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# Determination of Supersymmetric Particle Masses and Attributes with Genetic Divisors

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## ABSTRACT

Arithmetic conditions relating particle masses can be defined on the basis of (1) the supersymmetric conservation of congruence and (2) the observed characteristics of particle reactions and stabilities. Stated in the form of common divisors, these relations can be interpreted as expressions of genetic elements that represent specific particle characteristics. In order to illustrate this concept, it is shown that the pion triplet ($\pi^{\pm}$, $\pi^{o}$) can be associated with the existence of a greatest common divisor $d_{o\pm}$ in a way that can account for both the highly similar physical properties of these particles and the observed $\pi^{\pm}/\pi^{o}$ mass splitting. These results support the conclusion that a corresponding statement holds generally for all particle multiplets. Classification of the respective physical states is achieved by assignment of the common divisors to residue classes in a finite field $\mathbb{F}_{p_\alpha}$ and the existence of the multiplicative group of units $\mathbb{F}^*_{p_\alpha}$ enables the corresponding mass parameters to be associated with a rich subgroup structure. The existence of inverse states in $\mathbb{F}_{p_\alpha}$ allows relationships connecting particle mass values to be conveniently expressed in a form in which the genetic divisor structure is prominent. An example is given in which the masses of two neutral mesons (K° and $\pi$°) are related to the properties of the electron (e), a charged lepton. Physically, since this relationship reflects the cascade decay $K^\circ \rightarrow \pi^\circ + \pi^\circ / \pi^\circ \rightarrow e^+ + e^-$ , in which a neutral kaon is converted into four charged leptons, it enables the genetic divisor concept, through the intrinsic algebraic structure of the field, to provide a theoretical basis for the conservation of both electric charge and lepton number. It is further shown that the fundamental source of supersymmetry can be expressed in terms of hierarchical relationships between odd and even order subgroups of $\mathbb{F}^*_{p_\alpha}$, an outcome that automatically reflects itself in the phenomenon of fermion/boson pairing of individual particle systems. Accordingly, supersymmetry is best represented as a group rather than a particle property. The status of the Higgs subgroup of order 4 is singular; it is isolated from the hierarchical pattern and communicates globally to the mass scale through the seesaw congruence by (1) fusing the concepts of mass and space and (2) specifying the generators of the physical masses.

## I. Introduction

It has been shown [1] that the construction of inverse states in a finite field $\mathbb{F}_{P_\alpha}$ enables the organization of the mass scale with fundamental octets in an eight-dimensional index space that classifies particle mass states with residue class designations. Conformance with both CPT invariance and the concept of supersymmetry follows as a direct consequence of this formulation. Based on two parameters ($P_\alpha$ and $g_\alpha$) that are anchored on a concordance of physical data embracing both large scale cosmic parameters and small scale couplings [2–4], this analysis leads to (1) prospective values of the masses for the electron and muon neutrinos of ~ 0.808 meV and ~ 27.68 meV, respectively, (2) a magnitude for the unified strong-electroweak coupling constant $\alpha^* = (34.26)^{-1}$ that is physically defined by the electron and muon neutrino mass ratio, and (3) a seesaw congruence connecting the Higgs, the electron neutrino, and the muon neutrino masses. Specific evaluation of the masses of the corresponding supersymmetric Higgs pair revealed that both particles are superheavy (>$10^{18}$ GeV). The stated values of this predicted set of fundamental parameters are consistent with both the ranges of their potentially expected magnitudes [5−10] and all currently available experimental data. No renormalization of the Higgs masses was necessary, since the theoretical approach using a finite field [11] intrinsically excludes the possibility of divergences. Further, the Higgs fulfills its conjectured role through the seesaw relation as the particle defining the origin of all particle masses, since the mass parameters of the electron neutrino and muon neutrino systems, together with their supersymmetric partners, are the generators of the particle mass scale and give the founding basis of the corresponding index space [1]. Finally, since the computation of the Higgs masses is entirely determined by the modulus $P_\alpha$, which is fully defined by the large-scale parameters of the universe [1–4] through the value of the universal gravitational constant G and the requirement for perfect flatness ($\Omega_{total}$ = 1.0), the cosmic seesaw congruence provides an explicit unifying statement for the concepts of mass [12,13] and space [14,15]. Indeed, the overarching conclusion that follows from the form of the seesaw relation is <u>that fusion of these two physical concepts creates a single new archetype</u>.

## II. Determination of the Physical Masses

### A. General Conditions

The determination of the set of representatives of the residue classes $\{[B]_{P_\alpha}\}$ that corresponds to the <u>physically observed</u> masses requires the incorporation of information on particle interactions and stabilities. Previous analysis based on this theoretical picture has shown that exact arithmetic relationships representing the conservation of congruence [16] must be satisfied for open reaction channels. Specifically, for a general 4-body reaction

$$M_o \rightarrow m_x + m_y + m_z \tag{1}$$

involving particles with the corresponding integer mass numbers $B_o$, $B_x$, $B_y$, and $B_z$, these conditions are given [16] by the expression governing the conservation of energy

$$2B_o = d_x + d_y + d_z + \frac{B_x^2}{d_x} + \frac{B_y^2}{d_y} + \frac{B_z^2}{d_z} \tag{2}$$

and the corresponding statement of momentum conservation

$$\frac{B_x^2}{d_x} + \frac{B_y^2}{d_y} \geq \frac{B_z^2}{d_z} + d_x + d_y - d_z \geq \frac{B_x^2}{d_x} - \frac{B_y^2}{d_y} + d_x + d_y, \tag{3}$$

in which $d_x$, $d_y$, and $d_z$ respectively represent the appropriate divisors of $B_x^2$, $B_y^2$, and $B_z^2$, the squares of the corresponding mass parameters. Founded on the conservation of energy and momentum [16] for flat space ($\Omega_{total} = 1.0$), these two statements define explicit conditions on the three sets of divisors $\{d_x\}$, $\{d_y\}$, and $\{d_z\}$ in relation to the mass parameter $B_o$ of the initial particle. Since the theoretical structure [1] is comprehensively expressed with a finite field [11], Eqs. (2) and (3) represent relations among the residue classes $[x]_{p_\alpha}$ of the governing field $\mathbb{F}_{p_\alpha}$. Hence, the divisors $d_x$, $d_y$, and $d_z$ are corresponding representatives of the appropriate residue classes.

The nature of the constraints embodied in Eqs.(2) and (3) and their consequences on the particle mass spectrum can be illustrated by comparing the properties of the proton and the neutron. Since the proton is physically stable [17], the process

$$p \not\rightarrow e^+ + e^+ + e^- \tag{4}$$

has a vanishing amplitude; the channel is closed. The corresponding mathematical statement is achieved by requiring that the mass parameter [8] of the proton $B_p$ and the respective divisors of the square of the mass number of the electron $B_e$ be such that Eqs.(2) and (3) admit no solution [16]. Since we know from CPT invariance that the mass parameters of the electron and positron are equal, the computational scale of the analysis of this condition on the proton decay channel given by Eq.(4) is considerably reduced over the general case for which the particles in the exit channel have unequal masses.

Conversely, the commonly observed reaction of neutron decay [17]

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{5}$$

represents an open channel which yields a broad distribution of particle energies. In this case, the constraints on the mass number of the neutron $B_n$ and the mass numbers and divisors associated with the proton, the electron, and the electron neutrino must be such that the number of representations $N(2B_n)$ of the integer $2B_n$, given by the form of Eq.(2) and consistent with satisfaction of Eq.(3), is high [16]. A large multiplicity of solutions must exist for correspondence with the wide distribution of particle energies physically observed in neutron decay.

Taken together, Eqs.(2) and (3) establish a complex mutual arithmetic dependence on the mass numbers and divisors of the five participating particles in the reactions given by Eqs.(4) and (5). Consequently, the physical mass states correspond to that set of residues which, under the conditions specified by Eqs.(2) and (3), conform to the observed properties of open channel reactions and particle stabilities [4].

Since the squares of suitably smooth integer mass parameters for typical particles like the electron and proton are estimated to have more than $10^{12}$ divisors [18], the determination of the general solution of Eqs.(2) and (3) for arbitrary mass numbers is an overwhelmingly massive computation. However, it is known [16] that there exists a physically significant subclass of reactions of considerably reduced complexity that is identified by the condition

$$B_z = 1 \ , \tag{6}$$

namely, the category that corresponds to two-body decay amplitudes [16] of the observed form

$$M_o \rightarrow m_x + m_y \ , \tag{7}$$

an example of which is the pion decay process

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \ . \tag{8}$$

Specifically, with $B_z = 1$, Eqs.(2) and (3) are respectively modified to read

$$d_x + d_y + \frac{B_x^2}{d_x} + \frac{B_y^2}{d_y} = 2(B_o - 1) \tag{9}$$

and

$$d_x - d_y - \frac{B_x^2}{d_x} + \frac{B_y^2}{d_y} = 0 \ , \tag{10}$$

results which lead immediately to the statement

$$B_x^2 - B_y^2 = (d_x - d_y)(B_o - 1) \ . \tag{11}$$

With the identification of the factor $(d_x - d_y)$ with the modulus index

$$k = d_x - d_y, \tag{12}$$

Eq. (11) can be written as the congruence

$$B_x^2 \equiv B_y^2 \,(\mathrm{mod}(B_o - 1)). \tag{13}$$

Significantly, this form is identical to Eq.(14) in Ref. [16] under the transformation of the modulus $B_o \rightarrow B_o - 1$.

It follows directly from Eqs.(9) and (10) that

$$B_o = d_{xo} + d_{yo} + 1, \tag{14}$$

in which $d_{xo}$ and $d_{yo}$ respectively represent specific residue class members of the sets of divisors $\{d_x\}$ and $\{d_y\}$ of the integers $B_x^2$ and $B_y^2$. On physical grounds, since the conservation of energy and momentum legislates a single kinetic distribution of these quantities for two-body decay, Eq.(14) must also be the unique solution corresponding to the appropriate particle energies of the reaction. Hence, a strong constraint on the sets of divisors $\{d_x\}$ and $\{d_y\}$ for the existence of an open two-body decay channel is established. Conversely, if Eq.(14) cannot be satisfied by any of the respective members of the sets $\{d_x\}$ and $\{d_y\}$, the corresponding process represents a closed channel. Consequently, this outcome defines an explicit condition of stability against two-body decay for the system with mass parameter $B_o$.

### B. Divisor Residue Class Relations among Two-Body Decay Amplitudes

An examination of the five allowed mesonic decay reactions [17]

$$K^\circ \rightarrow \pi^\circ + \pi^\circ \ , \tag{15}$$

$$\pi^\circ \rightarrow e^+ + e^- \ , \tag{16}$$

$$K^\circ \rightarrow \pi^+ + \pi^- \ , \tag{17}$$

$$\pi^- \rightarrow e^- + \bar{\nu}_e \ , \tag{18}$$

and

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \tag{19}$$

illustrates several important consequences concerning the divisors that immediately flow from Eq.(14). Since CPT invariance demands that particles and corresponding antiparticles have identical masses, by inspection, we obtain from Eqs.(15) through (19)

$$d_{\pi^o} = \frac{B_{K^o} - 1}{2} \ , \tag{20}$$

$$d_e = \frac{B_{\pi^o} - 1}{2} \ , \tag{21}$$

$$d_{\pi^-} = \frac{B_{K^o} - 1}{2} \ , \tag{22}$$

$$d_e' + d_{\nu_e} = B_{\pi^-} - 1, \tag{23}$$

and

$$d_\mu + d_{\nu_\mu} = B_{\pi^-} - 1, \tag{24}$$

in which $d_e$ and $d_e'$ are potentially distinct $(d_e \neq d_e')$ divisors of the square of the electron mass parameter $B_e$.

We observe from Eqs. (20) and (22) that the $K^o$ decay channels establish the condition

$$d_{\pi^o} = d_{\pi^-} = \frac{B_{K^o} - 1}{2}. \tag{25}$$

Therefore, although the physical $\pi^\circ$ and $\pi^\in$ masses exhibit [17] a considerable splitting (~ 4.6 MeV), the corresponding mass parameters of these three particles must possess a common arithmetic divisor

$$d_{o\pm} < d_{\pi^o} , \tag{26}$$

since the kaon/pion mass ratio is considerably greater than two. Subject to the constraint expressed by Eq.(26), let $d_{o\pm}$ be defined as the greatest common divisor (gcd) of the $\pi^o$ and $\pi^\pm$ mass parameters derivable from $d_{\pi^o}$. In this sense, a specific residue class identity is explicitly expressed for the pion triplet ($\pi^\pm$, $\pi^o$) by an element of the internal structure ($d_{o\pm}$) of the corresponding mass parameters that arises from the common coupling to the $K^o$ meson. Indeed, if the integer $d_{o\pm}$ is composite, it defines a set of common divisors $\{d_\varepsilon\}_{o\pm}$ for the corresponding $\pi^o$ and $\pi^\pm$ mass parameters. Each of the elements $d_\varepsilon$ of the set $\{d_\varepsilon\}_{o\pm}$ specifies a respective residue class designation [4].

In recapitulation, the existence of the $K^o$ decay channels [17] given by Eqs.(15) and (17) leads to the statement

$$d_{\pi^o} = d_{\pi^-} = \frac{B_{K^o} - 1}{2} \Rightarrow d_{o\pm} \Rightarrow \{d_\varepsilon\}_{o\pm}, \tag{27}$$

a sequence which defines a divisor set $\{d_\varepsilon\}_{o\pm}$ that is commonly shared by the mass numbers of all three pions. We will consider the elements $d_\varepsilon$ of this set as genetic components (genes) that identify the common properties of these particles. Simultaneously, the nonequivalence of the masses of the members of the pion triplet ($\pi^\pm$, $\pi^o$) can also be expressed. For example, since different supplementary factors can be present in the respective mass numbers, this mode of identification through residue classes does not require that the $\pi^\pm/\pi^o$ mass splitting vanish. Accordingly, this mass difference is free to be established by other considerations.

### C. Role of Inverse States

The existence of inverses in a finite field $\mathbb{F}_{P_\alpha}$ facilitates the construction of important additional relationships among particle masses [1]. Specifically, for any prime integer P, we recall [11] that a field $\mathbb{F}_P$ consists of the set of residue classes

$$\{[0]_P, [1]_P, [2]_P, \ldots [P-1]_P\} \ . \tag{28}$$

Computations performed with elements $[x]_P$ of $\mathbb{F}_P$ are conducted with the customary rules of arithmetic and reduction modulo the prime P. Further, since all primes possess primitive roots [18], power residue systems of enumeration in $\mathbb{F}_P$ can be established [19]. In particular, the existence of the operation of division [11] in a finite algebraic field $\mathbb{F}_P$ enables the definition of an inverse $[x]_P^{-1}$ for every element $[x]_P$ of $\mathbb{F}_P$ except $[0]_P$. Hence, the operation

$$[x]_P \, [x]_P^{-1} = [1]_P \tag{29}$$

is defined for all nonvanishing values of x. The inverse is generally computed [20] by

$$[x]_P^{-1} = [x]_P^{P-2}, \quad (30)$$

a relation that follows immediately from Fermat's little theorem and explicitly shows that an inverse $[x]_P^{-1}$ generally depends on both the value of x and the modulus P.

The existence of inverse states enables relationships among particle masses to be constructed in which the divisor structure is prominent. Consider the $m_{K^o} / m_{\pi^o}$ mass ratio which is given by

$$\frac{m_{K^o}}{m_{\pi^o}} = \frac{[B_{K^o}]_{P_\alpha}}{[B_{\pi^o}]_{P_\alpha}}, \quad (31)$$

a statement in which the residue class designations of the respective mass numbers are explicitly identified. Evaluation of the right hand side of Eq.(31) with the $K^o$ and $\pi^o$ mass parameters would yield the physically measured $m_{K^o} / m_{\pi^o}$ mass ratio [17], a non-integral value with a magnitude of ~ 3.687.

The use of inverses in the field $\mathbb{F}_{P_\alpha}$ enables the physical quantity represented by Eq.(31) to be presented in a more informative alternative integral form. Specifically, from Eqs.(20), (21), and (22) we can reexpress the sense of Eq.(31) as

$$[B_{K^o}]_{P_\alpha}[B_{\pi^o}]_{P_\alpha}^{-1} = [B_{K^o}]_{P_\alpha}[B_{\pi^o}]_{P_\alpha}^{P_\alpha - 2} = [2d_{\pi^o} + 1]_{P_\alpha}[2d_e + 1]_{P_\alpha}^{-1} = [2d_{\pi^-} + 1]_{P_\alpha}[2d_e + 1]_{P_\alpha}^{-1}, \quad (32)$$

a statement that relates the masses of two neutral mesons ($K^\circ$and $\pi^\circ$) to a property ($d_e$) of a charged lepton ($e^-$). Since Eq.(32) involves the divisors associated with the respective mass parameters, it is a specific relation that couples corresponding genetic elements. Moreover, since the physical basis of the relationship is the existence of the cascade decay $K^\circ \rightarrow \pi^\circ + \pi^\circ / \pi^\circ \rightarrow e^+ + e^-$, in which the neutral kaon is transformed into charged leptons, we are thereby presented with the conclusion that the conservation of both electric charge and lepton number must be embedded in the intrinsic divisor structure expressed through the modular algebra of Eq.(32). Further, the dependence of Eq. (32) on the modulus $P_\alpha$, a parameter fully determined by the large-scale parameters of the universe [1–4], also couples these three particles ($K^o$, $\pi^\circ$, $e^-$) to the cosmic dimension and, consequently, to the Higgs system which designates the generators of the mass scale through a seesaw relation [1]. Hence, the finding stated in Eq.(32) is an explicit mathematical expression that establishes a direct connection between the cosmic and micro scales. Therefore, by firmly linking the big and the small, the consolidation illustrated by Eq.(32) recreates in modern mathematical form an ancient and everlasting idea that has undergone repeated origination, independent of cultural origin, throughout the entirety of human history [21–25].

## III. Group Theoretical Organization of Genetic Residue Classes

### A. Group and Subgroup Structures

The concept of genetic divisors introduced in **Section II.B** above can be organized in a group structure. The set of units of a finite field $\mathbb{F}_P$ with P a prime defines a cyclic group [11,20,26–30] of P − 1 elements designated as $\mathbb{F}_P^*$. It is a general and elementary property of finite groups that the order of any element of the group must be a divisor of the number of elements comprising the group.

Accordingly, the order δ of any $[x]_P \in \mathbb{F}_P^*$ divides P − 1 and the divisors of P − 1 constitute the orders of the subgroups contained within $\mathbb{F}_P^*$. Further, each subgroup has a generator $[\alpha_\delta]_P$ from which all elements of the subgroup can be produced.

The mathematical scale invariance of the general group properties permits illustration of the relevant subgroup structure with a small prime (P = 37) of the form P ≡ 1(mod 4), the physically motivated class [1–4, 16] of groups $\mathbb{F}_P^*$. Since $P - 1 = 36 = 2^2 \cdot 3^2$ in this case, the divisor set of P − 1 contains the nine members given by

$$\{d_x\}_{P-1} = \{d_x\}_{36} = \{1,2,3,4,6,9,12,18,36\}. \tag{33}$$

We note that $\{d_x\}_{P-1}$ plays the role in this example of the special subset of residue classes $\{B_x\}$ identified in a previous study [1] analyzing the properties of inverse states and that the restricted set of primitive roots $\{g_x\}$ therein defined [1] is not empty. The outcome that $\{g_x\} \neq \phi$ is also physically motivated [4]. In the divisor set given by Eq.(33), only 2 and 18 are primitive roots of 37, hence, $\{g_x\} = \{2,18\}$.

We present in Table I for P = 37 the explicit subgroup structures for orders 2,3,4,6,9,12, and 18 in order to exhibit the organizational pattern that emerges.

| **Order (δ)** | **Group Elements $\{x\}_\delta$** | **Generator $\{x_\delta\}_P$** |
|---|---|---|
| 2 | $\{1,36\}_2$ | $[36]_{37}$ |
| 3 | $\{1,10,26\}_3$ | $[10]_{37}$ |
| 4 | $\{1,31,36,6\}_4$ | $[31]_{37}$ |
| 6 | $\{1,11,10,36,26,27\}_6$ | $[11]_{37}$ |
| 9 | $\{1,7,12,10,33,9,26,34,16\}_9$ | $[7]_{37}$ |
| 12 | $\{1,8,27,31,26,23,36,29,10,6,11,14\}_{12}$ | $[8]_{37}$ |
| 18 | $\{1,3,9,27,7,21,26,4,12,36,34,28,10,30,16,11,33,25\}_{18}$ | $[3]_{37}$ |

*Table I: Structure of subgroup elements, written in order of ascending power index, and corresponding generators for orders 2,3,4,6,9,12, and 18 for the modulus P = 37. The group $\{\ \}_{36}$ of order 36 is trivial, since it contains by definition all primitive roots of the modulus and, as a consequence of Fermat's little theorem, all remaining non-primitive elements. Any primitive root can serve as the generator of this group.*

The odd and even order subgroups appearing in Table I illustrate contrasting structures if the concept of supersymmetry [31] developed in earlier work [1,3,4] is considered. For two residues x and y, the condition of supersymmetric pairing is expressed by the statement

$$x + y \equiv 0 \pmod{P}. \tag{34}$$

## B. Odd Order Subgroups

By definition of a group, every residue $[\alpha]_P$ is accompanied by its inverse $[\alpha]_P^{-1}$. It is seen from Table I, however, that no supersymmetric pairs are present; Eq.(34) is unfulfilled for each individual element of the odd order subgroups. However, the <u>sum</u> over all subgroup elements respects the condition

$$\sum_{\text{elements}} = [\alpha_i]_P \equiv 0 (\text{mod } P) . \tag{35}$$

Physically, the odd subgroups are comprised of particles and inverses with no representation of corresponding supersymmetric states. Explicit verification of these relationships can be made by reference to the entries in Table I for the subgroup orders 3 and 9.

## C. Even Order Subgroups

The elements of even order subgroups form two associative structures grouping into both (a) supersymmetric and (b) inverse state pairs. Specifically, for each residue $[\alpha]_P$, both $[\alpha]_P^{-1}$ and $[\beta]_P$ are present with $[\alpha]_P + [\beta]_P \equiv 0(\text{mod } P)$; Eq.(34) is satisfied. With the exception of order 4, which represents the unique case of the Higgs degeneracy [1]

$$[\alpha]_P^{-1} = [\beta]_P \ , \tag{36}$$

the condition

$$[\alpha]_P^{-1} \neq [\beta]_P \tag{37}$$

holds. Eq.(35) is again respected and $[1]_P$ and $[P-1]_P$ serve as self-inverses. In physical terms, particles P, supersymmetric partners $P_{ss}$, and corresponding inverse states $P_{in}$ and $(P_{ss})_{in}$ form a group. These characteristics are explicitly represented in Table I for the subgroups with the even orders 2, 6, 12, and 18.

## D. Supersymmetric Relations Between Odd and Even Order Subgroups

The condition for supersymmetry stated in Eq.(34) links subgroups of odd and even order. For example, it is evident from Table I that the supersymmetric partners to all elements of the subgroup $\{\ \}_3$ of odd order 3 are members of the even subgroup $\{\ \}_6$ of order 6, since

$$\{\ \}_6 \ ( \ \{\ \}_3 . \tag{38}$$

Accordingly, the supersymmetric residues corresponding to subgroup $\{\ \}_9$ in Table I are contained in $\{\ \}_{18}$, an outcome that is also readily apparent. Further, for every odd order subgroup there exists at least one even order subgroup containing it for a prime modulus $P \geq 3$, since 2 will always be a divisor of $P-1$.

The relationships of the seven subgroups presented in Table I can be represented in the planar diagram [32,33] illustrated in Fig. (1). We see that a pattern with a two-fold axis is produced by the hierarchy of subgroup orders and corresponding inclusions. The inclusions involving the pairing of supersymmetric residues in accord with Eq.(34) are specified with the notation (ss). Hence, the principle of supersymmetry [6,10,31], which rests on the hypothesized basis of individual particle fermion/boson pairing [31], exhibits a complementary expression through the pairing of subgroups of even and odd order. In fact, the subgroup structure can be considered as the fundamental origin of the supersymmetric principle. Accordingly, supersymmetry becomes a group property and the fermion/boson pairing of particles is simply a reflection of the basic underlying subgroup structure. This pairing respects the condition that the order of the odd subgroup must be a divisor of the order of the even subgroup including it. It follows that the minimum even order of this pairing is twice the value of the odd order subgroup. A further important consequence is the isolation of the Higgs subgroup [1] from this mode of pairing, since it has order 4 and accordingly cannot contain a non-trivial odd subgroup of lower order. In contrast to a hierarchical relationship, the Higgs system is communicated globally to the mass scale through the seesaw congruence [1] which specifies the generators of all particle masses.

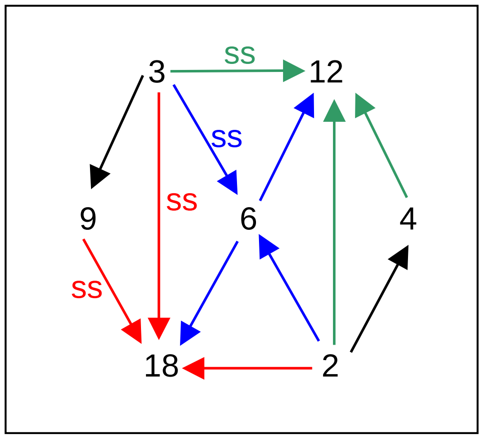


*Fig. (1): Two-fold hierarchical pattern of subgroup inclusion relationships for the seven subgroups given in Table I. The vertices identify the subgroup orders, the arrows indicate subgroup inclusion, and the symbol (ss) specifies the existence of supersymmetric pairing of the group elements. The directed graph (digraph) shown indicates that the concepts of supersymmetry expressed for individual residues by Eq.(34) can be based on relationships between even and odd order subgroups of* $\mathbb{F}_p^*$. *Supersymmetry then becomes a group rather than a particle property.*

We note that the planarity of the graph presented in Fig. (1) is not a general property. On the basis of a theorem of Kuratowski [34,35], which can be demonstrated with the Euler polyhedral formula, the graph shown is planar, since it does not contain graphs of the type $K_5$ or $K_{3,3}$. Nonplanar configurations arise inevitably for a sufficiently large value of the arithmetic function $d(P-1)$, the function [36,37] which specifies the number of divisors of $P-1$. For example, planarity is no longer possible for $P = 181$, a prime which gives $d(180) = 18$. The existence of this transition to nonplanarity connects the subgroup structure to the topology of discrete spaces [38,39].

## IV. Conclusions

The supersymmetric conservation of congruence and the observed properties of particle interactions and stabilities combine to establish an interlocking set of conditions defining the particle masses. Stated in the form of common divisors of the corresponding mass parameters, these relations can be interpreted as expressions of genetic elements that represent specific particle characteristics. It has been shown that the pion triplet ($\pi^{\pm}$, $\pi^{o}$) can be identified by the existence of a common divisor $d_{o\pm}$ in a way that can also account for the observed $\pi^{\pm}/\pi^{o}$ mass splitting. The results indicate that a corresponding statement would hold for all properly classified particle multiplets. With the introduction of inverse states made possible by the use of the mathematical structure of a finite field $\mathbb{F}_{p_\alpha}$, a relationship involving the $K^o/\pi^o$ mass ratio was expressed in a form that explicitly reveals the divisor structure and related those two neutral mesons to a property of a charged lepton, the electron. This finding (1) demonstrated the existence of relations among particle families based on the pattern of observed reactions, (2) illustrated how these relationships can be expressed in terms of genetic elements (divisors), (3) indicated how the genetic divisor concept can express the conservation of both electric charge and lepton number through the intrinsic algebraic structure of the field, and (4) made an explicit connection between the cosmic and microscales, an ancient universal idea that arose uniformly in human prehistory. Finally, it has been shown that the concept of supersymmetry can be expressed in terms of fundamental hierarchical relationships between odd and even order subgroups of the multiplicative group of units $\mathbb{F}_p^*$. Supersymmetry is thereby identified as a group property from which the fermion/boson pairing of individual particle states follows as a necessary consequence. In contrast to all other subgroups, since it has order 4, the Higgs system is isolated from the hierarchical pattern and is expressed globally to the mass scale through the seesaw congruence.

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