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I. Introduction

Forecasting can be broadly considered as a method or a technique for estimating many future aspects of a business or other operations. All forecasting methods can be divided into two broad categories: qualitative and quantitative. Many forecasting techniques use past or historical data in the form of time series. A time series is simply a set of observations measured at successive points in time or over successive periods of time and forecasts essentially provide future values of the time series on a specific variable such as sales volume. Exponential smoothing is one the time series techniques which is widely used in forecasting. Exponential smoothing gives greater weight to more recent observations and takes into account all previous observations. In ordinary terms, an exponential weighting scheme assigns the maximum weight to the most recent observation and the weights decline in a systematic manner as older and older observations are included. Weight in the exponential smoothing technique is given by exponential smoothing constant (α). Forecast values are varied with the values of this constant. So, forecast errors are also depended on α. Many authors used exponential smoothing method in forecasting. Snyder et al. (2002) has shown that exponential smoothing remains appropriate under more general conditions, where the variance is allowed to grow or contract with corresponding movements in the underlying level. Taylor (2003) investigated a new damped multiplicative trend approach. Gardner (2006) reviewed the research in exponential smoothing since the original work by Brown and Holt and brought the state of the art up to date and invented a complete statistical rationale for exponential smoothing based on a new class of state-space models with a single source of error. McKenzie and Gardner (2010) provided a theoretical rationale for the damped trend method based on Brown's original thinking about the form of underlying models for exponential smoothing. Hyndman (2002) provided a new approach to automatic forecasting based on an extended range of exponential smoothing methods. Corberan-Vallet et al. (2011) presented the Bayesian analysis of a general multivariate exponential smoothing model that allows us to forecast time series jointly, subject to correlated random disturbances.

No research paper has found which determine the optimal value of exponential smoothing constant. In this paper, mean square error (MSE) and mean absolute deviation (MAD) are considered to determine the forecast error and to minimize MSE and MAD, optimal values of exponential smoothing constant are determined.

II. Mathematical Formulation for MSE and MAD

Different forecasting techniques and time series analysis are described by Montgomery and Johnson (1976). From Montgomery and Johnson (1976), forecast at period t is given for exponential smoothing technique is given by,

$F_1 = A_0$; When $t = 1$

And

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \dots\dots\dots\dots\dots + \alpha(1-\alpha)^{t-1}A_0 \qquad (1)$$

When $t > 1$

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Where, F_t = Forecast at period t
A_{t-1} = Actual value of period $t-1$
α = Exponential smoothing constant
From (1), for different period, it can be written as,

$$F_1 = A_0 \quad (2)$$

$$F_2 = \alpha A_1 + \alpha(1-\alpha)A_0 \quad (3)$$

$$F_3 = \alpha A_2 + \alpha(1-\alpha)A_1 + \alpha(1-\alpha)^2 A_0 \quad (4)$$

$$F_4 = \alpha A_3 + \alpha(1-\alpha)A_2 + \alpha(1-\alpha)^2 A_1 + \alpha(1-\alpha)^3 A_0 \quad (5)$$

$$F_5 = \alpha A_4 + \alpha(1-\alpha)A_3 + \alpha(1-\alpha)^2 A_2 + \alpha(1-\alpha)^3 A_1 + \alpha(1-\alpha)^4 A_0 \quad (6)$$

And so on…..
The most commonly used measures of forecast accuracy are discussed by Mentzer and Kahn (1995). In this paper, mean square error and mean absolute deviation are considered to measure forecast accuracy.
Now Mean Square Error (MSE) can be determined as,

$$MSE = \sum_{i=1}^{t}(F_i - A_i)^2 / t$$

This can be written as,

$$MSE = [(F_1 - A_1)^2 + (F_2 - A_2)^2 + (F_3 - A_3)^2 + \ldots\ldots\ldots\ldots\ldots + (F_t - A_t)^2]/t$$

Putting the value of $F_1, F_2, F_3, \ldots\ldots\ldots, F_t$ equation of MSE is found,

$$MSE = [\{A_0 - A_1\}^2 + \{\alpha A_1 + \alpha(1-\alpha)A_0 - A_2\}^2 + \{\alpha A_2 + \alpha(1-\alpha)A_1 + \alpha(1-\alpha)^2 A_0 - A_3\}^2 + \ldots\ldots\ldots + \{\alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots\ldots\ldots\ldots + \alpha(1-\alpha)^{t-1} A_0 - A_t\}^2]/t \quad (7)$$

Again, Mean Absolute Deviation (MAD) is given by,

$$MAD = \sum_{i=1}^{t}|(F_i - A_i)| / t$$

This can be written as,

$$MAD = [|(F_1 - A_1)| + |(F_2 - A_2)| + |(F_3 - A_3)| + \ldots\ldots\ldots\ldots\ldots + |(F_t - A_t)|]/t$$

Putting the value of $F_1, F_2, F_3, \ldots\ldots\ldots, F_t$ equation of MAD is found

$$MAD = [|\{A_0 - A_1\}| + |\{\alpha A_1 + \alpha(1-\alpha)A_0 - A_2\}| + |\{\alpha A_2 + \alpha(1-\alpha)A_1 + \alpha(1-\alpha)^2 A_0 - A_3\}| + \ldots\ldots\ldots + |\{\alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots\ldots\ldots\ldots + \alpha(1-\alpha)^{t-1} A_0 - A_t\}|]/t \quad (8)$$

Now Trial and Error method is used to find minimum value of exponential smoothing constant. Putting the different values within 0 to 1 for exponential smoothing constant, MSE and MAD are calculated.

III. Result and Discussion

The method is described by considering a problem. A company's historical sales data are given for some previous periods. Actual values of sales are given in Table 1.

Table 1: Actual values sales data

Period (t)	Actual Value (A_t) (In thousands)
0	10
1	8
2	14
3	13
4	12
5	12.5

For different values of exponential smoothing constant, MSE and MAD are calculates. Mean square error for different values of exponential smoothing constant are determined using equation (7). Table 2 shows the MSE values for different α.

Table 2: MSE for different values of α

Exponential smoothing constant (α)	Mean Square Error (MSE)
0	133.85
0.1	76.374
0.2	45.241
0.3	28.3
0.4	18.95
0.5	13.685
0.6	10.687
0.7	9.02
0.8	8.211
0.82	8.13
0.84	8.074
0.85	8.054
0.86	8.04
0.87	8.033
0.88	8.031
0.89	8.034
0.9	8.044
0.95	8.175
0.97	8.267
1	8.45

Mean absolute deviation for different values of exponential smoothing constant are determined using equation (8). Table 3 presents the MAD values for different α.

Table 3: MAD for different values of α

Exponential smoothing constant (α)	Mean Absolute Deviation (MAD)
0	10.7
0.1	8.038
0.2	6.028
0.3	4.535
0.4	3.443
0.5	2.663
0.6	2.241
0.7	2.041
0.75	1.961
0.78	1.919
0.8	1.894
0.82	1.869
0.83	1.858
0.84	1.867
0.88	1.921
0.9	1.949
0.95	2.022
0.98	2.068
1	2.1

Values of MSE and MAD are differed with the values of α. MSE is decreased with increasing α up to 0.88 and after that MSE increases. Variation of MSE with α is shown in Figure 1. MAD is decreased with increasing α up to 0.83 and after that MAD is increased. Variation of MAD with α is shown in Figure 2. The value of exponential smoothing constant is 0.88 and 0.83 for minimum MSE and MAD respectively.

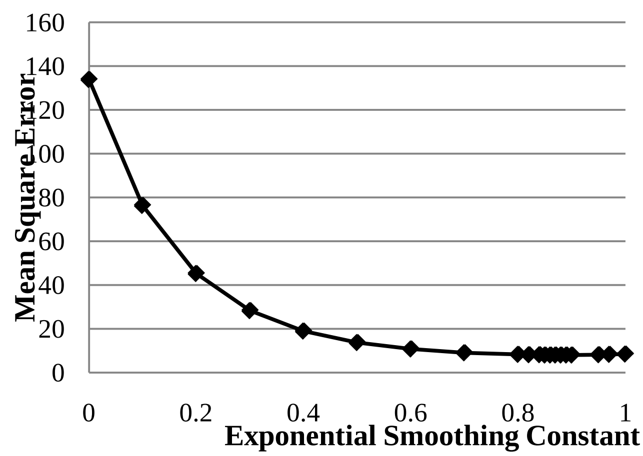

Figure 1: Variation of Mean Square Error for different values of α

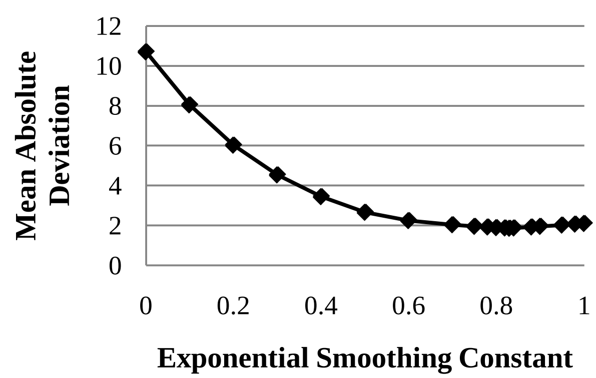

Figure 2: Variation of Mean Absolute Deviation for different values of α

To find the optimal value of exponential smoothing constant, minimum values of MSE and MAD are selected and corresponding value of exponential smoothing constant is the optimal value for this problem.

Minimum values of MSE and MAD and corresponding value of exponential smoothing constant is given in Table 4.

Table 4: Optimal Values of α for minimum MSE and MAD

Criteria	Minimum Value	Optimal value of α
Mean Square Error	8.031	0.88
Mean Absolute Deviation	1.858	0.83

From the optimal value of α, forecast values for different period are determined using equation (1) and it is shown is Table 5. From the five period actual values, a sale at sixth period is also forecasted. Forecasts at sixth period are 12.455 thousand and 12.068 thousands for minimum MSE and MAD respectively.

Table 5: Forecast values for minimum MSE and MAD

Period	Actual values (in thousands)	Forecast values (in thousands) for minimum MSE	Forecast values (in thousands) for minimum MAD
0	10	---	---
1	8	10	10
2	14	8.096	8.051
3	13	13.291	12.750
4	12	13.034	12.765
5	12.5	12.124	11.794
6	---	12.455	12.068

Figure 3 represents the actual values and corresponding forecast values for minimum mean square error. At period 2, forecast value differs significantly from actual value because forecast at period 1 is equal to actual value at period 0 and after that, forecast values are very close to actual values. The minimum mean absolute deviation is shown in Figure 4. actual values and corresponding forecast values for

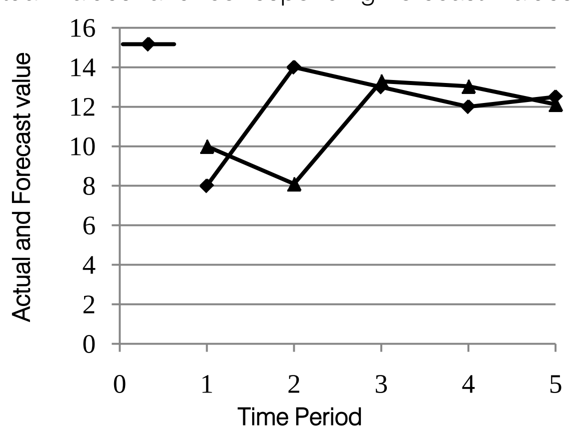

Figure 3: Comparison of actual values and forecast

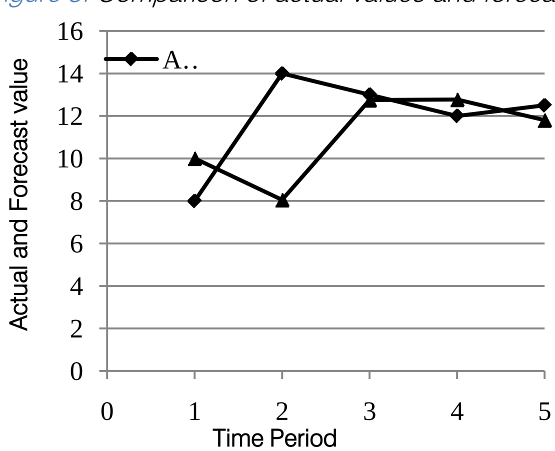

Figure 4: Comparison of actual values and forecast values to minimize MAD

IV. Conclusions

An exponential smoothing technique is widely used by many organizations to predict the future events. But problems are raised to assign the value of exponential smoothing constant. In this paper, this problem is solved by determining the optimal value of exponential smoothing constant. Mean square error and mean absolute deviation are minimized to get optimal value of the constant and optimal values are 0.88 and 0.83 for minimum mean square error and mean absolute deviation respectively. In this method, any organization can compute the optimal value of exponential smoothing constant to enhance the accuracy of forecasting. This work can be extended to minimize some other forecast errors such as mean absolute percent error (MAPE), cumulative forecast error (CFE) etc.

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