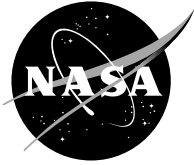# Determination of Rolling-Element Fatigue Life From Computer Generated Bearing Tests

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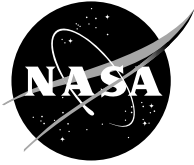
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SUMMARY

Two types of rolling-element bearings representing radial loaded and thrust loaded bearings were used for this study. Three hundred forty (340) virtual bearing sets totaling 31400 bearings were randomly assembled and tested by Monte Carlo (random) number generation. The Monte Carlo results were compared with endurance data from 51 bearing sets comprising 5321 bearings. A simple algebraic relation was established for the upper and lower L_{10} life limits as function of number of bearings failed for any bearing geometry. There is a fifty percent (50%) probability that the resultant bearing life will be less than that calculated. The maximum and minimum variation between the bearing resultant life and the calculated life correlate with the 90-percent confidence limits for a Weibull slope of 1.5. The calculated lives for bearings using a load-life exponent p of 4 for ball bearings and 5 for roller bearings correlated with the Monte Carlo generated bearing lives and the bearing data. STLE life factors for bearing steel and processing provide a reasonable accounting for differences between bearing life data and calculated life. Variations in Weibull slope from the Monte Carlo testing and bearing data correlated. There was excellent agreement between percent of individual components failed from Monte Carlo simulation and that predicted.

NOMENCLATURE

C, C_D	dynamic load capacity, N (lbf)
c	stress-life exponent
e	Weibull slope
h	exponent
L	life, number of stress cycles or hr
L_{10}	10-percent life or life at which 90 percent of a population survives, number of stress cycles or hr
L_β	characteristic life or life at which 63.2 percent of population fails, number of stress cycles or hr
N	life, number of stress cycles
n	number of failed bearings or number of elements in population
P, P_{eq}	equivalent radial load, N (lbf)
p	load-life exponent
S	probability of survival, fraction or percent
V	stressed volume, m^3, (in.^{3})
Z_o	depth to the orthogonal shearing stress, m (in.)
τ_o	orthogonal shearing stress, GPa (ksi)

Subscripts

i	ith component or bearing
ir	inner race
L	lower limit
max	maximum
min	minimum
n	number of components
or	outer race

re   rolling elements
sys   system
up   upper limit

Definitions
calculated life   the life obtained using the Lundberg-Palmgren life equations
resultant life   the life obtained from the Weibull analysis of bearing systems generated by a Monte Carlo technique

INTRODUCTION

Predicting and verifying rolling-element bearing life is a complex task. Accurate prediction of bearing lives is necessary to predict replacement rates, maintain rotating machinery and establish warranty limits on manufactured goods. Complicating the issue is the fact that fatigue failure is extremely variable and dependent upon materials, processing, and operating conditions.

Rolling-bearing fatigue life analysis is based on the initiation or first evidence of fatigue spalling on either a bearing race or a rolling element (ball or roller). This spalling phenomenon is load cycle dependent. Generally, the spall begins in the region of maximum shear stresses, which is located below the contact surface, and propagates into a crack network. Failures other than those caused by classical rolling-element fatigue are considered avoidable if the bearing is properly designed, handled, installed, lubricated and not overloaded (1). However, under low elastohydrodynamic (EHD) lubricant film conditions, rolling-element fatigue can be surface or near-surface initiated with the spall propagating into the region of maximum shearing stresses.

If a number of apparently identical bearings are tested to fatigue at a specific load, there is a wide dispersion in life among the various bearings. For a group of 30 or more bearings the ratio of the longest to the shortest life may be 20 or more (1).

In 1939, Weibull (2-4) developed a method and an equation for statistically evaluating the fracture strength of materials based upon small population sizes. This method can be and has been applied to analyze, determine, and predict the cumulative statistical distribution of fatigue failure or any other phenomenon or physical characteristic that manifests a statistical distribution.

Based upon the work of Weibull (2), Lundberg, and Palmgren (5), in 1947, showed that the probability of survival S could be expressed as a power function of the orthogonal shear stress τ_o, life N, depth to the maximum orthogonal shear stress Z_o, and stressed volume V. That is

$$\ln \frac{1}{S} \sim \tau_o \frac{N^e}{Z_o^h} V \tag{1}$$

From Eq. (1), Lundberg and Palmgren (5) derived the following relation

$$L_{10} = [C_D / Peq]^p \tag{2}$$

where C_D, the basic dynamic load capacity, is defined as the load that a bearing can carry for one million inner-race revolutions with a 90-percent probability of survival, Peq is the equivalent bearing load, and p is the load life exponent. The derivation of Eq. (2) is discussed in Zaretsky et al (6).

The term "basic rating life," as used in bearing catalogs, usually means the fatigue life exceeded by 90 percent of the bearings or the time before which 10 percent of the bearings fail. This basic rating life is referred to as the "L_{10} life" (sometimes called the B_{10} life or 10-percent life). The 10-percent life is approximately one-seventh of the mean life or MTBF (mean time between failure), for a bearing life dispersion curve (1).

Harris (7,8) analyzed 62 rolling-element bearing endurance sets. These data were obtained from four bearing manufacturers, two helicopter manufacturers, three aircraft engine manufacturers, and U.S. Government agency-sponsored technical reports. The data sets comprised deep-groove radial ball bearings, angular-contact ball bearings, and cylindrical roller bearings totaling 7935 bearings.

Using the Harris data (7,8), Zaretsky, Poplawski, and Miller (9) compared the ratio of the L_{10} lives of the field and laboratory bearing life data to that predicted by various life theories discussed in Ref. (6). For the Lundberg-Palmgren equations discussed above, the mean ratios of the L_{10} actual lives divided by the L_{10} predicted lives (determined using STLE life factors (1)) were 14.5, 3.5, and 20.1 for angular-ball bearings, deep-groove ball bearings, and cylindrical roller bearings, respectively. While it is probable that all design and operating parameters

necessary to more accurately calculate bearing life were not available to Harris (7,8), these bearing data are the best compilation in the open literature.

An issue that occurs from these data and analysis is the variation between bearing life calculations and the actual endurance characteristics of the bearings. Experience has shown that endurance tests of groups of identical bearings under identical conditions can produce variation in L_{10} life from group to group that exceeds reasonable engineering expectations, that is, where the life is significantly less or more than that calculated. This is an important issue for product warranty, comparison of bearings from different sources and variation in life from lot-to-lot from the same source.

In view of the aforementioned, and using the Lundberg-Palmgren analysis (5) as the basis, the objectives of the work reported herein are: (a) to determine the variation in rolling-element bearing lives and distribution parameters as a function of sample size (number of bearings tested); (b) compare the statistical variation in bearing life due to finite sample size to the Harris rolling-element bearing data (7,8); and (c) determine the most likely value of the load-life exponent based upon a comparison of the field data of Harris (7,8) to the upper and lower limits (or 90% confidence limits) obtained for a Weibull-based Monte Carlo prediction of bearing life.

PROCEDURE

Bearing Life Analysis

G. Lundberg and A. Palmgren (5), using the Weibull equation (2–4), first derived the relationship between individual component lives and system life where

$$\left[1 / L_{sys}\right]^e = \sum_{i=1}^{n} \left[1 / L_i\right]^e \tag{3}$$

Using Eq. (1), Lundberg and Palmgren (5) develop equations for the lives of the inner and outer races of a bearing and combine them using Eq. (3) to determine the bearing life at a 10-percent probability of failure or the time beyond which 90 percent of the bearings will survive where

$$[1/L_{10}]^e = [1/L_{ir}]^e + [1/L_{or}]^e \tag{4}$$

Unfortunately, Lundberg and Palmgren (5) do not directly calculate the lives of the rolling element (ball or rollers) set of the bearing. However, through the benchmarking of the equations with bearing life data by use of a material-geometry factor, the life of the rolling elements are implicitly included in the life calculation of Eq. (4).

The rationale for not including the rolling elements in Eq. (4) appears in the 1945 edition of A. Palmgren's book (10) wherein he states that, "…the fatigue phenomenon which determines the life (of the bearing) usually develops on the raceway of one ring or the other. Thus, the rolling elements are not the weakest parts of the bearing … ." The data base that Palmgren used to benchmark his and later the Lundberg-Palmgren equations were obtained under radially loaded conditions. Under these conditions the life of the rolling elements as a system will be equal or greater than the outer race. As a result, failure of the rolling elements in determining bearing life was not initially a consideration by Palmgren. Equation (4) should be written as follows

$$[1/L_{10}]^e = [1/L_{ir}]^e + [1/L_{re}]^e + [1/L_{or}]^e \tag{5}$$

where the Weibull slope e is the same for each of the components as well as the bearing as a system.

Comparing Eq. (5) with Eq. (4), the value of the L_{10} bearing life will be the same. However, the values of the L_{ir} and L_{or} between the two equations will not be the same, but, the ratio of L_{or}/L_{ir} will remain unchanged.

In order to account for material and processing variations between the rolling elements and the races, it is important to break out the ball or roller life from that of the inner and outer races using Eq. (5). This can be accomplished using *Zaretsky's Rule (1)* as follows

For radially loaded ball and roller bearings, the life of the rolling element set is equal to or greater than the life of the outer race. Let the life of the rolling element set (as a system) be equal to that of the outer race.

From Eq. (5)

$$[1/L_{10}]^e = [1/L_{ir}]^e + 2[1/L_{or}]^e \qquad (6)$$

where

$$L_{re} = L_{or}$$

For thrust loaded ball and roller bearings, the life of the rolling element set is equal to or greater than the life of the inner race but less than that of the outer race. Let the life of the rolling element set (as a system) be equal to that of the inner race.

From Eq. (5)

$$[1/L_{10}]^e = 2[1/L_{ir}]^e + [1/L_{or}]^e \qquad (7)$$

where

$$L_{re} = L_{ir}$$

Examples for using Eqs. (5) to (7) are given in Zaretsky (1). As previously stated, the resulting values for L_{ir} and L_{or} from these equations are not the same as those from Eq. (4).

Bearing Type, Operating Conditions and Calculated Lives

Two types of rolling-element bearings representing radial loaded and thrust loaded bearings were used for this study. They were a 6010-size (50-mm bore) deep-groove ball bearing and a 7010-size (50-mm bore) angular-contact ball bearing, respectively. The bearing specifications and geometry are summarized in Table 1. For purposes of this analysis all life factors such as for material and processing were set to unity since we were interested primarily in the qualitative results. However, a lubricant life factor was used as a function of lubricant film parameter from Zaretsky (1) for these operating conditions since its effect on the resulting lives of the inner and outer races can be different.

Table 1. Bearing specifications, operating conditions, and lives used in assembly and Monte Carlo testing

Bearing type		Deep-groove ball bearing		Angular-contact ball bearing	
Bore size, mm		50		50	
Curvatures, percent	Inner race	52		52	
	Outer race	52		52	
Ball diameter, mm (in.)		8.73 (11/32)		8.73 (11/32)	
Number of balls		14		19	
Contact angle, deg		0		25	
Load, N (lbs)		950 (214) radial		2800 (630) thrust	
Maximum Hertz stress, GPa (ksi)		1.55 (225)		1.55 (225)	
Lubricant type		MIL–L–23699		MIL–L–23699	
Surface finish, rms µ (µm)	Inner race	7.62×10^{-2} (3)		7.62×10^{-2} (3)	
	Outer race	7.62×10^{-2} (3)		7.62×10^{-2} (3)	
	Balls	2.54×10^{-2} (1)		2.54×10^{-2} (1)	
Operating temperature, ºC (ºF)		135 (275)		135 (275)	
Lubricant life factors	Inner race	0.75		0.79	
	Outer race	1.05		1.04	
Life, hrs (see Fig. 1)	Component	L_{10}	L_{50}	L_{10}	L_{50}
	Inner race[a]	9547	52123	1974	10775
	Outer race[a]	38188	208448	7885	43040
	Ball[a]	38118	208448	1974	10775
	Bearing[b]	6912	37729	964	5262
Weibull slope, *e*		1.11		1.11	

[a]Life based on Zaretsky's rule and lubricant life factor (from Ref. (1)).
[b]Life based on Lundberg-Palmgren equations (from Ref. (5)) and lubricant life factor (from Ref. (1)).
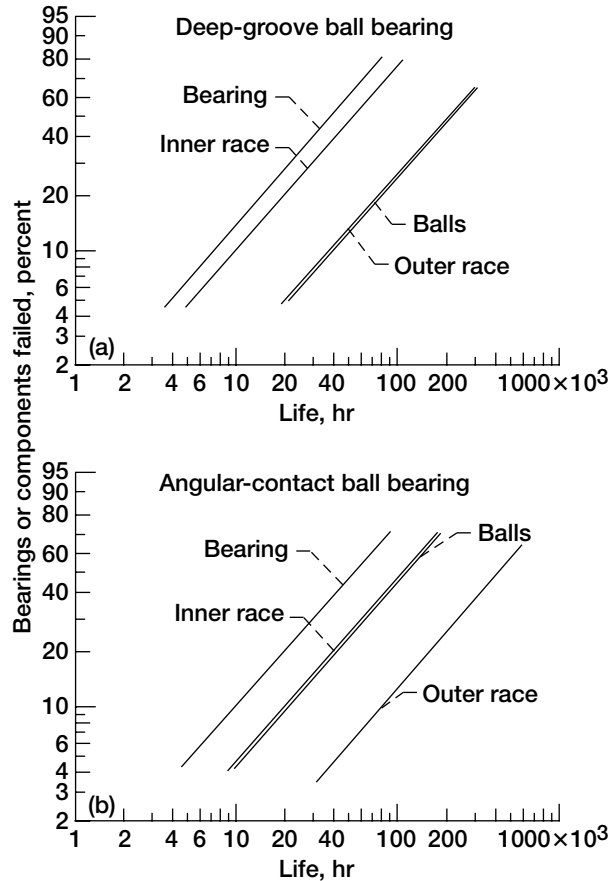
Fig. 1.—Weibull plots for lives of deep-groove and angular-contact ball bearings and their respective components. Bore size, 50 mm: maximum Hertz stress, 1.55 GPa (225 ksi); speed, 10 000 rpm. (a) Deep-groove ball bearing; radial load, 950N (214 lb). (b) Angular-contact ball bearing; thrust load, 2800N (630 lb).

Operating conditions for both bearing types were assumed to be 10000 rpm using a MIL–L–23699 (tetraester based) lubricant at 135 °C (275 °F). The respective loads applied to both bearings were calculated to result in a maximum Hertz stress on the inner race of each bearing of 1.55 GPa (225 ksi). These operating conditions are summarized in Table 1.

The bearing lives were calculated according to Lundberg-Palmgren Eqs. (1) and (4) with a lubricant life factor. The lives of the inner and outer races were calculated together with the lives of the balls using Eqs. (6) and (7). These results are also summarized in Table 1. Weibull plots of the bearing and their individual component lives are shown in Fig. 1.

Virtual Bearing Testing

A rolling-element bearing is composed of 4 components. These are the inner and outer races and a plurality of rolling elements that are positioned and retained by a separator (cage) between the two races. The life of a bearing is probabilistic and is calculated based upon rolling-element fatigue (spalling failure) of either the inner or outer races and/or the rolling elements. Upon the formation of a spall on anyone of these components, the bearing is no longer fit for its intended purpose and is subject to being replaced. The separator is assumed not to fail under normal operating conditions. The variables that affect bearing life are discussed in detail in Ref. (1).

The cumulative distribution of the individual components of the two bearing types is shown in the two-parameter Weibull plots of Fig. 1. The general equation representing these plots is as follows

$$\ln\ln\frac{1}{S} = e\,\ln\left(\frac{L}{L_\beta}\right) \quad \text{where } 0 < L < \infty; \quad 0 < S < 1 \tag{8}$$

The Weibull plots shown in Fig. 1 are the $\ln\ln\dfrac{1}{S}$ graduated in percent of bearings or components failed as the ordinate as a function of $\ln L$, the log of the time or cycles to failure as the abscissa. The tangent of the line is designated as the Weibull slope e. The Weibull slope e is indicative of the shape of the cumulative distribution of the data. Based upon their database, Lundberg and Palmgren (5) use a value for the Weibull slope e of 1.11. This results in an approximately exponential distribution of the bearing failure data. In Eq. (8), L_β is the characteristic life or the life at which 63.2 percent of the bearings fail.

It was assumed that for each of the two bearing types described in Table 1 there are three virtual bins containing components from which the bearing was assembled. As in a realistic manufacturing process, each of the component parts of the respective bearings are grouped in separate bins. Each bin contains either 1000 inner rings, outer rings or ball sets. Each component part and ball set is assigned an order number (1, 2, 3, ….1000) corresponding to its life correlated to the respective Weibull plots for the components shown in Fig. 1.
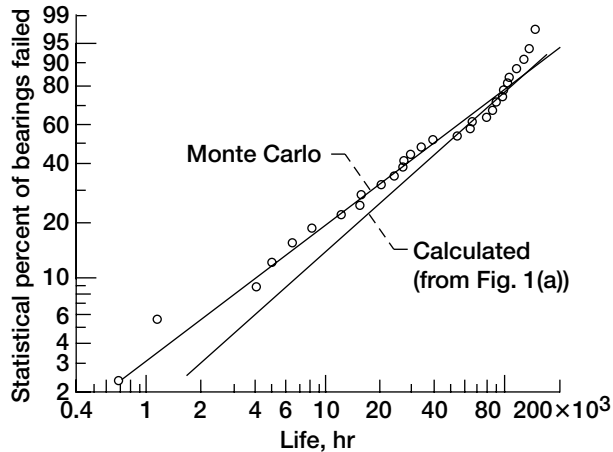
Fig. 2.—Representative Weibull plot for Monte Carlo bearing tests of 50-mm bore deep-groove ball bearings compared to Weibull plot of calculated lives. Bore size, 50 mm: maximum Hertz stress, 1.55 GPa (225 ksi); speed, 10 000 rpm; L_{10} life, 3938 hr; L_{50} life, 37 087 hr; Weibull slope e, 0.84; failure index, 30 out of 30.

Using Monte Carlo techniques, bearings were randomly assembled from the three virtual part bins for each bearing type. A 3-by-n matrix was randomly generated using the spreadsheet RAND function where **n** was the desired number of bearings to be assembled. The life of each individual bearing based upon the weak link theory was determined as being the lowest life of the randomly selected component of that bearing. Values of **n** were arbitrarily selected to be 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 35, 40, 45, 50, 100, 200, and 1000. All bearings in each set failed; there were no suspensions or censoring. Using the method of Johnson (11), the resulting lives of each group of **n** bearings were plotted on Weibull plots. A straight line was fitted to the data points using the method of least squares. The Weibull slope and the L_{10} and L_{50} lives were determined from the resultant line. A representative Weibull plot comprising 30 bearings is shown in Fig. 2. For each value of **n**, the procedure was repeated 10 times to estimate variation between trials and to determine the maximum and minimum values for the series of 10 trials.

RESULTS AND DISCUSSION

Endurance Life Variation

A. Palmgren in the 1945 edition of his book (10) presents an excellent discussion of bearing life. If a bearing is properly designed, lubricated, operated, and maintained, its service life is limited by rolling-element fatigue. In the classical sense, a crack initiates in the subsurface zone of maximum shearing stresses below the running track of one of its elements and propagates into a pit or spall that is initially limited in area to that of the Hertzian contact area and to the depth of the maximum shearing stresses (1).

The time it takes for this spall to occur is the measure of the bearing life. According to Palmgren (10), bearing life is measured in number of revolutions of the bearing or in the number of hours of bearing operation at a given speed. Palmgren (10) states that "it is necessary to weigh, in a suitable manner, the contrary requirements of reliable service and low cost. Therefore, it has been decided to define the 'estimated life' as that number of bearing revolutions or that number of working hours at a certain speed of rotation, which will be reached by 90 percent of all bearings." Palmgren first proposed this definition of bearing life in 1924 (12,13). Today, this is the universally accepted definition for most bearing life calculations. As far as we can determine, it is the first probabilistic approach to life prediction of machine elements.

For our study bearings were randomly assembled from the three virtual part bins for each of the two bearing types by Monte Carlo (random) number generation. The life of each individual bearing based upon the weak link theory was determined as being the life of the lowest lived randomly selected component of that bearing. Using the method of Johnson (11), the resulting lives of each group of **n** bearings were plotted on Weibull plots and the Weibull slope and lives were determined (Fig. 2). For each value of **n**, the procedure was repeated 10 times to determine maximum values, minimum values, and the degree of variation between trials. The results are summarized in Table 2.

Table 2 contains the maximum and minimum values of the virtual L_{10} bearing lives of 10 trials of each group of **n** bearings with corresponding values for Weibull slope e and L_{50} bearing life. The calculated value was determined using the Palmgren-Lundberg equation and a life factor for lubrication—all other life factors were set to unity. The maximum and minimum values of the L_{10} lives as a percent of the calculated L_{10} life for each group of **n** bearings was determined as follows

$$\textit{Maximum Variation from Calculated } \boldsymbol{L_{10}} \textit{ Life} = \frac{\textit{Maximum } \boldsymbol{L_{10}} - \textit{Calculated } \boldsymbol{L_{10}}}{\textit{Calculated } \boldsymbol{L_{10}}} \times 100 \textit{ percent} \qquad (9a)$$

$$\textit{Minimum Variation from Calculated } \boldsymbol{L_{10}} \textit{ Life} = \frac{\textit{Minimum } \boldsymbol{L_{10}} - \textit{Calculated } \boldsymbol{L_{10}}}{\textit{Calculated } \boldsymbol{L_{10}}} \times 100 \textit{ percent} \qquad (9b)$$

Table 2. Summary of minimum and maximum life values from Monte Carlo simulation of assembly and testing
of 340 sets of 50-mm bore, deep-groove, and angular-contact ball bearings

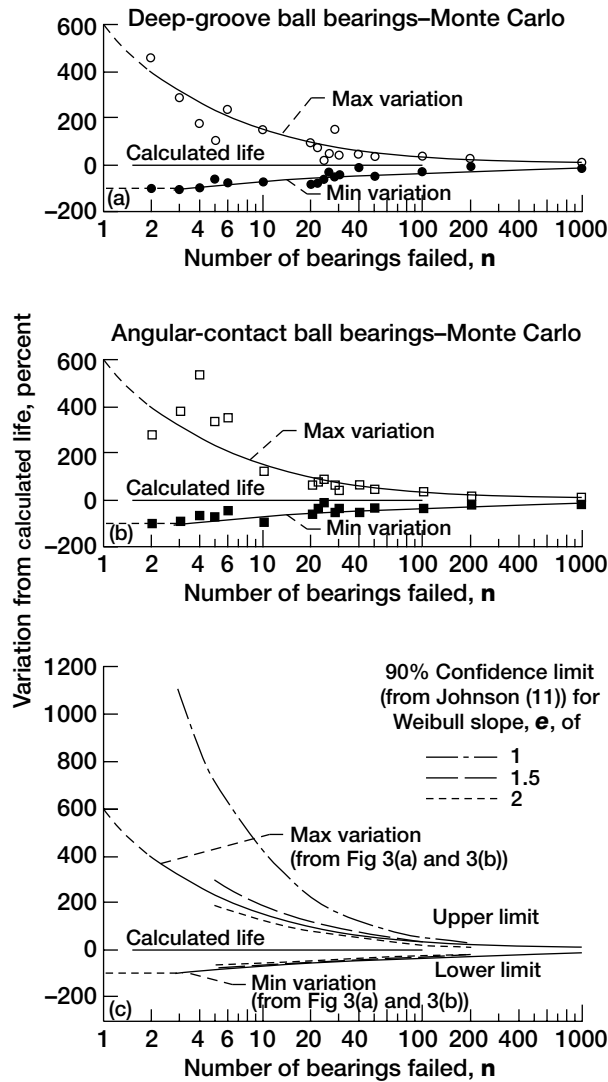| Number of bearings in a set	Maximum and minimum values of bearing life, hrs, and Weibull slope, e						L_{10} lives below that calculated, percent	Variation from calculated L_{10} life, percent	
	Maximum L_{10}	Corresponding		Minimum L_{10}	Corresponding			Maximum	Minimum
		L_{50}	e		L_{50}	e			
Deep-groove ball bearings									
2	38709	66136	3.52	375	13478	0.53	40	461	−95
3	26767	65208	2.12	272	16186	0.46	70	286	−96
4	19331	78650	1.34	279	8500	0.55	60	180	−96
5	14091	63944	1.25	2619	15980	1.04	70	104	−59
6	23138	46278	2.72	270	14610	0.47	40	235	−96
10	10419	46713	1.26	1927	13845	0.96	80	51	−72
20	13538	55996	1.33	1176	13118	0.78	70	96	−82
22	12750	47314	1.44	1858	30048	0.68	60	84	−73
24	8526	41966	1.18	2886	27613	0.83	60	23	−58
26	10224	42022	1.33	4961	36330	0.95	40	48	−28
28	17635	54305	1.68	3946	29727	0.93	70	155	−43
30	10362	43021	1.32	3940	37090	0.84	50	50	−43
40	10318	36939	1.48	6769	34633	1.15	20	49	−11
50	9469	41669	1.27	3956	29150	0.94	50	37	−43
100	9936	51583	1.14	5033	32938	1.00	50	44	−27
200	9217	42421	1.23	6663	37852	1.08	30	33	−4
1000	7754	40660	1.14	6390	34511	1.18	10	12	−11
Angular-contact ball bearings									
2	3696	4269	13.08	39	2906	0.44	60	283	−95
3	4612	17330	1.42	96	5026	0.48	60	378	−90
4	6194	15929	2.00	293	3674	0.75	40	542	−70
5	4203	7164	3.53	279	6267	0.61	40	336	−71
6	4404	10753	2.11	565	5013	0.86	40	357	−41
10	2179	8112	1.43	105	2648	0.58	40	126	−89
20	1612	6005	1.43	341	3908	0.77	70	67	−65
22	1756	6799	1.39	241	4205	0.66	80	82	−75
24	1903	8741	1.24	465	3746	0.90	40	97	−52
26	1431	5526	1.39	463	4551	0.84	70	48	−52
28	1560	5860	1.42	579	5246	0.86	70	62	−40
30	1423	5651	1.37	609	4289	0.97	50	48	−37
40	1701	7281	1.30	476	4835	0.81	80	76	−51
50	1406	7275	1.15	708	4785	0.99	40	46	−27
100	1471	6889	1.22	751	4708	1.03	40	53	−27
200	1179	5597	1.21	852	5210	1.04	60	22	−12
1000	1101	5429	1.18	917	5067	1.10	30	14	−5

Fig. 3.—Maximum and minimum variation of L_{10} lives as percent of calculated L_{10} for each group of n bearings compared with 90% confidence limits based on Weibull slope and number of bearings failed n. (a) 50-mm bore deep-groove ball bearing. (b) 50-mm bore angular-contact ball bearing. (c) 90% confidence limits.

The life results and variations are summarized in Table 2. The variations were plotted on semi-log paper and are shown in Figs. 3(a) and 3(b) for the deep-groove and angular-contact ball bearings, respectively. Best-fit curves obtained using the linear regression package of a commercial spreadsheet were fitted through the points for the minimum and maximum values shown in each of the plots. The results for both sets of bearings were nearly identical. From these curve fits and Eqs. (9a) and (9b), the life equations from each set of the respective curves were as follows

Maximum L_{10} Life = Calculated L_{10} Life $(1 + 6n^{-0.6})$
$$(10a)$$

Minimum L_{10} Life = Calculated L_{10} Life $(1 - 1.5n^{-0.33})$
where $n > 3$
$$(10b)$$

Minimum L_{10} Life $\rightarrow 0$ where $n \leq 3$

These life equations hold for any bearing geometry with a known calculated L_{10} life.

Relating these results to experimental research, the resultant L_{10} life should fall between these values if the true life is no different from the analytically predicted (calculated) life. If, however, the L_{10} life is greater than the *Maximum Variation L_{10} Life*, then the true life is probably greater than that calculated. If the L_{10} life is less than the *Minimum Variation L_{10} Life*, it must be reasonably concluded that the true L_{10} life is probably less than that calculated.

The data were studied to determine if the number of bearings tested affects whether the resultant L_{10} life will be less or more than that calculated. Out of the 170 trials comprising 15700 angular-contact ball bearings, the resultant L_{10} life was less than that calculated 54 percent of the time. For 170 trials comprising 15700 deep-groove ball bearings, the resultant L_{10} life was less than that calculated 51 percent of the time. The variation was random and independent of the number of bearings tested.

From this Weibull-based Monte Carlo study, for the thrust-loaded angular-contact ball bearings it was found that 45.4 percent of the failures occur on the inner race, 45.2 percent occur on the balls, and 9.4 percent on the outer race. For the radially loaded deep-groove ball bearings, 70.1 percent of the failures occur on the inner race, 15.1 percent on the balls, and 14.8 percent on the outer race. The failure locations for radially loaded cylindrical roller bearings are expected to be similar to those of the deep-groove ball bearing.

Similar failure trends in the percentage of individual components failed with respect to total system can be derived from the Lundberg-Palmgren model for system failure (5). The percentage of the bearing failures that are due to failure of the inner race can be derived using Eq. (5) and expressed as

$$percent\ inner\text{-}race\ failures = \left[\frac{L_{10\ sys}}{L_{10\ ir}}\right]^{e} x100 = \frac{1}{\left[\dfrac{L_{10\ ir}}{L_{10\ re}}\right]^{e} + 1 + \left[\dfrac{L_{10\ ir}}{L_{10\ or}}\right]^{e}} x100 \qquad (11)$$

Table 3. Comparison of bearing component failure distributions based upon a Weibull-based Monte Carlo method and calculated from Lundberg-Palmgren system life equation for deep-groove and angular-contact ball bearings

Bearing Type	Component	Percent Failure	
		Weibull-based Monte Carlo result	Calculated from Lundberg-Palmgren Eq. (5)
Deep-Groove Ball Bearing	Inner Race	70.1	69.9
	Rolling Element	14.8	15.0
	Outer Race	15.1	15.0
Angular Contact Ball Bearing	Inner Race	45.4	45.1
	Rolling Element	45.2	45.1
	Outer Race	9.4	9.7

where $L_{10\,sys}$ is the life at which 10 percent of the assembled systems have failed, $L_{10\,ir}$ is the life at which 10 percent of the inner rings have failed, $L_{10\,or}$ is the life at which 10 percent of the outer rings have failed, and e is the Weibull slope. A derivation of Eq. (11) is provided in the appendix. The percentage of rolling elements failed or the percentage of outer races failed can be expressed similarly (see the appendix). Table 3 is a summary of the percentage of inner race, rolling element, and outer race failures obtained from the Weibull-based Monte Carlo method and Eq. (11). There is excellent agreement between these techniques.

Assuming that the calculated L_{10} life is exact and not subject to unknown manufacturing or operating variables, there is an even (50 %) probability that the resultant L_{10} life for any randomly selected bearing test group will be less than that calculated. As the number of bearings in a test group increases, the resultant life approaches the predicted life but with half the lives still being less than that calculated.

Confidence Limits

The results of the above analysis were compared by us to that of L. Johnson (11) who determined the 90-percent confidence limits as a function of Weibull slopes of 1, 1.5, and 2 and the number of failed bearings **n** in a set. These confidence limits for the L_{10} life are plotted in Fig. 3(c). The confidence limits designate that the true population of the L_{10} lives will fall between the upper and lower values in 90 percent of all possible cases or tests (11). An approximate curve fit of the Johnson analysis results in the following approximate equations for the upper and lower values of the L_{10} life 90-percent confidence limits

For Weibull slope = 1

$$\text{Upper limit: } L_{10up} = \text{calculated } L_{10} + 26.5 \text{ calculated } L_{10}/\mathbf{n}^{0.8} \tag{12a}$$

$$\text{Lower limit: } L_{10L} = \text{calculated } L_{10} - \text{calculated } L_{10}(1.2 - 0.17\ln\mathbf{n}) \tag{12b}$$

$$L_{10L} \rightarrow 0 \text{ where } \mathbf{n} \leq 3 \tag{12c}$$

For Weibull slope = 1.5

$$\text{Upper limit: } L_{10up} = \text{calculated } L_{10} + 9.5 \text{ calculated } L_{10}/\mathbf{n}^{0.7} \tag{13a}$$

$$\text{Lower limit: } L_{10L} = \text{calculated } L_{10} - \text{calculated } L_{10}(1 - 0.15\ln\mathbf{n}) \tag{13b}$$

$$L_{10L} = \text{calculated } L_{10} \text{ where } \mathbf{n} \geq 786 \tag{13c}$$

For Weibull slope = 2

$$\text{Upper limit: } L_{10up} = \text{calculated } L_{10} + 5.5 \text{ calculated } L_{10}/\mathbf{n}^{0.6} \tag{14a}$$

$$\text{Lower limit: } L_{10L} = \text{calculated } L_{10} - \text{calculated } L_{10}(0.87 - 0.13\ln\mathbf{n}) \tag{14b}$$

$$L_{10L} = \text{calculated } L_{10} \text{ where } \mathbf{n} \geq 800 \tag{14c}$$

The results from Figs. 3(a) and 3(b) were superimposed in Fig. 3(c). What is significant is that the minimum variation values of the L_{10} life coincide with the lower 90-percent confidence limits for a Weibull slope of 1. However, there are small differences in life between the lower life limits for Weibull slopes of 1, 1.5, and 2. The 90-percent confidence limits of L_{10} life for a Weibull slope of 1.5 best correlated with the Monte Carlo results for the upper and lower life variations as a function of the number of bearings tested to failure and is independent of whether the bearing is thrust or radially loaded.

Maximum Likelihood Estimators (MLE)

The curve fits of the upper and lower bounds of the Monte Carlo generated bearing lives (Eqs. (10a) and (10b)) and the tabulated 90% confidence limits for a Weibull slope of 1.5 from Johnson (11) (Eqs. (13a) and (13b)) are in excellent agreement. These curves are expressed in easy to apply algebraic equation defining bearing life variation. As will be discussed, bearing data available in the open literature (7,8) reasonably fall between these easily established limits. Another method for calculating these upper and lower bounds is maximum likelihood estimators (MLE).

MLE can be obtained from the methods of Cohen (14), Harter and Moore (15), and McCool (16), from which confidence limits can be established (17). Figure 4 includes the upper and lower bounds established from the Weibull-based Monte Carlo technique (Eqs. (10a) and (10b)), the 90%-confidence limits based upon Johnson (Eqs. (14a) and (14b)) and the confidence limits based upon MLE. In general, there is good agreement among these techniques.

The confidence levels based upon the MLE are more complicated to calculate than either Eqs. (10a) and (10b) or (14a) and (14b). The MLE is an iterative process. It can be sensitive to the choice of starting values, and the calculation is usually non-trivial, tending to require the use of computational software. It should be noted that the maximum likelihood limits can be biased for small sample sizes. Additionally, the information available in the open literature (17) limits the assignment of confidence limits and the application of these techniques to several narrowly defined cases of limited engineering application.

Of the 340 bearing population studied by us and bounded by both the Monte Carlo results (Eqs. (10a) and (10b)) and the 90% confidence limit curve fits of Johnson (Eqs. (13a) and (13b)), the confidence limits based upon MLE can be determined for only 8 cases without extensive additional Monte Carlo simulations. It is worth noting that an engineering approach to confidence intervals has been proposed by Houpert (18), and is based upon a linear regression curve fitting technique. The technique is more complex than the algebraic equations (Eqs. (10a), (10b), (13a), and (13b)) presented by us. A comparison between our technique and that of Houpert (18) is beyond the scope of this paper.
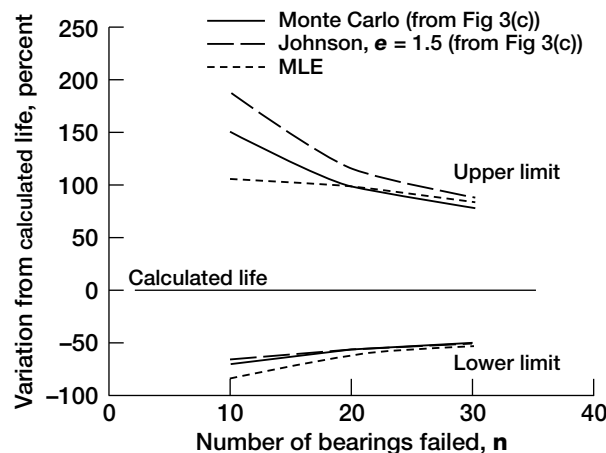

Fig. 4.—Comparison of 90% confidence limits from maximum likelihood estimates (MLE) (14,17) of maximum and minimum variation of L_{10} lives as percent of calculated L_{10} to that of Johnson (11) and Monte Carlo method for 50-mm bore deep-groove ball bearing.
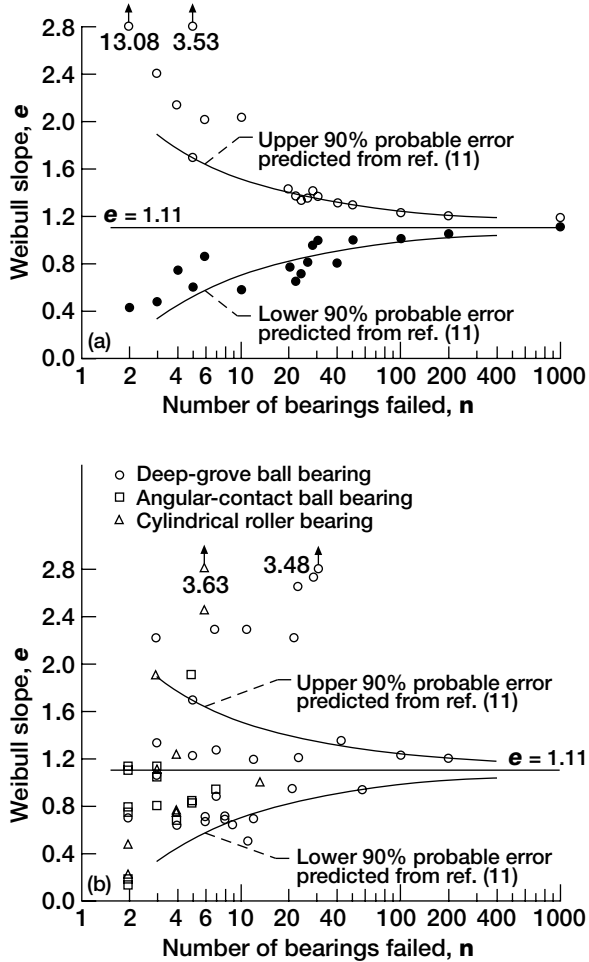
Weibull Slope Variation

Fig. 5.—Variation of Weibull slope *e* compared to predicted 90% probable error. (a) Extremes of Weibull slope from Monte Carlo testing for each group of ten bearing trials of **n** bearings. (b) Weibull slopes from 51 sets of ball and roller bearing data from ref. (7,8).

Lundberg and Palmgren (5) assumed the value of the Weibull slope *e* in Eq. (1) to be 1.11. This value was necessary in their analysis because it approximated that value exhibited by their experimental data and it made the end result of their life prediction analysis correlate with their bearing life database at that time. Experience has shown that most rolling bearing life data exhibit Weibull slopes between 1 and 2. For the analysis performed by us, we too assumed a Weibull slope of 1.11 for all of the components for each bearing. This should theoretically result in a bearing Weibull slope of 1.11 as shown in Fig. 1 for the deep-groove and angular-contact ball bearings.

Johnson (11) analyzed the probable variation of the Weibull slope as a function of the number of bearings tested to failure. Based on the Johnson analysis (11), in 90 percent of all possible cases the resultant Weibull slope will be within the limits shown in Fig. 4 based upon a Weibull slope of 1.11. Based on Johnson (11), the approximate relation for the number of bearings failed **n** and the limits of the value of Weibull slope *e* equal to 1.11 are as follows:

$$\text{Maximum Weibull slope} = 1.11 + 1.31\ \mathbf{n}^{-0.5} \tag{15a}$$

$$\text{Minimum Weibull slope} = 1.11 - 1.31\ \mathbf{n}^{-0.5} \tag{15b}$$

The results of the extremes in the Weibull slopes for each group of the ten bearing trials of **n** bearings are compared with the Johnson analysis in Fig. 5(a). Note that the Weibull slopes for the data summarized in Table 2 for the maximum and minimum bearing lives are not necessarily the same as the maximum and minimum values of the Weibull slopes for each of trials of **n** bearings. For the data reported in Table 2 the relation between the number of bearings tested and the limits of the Weibull slope are as follows:

$$\text{Maximum Weibull slope} = 1.2 + 5(\ln\mathbf{n})^{-3} \tag{16a}$$

$$\text{Minimum Weibull slope} = 1.11 - 0.95\mathbf{n}^{-0.33} \tag{16b}$$

Where the number of bearings failed is 10 or greater, there is a reasonably good correlation between the limits of the slopes generated from the Johnson analysis (11) and those from our Monte Carlo bearing tests. Where the number of failed bearings is below 10, there are differences between the extremes in Weibull slope between the Monte Carlo bearing tests and those of Johnson, especially at the upper limits for the Weibull slopes.

Comparison With Bearing Data

Harris (7,8) analyzed 62 rolling-element bearing endurance sets. These data were obtained from four bearing manufacturers, two helicopter manufacturers, three aircraft engine manufacturers, and U.S. Government agency-sponsored technical reports. The data sets comprised deep-groove radial ball bearings, angular-contact ball bearings, and cylindrical roller bearings for a total of 7935 bearings. Of these, 5321 bearings comprised one sample size for a single cylindrical roller bearing leaving 2614 bearings distributed among the remaining bearing types and sizes. Among the 62 rolling-element bearing endurance sets, 11 had one or no failure and could not be used for our analysis. These data are summarized in Table 4. A discussion of the Harris data can be found in Refs. (6) and (9).

Table 4. Summary of rolling-element bearing life data for three bearing types
(data from Ref. (7))

Bearing set identification number[a]	Maximum Hertz stress, GPa (ksi)	Ratio of actual L_{10} life to calculated[b]	Weibull slope, e	Failure index[c]	L_{10} life variation from calculated, percent[d]	Steel and processing
Deep-groove ball bearing; load-life exponent, p = 3						
6	3.13 (454)	3.42	2.29	11/18	242	VAR[e] AISI M–50
18	4.03 (584)	8.77	2.22	3/6	777	CVD[f] AISI 52100
28	3.41 (495)	4.88	2.29	7/30	388	VIM-VAR[g] AISI M–50
29	3.41 (495)	9.88	1.06	3/28	888	VIM-VAR AISI M–50
30	3.55 (515)	4.67	0.72	6/37	367	VIM-VAR AISI M–50
40	3.41 (495)	0.89	0.51	11/40	−11	CVD AISI 52100
41	3.41 (495)	6.85	0.70	2/41	585	CVD AISI 52100
42	3.41 (495)	3.80	2.65	23/37	280	CVD AISI 52100
43	3.88 (563)	0.89	1.28	7/11	−11	CVD AISI 52100
44	3.41 (495)	1.51	0.89	7/37	51	CVD AISI 52100
46	3.88 (563)	2.99	2.22	22/40	199	CVD AISI 52100
47	3.41 (495)	2.53	0.65	4/33	153	CVD AISI 52100
48	3.41 (495)	0.52	3.48	33/40	−48	VIM-VAR AISI M–50
49	3.41 (495)	1.50	0.68	6/40	50	VIM-VAR M–50NiL
50	3.41 (495)	4.98	1.23	5/40	398	VIM-VAR M–50NiL
51	3.1 (539)	3.16	1.33	3/28	216	CVD AISI 52100
53	3.12 (452)	1.53	0.65	9/67	53	CVD AISI 52100
54	3.72 (539)	2.11	0.93	21/60	111	CVD AISI 52100
55	3.72 (539)	1.44	0.95	57/57	44	CVD AISI 52100
56	3.72 (539)	5.21	0.72	8/30	421	CVD AISI 52100
57	3.72 (539)	2.32	0.70	12/30	132	CVD AISI 52100
58	3.72 (539)	4.85	0.69	8/29	385	CVD AISI 8620
59	3.72 (539)	5.88	1.20	12/29	488	CVD AISI 8620
60	3.72 (539)	0.38	2.75	29/29	−62	CVD AISI 52100
61	2.97 (431)	3.27	1.36	43/103	227	CVD AISI 52100

Bearing set identification number[a]	Maximum Hertz stress, GPa (ksi)	Ratio of actual L_{10} life to calculated[b]	Weibull slope, e	Failure index[c]	L_{10} life variation from calculated, percent[d]	Steel and processing
62	2.97 (431)	1.45	1.21	23/79	45	CVD AISI 52100
Angular-contact ball bearings; load-life exponent, p = 3							
1	3.65 (530)	7.63	0.14	2/12	663	VIM-VAR M–50NiL
2	1.69 (245)	96	1.15	3/199	9500	VIM-VAR AISI M–50
4	2.01 (292)	3.26	0.84	5/17	226	VAR AISI M–50
8	2.07 (300)	4.46	1.14	2/10	346	VAR AISI M–50
9	2.34 (339)	1.11	0.84	5/10	11	VAR AISI M–50
10	2.34 (339)	1.04	0.69	4/10	4	VAR AISI M–50
12	2.70 (391)	3.37	1.91	5/8	337	VAR AISI M–50
32	2.23 (324)	6.33	1.08	3/30	533	VIM-VAR AISI M–50
33	1.97 (286)	1.11	0.81	3/20	11	VIM-VAR AISI M–50
34	1.97 (286)	2.26	1.10	2/20	126	VIM-VAR AISI M–50
35	1.25 (181)	2.08	0.80	2/64	108	VIM-VAR AISI M–50
37	1.28 (185)	3.46	0.95	7/362	246	VIM-VAR AISI M–50
38	1.28 (185)	101.62	0.67	2/634	10062	VIM-VAR AISI M–50
39	1.28 (185)	12.85	0.20	2/33	1185	VIM-VAR AISI M–50
Cylindrical roller bearings; load-life exponent, p = 10/3							
3	1.22 (177)	36	1.00	13/5321	3500	VIM-VAR AISI M–50
5	2.15 (312)	90	0.21	2/19	8900	VAR AISI M–50
14	2.45 (356)	16	3.63	6/6	1500	CVD AISI 52100
15	2.45 (356)	2165	2.43	6/6	2065	VAR AISI M–50
16	2.39 (346)	1.57	0.74	4/7	57	CVD AISI 52100
17	2.39 (346)	6.49	0.48	2/8	549	VAR AISI M–50
20	2.23 (324)	6.94	1.08	3/6	594	CVD AISI 52100
22	2.34 (339)	4.78	1.90	3/6	378	CVD AISI 52100
24	2.29 (332)	10.5	0.75	4/6	905	CVD AISI 52100
26	2.35 (341)	11.23	1.23	4/6	1023	CVD AISI 52100

Table 4. Summary of rolling-element bearing life data for three bearing types
(data from Ref. (7)) (concluded)

Bearing set identification number[a]	Maximum Hertz stress, GPa (ksi)	Ratio of actual L_{10} life to calculated[b]	Weibull slope, e	Failure index[c]	L_{10} life variation from calculated, percent[d]	Steel and processing
27	2.35 (341)	7.63	0.67	4/6	663	VAR AISI M–50

[a]Refers to bearing sets as identified in Refs. (7) and (8)
[b]Life calculation based on Lundberg-Palmgren equations with STLE life factors
[c]Number of bearings failed out of number of bearings in a set
[d]Refer to Eqs. (2) and (9)
[e]Vacuum arc remelting
[f]Carbon vacuum degassing
[g]Vacuum induction melting, vacuum arc remelting
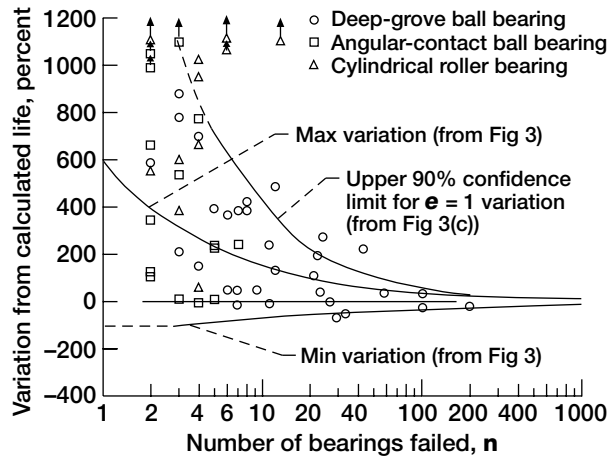
Fig. 6.—Variation between actual and calculated
L_{10} bearing lives for 51 sets of deep-groove and
angular-contact ball bearings and cylindrical roller
bearings from ref. (7,8) compared to Monte Carlo
variations and 90% confidence limit.

There is insufficient technical information reported in Refs. (7) and (8) regarding bearing geometry, operating conditions, fit-ups, and material properties as well as individual data points and Weibull plots to independently evaluate the life results reported. However limited, these results are the best compilation of bearing life data in the open literature.

The data of Table 4 are plotted in Fig. 6 using the number of bearings failed rather than the number of bearings in a set. These data are compared to the maximum and minimum life variations of Fig. 3 and the 90-percent confidence upper limit for a Weibull slope of 1 from Fig. 3(c) and Eq. (13a). Figure 6 consists of 51 sets of bearing data. These data suggest a greater variation between calculated and resultant L_{10} lives than that of the Monte Carlo virtual bearing tests of Figs. 3(a) and 3(b).

Of these data, 39 percent fall between the maximum and minimum life variations suggesting that the statistical variations of these lives are within that predicted. Four bearing sets representing 8 percent of the bearing sets had lives less than that predicted. Thirty bearing sets or 59 percent of the bearing sets exceeded the maximum life variation of this Monte Carlo study. Eight of these bearing sets or 16 percent exceeded the 90-percent confidence upper limit of Johnson. However, only one bearing set representing 2 percent of the bearing sets fall below the lower life limit. Therefore, it can be reasonably concluded that 98 percent of the bearing sets have acceptable life results using the Lundberg-Palmgren equations with the life adjustment factors from Ref. (1) to predict bearing life.
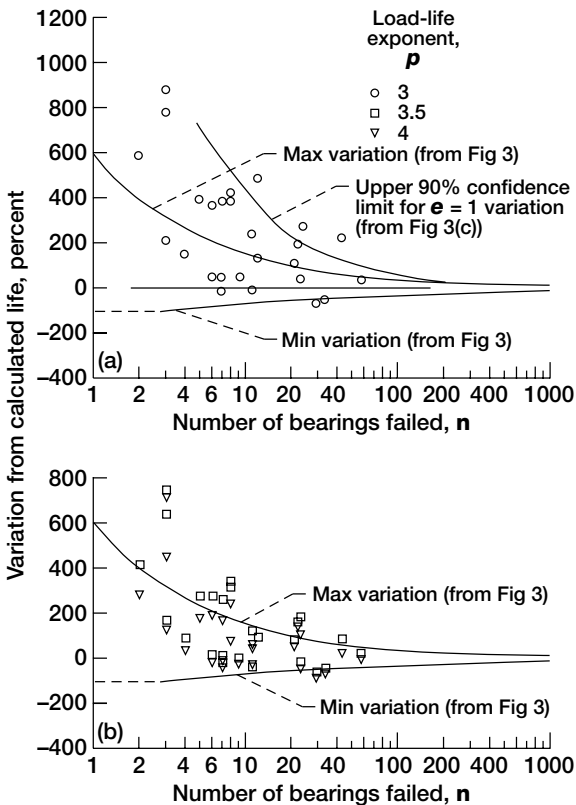
Fig. 7.—Variation between actual and calculated L_{10} bearing lives for deep-groove ball bearings as a function of load-life exponent p. (a) Load-life exponent $p = 3$ (from Fig. 6). (b) Load-life exponent $p = 3.5$ and 4.
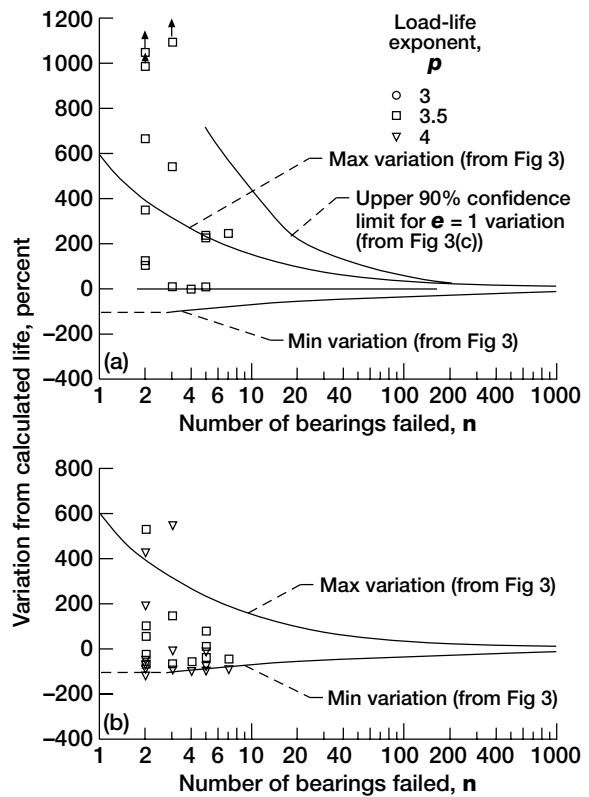
Fig. 8.—Variation between actual and calculated L_{10} bearing lives for angular-contact ball bearings as a function of load-life exponent p. (a) Load-life exponent $p = 3$ (from Fig. 6). (b) Load-life exponent $p = 3.5$ and 4.

Figure 7(a) consists only of the deep-groove ball bearings from Fig. 6. There are 26 deep-groove ball bearing sets comprising 51 percent of all bearing sets. Of these 11 bearings sets or 42 percent are between the maximum and minimum life values. One bearing set representing 4 percent of the deep-groove ball bearings falls below the lower life value. Fourteen or 54 percent of the bearing sets exceeded the maximum life value. Three of these bearing sets or 12 percent exceeded the 90-percent confidence upper limit. For the deep-groove bearing sets, it can also be concluded that by using the Lundberg-Palmgren equations with life adjustment factors to predict bearing life, 96 percent of the bearing sets have acceptable life results.

Figure 8(a) consists only of the angular-contact ball bearings from Fig. 6. There are 14 angular-contact ball bearing sets comprising 27 percent of all bearing sets. All of the bearing sets equaled or exceeded the predicted life value. Of these 8 bearings sets or 57 percent are between the predicted and maximum life variation. Six or 43 percent of the bearing sets exceeded the maximum life variation. Three of these bearing sets or 14 percent exceed the 90-percent confidence upper limit. For the angular-contact ball bearing sets, it can also be concluded that by using the Lundberg-Palmgren equations with life adjustment factors used to calculate bearing life, 100 percent of the bearing sets have acceptable life results.

Based upon the work of Zaretsky, Poplawski, and Peters (6, 19, 20) it can be concluded that the life of these bearings are under-predicted. They suggest that for ball bearings the load-life exponent p from Eq. (2) should be 4 instead of 3 used by Lundberg and Palmgren (5) and reflected in the data of Figs. 7(a) and 8(a). The life data for the ball bearings of Figs. 7(a) and 8(a) were recalculated using load-life exponent values p of 3.5 and 4. The results are shown and compared in Figs. 7(b) and 8(b) for the deep-groove and angular-contact bearings, respectively. A load-life exponent of 4 best reflects the variation in the ratio of the actual life to the predicted life and $p = 3$ is conservative commensurate with good engineering practice.
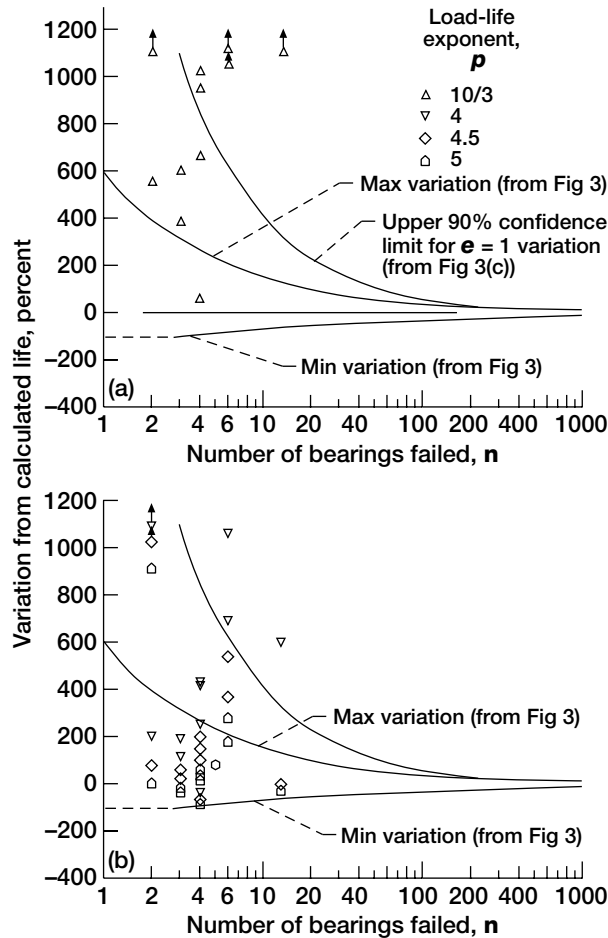
Fig. 9.—Variation between actual and calculated L_{10} bearing lives for cylindrical roller bearings as a function of load-life exponent p. (a) Load-life exponent p = 10/3 (from Fig. 6). (b) Load-life exponent p = 4, 4.5, and 5.

Figure 9(a) comprises only the cylindrical roller bearings from Fig. 6. There are 11 cylindrical roller bearing sets comprising 22 percent of all bearing sets. Of these, one bearing set or one percent is between the maximum and minimum life variation. No bearing set falls below the lower life variation. Ten or 91 percent of the bearing sets exceeded the maximum life variation and 6 of these bearing sets or 55 percent exceeded the 90-percent confidence upper limit. For the cylindrical roller bearing sets, it can also be concluded that by using the Lundberg-Palmgren equations with life adjustment factors to predict bearing life, 100 percent of the bearing sets have acceptable life results.

Referring again to Eq. (2), the value for the load-life exponent p for cylindrical roller bearings as used by Lundberg and Palmgren is 10/3 or 3.333. Poplawski, Peters and Zaretsky (19,20) stated that based upon their experience and analysis, the load-life exponent p of 10/3 is incorrect and will under predict roller bearing life. It was their recommendation that this value be revised to 4 with consideration given to increasing it to 5. Based their recommendation, the data for the cylindrical roller bearings from Fig. 9(a) were recalculated by us using values of p equal to 4, 4.5, and 5. These results are shown in Fig. 9(b). A load-life exponent of p = 5 best reflects the cylindrical bearing life results that are reported. However, by using a value of p = 4 a more conservative life prediction results that may be more commensurate with good engineering practice.

Table 5 shows the effect on bearing life using different values of the load-life exponent p for ball and roller bearings at three load conditions. The table was normalized to the light load condition where Peq = 0.05 C_D. What is apparent from this table and

Table 5. Effect of load-life exponent, p, on bearing life prediction

Bearing load Peq, percent dynamic load capacity C_D	Relative life						
	Load-life exponent, p						
	Ball bearings			Roller bearings			
	3[a]	3.5	4	10/3[b]	4	4.5	5
Light load, 5	1	4.47	20	1	7.4	33	148
Normal load, 10	0.13	0.40	1.25	0.10	0.46	1.46	4.61
Heavy load, 20	0.02	0.03	0.08	0.01	0.03	0.06	0.14
Life factor							
Light load, 5	20[c]	4.5	1	148[d]	20	4.5	1
Normal load, 10	10	3.2	1	46	10	3.2	1
Heavy load, 20	5	2.2	1	15	5	2.2	1

[a]Normalized to load-life exponent p = 3 and light load
[b]Normalized to load-life exponent p = 10/3 and light load
[c]Normalized to load-life exponent p = 4
[d]Normalized to load-life exponent p = 5

the bearing data is that there is built into the Lundberg-Palmgren life calculations a very conservative safety factor depending on the load. For lightly loaded ball bearings, this factor could be as high as 20 and for heavy loaded ball bearings as much as 4. For roller bearings, the factor for lightly loaded bearings is 149 and heavy loaded bearings as much as 14.

Referring to the bearing data of Table 4 and Fig. 6, only 4 sets of data are less than that predicted and only two significantly. This results in 92 percent of the bearing sets equaling and/or exceeding the currently predicted life. Should the load-life exponent p be changed to its correct value and more correctly predict bearing life, there is a probability that 50 percent of the bearings in small subsets would not reach their predicted life in actual application. From a design engineering approach, it would be prudent engineering practice to maintain the load-life exponent p = 3 for ball bearings. However, we suggest that the load-life exponent p = 10/3 for roller bearings be changed to p = 4. This would result in each bearing type having similar life factors for purposes of design as shown in Table 5.

The Weibull slopes for each of the bearings are plotted in Fig. 5(b). While most of the scatter falls within the predicted range, approximately 25 percent of the Weibull slopes for the bearing data fall outside that predicted. These results correlate with the Weibull slopes obtained with the Monte Carlo bearing virtual data. It can be concluded that larger deviations in bearing Weibull slope can occur than predicted by Johnson's method based on a Weibull slope of 1.11, even with bearing sets of **n** equal 30.

Material and Processing Effects

The life calculations for the data of Table 4 and Fig. 6 have material and steel processing life factors incorporated in them from Ref. (1). Table 6 breaks down and summarizes these life factors for each of the materials listed in Table 4. The data of Table 4 and Fig. 6 are broken down and plotted in part (a) of Figs. 10 through 13 based on material and steel processing variables. These data were adjusted for a load-life exponent p of 4 for ball bearings and 5 for roller bearings and are shown in part (b) of Figs. 10 through 13. The adjusted life results correlated with those of the Monte Carlo tests shown in Fig. 3. Based upon these material and processing life factors and load-life exponents, each bearing data set appears consistent with the other. However, for AISI 8620 steel, these data suggest that the material factor should be increased from 1.5 to 2.

Table 6. Life factors for bearing steels and processing (from Ref. (1))

Material and process	Life factor		
	Material	Process	Resultant
CVD[a] AISI 52100	3	1.5	4.5
CVD[a] AISI 8620	1.5	1.5	2.25
VAR[b] AISI M–50	2	3	6
VIM-VAR[c] AISI M–50	2	6	12
VIM-VAR[c] M–50NiL	4	6	24

[a]Carbon vacuum degassing
[b]Vacuum arc remelting
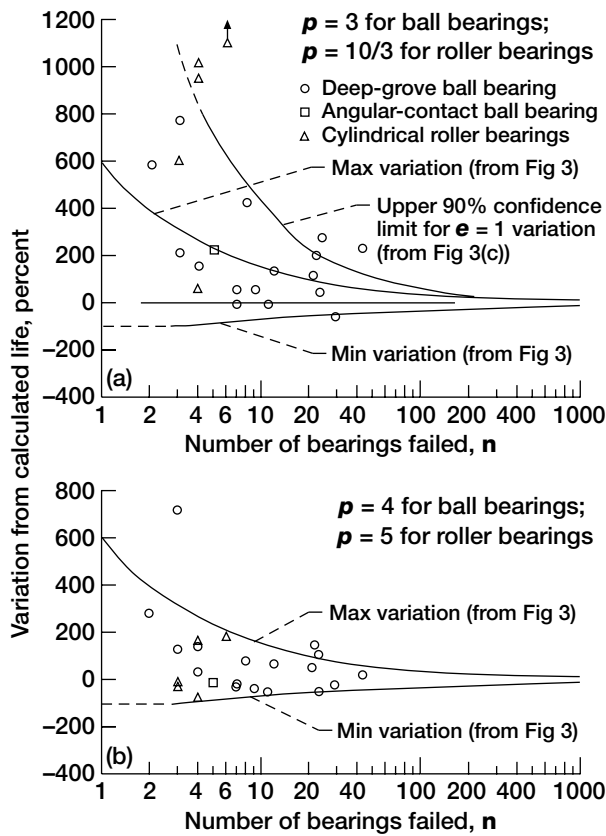[c]Vacuum induction melting, vacuum arc remelting

Fig. 10.—Effect of CVD AISI 52100 steel and load-life exponent on bearing life. (a) Load-life exponent **p**, 3 for ball bearings; 10/3 for cylindrical roller bearings (from Fig. 6). (b) Load-life exponent **p**, 4 for ball bearings; 5 for cylindrical roller bearings.
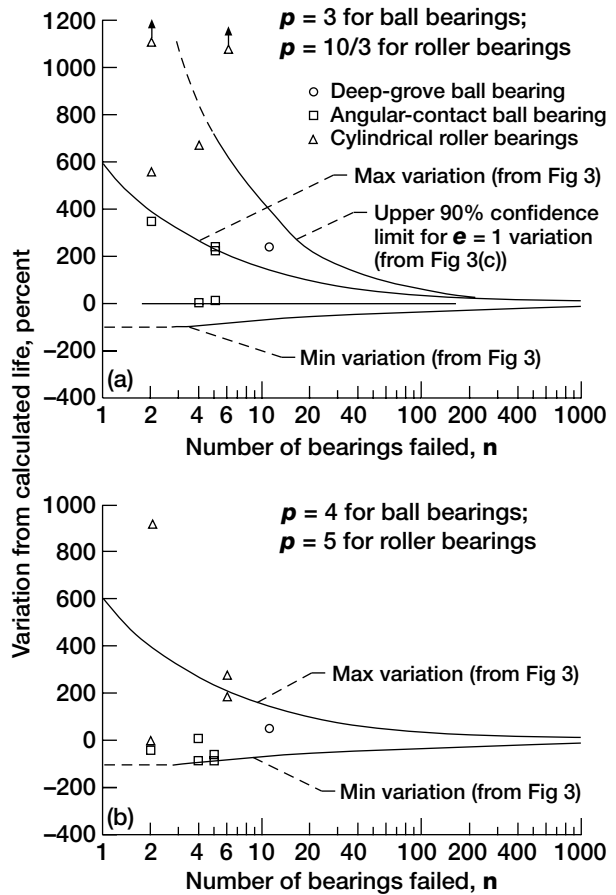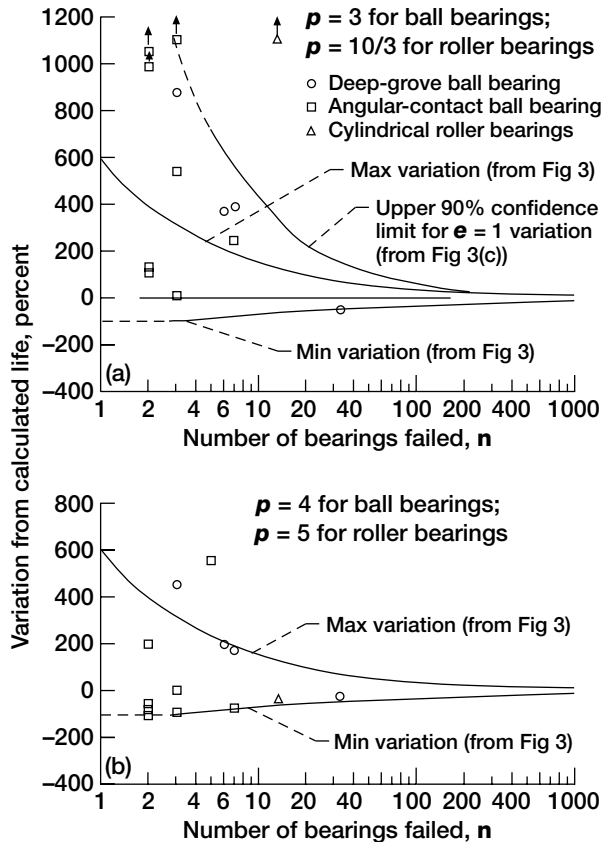
Fig. 11.—Effect of VAR AISI M-50 steel and load-life exponent on bearing life. (a) Load-life exponent **p**, 3 for ball bearings; 10/3 for cylindrical roller bearings (from Fig. 6). (b) Load-life exponent **p**, 4 for ball bearings; 5 for cylindrical roller bearings.

Fig. 12.—Effect of VIM-VAR AISI M-50 steel and load-life exponent on bearing life. (a) Load-life exponent **p**, 3 for ball bearings; 10/3 for cylindrical roller bearings (from Fig. 6). (b) Load-life exponent **p**, 4 for ball bearings; 5 for cylindrical roller bearings.
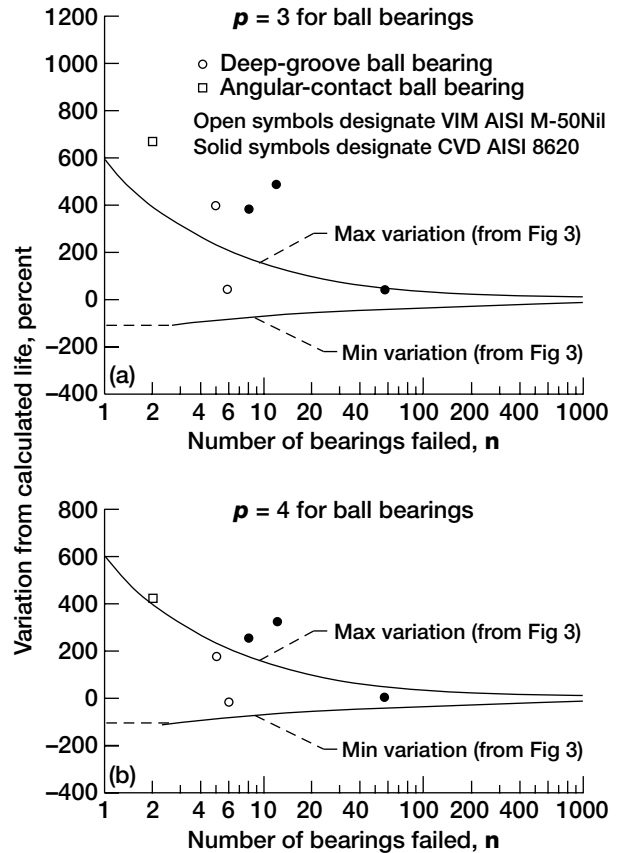
Fig. 13.—Effect of VIM-VAR M-50Nil and CVD AISI 8620 steels and load-life exponent on bearing life. (a) Load-life exponent **p**, 3 for ball bearings (from Fig. 6). (b) Load-life exponent **p**, 4 for ball bearings.

Comparing Life Results to Prediction

Rules can be implied from the results of this paper to compare and distinguish tests of identical bearings either from two or more sources or made from different manufacturing methods. The following rules are suggested to determine if the bearings are acceptable for their intended application or if there are significant differences between the two groups of bearings.

1. If the L_{10} lives of both bearing tests are between the Maximum and Minimum L_{10} life variations, there can be no conclusion that there is a significant difference between the two sets of bearings regardless of the ratio of the L_{10} lives. The bearing sets are acceptable for their intended application (Fig. 14(a)).
2. If the L_{10} life of one set of bearings is greater than the maximum variation and the second set is less the minimum value, there exists a significant difference between the bearing sets. Only, one bearing set is acceptable for its intended application (Fig. 14(b)).
3. If the L_{10} lives of both sets of bearings exceed the maximum variation, the bearing life differences may or may not be significant and should be evaluated based upon calculation of confidence numbers according to the method of Johnson (11). Both sets of bearings are acceptable for their intended application (Fig. 14(c)).

4. If the L_{10} lives of both sets of bearings are less than the minimum variation, the bearing life differences may or may not be significant. However, both sets of bearings are not acceptable for their intended application (Fig. 14(d)).
5. If the L_{10} life of one set of bearings exceed the maximum variation and the other set is between the maximum and minimum variation, the bearing life differences may or may not be significant and should be evaluated based upon calculation of confidence numbers according to the method of Johnson (11). Both sets of bearings are acceptable for their intended application (Fig. 14(e)).
6. If the L_{10} life of one set of bearings is less than the minimum variation and the other set is between the maximum and minimum variation, there exists a significant difference between the bearing sets. Only, one set of bearings is acceptable for its intended application (Fig. 14(f)).
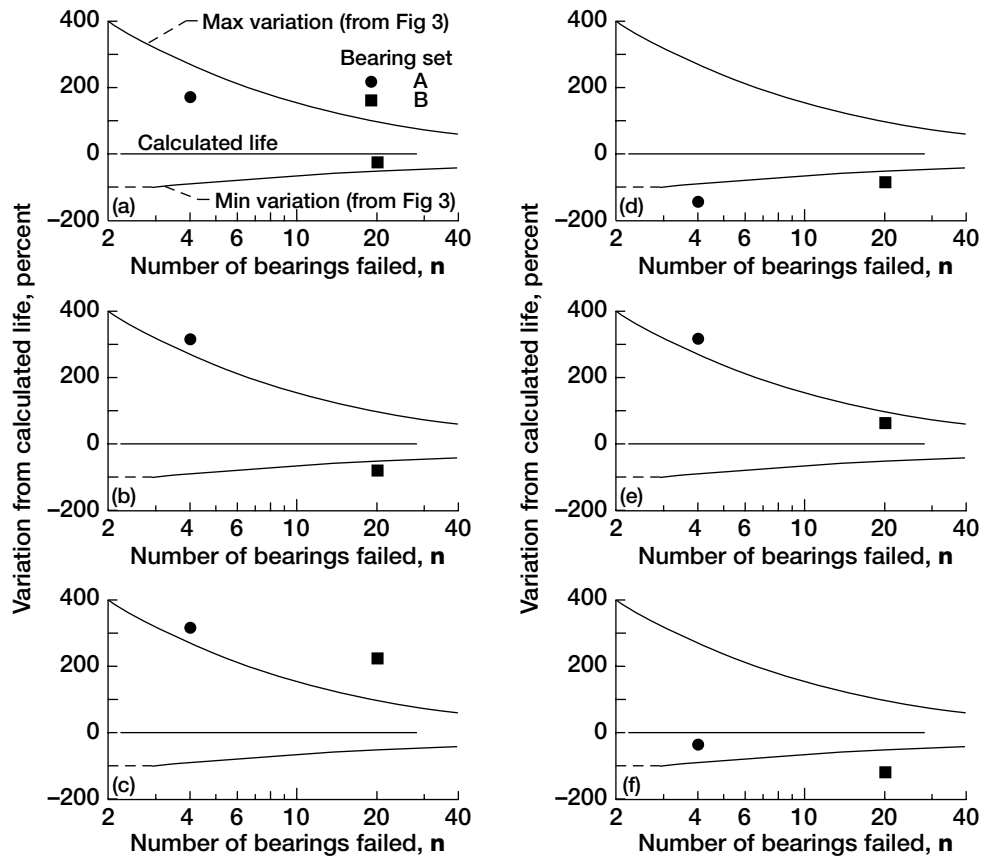

Fig. 14.—Rules for comparing bearing life results to calculated life. (a) A and B bearing sets are acceptable. (b) A bearing set is acceptable. B bearing set not acceptable. (c) A and B bearing sets are acceptable. (d) A and B bearing sets are not acceptable. (e) A and B bearing sets are acceptable. (f) A bearing set is acceptable. B bearing set not acceptable.

SUMMARY OF RESULTS

Two types of rolling-element bearings representing radial loaded and thrust loaded bearings were used for this study. Three hundred forty (340) virtual bearing sets totaling 31400 bearings were randomly assembled from three virtual part bins for each of the bearing types by Monte Carlo (random) number generation. The life of each individual bearing based upon the weak link theory was determined as being the life of the lowest lived randomly selected component of that bearing. Using the method of Johnson (11), the individual bearing lives for each group of **n** bearings were plotted on Weibull plots and the L_{10} lives and Weibull slopes were determined. The Monte Carlo results were compared with endurance data from 51 bearing sets comprising 5321 bearings. The following results were obtained:

1. A simple algebraic relationship was established for the upper and lower bearing L_{10} life limits as function of number of bearings failed for any bearing geometry.
2. Assuming an ideal and accurate bearing life prediction procedure, randomly assembled and selected bearings from a carefully controlled large bearing population will result in a fifty percent (50 %) probability that the resultant bearing life will be less than that calculated regardless of the number of failed bearings making up the randomly selected group. However, the variation of the resultant life from that calculated will decrease as the number of failed bearings in the randomly selected group increases.
3. The maximum and minimum variation between the bearing resultant life and the calculated life was found to correlate with the 90-percent confidence limits for a Weibull slope of 1.5 and is independent of whether the bearing is thrust or radially loaded.
4. Recalculating the lives for the actual bearing data using a load-life exponent **p** of 4 for ball bearings and 5 for roller bearings results in a reasonable correlation between the maximum and minimum values of the Monte Carlo generated bearing lives and the actual bearing life data. For design purposes, the Lundberg-Palmgren life calculations incorporating a load-life exponent **p** of 3 for ball bearings should be retained. However, the load-life exponent **p** for roller bearings should be changed from 10/3 to 4.
5. The STLE life factors for bearing steel and processing in conjunction with the Lundberg-Palmgren life equation provide a reasonable accounting of differences in the material chemistry and processing. However, the material life factor for AISI 8620 should be changed from 1.5 to 2.
6. Maximum and minimum variations in Weibull slope from the Monte Carlo testing and bearing endurance data correlated with predicted values and was a function of the number of bearing failures. The greater the number of failures the less variation in the Weibull slope from that predicted.
7. There is excellent agreement between the percentage of individual components failed from Monte Carlo simulation and that predicted from the method of Johnson.

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APPENDIX

G. Lundberg and A. Palmgren (5) using the Weibull equation (2–4) first derived the relationship between individual component lives and system life where

$$\left[1/L_{sys}\right]^e = \sum_{i=1}^{n}\left[1/L_i\right]^e \tag{3}$$

Therefore, the system life of a bearing composed of three components (inner race, rolling element, and outer race) that can fail is expressed as

$$[1/L_{sys}]^e = [1/L_{ir}]^e + [1/L_{re}]^e + [1/L_{or}]^e \tag{Eq. (5)}$$

where L_{ir} is the life of the inner race, L_{re} is the life of the rolling element, L_{or} is the life of the outer race and e is the Weibull slope. The fraction of failures due to the failure of a component of a system is expressed by Johnson (11) as

$$fraction\ of\ ir\ failures = \left[\frac{L_{sys}}{L_{ir}}\right]^e \tag{Eq. (A–1a)}$$

$$fraction\ of\ re\ failures = \left[\frac{L_{sys}}{L_{re}}\right]^e \tag{Eq. (A–1b)}$$

$$fraction\ of\ or\ failures = \left[\frac{L_{sys}}{L_{or}}\right]^e \tag{Eq. (A–1c)}$$

Therefore, to determine the fraction of inner-ring (*ir*) failures Eq. (5), which is a definition of the total system life, is multiplied by L_{ir}^e.

$$\left[L_{ir}\right]^e\left[\frac{1}{L_{sys}^e}\right] = \left[L_{ir}\right]^e\left[\frac{1}{L_{ir}^e} + \frac{1}{L_{re}^e} + \frac{1}{L_{or}^e}\right] \tag{Eq. (A–2)}$$

Thus, from Eqs. (A–1a) and (A–2)

$$fraction\ of\ ir\ failures = \left[\frac{L_{sys}}{L_{ir}}\right]^e = \frac{1}{\left[\frac{L_{ir}}{L_{ir}}\right]^e + \left[\frac{L_{ir}}{L_{re}}\right]^e + \left[\frac{L_{ir}}{L_{or}}\right]^e} = \frac{1}{1 + \left[\frac{L_{ir}}{L_{re}}\right]^e + \left[\frac{L_{ir}}{L_{or}}\right]^e} \tag{Eq. (A–3a)}$$

Subsequently, the fraction of rolling-element (*re*) failures can be expressed as

$$fraction\ of\ re\ failures = \left[\frac{L_{sys}}{L_{re}}\right]^e = \frac{1}{\left[\frac{L_{re}}{L_{ir}}\right]^e + 1 + \left[\frac{L_{re}}{L_{or}}\right]^e} \tag{Eq. (A–3b)}$$

and the fraction of outer-ring (*or*) failures can be expressed as

$$fraction\ of\ or\ failures = \left[\frac{L_{sys}}{L_{or}}\right]^e = \frac{1}{\left[\frac{L_{or}}{L_{ir}}\right]^e + \left[\frac{L_{or}}{L_{re}}\right]^e + 1} \tag{Eq. (A–3c)}$$

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