

Determination of optimum parameters for multi-performance characteristics in turning by using grey relational analysis

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Abstract Optimization of multi-criteria problems is a great need of producers to produce precision parts with low costs. Optimization of multi-performance characteristics is more complex compared to optimization of single-performance characteristics. The theory of grey system is a new technique for performing prediction, relational analysis, and decision making in many areas. In this paper, the use of grey relational analysis for optimizing the turning process parameters for the workpiece surface roughness and the chip thickness is introduced. Various turning parameters, such as cutting speed, feed rate, tool nose radius, and concentration of solid–liquid lubricants (minimum-quantity lubricant) were considered. A factorial design with eight added center points was used for the experimental design. Optimal machining parameters were determined by the grey relational grade obtained from the grey relational analysis for multi-performance characteristics (the surface roughness and the chip thickness). The results of confirmation experiments reveal that grey relational analysis coupled with factorial design can effectively be used to obtain the optimal combination of turning parameters. Experimental results have shown that the surface roughness and the chip thickness in the turning process can be improved effectively through the new approach. The minimum surface roughness and smallest chip thickness are 9.83 and 0.32 mm, respectively, obtained at optimal conditions of cutting speed, 1,200 rpm; feed rate, 0.06 mm/rev; nose radius, 0.8 mm; and concentration of solid–liquid lubricant (10% boric acid + SAE-40 base oil).

Keywords Optimization · Grey relational analysis · Turning · Surface roughness

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1 Introduction

Machining by turning involves the use of a lathe and is used primarily to produce cylindrical or conical parts. With common attachments, flat faces curved surfaces grinding, and boring can be done on a lathe. Therefore, it is valuable to increase tool life, to improve surface accuracy, to reduce cutting force and chip thickness in turning operations through an optimization study. Among these four characteristics, surface roughness and chip thickness play the most important roles in the performance of a turned part. Cutting speed, feed rate, depth of cut, tool-workpiece material, tool geometry, and coolant conditions are the turning parameters which highly affect the performance measures. In order to improve machining efficiency, reduce the machining cost, and improve the quality of machined parts, it is necessary to select the most appropriate machining conditions. The setting of turning parameters relies strongly on the experience of operators. It is difficult to achieve the highest performance of a machine because there are too many adjustable machining parameters. In order to minimize these machining problems, there is a need to develop scientific methods to select cutting conditions and tool geometry for free machining of metals. In this article, the use of factorial design with eight added center points and grey relational analysis to optimize the turning parameters with multiple performance characteristics, including surface roughness and chip thickness, is reported. Turning process parameters such as cutting tool geometry and materials, depth of cut, cutting speed feed rates, as well as the use of cutting fluids, will impact the material removal rates and the machining quality such as surface roughness and chip thickness [1, 2]. Proper selection of the cutting parameters can produce higher tool life, better surface roughness, and minimum chip thickness. Yang et al. [3] used the Taguchi method to investigate the cutting characteristics of S45C steel bars using

tungsten carbide cutting tools. The optimal cutting parameters, namely, cutting speed, feed rate, and depth of cut for turning operations, with regard to performance indexes such as tool life and surface roughness were considered. In their study, cutting speed, feed rate, and depth of cut were the primary factors investigated. One of the most important parameters in tool geometry is the tool nose radius. It strengthens the tool point. It also produced better surface finish, because tool marks are not deep as formed by sharp tools. In this study, tool nose radius has been taken into account along with cutting speed, feed rate, and depth of cut. Kopac et al. [4] investigated the optimal machining parameters for achieving good surface roughness in fine turning of cold pre-formed steel C15E4 (ISO). Taylor [5] introduced the concept of optimum speed for metal-cutting operations. Since then, many approaches have been proposed for optimizing machining parameters for better economic performance. Trang et al. [6] utilized a fuzzy logic Taguchi method to optimize the process parameters of the submerged arc welding in hard facing. Bhattacharyya [7] used the Lagrangian function method in searching for optimum cutting parameters. Modeling and optimization are necessary for the control of the steel turning process, to achieve improved product quality, high productivity, and low cost. Suresh et al. [8] developed a surface roughness prediction model for turning mild steel using response surface methodology. Surface roughness prediction model has also been optimized by using genetic algorithms [8]. Abhang et al. [9] developed a surface roughness prediction model for dry turning of EN-31 steel alloy using a response surface methodology. Surface roughness prediction model has also been optimized by using lingo-solver approach. Nihat Tosun [10] used the grey relational analysis technique and determined the optimum drilling process parameters. Various drilling parameters such as feed rate, cutting speed, drill type, and point angles were considered and optimized by the grey relational grade obtained from the grey relational analysis for multi-performance characteristics (surface roughness and the burr height). Lin [11] investigated the tool life, surface roughness, and burr formation in high-speed drilling of stainless steel using tin-coated carbide drill. Kao et al. [12] obtained grey relational grade using grey relational analysis while electrochemical polishing of the stainless steel. Optimal machining parameters were determined by the grey relational grade as the performance index. They observed that the performance characteristics such as surface roughness and passivation strength are improved.

In the present study, the minimum quantity lubrication is provided with solid lubricant mixed with SAE-40 base oil (10% boric acid, 10% MoS₂, and 10% graphite powder mixed with SAE-40 base oil by weight separately). Minimum quantity of lubrication without formation of foam is applied to the workpiece at approximately 4 to 5 ml/min,

which is not in contact with the tool. It is also required to see the effect of increasing concentration of solid lubricant with SAE-40 base oil on the cutting force for the machining of EN-31 steels. After running pilot experiments of different concentrations of solid lubricant (MoS₂, boric acid, and graphite with base oil SAE 40, i.e., 2%, 3%, 5%, 10%, 15%, 20%, and 25% by weight) at a cutting speed of 710 rpm, feed rate 0.10 mm/rev, depth of cut 0.4 mm, and tool nose radius 0.8 mm [13], 10% of each solid lubricant with SAE-40 base oil gave minimum (stable) cutting force during pilot experiments [13]. Therefore, we have selected 10% solid lubricant with SAE-40 base oil during performance of the experiments. Machining with 10% concentration of solid lubricant (MoS₂, boric acid, and graphite powder) mixed with SAE-40 base oil is an environmentally safe alternative to conventional metal cutting. Hence, an attempt has been made in the present work to investigate and select the proper minimum quantity lubricants in metal cutting; SAE-40 base oil is chosen as the mixing medium, due to its higher viscosity and hence improved lubricating properties. Minimum quantity lubricants (solid lubricants, MoS₂, boric acid, and graphite) exhibit minimal friction and provide effective separation between workpiece and tool surfaces. Several studies related to the lubrication properties of solid lubricants (boric acid) are carried out over the past several decades [14]. Another study focused on the use of solid lubricant (boric acid and MoS₂) as a lubricant in forming and drilling [15]. In metal-forming applications [15], it is shown that the boric acid provided very low friction between an aluminum workpiece and steel-forming tool. Shaji et al. [16] investigated the possibility of using graphite as a lubricating medium to reduce the heat generated in the grinding zone in surface grinding. Different process parameters like cutting forces, cutting temperature, specific energy, and surface roughness were observed and reported to be reduced when compared to those in grinding with conventional coolant. Graphite and molybdenum disulphide-assisted end milling process resulted in considerable improvement in the process performance as compared to that of machining with cutting fluid in terms of cutting forces, surface quality, and specific energy [17]. Solid lubricants like MoS₂, MoS₂-based grease, graphite-based grease, and silicon compound mixed with SAE-20 oil have been reported to improve surface quality at different proportions while machining aluminum and brass [18]. The feasibility of application of graphite as a solid lubricant in surface grinding was investigated by applying it in a suitable paste form to the workpiece surface of the wheel [19]. Ingole et al. [20] studied the effect of lubricants on the surface finish in burnishing of En8 specimens. Using 2³ factorial designs, surface roughness, model equations were developed. The burnishing parameters considered were speed, feed, and force, and the other parameters were

constant. The lubricants studied were SAE-30, grease, and a mixture of the two. Out of these, SAE-30 was found to be better. Venugopal et al. [21] investigated the use of graphite as a lubricating medium in the grinding process to reduce the heat generated at the grinding zone. The effective role of graphite as lubricant is evident from the overall improvement in the grinding process. Different process performance parameters like cutting forces, cutting zone temperatures specific energy, and surface roughness were observed and reported to be reduced when compared to those with grinding with conventional coolant. Shirsat et al. [22] studied the influence of burnishing parameters on surface finish in burnishing of aluminum specimens. The finishing parameters considered were speed, feed rate, and burnishing force. It was found that the surface roughness improves initially with an increase in these parameters. After a certain stage, the surface finish deteriorates, and fatigue life decreases. The lubricant studied were kerosene, SAE-30 oil, 5% graphite by weight in SAE-30 oil, and 10% graphite by weight in SAE-30 oil. Out of these, kerosene was found to be better.

The grey system theory initiated by Deng [23] in 1982 has been proven to be useful for dealing with poor, incomplete, and uncertain information. The grey relational based on the grey system theory can be used to solve the complicated interrelationships among the multiple performance characteristics effectively [24]. Palanikumar et al. [25] applied Taguchi and response surface methodologies for optimizing machining conditions in turning of Al/sic particulate metal matrix composites. They concluded that feed rate is a factor which has greater influence on surface roughness (R_a), followed by cutting speed and percentage of volume fraction of Sic. Lin [26] used grey relational analysis to optimize turning operations with multiple performance characteristics. He analyzed tool life, cutting force, and surface roughness in turning operations. Chaudhury et al. [27] had predicted surface roughness parameter R_a using response surface methodology and 2^3 factorial designs when turning high-strength steel. Lin et al. [28] used the grey relational analysis based on an orthogonal and fuzzy-based Taguchi method for optimizing a multi-response electrical discharge machining process. Brahmanekar et al. [29] used a new combination of response surface method and grey relational analysis to optimize electro-discharge machining parameters with multi-performance characteristics.

The purpose of the present work is to introduce the use of grey relational analysis in selecting optimum turning conditions on multi-performance characteristics, namely the surface roughness and chip thickness. To the best knowledge of the authors, there is no published work evaluating the optimization and the effect of metal-cutting parameters on the multi-performance characteristics in turning process by using grey relational analysis. The setting of turning parameters was accomplished using the factorial design with eight added center point (2^4+8) composite design [30]. In addition, the

most effective factor and the order of importance of the controllable factors to the multi-performance characteristics in the turning process were determined. Thus, by properly adjusting the control factors, we can improve work efficiency and produce quality parts.

2 Experimental conditions and procedure

In the metal-cutting process, the surface finish and chip thickness depend upon many parameters. Out of these parameters, cutting conditions and tool geometry play a major role in deciding the chip and surface quality. The range of each parameter is set at three different levels, namely low, medium, and high based on industrial practice. The initial cutting parameters were cutting speed of 710 rpm, a feed rate of 0.10 mm/rev, tool nose radius of 0.8 mm, and concentration of lubricants of 10% MoS_2 +SAE-40 base oil. To perform the experimental design, three levels of the cutting parameters were selected and listed in Table 1. Since the considered variables are multilevel variables and their outcome effects are not linearly related, it has been decided to use three level tests for each factor. The depth of cut 0.4 mm is kept constant throughout the experiments. In full factorial design, the number of experimental runs exponentially increases as the number of factors, and their levels. This results in a huge experimentation cost and considerable time periods. Fewer trials imply that time and cost are reduced, for example, for an experiment with four factors at three levels; a full factorial design would require $3^4=81$ trials. With three replications, the number of trials would be 243. So, in order to compromise these two adverse factors and to search for the optimal process condition through a limited number of experimental runs, composite factorial (2^4+8) consisting of 24 sets of data was selected to optimize the multiple performance characteristics of the turning process. Experiments were conducted with the process parameters given in Table 1, to obtain the machined surface on the EN-31 steel. A high-precision (LTM-20) heavy-duty lathe machine was used for experimentation as shown in Fig. 1. The workpiece material used for experimentation was EN-31 steel (size 500 mm in length and 51 mm in diameter). This material is suitable for a wide variety of automotive-type applications like axle, roller bearings, ball bearings, shear blades, spindle mandrels, forming and molding dies, rollers, blanking and forming tools, knurling tools, and spline shafts. These are produced using this material by turning process. The cutting tools used for experimentation were CNMA 120404, CNMA 120408, CNMA 120412, and a diamond-shaped carbide. The tool holder used for experimentation was WIDAX SCLCR 1212Fo9T3 (ISO-designated). Twenty-four experiments were conducted by

Table 1 Experimental conditions

| Levels of turning parameters | <i>A</i> Cutting speed (rpm) | <i>B</i> Feed rate (mm/rev) | <i>C</i> Tool nose radius (mm) | <i>D</i> Concentration of lubricants (%) |
|------------------------------|---------------------------------|--------------------------------|-----------------------------------|---|
| Low 1 (-1) | 250 | 0.06 | 0.4 | 10% graphite + SAE-40 base oil |
| Middle 2 (0) | 710 | 0.10 | 0.8 | 10% MOS ₂ + SAE-40 base oil |
| Higher 3 (+1) | 1200 | 0.15 | 1.2 | 10% boric Acid + SAE-40 base oil |

varying all the parameters identified to study the influence and optimization of these parameters on surface roughness and chip thickness. To obtain a more accurate result, each combination of experiments was repeated three times. In order to prevent a sudden increase of the cutting forces due to the dullness of the cutting edge, the carbide tool was changed after three repetitions of each experiment. Table 2 shows the selected design matrix based on composite factorial design consisting of 24 sets of coded conditions and the experimental results for the responses of surface roughness and chip thickness. All these data were utilized for the analysis and evaluation of the optimal parameter combination required to achieve the desired quality within the experimental domain. An optical surface roughness-measuring microscope was used to measure the surface roughness (Ra) of the machined components as shown in Fig. 2. The surface roughness was measured at three equally spaced locations around the circumference of the workpiece to obtain the statistically significant data for each test.

The chip thickness is a parameter that is used to understand the basic metal-cutting process. The comparison of chip produced is one of the major parameters influencing productivity in the metal-cutting industry. A lower chip thickness implies better lubrication at the chip tool interface and formation of chips of thinner sections, i.e., if the chip thickness decreases, the process efficiency goes up. The chips were collected at the end of each experiment, and the chip thickness was measured using a slider caliper. The chips were produced during machining with different lubricants as shown in Fig. 3a–c. The chip thickness values are the mathematical average of three measurements taken from the different specimens turned in the same experimental conditions.

**Fig. 1** Experimental setup on HMT heavy-duty lathe machine

3 Grey relational analysis

In the recent years, Deng proposed application of the principles of grey relational analysis [31]. Grey relational analysis is a method of measuring the degree of approximation among sequences according to the grey relational grade. The theories of grey relational analysis have already attracted the interest of researchers. In the grey relational analysis, the measured values of the experimental results of surface finish and chip thickness were first normalized in the range between zero and one, which is also called grey relational generation. Next, the grey relational coefficients were calculated from the normalized experimental results to express the relationship between the desired and actual experimental results. Then, the grey relational grades were computed by averaging the grey relational coefficient corresponding to each performance characteristic [32]. The overall equation of the multiple performance characteristic is based on the grey relational grade. As a result, optimization of the complicated multiple performance characteristics can be converted into optimization of a single grey relational grade. The optimal level of the process parameters is the level with highest grey relational grade. With the grey relational analysis, the optimal combination of the process parameters can be predicted.

Based on the above discussion, the use of the factorial design with grey relational analysis to optimize the turning parameters with multiple performance characteristics includes the following steps [26]:

- Identify the performance characteristics and cutting parameters to be evaluated.
- Determine the number of levels for the turning parameters.
- Select the factorial design matrix and assign the turning parameters.
- Conduct the experiments based on the factorial design of the experiment.
- Normalize the experimental results of surface roughness and chip thickness.
- Perform the grey relational generating and calculate the grey relational coefficient.
- Calculate the grey relational grade by averaging the grey relational coefficient.
- Analyze the experimental results using the grey relational grade.
- Select the optimal levels of turning parameters.

Table 2 Design matrix

| Exp. no. | A Cutting speed (RPM) | B Feed rate (mm/rev) | C Nose radius (mm) | D Concentration of lubricants (%) | Surface roughness (Ra) (μm) ^a | Chip thickness (mm) ^a |
|----------|-----------------------------|----------------------------|--------------------------|---|---|-------------------------------------|
| 1 | -1 | -1 | -1 | -1 | 11.86 | 0.41 |
| 2 | -1 | -1 | -1 | +1 | 10.43 | 0.34 |
| 3 | -1 | -1 | +1 | -1 | 12.30 | 0.38 |
| 4 | -1 | -1 | +1 | +1 | 11.53 | 0.35 |
| 5 | -1 | +1 | -1 | -1 | 12.79 | 0.50 |
| 6 | -1 | +1 | -1 | +1 | 10.58 | 0.40 |
| 7 | -1 | +1 | +1 | -1 | 13.84 | 0.51 |
| 8 | -1 | +1 | +1 | +1 | 12.80 | 0.42 |
| 9 | +1 | -1 | -1 | -1 | 11.82 | 0.43 |
| 10 | +1 | -1 | -1 | +1 | 9.94 | 0.31 |
| 11 | +1 | -1 | +1 | -1 | 10.86 | 0.40 |
| 12 | +1 | -1 | +1 | +1 | 9.86 | 0.30 |
| 13 | +1 | +1 | -1 | -1 | 13.87 | 0.46 |
| 14 | +1 | +1 | -1 | +1 | 11.60 | 0.45 |
| 15 | +1 | +1 | +1 | -1 | 11.80 | 0.49 |
| 16 | +1 | +1 | +1 | +1 | 10.44 | 0.46 |
| 17 | 0 | 0 | 0 | 0 | 12.49 | 0.48 |
| 18 | 0 | 0 | 0 | 0 | 11.50 | 0.39 |
| 19 | 0 | 0 | 0 | 0 | 10.55 | 0.49 |
| 20 | 0 | 0 | 0 | 0 | 11.70 | 0.44 |
| 21 | 0 | 0 | 0 | 0 | 11.20 | 0.35 |
| 22 | 0 | 0 | 0 | 0 | 10.08 | 0.37 |
| 23 | 0 | 0 | 0 | 0 | 9.85 | 0.45 |
| 24 | 0 | 0 | 0 | 0 | 12.23 | 0.32 |

^aAverage of three experiment results

3.1 Data preprocessing

Data preprocessing is normally required since the range and unit in one data sequence may differ from the others. Data preprocessing is also necessary when the sequence scatter range is too large or when the directions of the target in the sequences are different. Data preprocessing is a means of transferring the original sequence to a comparable sequence. Depending on the characteristics

of a data sequence, there are various methodologies of data preprocessing available for the gray relational analysis [33].

If the target value of the original sequence is infinite, then it has a characteristic of the “higher is better.” The original sequence can be normalized as follows:

$$x_i^*(k) = \frac{x_i^o(k) - \min x_i^o(k)}{\max x_i^o(k) - \min x_i^o(k)} \tag{1}$$

When the “lower is better” is a characteristic of the original sequence, then the original sequence should be normalized as follows:

$$x_i^*(k) = \frac{\max x_i^o(k) - x_i^o(k)}{\max x_i^o(k) - \min x_i^o(k)} \tag{2}$$

However, if there is a definite target value (desired value) to be achieved, the original sequence will be normalized in from:

$$x_i^*(k) = 1 - \frac{|x_i^o(k) - x_i^o|}{\max x_i^o(k) - x_i^o} \tag{3}$$



Fig. 2 Surface roughness-measuring microscope with workpiece EN-31 steel

Fig. 3 Chips produced with different lubricant during experiments. **a** 10% graphite + SAE-40 oil, **b** 10% MoS₂ + SAE-40 oil, **c** 10% boric acid + SAE-40 oil



Or, the original sequence can be simply normalized by the most basic methodology, i.e., let the value of the original sequence be divided by the first value of the sequence:

$$x_i^*(k) = \frac{x_i^o(k)}{x_i^o(1)} \tag{4}$$

Where $i=1, \dots, m$; $k=1, \dots, n$. m is the number of experimental data items, and n is the number of parameters. $x_i^o(k)$ denotes the original sequence, $x_i^*(k)$ the sequence after the data preprocessing, $\max. x_i^o(k)$ the largest value of $x_i^o(k)$, $\min. x_i^o(k)$ the smallest value of $x_i^o(k)$, and x_i^o is the desired value of $x_i^o(k)$.

3.2 Gray relational coefficient and gray relational grade

In gray relational analysis, the measure of the relevancy between two systems or two sequences is defined as the gray relational grade. When only one sequence, $x_o(k)$, is available as the reference sequence, and all other sequences serve as comparison sequences, it is called a local gray relation measurement. After data preprocessing is carried out, the gray relation coefficient $\xi_i(k)$ for the k th performance characteristics in the i th experiment can be expressed as follows [31, 32]

$$\xi_i(k) = \frac{\Delta_{\min} + \zeta \cdot \Delta_{\max}}{\Delta_{oi}(k) + \zeta \cdot \Delta_{\max}} \tag{5}$$

$$\Delta_{oi}(k) = |x_o^*(k) - x_i^*(k)|,$$

$$\Delta_{\max} = 1.00, \quad \Delta_{\min} = 0.00$$

Where, $\Delta_{oi}(k)$ is the deviation sequence of the reference sequence $x_o^*(k)$ and the comparability sequence $x_i^*(k)$. ζ is the distinguishing or identification coefficient which is defined in the range $0 \leq \zeta \leq 1$ (the value may be adjusted based on the practical needs of the system). A value of ζ is the smaller, and the distinguished ability is the larger.

The purpose of defining this coefficient is to show the relational degree between the reference sequence $x_o^*(k)$ and the comparability 24 sequences $x_i^*(k)$, where, $i=1, 2, \dots, m$ and $k=1, 2, \dots, n$ with $m=24$ and $n=2$ in this study.

The definition of grey relational grade in the course of grey relational analysis is to reveal the degree of relation between the 24 sequences $[x_o^*(k) \text{ and } x_i^*(k), i=1, 2, 3, \dots, 24]$.

After the grey relational coefficient is derived, it is usual to take the average value of the grey relational coefficients as the grey relational grade [31, 32]. The grey relational grade is defined as follows:

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k) \tag{6}$$

However, in a real engineering system, the relative importance of various factors varies. In the real condition of unequal weight being carried by the various factors, the grey relational grade in Eq. 6 was extended and defined as recommended by Deng [31] and Lin et al. [32].

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n w_k \xi_i(k) \quad \sum_{k=1}^n w_k = 1 \tag{7}$$

Where w_k denotes the normalized weight of factor k . Given the same weight, Eqs. 6 and 7 are equal.

Here, the grey relational grade γ_i represents the level of correlation between the reference sequence and the comparability sequence. If the two sequences are identical by coincidence, then the value of grey relational grade is equal to 1. The grey relational grade also indicates the degree of influence that the comparability sequence could exert over the reference sequence. Therefore, if a particular comparability sequence is more important than the other comparability sequences to the reference sequence, then the grey relational grade for that comparability sequence and reference sequence will be higher than other grey relational grades [33]. Grey relational analysis is actually a measurement of absolute value of data difference between sequences, and it could be used to measure approximation correlation between sequences.

4 Analysis and discussion of experimental results

In the present study, the workpiece surface roughness and chip thickness values in different turning parameters are listed in Table 2. In the turning process, lower surface roughness and chip thickness are indications of better performance. For data preprocessing in the grey relational analysis process, both surface roughness and chip thickness are taken as “lower is the better.” Let the results of 24 experiments be

the comparability sequences $x_i^o(k)$, $i=1-24$, $k=1$. All the sequences after data preprocessing using Eq. 2 are listed in Table 3 and denoted as $x_o^*(k)$ and $x_i^*(k)$ for reference sequence and comparability sequence, respectively.

The deviation sequences Δ_{oi} can be calculated as follows.

$$\begin{aligned} \Delta_{oi}(1) &= |x_o^*(1) - x_i^*(1)| = |1.00 - 0.5| = 0.5 \\ \Delta_{oi}(2) &= |x_o^*(2) - x_i^*(2)| = |1.00 - 0.476| = 0.524 \\ \text{So } \Delta_{oi} &= (0.5, 0.524). \end{aligned}$$

The same calculation method was performed for $i=1-24$, and the results of all Δ_{oi} for $i=1-24$ are given in Table 4. Using Table 4, Δ_{max} and Δ_{min} , can be found as follows.

$$\begin{aligned} \Delta_{max} &= \Delta_{01}(1) = \Delta_{24}(2) = 1.00 \\ \Delta_{min} &= \Delta_{24}(1) = \Delta_{24}(2) = 0.00 \end{aligned}$$

The distinguishing coefficient ζ can be substituted into Eq. 5 to produce the grey relational coefficient. The value for ζ is taken as 0.5 since both the process parameters are of equal weight. The grey relational coefficients and grade values of each experiment of the factorial design were

Table 3 The sequence after data preprocessing

| Reference/comparability sequence | Surface roughness | Chip thickness |
|------------------------------------|-------------------|----------------|
| Reference sequence, comp. sequence | 1.0000 | 1.0000 |
| Exp. no. 1 | 0.500 | 0.476 |
| Exp. no. 2 | 0.856 | 0.810 |
| Exp. no. 3 | 0.391 | 0.619 |
| Exp. no. 4 | 0.582 | 0.762 |
| Exp. no. 5 | 0.269 | 0.048 |
| Exp. no. 6 | 0.818 | 0.524 |
| Exp. no. 7 | 0.007 | 0.000 |
| Exp. no. 8 | 0.266 | 0.429 |
| Exp. no. 9 | 0.510 | 0.381 |
| Exp. no. 10 | 0.978 | 0.952 |
| Exp. no. 11 | 0.749 | 0.524 |
| Exp. no. 12 | 0.998 | 1.000 |
| Exp. no. 13 | 0.000 | 0.238 |
| Exp. no. 14 | 0.565 | 0.286 |
| Exp. no. 15 | 0.515 | 0.095 |
| Exp. no. 16 | 0.853 | 0.238 |
| Exp. no. 17 | 0.343 | 0.143 |
| Exp. no. 18 | 0.590 | 0.571 |
| Exp. no. 19 | 0.826 | 0.095 |
| Exp. no. 20 | 0.540 | 0.333 |
| Exp. no. 21 | 0.664 | 0.762 |
| Exp. no. 22 | 0.943 | 0.667 |
| Exp. no. 23 | 1.00 | 0.286 |
| Exp. no. 24 | 0.408 | 0.905 |

Table 4 The deviation sequence

| Deviation sequences | $\Delta_{oi}(1)$ | $\Delta_{oi}(2)$ |
|---------------------|------------------|------------------|
| Exp. no. 1 | 0.500 | 0.524 |
| Exp. no. 2 | 0.144 | 0.190 |
| Exp. no. 3 | 0.609 | 0.381 |
| Exp. no. 4 | 0.418 | 0.238 |
| Exp. no. 5 | 0.731 | 0.952 |
| Exp. no. 6 | 0.182 | 0.476 |
| Exp. no. 7 | 0.993 | 1.000 |
| Exp. no. 8 | 0.734 | 0.571 |
| Exp. no. 9 | 0.490 | 0.619 |
| Exp. no. 10 | 0.022 | 0.048 |
| Exp. no. 11 | 0.251 | 0.476 |
| Exp. no. 12 | 0.002 | 0.000 |
| Exp. no. 13 | 1.000 | 0.762 |
| Exp. no. 14 | 0.435 | 0.714 |
| Exp. no. 15 | 0.485 | 0.905 |
| Exp. no. 16 | 0.147 | 0.762 |
| Exp. no. 17 | 0.657 | 0.857 |
| Exp. no. 18 | 0.410 | 0.429 |
| Exp. no. 19 | 0.174 | 0.905 |
| Exp. no. 20 | 0.460 | 0.667 |
| Exp. no. 21 | 0.336 | 0.238 |
| Exp. no. 22 | 0.057 | 0.333 |
| Exp. no. 23 | 0.000 | 0.714 |
| Exp. no. 24 | 0.592 | 0.095 |

calculated by applying Eqs. 5 and 7 (Table 5). Table 5 shows the grey relational grade for each experiment using factorial design. The higher grey relational grade represents that the corresponding experimental result is closer to the ideally normalized value. Experiment 12 has the best multi-performance characteristics among 24 experiments because it has the highest grey relational grade as shown in Table 5 and Fig. 4. It can be seen that in the present study, optimization of the complicated multi-performance characteristics of turning En-31 steel alloy has been converted into optimization of a grey relational grade.

In addition to the determination of optimum turning parameters for surface roughness and chip thickness, the response table for the factorial design method was used to calculate the average grey relational grade for each level of the turning parameters. The procedure is:

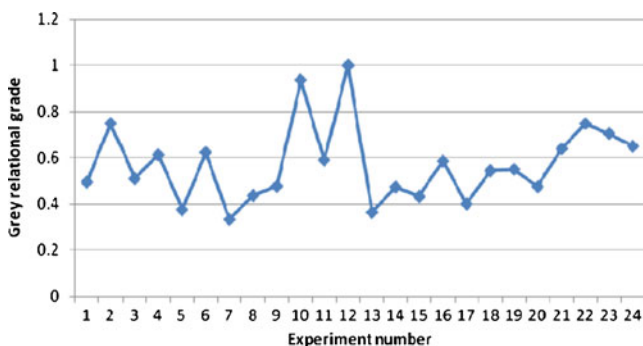
1. Group the grey relational grades by factor level for each column in the factorial design.
2. Take their average; for example, the grey relational grade for factor A at level 1 can be calculated as follows.

$$\begin{aligned} \text{Level1}(A) &= 1/8 (0.494 + 0.750 + 0.509 + 0.611 + 0.375 \\ &\quad + 0.622 + 0.334 + 0.436) = 0.516 \end{aligned}$$

Table 5 The calculated grey relational coefficient and grey relational grade for 24 comparability sequences

| Exp. no. | Grey relational coeff. Ra (μm) | Grey relational coeff. Tc(mm) | Grey relational grade |
|-------------|---|-------------------------------|-----------------------|
| Exp. no. 1 | 0.500 | 0.488 | 0.494 |
| Exp. no. 2 | 0.776 | 0.724 | 0.750 |
| Exp. no. 3 | 0.451 | 0.568 | 0.509 |
| Exp. no. 4 | 0.545 | 0.678 | 0.611 |
| Exp. no. 5 | 0.406 | 0.344 | 0.375 |
| Exp. no. 6 | 0.733 | 0.512 | 0.622 |
| Exp. no. 7 | 0.335 | 0.333 | 0.334 |
| Exp. no. 8 | 0.405 | 0.467 | 0.436 |
| Exp. no. 9 | 0.505 | 0.447 | 0.476 |
| Exp. no. 10 | 0.958 | 0.912 | 0.935 |
| Exp. no. 11 | 0.666 | 0.512 | 0.589 |
| Exp. no. 12 | 0.996 | 1.000 | 0.998 |
| Exp. no. 13 | 0.333 | 0.396 | 0.364 |
| Exp. no. 14 | 0.535 | 0.412 | 0.473 |
| Exp. no. 15 | 0.508 | 0.356 | 0.432 |
| Exp. no. 16 | 0.773 | 0.396 | 0.584 |
| Exp. no. 17 | 0.432 | 0.368 | 0.400 |
| Exp. no. 18 | 0.549 | 0.538 | 0.543 |
| Exp. no. 19 | 0.742 | 0.356 | 0.549 |
| Exp. no. 20 | 0.521 | 0.428 | 0.474 |
| Exp. no. 21 | 0.598 | 0.678 | 0.638 |
| Exp. no. 22 | 0.898 | 0.600 | 0.749 |
| Exp. no. 23 | 1.000 | 0.412 | 0.706 |
| Exp. no. 24 | 0.458 | 0.840 | 0.649 |

The mean of the grey relational grade values for each level of the turning parameters was calculated using the same method. The grey relational grade represents the level of correlation between the reference sequence and the comparability sequence, the greater value of the grey relational grade means that the comparability sequence has a stronger correlation to the reference sequence [32]. The mean of the grey relational grade for each level of the turning parameters is summarized and shown in the multi-response performance

**Fig. 4** Grey relation grades for the minimum surface roughness and chip thickness

index (Table 6). Figure 5 shows the grey relational grade obtained for different turning parameters. Basically, the larger the grey relational grade is, the closer will be the product quality to the ideal value. Thus, a larger grey relational grade is desired for optimum performance. Therefore, the optimal level of machining parameters setting for improved surface quality and minimum chip thickness is ($A_3, B_1, C_2,$ and D_3) as given in Table 6. The optimal level of the turning parameters is the level with the highest grey relational grade. An asterisk (*) indicates that the level value represents in a better turning performance. Based on the grey relational grade values given in Table 6, the optimal machining performance for both the surface roughness and the chip thickness was obtained for 10% boric acid lubricant (level 3), 1,200 rpm cutting speed (level 3), 0.06 mm/rev feed rate (level 1), and 0.8 mm tool nose radius (level 2) combination. The greater values in Fig. 5 give the smaller chip thickness and good surface finish quality. Therefore, experiment 12, as shown in Table 5 and Fig. 4, may be considered as very close to fit the optimal process conditions.

As listed in Table 6, the difference between the maximum and the minimum value of the grey relational grade of the turning parameters is as follows: 0.09 for cutting speed, 0.218 for feed rate, 0.028 for tool nose radius, and 0.230 for lubricant type. The most effective factor affecting performance characteristics is determined by comparing these values. This comparison will give the level of significance of the controllable factors over the multi-performance characteristics. The most effective controllable factor was the maximum of these values. Here, the maximum value among 0.09, 0.218, 0.028, and 0.230 is 0.230. The value indicates that the lubricant type has the strongest effect on the multi-performance characteristics among the other turning parameters. On the other hand, the significance of the role that every controllable factor plays over the multi-performance characteristics can be obtained by examining these values. The order of importance of the controllable factors to the multi-performance characteristics in the turning parameters, in sequence, can be listed as: factor D (lubricant type), B (feed rate), A (cutting speed), and C (tool nose radius) (i.e., $0.230 > 0.218 > 0.09 > 0.028$). Factor D (lubricant type) was the most effective factor to the performance. This indicates that the

Table 6 Response table for grey relational grade

| Turning parameters | Level 1 (-1) | Level 2 (0) | Level 3 (+1) | Max-min |
|--------------------|--------------------|--------------------|--------------------|---------|
| A (V) m/min | 0.516 | 0.589 | 0.606 ^a | 0.090 |
| B (F) mm/rev | 0.670 ^a | 0.589 | 0.453 | 0.217 |
| C (R) mm | 0.561 | 0.589 ^a | 0.562 | 0.028 |
| D (lubricant) | 0.446 | 0.589 | 0.676 ^a | 0.230 |

Average grey relational grade by factor level

^a Shows optimal turning parameters

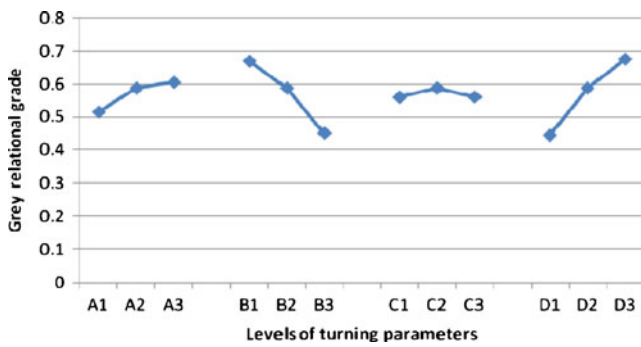


Fig. 5 Effect of turning parameters on the multi-performance characteristics

turning performance was strongly affected by the lubricant type.

4.1 Effect of turning parameters on performance measures

The multi-performance characteristic called grey relational grade was found to be significantly affected by lubricant concentration (lubricant type), feed rate, cutting speed, and tool nose radius. Figure 5 shows the effects of turning parameters on the multi-performance characteristics (the surface roughness and chip thickness) and the response graph of each level of the turning parameters for the performance. The response surfaces are developed by using the response surface methodology referred by Montgomery [30]. Basically, the larger the grey relational grade, the better are the multi-performance characteristics. The greater values in Figure 5 give the lower chip thickness and good surface finish quality. Figure 6 shows the response surface of grey relational grade. It is clear from Fig. 6 that the cutting speed and feed rate are the most significant factors that affect the grey relational grade. With an increase in cutting speed, both surface roughness and chip thickness decrease which increases grey relational grade. While an increase in surface roughness and chip thickness with increase in feed rate results into reduction of grey relational grade.

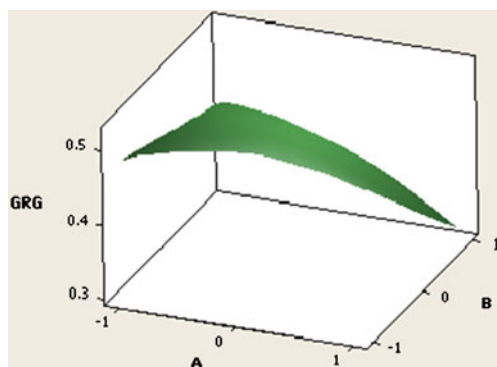


Fig. 6 Response surface of grey relational grade for combined effect of cutting speed and feed rate

Figure 7 shows the effect of turning parameters on surface roughness value. From this figure, a smoother surface can be produced by 10% boric acid+SAE-40 base oil (level 3), smaller feed rate (level 1), or using higher cutting speed (level 3) followed by nose radius at middle level. Variation in surface roughness is very small in the range of 0.4, 0.8, and 1.2 mm nose radii. For nose radii beyond 0.8 mm, it was found that the self-excited vibrations tend to deteriorate the surface roughness. The best results were obtained in the range of 0.4 to 0.8 mm nose radii [13]. The reason for improved surface finish at the range 0.4 to 0.8 mm nose radii is that the chip thickness is reduced at the nose region of the tool. If the nose radius is excessive, chatter may be introduced and small particles of work material may form a burr on the tool at the trailing edge of the nose causing a breakout of tool material. Under such conditions, the surface roughness tends to deteriorate.

Figure 8a and b shows the estimated response surface for the combined effects of turning parameters on Ra values at selected levels. The height of the surface represents the value of the Ra. It can be realized that the combination between high cutting speed and low feed rate results in a considerable reduction in surface roughness (Ra) and also between a high level of lubricant type concentration (10% boric acid + SAE-40 oil) and low feed rate results in a considerable reduction in surface roughness (Ra). The response surface plot (Fig. 8a) indicates that the minimum surface roughness is at about 1,200 rpm and 0.06 mm/rev. The response surface plot (Fig. 8b) indicates that the minimum surface roughness (Ra) is at about 10% boric acid + SAE-40 oil) (level 3), and 0.06 mm/rev.

Figure 9 shows the effect of turning parameters on chip thickness. From this figure, a minimum chip thickness can be produced by lubricant concentration at higher level (10% boric acid + SAE-40 oil), lower feed rate, and higher cutting speed followed by tool nose radius. Figure 10a and b shows the response surface of chip thickness. It clearly shows an increase in chip thickness with increase in the feed rate and also a decrease in chip thickness with the increase in cutting

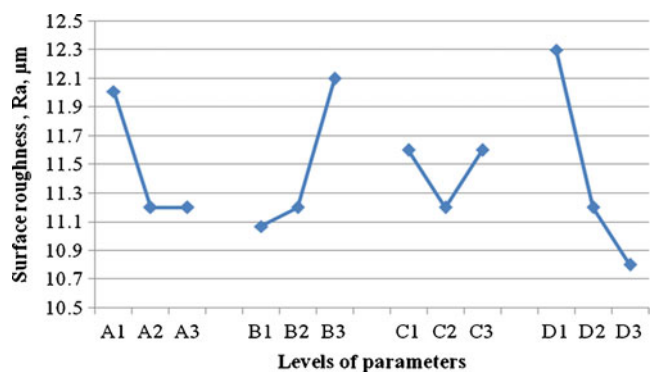


Fig.7 Effect of turning parameters on surface roughness

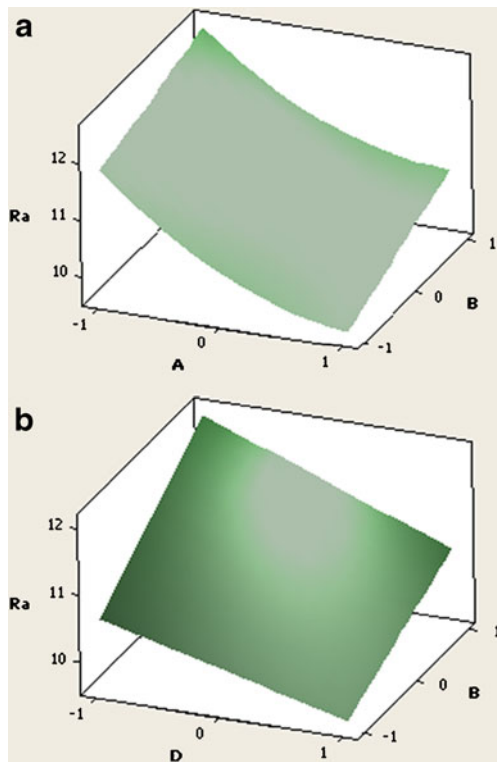


Fig. 8 **a** Response surface of combined effect of cutting speed and feed rate on surface roughness. **b** Response surface of combined effect of lubricant type and feed rate on surface roughness

speed [13]. The reduction in chip thickness was observed and is maximum with 10% boric acid + SAE-40 base oil lubrication (level 3) as compared to other solid–liquid (10% graphite + SAE-40 , 10% MoS₂ + SAE-40) lubricant. This is because of better lubrication effect produced by 10% boric acid + SAE-40 oil at the chip tool interface due to the formation of fluid cushion. Reduced chip thickness in 10% boric acid + SAE-40 oil lubricant machining results from the lowered cutting temperature and reduced adhesion between the tool and chip [13–15]. The variations in the surface roughness and chip thickness values were approved by these counters (Figs. 8 and 10) more clearly.

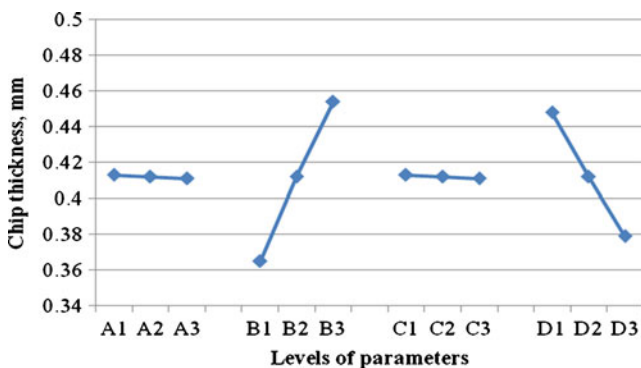


Fig. 9 Effect of turning parameters on chip thickness

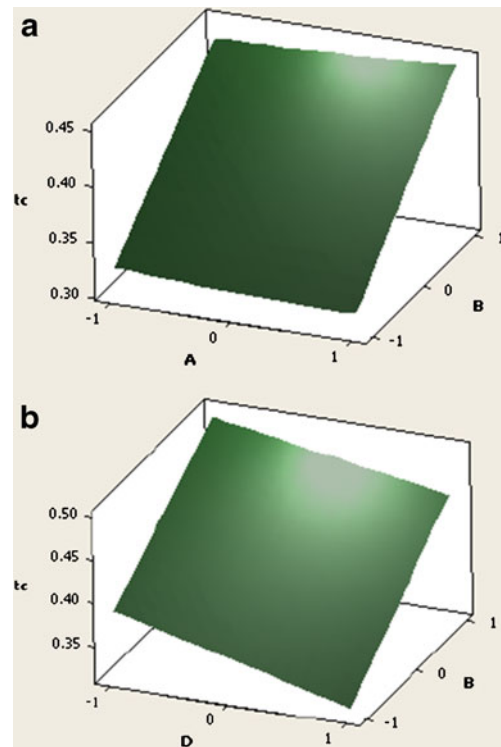


Fig. 10 **a** Response surface of combined effect of cutting speed and feed rate on chip thickness. **b** Response surface of combined effect of lubricant type and feed rate on chip thickness

5 Confirmation test

After obtaining the optimal level of the turning parameters, the next step is to verify the improvement of the performance characteristics using this optimal combination. Table 7 compares the results of the confirmation experiments using the optimal turning parameters (A_3, B_1, C_2, D_3) obtained by the proposed method and with those of the initial turning parameters (A_2, B_2, C_2, D_2), which are often introduced into the confirmation experiment in many of the studies (10, 11, and 26) for comparison to the optimum parameters, are performed on the lathe and drilling. Three trials were conducted at optimal level, and the corresponding surface roughness and chip thickness values (average of three trials)

Table 7 Comparison between initial level and optimal level

| Name of design | Best combination | Surface roughness (μm) | Chip thickness (mm) |
|---|------------------|-------------------------------------|---------------------|
| Initial design | $A_2B_2C_2D_2$ | 11.70 | 0.44 |
| Optimal design | $A_3B_1C_2D_3$ | 9.83 ^a | 0.32 ^a |
| Final gain at optimum combination of parameters | – | 16.06% | 27.30% |

^a Shows the average of three trial results

were measured and reported in Table 7. As shown in Table 7, surface roughness decreases from 11.70 to 9.83 μm and chip thickness decreases from 0.44 mm to 0.32 mm, or, in other words, the corresponding confirmation tests show that the values of surface roughness and chip thickness are greatly improved by 16.06% and 27.30%, respectively. Consequently, these confirmation tests reveal that the proposed method for solving the optimal combinations of the turning parameters in this work improves surface finish and chip thickness. It can be seen that the overall performance of turning process has been improved.

6 Conclusions

In this paper, the optimal turning parameters were determined for the multi-performance characteristics (surface roughness and chip thickness) in the turning process by using the grey relational analysis. The grey relational analysis, based on the factorial design matrix response table, was proposed as a way of studying the optimization of turning operation factors. The surface roughness and the chip thickness were selected to be the quality targets. From the response table of the average grey relational grade, the largest value of grey relational coefficient for the turning parameters was found. These values are the recommended levels of controllable turning parameters for the multi-performance characteristics. It was found that the lubricant type has the strongest effect among the other turning parameters used on the multi-performance characteristics. In other words, the most influential factor is lubricant type. The order of importance of the controllable factors to the multi-performance characteristics is lubricant type, feed rate, cutting speed, and tool nose radius. Experimental results have shown clearly that the surface roughness and the chip thickness in the turning operation can be improved effectively through the proposed approach. As a result, optimization of the complicated multiple performance characteristics can be greatly simplified through this approach. It is shown that the performance characteristics of the turning process such as surface finish and chip thickness are improved together by using the method proposed by this study. Optimum cutting conditions for minimum surface roughness and smallest chip thickness are minimum quantity of lubricant, 10% concentration of boric acid + SAE-40 base oil, feed rate 0.06 mm/rev, cutting speed 1,200 rpm, and 0.8 mm tool nose radius. The percentage reduction in surface roughness and the chip thickness at optimum combination of parameters are 16.06% and 27.30%, respectively. The effectiveness of this approach has been successfully established by confirmation experiment. Thus, the solutions from this method can be used by production/industrial engineers who are willing to search for an optimal solution of metal-cutting

operation. In the future, this study can be extended to different metal cutting operations with different work materials and machine tools.

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