable suspension and acoustic depth sounding measurements for both streambed conditions, the absolute error was assumed to be +/- 0.3 ft. The root mean square equations for conditions (B) and (C) were algebraically reduced to the form shown in table 1, which shows that the standard error approaches 2 percent as depth increases.

For the remaining conditions, (D), (E), and (F), the standard errors in table 1 are based on experience and are highly subjective. For condition (D), it is assumed that any individual depth measurement made in a stream with shifting sand and possibly moving dunes (or anti-dunes) could have a standard error of about 10 percent. For condition (F), which represents an unstable streambed, but one with high velocities and vertical angles, it was assumed that individual standard errors for depth are about 15 percent. For condition (E), a stable streambed with high velocities and vertical angles, it was assumed that individual standard errors for depth are about 5 percent. Conditions (E) and (F) apply only to cable suspension measurements of depth.

For purposes of comparison, the individual depth measurement standard errors were converted to feet for selected depths and are shown in table 2 and in figures 1 and 2 for the various streambed conditions and sounding methods. Values for rod suspension measurements have been rounded to hundredths, and values for cable suspension measurements have been rounded to tenths of a foot. Values for acoustic depth measurements are the same as those for cable suspension measurements, and are not included in table 2 and figure 2.

Table 2. Standard errors for individual depth measurements, based on the percentages, or formulas, given in table 1

[--, not applicable]

Depth measuring condition (table 1)	Standard error, in feet								
	Rod suspension					Cable suspension			
	Depth, in feet					Depth, in feet			
	0.5	1	2	3	4	3	10	30	50
(A)	0.01	0.02	0.04	0.06	0.08	0.1	0.2	0.6	1.0
(B)	.05	.05	.06	.08	.09	.3	.4	.7	1.0
(C)	.10	.10	.11	.12	.13	.3	.4	.7	1.0
(D)	.05	.10	.20	.30	.40	.3	1.0	3.0	5.0
(E)	--	--	--	--	--	.2	.5	1.5	2.5
(F)	--	--	--	--	--	.4	1.5	4.5	7.5

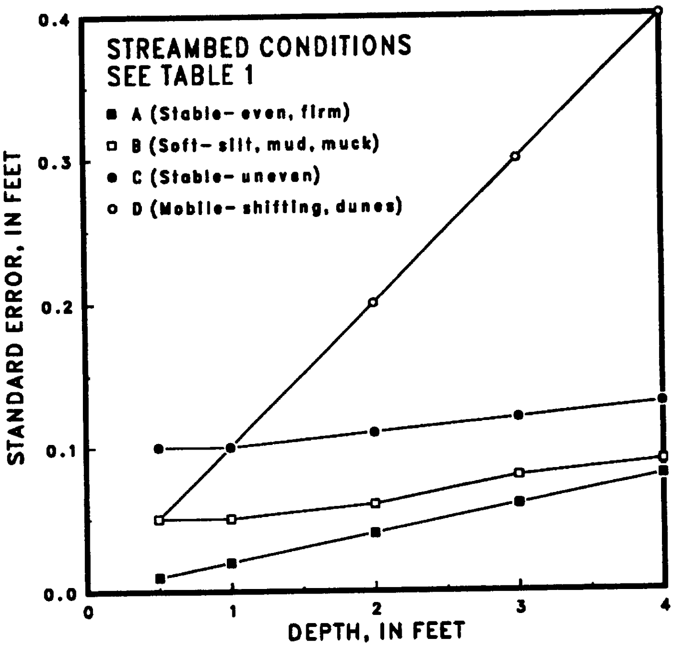

Figure 1.--Relation between depth and standard error for individual depth measurements made using a rod suspension system.

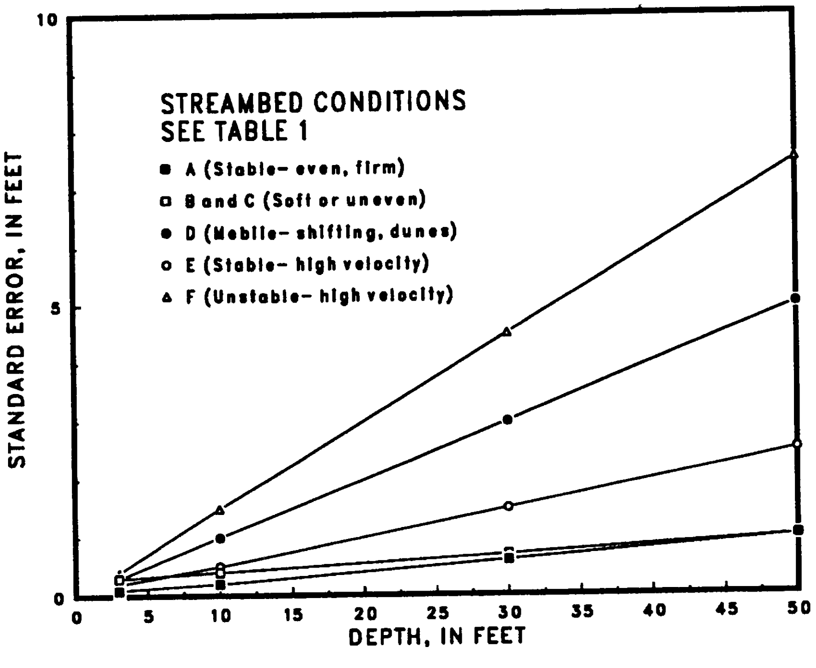

Figure 2.--Relation between depth and standard error for individual depth measurements made using a cable suspension system.

Uncertainties in Mean Velocity

Several factors enter into the determination of mean stream velocity, and most of these have been closely analyzed for error magnitude. The primary error sources for velocity are related to instrument errors, vertical velocity distribution, horizontal velocity distribution, velocity pulsation, oblique flow, and stream turbulence. Except for horizontal velocity distribution, these will be discussed and evaluated in this section. Horizontal velocity distribution will be discussed in the section on computational methods.

Instrument Errors

Two types of current meters, the Price AA and the Price Pygmy, are predominantly used in the United States for measuring stream velocity. These are both vertical-axis, cup-type meters. Although some use is made of horizontal-axis, vane, or propeller-type meters, these are not included in this analysis. Electromagnetic and ultrasonic meters also are not included because they are not used extensively, and little error data are available for them.

Instrument error for the two Price current meters has been defined by several investigators. For purposes of this study, data by Smoot and Carter (1968) are used in evaluating the error for the Price AA current meter with metal cups, for both individual and standard ratings. For the Price Pygmy meter, instrument error data given by Schneider and Smoot (1976) will be used.

The current meter errors reported by these investigators represent differences between different current meters, or groups of current meters. Although the reported errors are composed of both random and systematic components, the random component is considered very small. The systematic component becomes dominant because the usual practice is to use the same current meter throughout a discharge measurement. The current meter error, therefore, is treated as a systematic error when computing the overall discharge measurement error in this paper.

Smoot and Carter (1968) evaluated instrument error for the Price AA current meter and found no significant differences between the individually rated and standard rated meters. Likewise, they found no significant differences between new and used meters provided the meters were in good repair. The standard errors listed in table 3 are an average of their results for several different groups of meters, and for individual and standard ratings. Their results indicate that for velocities greater than about 2.3 feet per second (ft/s) instrument error is constant at about 0.3 percent. The standard errors for velocities from 0.25 to 2.2 ft/s appear to be logarithmically distributed and were thus used in a regression analysis to define an equation. The equation can probably be extrapolated down to about 0.1 ft/s. Based on this analysis, the instrument error for the Price AA current meter for velocities in the range of 0.1 to 2.3 has been defined as:

$$S_i = \frac{0.7}{V} \qquad (2)$$

where S_i is the instrument standard error, in percent, V is the mean velocity, in ft/s, and 0.7 is the regression constant. The relation between mean velocity and standard error is shown in figure 3.

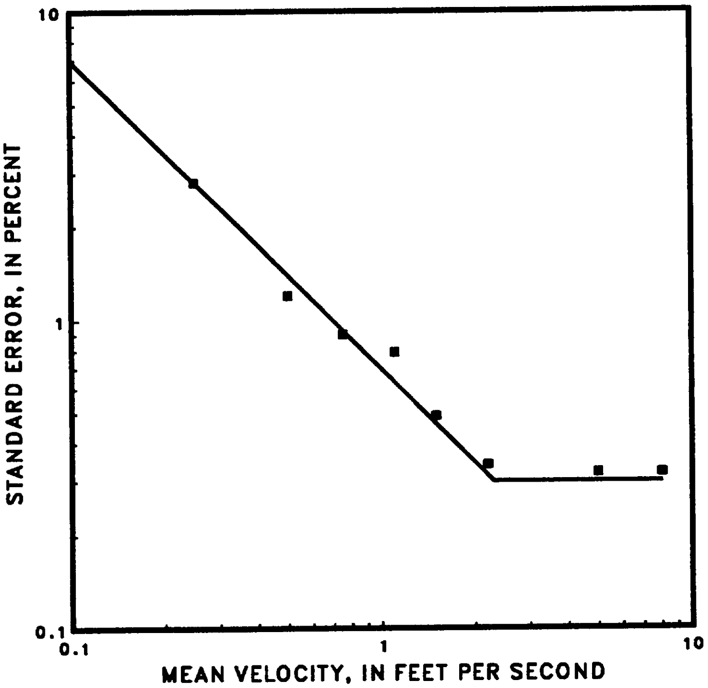

Figure 3.--Relation between mean velocity and standard error of individual measurements for Price AA current meters.

Table 3. Instrument error for Price AA current meters (from Smoot and Carter, 1968)

Velocity, in feet per second	Standard error, in percent
0.25	2.8
.50	1.2
.75	.90
1.1	.79
1.5	.49
2.2	.34
5.0	.32
8.0	.32

Instrument error for the Price Pygmy current meter was evaluated by Schneider and Smoot (1976) for new and used meters, as well as standard and individually rated meters. Their study is based on three groups of 50 meters and one group of 26 meters that were tested in a tow tank at speeds of 0.25 to 3.00 ft/s. The meters were equipped with metal cups and the standard beaded contact wire, except for the fourth group of 26 meters which were equipped with a straight unbeaded contact wire. One group of meters were new meters, and three groups were used meters. Each meter was rated individually, and a standard rating was developed for each group of meters.

The results of their studies indicate that for most of the velocity range there is a significant difference between standard rated and individually rated Pygmy meters. However, new meters for the most part show about the same error characteristics as used meters. The type of contact wire (beaded versus unbeaded) did not seem to make a significant difference. For purposes of this report, their results for all four groups of meters were averaged and used to define error functions. Separate error functions were defined for standard rated meters and individually rated meters. Table 4 presents the average standard errors as determined from the data presented by Schneider and Smoot (1976).

Table 4. Instrument error for standard and individually rated Price Pygmy current meters (from Schneider and Smoot, 1976)

Velocity, in feet per second	Standard error, in percent	
	Standard ratings	Individual ratings
0.25	5.14	4.20
.50	2.22	1.62
.75	1.73	1.14
1.50	1.51	.92
2.20	1.29	.60
3.00	1.42	.52

A logarithmic plot of the data in table 4 is presented in figure 4. This figure indicates that, for velocities less than about 0.5 ft/s, the error for Price Pygmy meters increases rapidly as velocity decreases. Equations 3, 4, 5, and 6 are based on the plots in figure 4 and can be used to estimate instrument standard error, S_i,in percent, for the Price Pygmy meter.

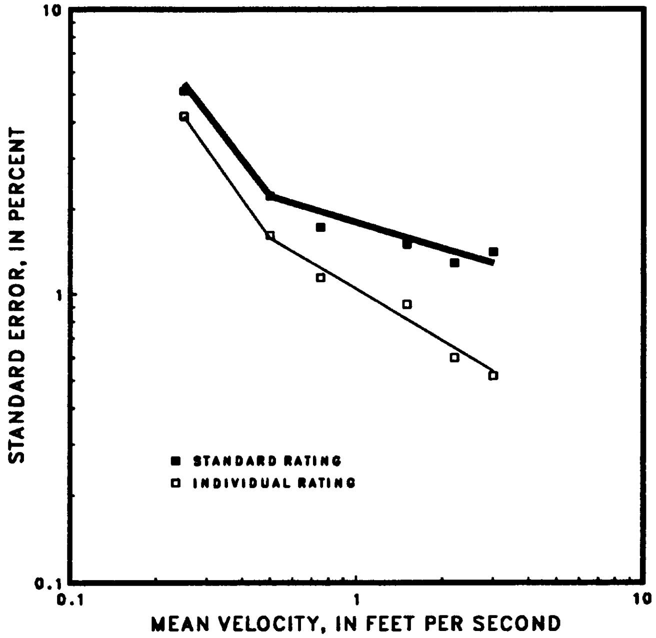

Figure 4.--Relation between mean velocity and standard error of individual measurements for standard and individually rated Pygmy current meters.

For individually rated meters and velocities (V) in the range of 0.1 to 0.5 ft/s,

$$S_i = 0.6V^{-1.4} \tag{3}$$

For individually rated meters and velocities in the range of 0.5 to 3.0 ft/s,

$$S_i = 1.05V^{-0.6} \tag{4}$$

For standard rated meters and velocities in the range of 0.1 to 0.5 ft/s,

$$S_i = 0.9V^{-1.3} \tag{5}$$

For standard rated meters and velocities in the range of 0.5 to 3.0 ft/s,

$$S_i = 1.8V^{-0.3} \tag{6}$$

Price Pygmy meters generally are not used for very slow velocities (less than about 0.1 ft/s), or for velocities greater than about 3 ft/s. Extrapolation of these equations above and below the defined limits may sometimes be required but should be avoided if possible.

Pulsation Errors

Water flowing in natural rivers and streams has a tendency to pulsate at any given point. An instantaneous measurement of the velocity at a point could be considerably different from the mean velocity at that point. By observing the velocity over a period of time, the pulsation differences are averaged and the mean velocity during the time of exposure approaches the true mean velocity. In general, the longer the time of exposure, the more accurate the mean velocity becomes.

Studies by Carter and Anderson (1963), using data for 23 different rivers, for time periods of 15 to 240 seconds, for depths of 2.4 to 26.7 ft, for velocities of 0.43 to 7.9 ft/s, and for observation points of 0.2-, 0.4-, 0.6-, and 0.8-depth, show that pulsation errors vary with time of exposure and with the observation depth. Table 5 lists the standard error of velocity measurements resulting from pulsation for individual observation points. These errors are logarithmically distributed as shown in figure 5 and can be represented by equations 7 and 8.

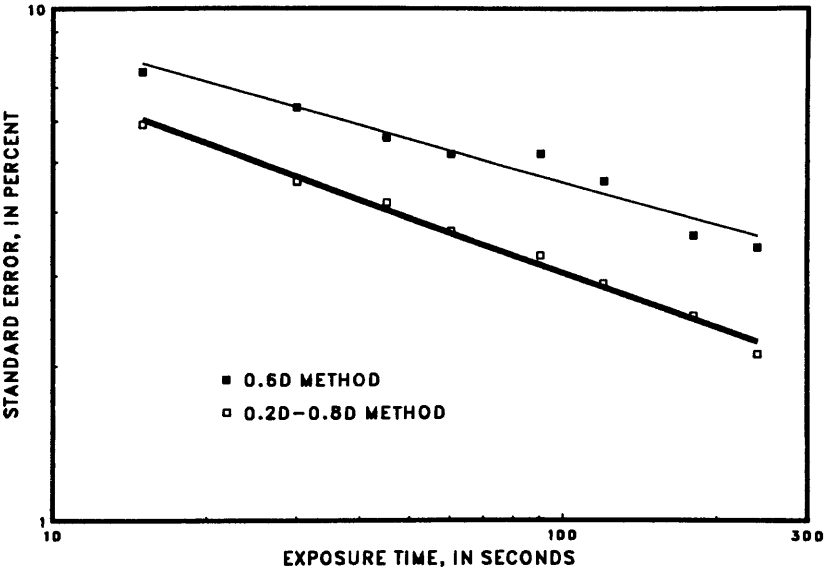

Figure 5.--Relation between standard error and exposure time for velocity pulsation for the 0.6-depth and the 0.2- and 0.8-depth measurement methods.

Table 5. Standard error for velocity pulsation

Depth of observation, as a fraction	Standard error, S_t, in percent, for indicated exposure time, in seconds							
of depth (D)	15	30	45	60	90	120	180	240
0.2D	5.7	4.7	4.2	3.8	3.3	2.9	2.5	2.0
.6D	7.5	6.4	5.6	5.2	5.2	4.6	3.6	3.4
.8D	10.3	8.0	7.2	6.4	5.6	5.0	4.4	3.6
0.2D-0.8D*	5.9	4.6	4.2	3.7	3.3	2.9	2.5	2.1

* The standard errors shown for the 0.2D-0.8D method are based on the standard error for 0.2D and 0.8D, and are computed as,

$$S_t = \frac{\sqrt{(S^2_{t_{0.2D}} + S^2_{t_{0.8D}})}}{2}$$

For the 0.6 depth (0.6D) method,

$$S_t = 16.6T^{-0.28} \tag{7}$$

For the 0.2 and 0.8 depth (0.2D-0.8D) method,

$$S_t = 16.0T^{-0.36} \tag{8}$$

where S_t is the standard error, in percent, for pulsation error, and T is the time of exposure, in seconds.

If methods other than the 0.6D or 0.2D-0.8D methods are used, or if a discharge measurement contains more than one method, then the pulsation error equation that most nearly fits the method used should be applied. For example, if the 0.5D method was used, then the equation for the 0.6D method should be applied. If both 0.6D and 0.2D-0.8D velocity observations were used in a discharge measurement, then the equation selected should correspond to the observation method used for most of the verticals in the discharge measurement.

Vertical Velocity Distribution Errors

The determination of the mean velocity in a vertical is usually based on the one-point method or the two-point method. The one-point method assumes that the mean velocity in the vertical equals the velocity measured at 0.6 of the depth (0.6D) below the water surface. The two-point method assumes that the mean velocity in the vertical equals the arithmetic mean of velocities measured at 0.2 of the depth (0.2D) and 0.8 of the depth (0.8D) below the water surface. Other methods are sometimes used which utilize more than two measured velocities in a vertical. The most accurate of these is referred to as the vertical velocity distribution method which is based on measured velocities at intervals of 0.1 of the depth from the surface to the streambed. This report, however, considers only the one-point and two-point methods, which are the commonly used methods for almost all discharge measurements.

Carter and Anderson (1963) used data from 1,800 verticals taken at more than 100 stream sites to show that the standard error, S_{rs}, of the mean velocity in a vertical averaged about 11.2 percent for the 1-point method, and 4.3 percent for the 2-point method. These errors were based on the difference between the mean velocity computed from the 1-point, or 2-point, method and the mean velocity computed from the 11-point vertical velocity profile, assumed to be the true mean. They did not define the standard error for methods using more than 2 observation points in the vertical.

Carter and Anderson (1963) also developed the following equation to compute the standard error due to error in the vertical velocity distribution over an entire cross section:

$$S_s = \frac{S_{rs}\sqrt{1 + (N-1)\bar{p}}}{\sqrt{N}} \qquad (9)$$

where S_s is the standard error, in percent, for the cross section, S_{rs} is the standard error, in percent, for a single vertical as defined in the preceding paragraph, N is the number of verticals in the cross section, and $\bar{p}$ is the average correlation coefficient for a cross section. They defined the value of $\bar{p}$ as 0.04.

Substituting values for S_{rs} and $\bar{p}$ in equation 9 yields the following equations for estimating S_s, the vertical velocity distribution error for an entire cross section, for the 1-point and 2-point methods:

For the 1-point method,

$$S_s = \sqrt{\frac{120.4}{N} + 5.02} \qquad (10)$$

and for the 2-point method,

$$S_s = \sqrt{\frac{17.75}{N} + 0.74} \qquad (11)$$

Oblique Flow Errors

Oblique flow (flow not perpendicular to the measurement section) can be either horizontal or vertical. The Price current meter is not affected by horizontal oblique flow and will generally register the same for flow parallel to the axis of the meter and for flow at moderate angles to the meter axis. Also, when the meter is suspended by cable, the meter will automatically align itself with the direction of flow. When a horizontal angle of flow exists, however, the velocity measured by the meter will be greater than the velocity component normal to the cross section. Therefore, a correction is applied by multiplying the measured velocity times the cosine of the angle between a line normal to the cross section and the direction of flow. Corrections for individual verticals may sometimes be large. The error for an individual vertical can be large because of inability to observe the angle accurately, inability to observe the angle at depths below the water surface, and fluctuation of the angle. Most measurements, however, have only a few verticals where horizontal angles are present, and the overall measurement error due to horizontal angles will be small and can probably be safely neglected. Where horizontal angles are present throughout most of a cross section, the overall error should be considered in the discharge measurement error. Because there is no information available to evaluate the magnitude of the standard error, S_h, for horizontal angle error, it is suggested that a standard error of 1 percent (roughly equivalent to an angular error of about 5 degrees) be used for cross sections where horizontal angles are present for most of the cross section.

Vertically oblique flow is not considered significant for purposes of this report. Although measurement errors can be introduced if the current meter is placed in the flow in such a way that the axis is inclined vertically with the direction of current, or the flow is not reasonably horizontal, these errors are generally thought to be small for most measurements. Vertical components of flow generally cannot be observed in the process of making a discharge measurement, and therefore adjustments are not made for vertically oblique flow. For purposes of this report, it has been assumed that errors due to vertically oblique flow are small and can be neglected.

Stream Turbulence Errors

There is confusion as to the effect of turbulence on the accuracy of velocity measurements with the Price current meter. Some investigators have indicated that the Price meter over-registers in turbulent flow, while others have studied the problem and concluded that the meter is not affected. Turbulence errors are also partly included in other velocity error components, such as velocity pulsation and vertical distribution. Carter and Anderson (1963) concluded that turbulence did not cause significant error, so for purposes of this report, it is assumed that no significant error is introduced by flow turbulence, and it will therefore be neglected.

Uncertainties In Computation Procedures

Two computational errors are considered, one being the method of computing the horizontal distribution of depth and velocity, and the other being the method used to compute flow between consecutive verticals. In some respects, these errors are related and will therefore be discussed together.

As discussed earlier in this report, a discharge measurement consists of measurements of depth and velocity at a number of verticals in a cross section, with discharge being computed for a segment represented by a vertical, or two adjacent verticals. Historically, two computation methods have been used, the mean section method and the mid-section method. The mean section method assumes a linear distribution of depth and velocity between verticals, and uses the mean depth and mean velocity of adjacent verticals to compute the discharge for the sub-area (segment) between two adjacent verticals. The mid-section method assumes the depth and velocity for a vertical applies to a sub-area (segment) extending halfway to the vertical on either side of the measured vertical. Different investigators have arrived at different conclusions regarding the accuracy of the two methods. The United States and Canada have adopted the mid-section method because of its simplicity and because the error due to the computation procedure is small.

The assumption of linearity and/or uniformity of depth and velocity between verticals has been studied by a number of investigators, including Carter and Anderson (1963) and Herschy (1971). The ISO (1979) recommends a standard, based on these investigations, for horizontal distribution errors. These errors are directly related to the number of verticals used for the discharge measurement. The standard error related to horizontal distribution, S_v, in percent, can be estimated from the following equation:

$$S_v = 32N^{-0.88} \tag{12}$$

This equation represents an average relation, as shown in figure 6, based on the data sets presented in the sources mentioned above.

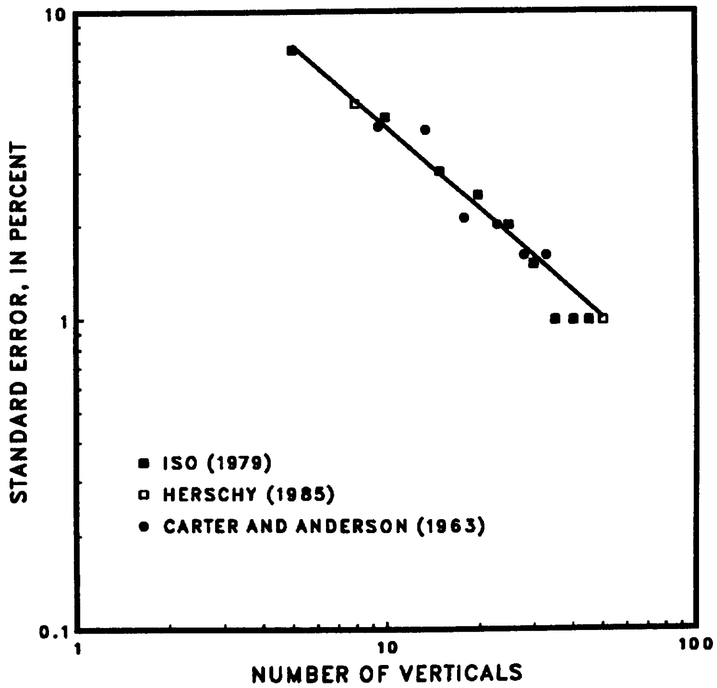

Figure 6.--Relation between number of verticals and standard error for horizontal distribution of velocity and depth.

Systematic Errors

All of the uncertainties mentioned to this point generally are referred to as random errors, meaning they can be either positive or negative and are randomly distributed throughout the discharge measurement. There are, in addition to the random errors, the possibility of systematic errors in the measurement of depth, width, and velocity. These are errors caused by improperly calibrated equipment, or improper use of such equipment, so that a systematic error (either positive or negative) is introduced. Such errors also can be referred to as biases. Most investigators have stated that systematic errors are small, generally less than 0.5 percent each for measurement of width, depth, and velocity. The systematic standard errors, as used in this report, are,

S_{sb} = 0.5 percent (for width),
S_{sd} = 0.5 percent (for depth),
and S_{sv} = 0.5 percent (for velocity).

Current-meter instrument error, S_i, was described earlier in this report as being composed of both random and systematic error components. Although it is treated as a systematic error, it is a separate error from the above systematic error for velocity.

Other Uncertainties

Uncertainties in the measurement of discharge can be caused by many factors for which the standard error cannot be readily assessed. Boundary effects, ice, flow obstructions, and wind can affect the flow and/or the individual measurements of depth and velocity so that errors are introduced. There has been little or no study of these factors to evaluate the magnitude of such errors. Improper measuring technique, which includes incorrect spacing of verticals, can result in a large percentage (greater than 10 percent) of the flow being measured in one or more verticals. Incorrect equipment and carelessness in making the measurement may introduce additional errors. For this analysis, however, it was assumed that the streamgager employs proper procedures and equipment in a careful manner, and that resulting uncertainties are small. Moderate to large changes in stage during the course of a discharge measurement will introduce uncertainties to the computed discharge and to the mean stage of the measurement, both of which affect the overall uncertainty of the discharge measurement. There is no known assessment of this error. To overcome rapid changes in stage, guidelines for making discharge measurements recommend that fewer verticals be used, that only one velocity measurement be made in each vertical, and that shorter times of exposure be used to measure point velocities. Each of these shortcuts is intended to decrease the duration of the discharge measurement, and hence to reduce the total change in stage during the measurement. Obviously, each shortcut also introduces additional uncertainty in the overall measurement.

Because there is little or no basis for assessing the errors mentioned in the preceding paragraph, it is recommended that standard errors not be computed for measurements having significant effects from boundary conditions, ice, flow obstructions, wind, improper equipment usage, or moderate to large changes in stage. If the standard error is computed for such a measurement, the resulting standard error should be stated as “greater than x percent.”

OVERALL DISCHARGE MEASUREMENT ERROR

The standard error, S_q, for an individual measurement of discharge can be estimated by determining the individual component errors that are considered significant as described in this report, and combining them into a root-mean-square error as follows:

$$S_q = \sqrt{\left(\frac{(S_d^2 + S_t^2)}{N}\right) + S_i^2 + S_s^2 + S_h^2 + S_v^2 + S_{sb}^2 + S_{sd}^2 + S_{sv}^2} \qquad (13)$$

This equation assumes that each of the error terms are independent of each other. It also assumes that the cross section is reasonably uniform so that average values of depth and velocity can be used. For cross sections that are not uniform, such as a main channel with an overflow channel, the standard error should be computed for subsections of the measurement where each subsection is reasonably uniform in depth and velocity. The subsection standard errors can then be combined by a root mean square computation to define the standard error for the entire discharge measurement.

The number of verticals, N, is used in equation 13 to account for the averaging effect of repeated measurements on errors caused by depth measurements (S_d) and pulsation of velocity (S_t). The standard error for vertical distribution of velocity (S_s) has already been adjusted for N because of the manner in which it is computed. The standard errors for current meters (S_i), oblique flow (S_h), and horizontal distribution of depth and velocity (S_v) apply directly to the entire cross section as described in the text. Each of the last three terms, S_{sb}, S_{sd}, and S_{sv}, are assumed to be 0.5 percent, and can therefore have that value substituted in the equation. The resulting equation for estimating discharge measurement error therefore reduces to,

$$S_q = \sqrt{\left(\frac{(S_d^2 + S_t^2)}{N}\right) + S_i^2 + S_s^2 + S_h^2 + S_v^2 + 0.75} \qquad (14)$$

The above equation can provide a useful estimate of standard error for most discharge measurements. The standard errors and related equations recommended for evaluating the terms in equation 14 are summarized in table 6. To illustrate the magnitude of error that might be expected for various measuring conditions, several hypothetical examples are presented in table 7. Note that standard errors can range from about 2 percent for measurements made under the best conditions (examples 2 and 6), to almost 20 percent for measurements made under very poor conditions using shortcut methods (example 5). Most measurements probably will fall in the range of 3 to 6 percent, which is typically considered a good measurement.

Table 6. Summary of discharge measurement uncertainty components
[V, velocity, in feet per second; D, depth, in feet; <, less than; >, greater than]

Type of uncertainty	Standard error symbol	Method of evaluation
Depth	S_d	See table 1
Velocity		
Instrument	S_i	Indeterminate (Price AA meter, $V < 0.1$) Equation 2 (Price AA meter, $0.1 < V < 2.3$) $S_i = 0.3$ percent (Price AA meter, $V > 2.3$) Equation 3 (Pygmy meter individual rating, $0.1 < V < 0.5$) Equation 4 (Pygmy meter individual rating, $0.5 < V < 3.0$) Equation 5 (Pygmy meter standard rating, $0.1 < V < 0.5$) Equation 6 (Pygmy meter standard rating, $0.5 < V < 3.0$)
Pulsation	S_t	Equation 7 (0.6D method) Equation 8 (0.2D-0.8D method)
Vertical distribution	S_s	Equation 10 (0.6D method) Equation 11 (0.2D-0.8D method)
Horizontal angles	S_h	$S_h = 0$ percent (for none or few verticals with horizontal angles) $S_h = 1$ percent (for many verticals with horizontal angles)
Computation of horizontal distribution of velocity and depth	S_v	Equation 12

Table 7. Examples of standard errors of measurement for various types of discharge measurements
[D, depth; Py(indv), Pygmy meter individual rating; Py(std), Pygmy meter standard rating; AA, Price AA meter]

Measurement variables and standard error	Measurement examples					
	(1) Good wading	(2) Good Cable	(3) Sluggish flow	(4) Shortcut methods	(5) Very poor overall	(6) Very good wading
Measurement variables						
Average depth, (ft)	1.8	10	10	15	5	2.2
Average velocity, (ft/s)	1.5	2.5	.15	5	.1	2.5
Average time of exposure, (seconds)	45	50	50	23	20	50
Number of verticals, N	25	28	28	6	10	30
Method	.6D	.2D-.8D	.2D-.8D	.6D	.6D	.2D-.8D
Horizontal angles	No	No	No	No	Yes	No
Suspension	Rod	Cable	Cable	Cable	Cable	Rod
Meter	Py(indv)	AA	AA	AA	Py(std)	AA
Depth measuring condition (table 1)	A	A	B	C	B	A
Standard error (table 6)						
S_d, percent	2.0	2.0	3.6	2.8	6.3	2.0
S_t, percent	5.7	3.9	3.9	6.9	7.2	3.9
S_i, percent	.8	.3	4.7	.3	18	.3
S_s, percent	3.1	1.2	1.2	5.0	4.1	1.2
S_h, percent	0	0	0	0	1.0	0
S_v, percent	1.9	1.7	1.7	6.6	4.2	1.6
S_q, percent (see equation 14)	4.0	2.4	5.3	8.9	19	2.3

Note that a good cable suspension measurement (example 2 in table 7) shows a significantly better standard error than a good wading measurement (example 1 in table 7). Some may question this apparent anomaly; however, for these two examples, the cable suspension measurement uses the two-point (0.2D-0.8D) method, whereas the wading measurement uses the one-point (0.6D) method. This makes a significant difference because the vertical velocity distribution uncertainty, S_s, is much less when the two-point method is used. Example 6 in table 7 is for a wading measurement similar to that in example 1 except that the two-point method is used. Here the standard error is about the same as that for the cable suspension measurement (example 2).

The standard errors, S_q, computed from equation 14 may not consider all sources of uncertainty, and therefore may not be highly accurate. However, they do provide a relative reference for comparison, and can be used as an aid when defining stage-discharge relations and shifting control conditions.

COMPUTER PROGRAM

A computer program has been developed for evaluating the overall standard error of individual discharge measurements. The program uses the methods for calculating estimates of standard errors described in this paper. It is designed for use on the Prime computer, but personal computer versions are also available. The program prompts the user for the necessary information, alerts the user to acceptable ranges of input data, and then automatically computes the standard error. An evaluation can be made for one or many measurements, and the results are stored in an ASCII file and labeled with a date and time. The file contains a summary of input data, computed standard error, and corresponding qualitative rating (excellent, good, fair, or poor), for each discharge measurement.

SUMMARY AND CONCLUSIONS

This report presents methods for estimating the standard error for most error components of a current-meter discharge measurement, namely the measurement of width, depth, and velocity, and the computation procedures. The individual error components then can be combined using a root-mean-square error analysis to define the overall standard error of the discharge measurement. This analysis can be used to evaluate the uncertainty of most discharge measurements, which then can be used in further analysis of stage-discharge ratings and computation of daily discharge. The procedures described herein should not be used to estimate standard errors of measurements if the accuracy of the measurements are affected by boundary conditions, ice, obstructions, wind, the use of improper procedures and equipment, carelessness, or moderate to large changes in stage. Under these conditions, the standard error probably will be greater than that computed by the methods described herein.

REFERENCES

Carter, R. W., and Anderson, I. E., 1963, Accuracy of current meter measurements: Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, HY 4, Part 1, p. 105-115.

Dickinson, W. T., 1967, Accuracy of discharge determinations: Hydrology Papers, Colorado State University, Fort Collins, CO, No. 20, June, p. 1-54.

Herschy, R. W., 1971, The magnitude of errors at flow measurement stations: Water Resources Board, Reading Bridge House, Reading, Berkshire, England, TN11, (Revised), 30 p.

-----, 1985, Accuracy-streamflow measurement: Elsevier Applied Science Publishing, Chapter 14, p. 474-510.

ISO, 1979, Liquid flow measurement in open channels - velocity area methods: International Organization for Standardization, Geneva, Switzerland, International Standard ISO 748, 23 p.

Pelletier, P. M., 1988, Uncertainties in the single determination of river discharge: a literature review: Canadian Journal of Civil Engineering, vol. 15, no. 5, p. 834-850.

Schneider, V. R., and Smoot, G. F., 1976, Development of a standard rating for the Price Pygmy current meter: U.S. Geological Survey Journal of Research, vol. 4, no. 3, p. 293-297.

Smoot, G. F., and Carter, R. W., 1968, Are individual current meter ratings necessary?: American Society of Civil Engineers Journal of the Hydraulics Division, 94 (HY2), p. 391-397.

WMO, 1980, Manual on stream gaging: vol. I, Fieldwork, vol. II, Computation of discharge: Secretariat of the World Meteorological Organization, Operational Hydrology Report No. 13, WMO No. 519.