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# DETERMINATION OF THE NATURAL PERIODS OF BUILDINGS

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ABSTRACT

Random wind excitation has been used to find the first three modes of vibration of a nineteen-story building about the two major translational axes and the torsional axis of the building. Vibration records were obtained with Willmore electromagnetic seismometers feeding into a multichannel magnetic tape recorder. A harmonic analysis was then performed with the aid of an analogue computer to determine the first few vibration modes. Concurrently with the experimental program the building modes were computed two ways using different simplifying assumptions regarding the lateral stiffness of the structure.

## Introduction

This study was prompted by the need for accurate information on the natural periods of vibration of buildings for comparison with values predicted from mathematical models. Studies of this type were initiated some years ago by the U. S. Coast and Geodetic survey but it was believed that more accurate and extensive information about wind-excited structures could be gained using modern methods of analysis. Accordingly, a trial program was undertaken on one building.

The building was a new Federal Government building, built for the Department of Public Works to house the Department of National Health and Welfare, located at Tunney's Pasture in Ottawa. The building itself is situated on an exposed parkland area with very few trees or buildings nearby. Comprising 19 stories (17 floors, ground level and basement) and shaped like a rectangular prism, it is 140 by 88 ft by 235 ft high. The structural steel columns of the building run the full height with the main structural changes occurring at the 4th and 14th floors. In the center of the structure is a steel and concrete core that houses the elevator shafts and stairways. Of more massive construction than most modern office blocks, the building has 11-in. reinforced concrete floor slabs and outside walls composed of heavy precast concrete window sections, each weighing approximately 2 tons. The weight of these window sections at each floor level is borne by the floor slab itself so that the outside walls are not considered as a single structural unit. Figure 1 shows a general view of the building; figure 2 indicates the layout of the structure.

## Instrumentation

The heart of any measuring system is the sensing element. Since the periods of vibration of structures that are of interest lie in the same range as those periods observed in earthquakes, generally in the range 0.05 to 10 seconds, it was obvious that a transducer such as is used for recording earthquakes could be used to record structural vibrations. Not only is it sensitive in the correct frequency range, but because of its high sensitivity it is ideal for recording small motions of buildings induced by wind excitation.

The Willmore Mark II seismometer was used in this study. This is an electromagnetic transducer with a fixed coil and a heavy magnet that acts as the moving mass. It can be set to measure in either the vertical or horizontal direction. The natural period of the suspension can be adjusted from 0.6 to 5.0 sec, and it was set at 2.5 sec to measure horizontally. When damped electrically to 0.65 of critical damping the velocity response is flat over the desired period range of 0.05 to 2.0 sec.

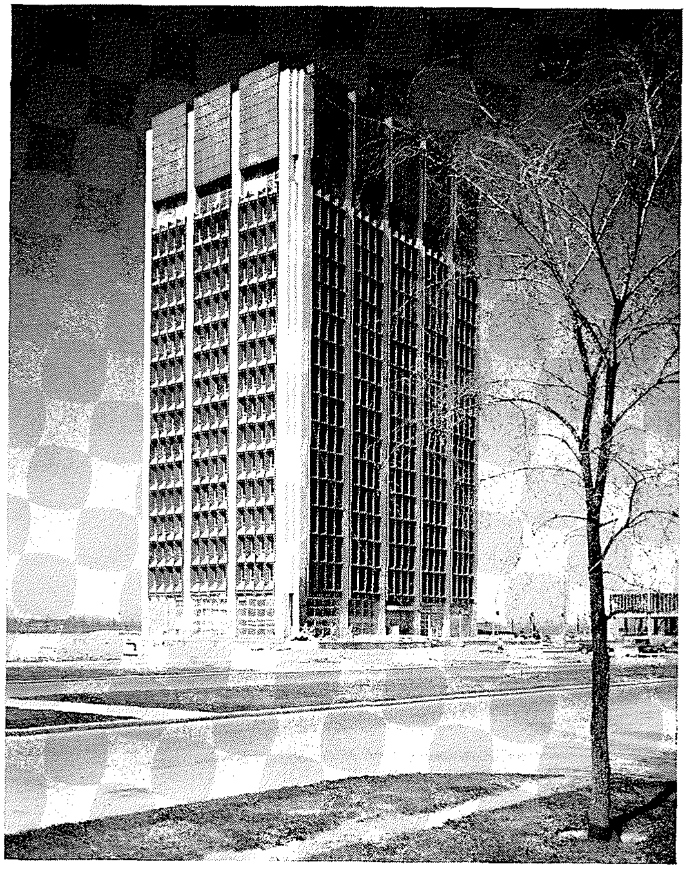

FIG. 1. The National Health and Welfare Headquarters Building, Tunney's Pasture, Ottawa. *Consulting Architects:* Balharrie, Helmer and Associates, Ottawa; Greenspoon, Freedlander and Dunne, Montreal. *Consulting Structural Engineers:* Marshall and Marshall, Montreal. *Consulting Engineer In Charge Of Supervision and Construction:* Mr. G. Pack, Balharrie, Helmer and Associates.

The electrical outputs from the Willmore seismometer were fed through dc amplifiers into a portable 7-channel FM tape recorder. Paper records using a multichannel galvanometer recorder with direct writing paper were obtained simultaneously with the tape recordings so that a check could be kept on the vibration levels.

The magnetic tape recordings so obtained contain the vibration data en masse in the building in the frequency range of the transducer; the problem is to determine the amount of energy present at each frequency and also its origin. To this end a harmonic wave analyser giving a Fourier analysis of a complex waveform was

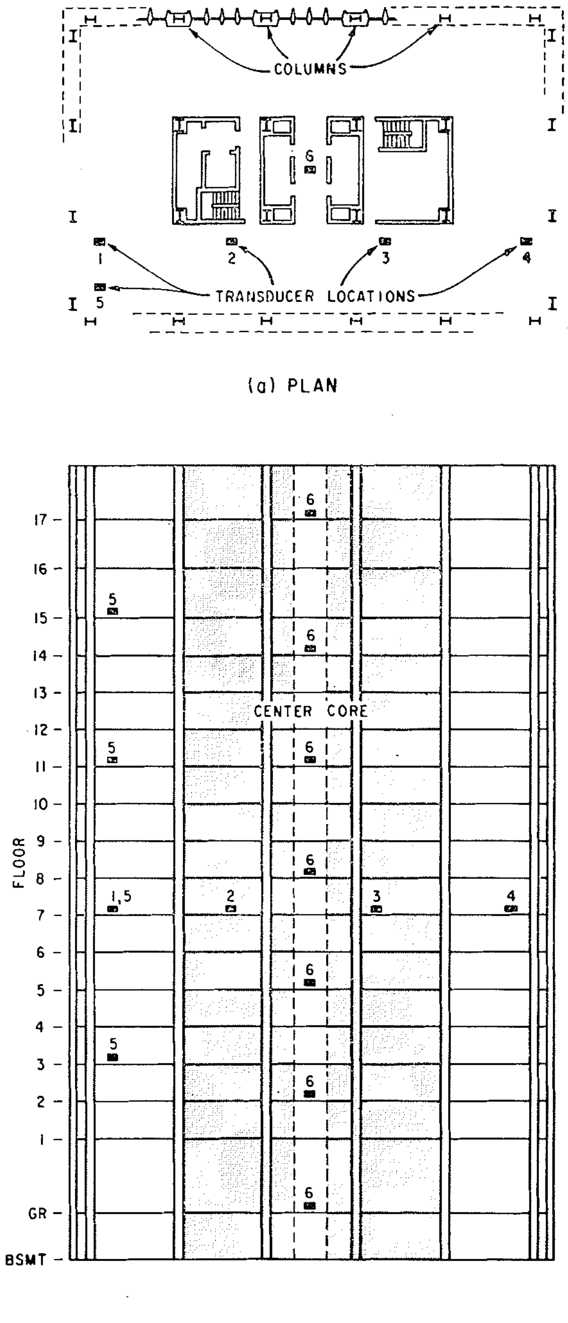


FIG. 2. (a) Plan view of the National Health and Welfare Building showing columns, core and transducer locations. (b) Front elevation showing floor levels, columns, core, and transducer locations.

employed. In this study, power spectral density curves were obtained for each recording, giving a measure of the average distribution of vibration energy per unit band width over the frequency spectrum for the length of the recording. The original recordings obtained from the structural vibrations could not be analysed directly as the frequencies involved were outside the frequency range of the harmonic analyser. To rectify this, the original records, taken with a tape speed of $1\frac{7}{8}$ ips, were played back at 30 ips and re-recorded at $1\frac{7}{8}$ ips on a tape loop. This loop was in turn played back at 30 ips, giving a final speed-up factor of 256, and the output fed to the harmonic analyser. Thus a typical structural vibration of about 1 cps appears as 256 cps to the wave analyser. The maximum band width used was 20 cps which corresponds to 0.08 cps in real time. The block diagrams in fig. 3 indicate

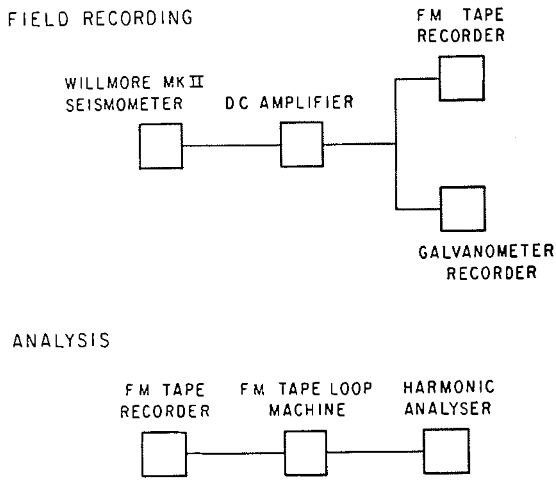


FIG. 3. Block diagrams of the instrumental systems.

the recording and analysis systems. Figures 5 to 7 show the typical power spectral density curves for the structural vibrations from wind excitation.

The preliminary experimental work on this building provided many useful results indicating the best approach for future studies. Initially, transducers were placed on the 7th floor at various locations and simultaneous recordings made parallel to the short dimension of the building and then parallel to the long dimension of the building. Figure 2 shows the transducer locations, numbered 1 to 6. Typical paper records (fig. 4) show the vibrations at locations 1, 2, 3, 4, and 6 parallel to the short dimension. The modes of lateral vibration were obtained by placing the transducers at locations 2 and 3 near the center of the building and the rotational modes were picked up toward the outer ends of the building at locations 1 and 4. Power spectral density curves for some of these recordings are shown in fig. 5, and it can be seen that the distinct peaks at certain frequencies correspond to the various modes of vibration. Later, the transducers at locations 1 and 4 were connected in series but oriented 180° out of phase, so that the translational motion of the building would tend to cancel out leaving the rotational motion. The bottom curves in fig. 5 show the value of this technique in accentuating the rotational modes.

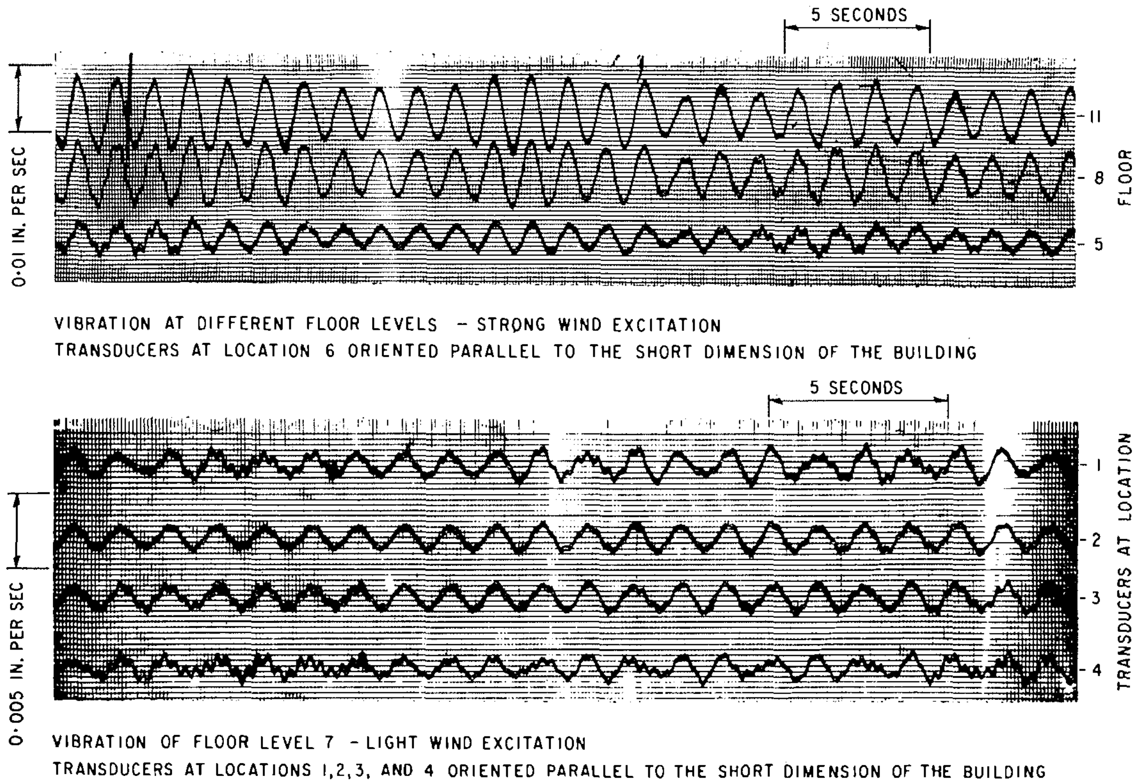


FIG. 4. Typical paper vibration records.

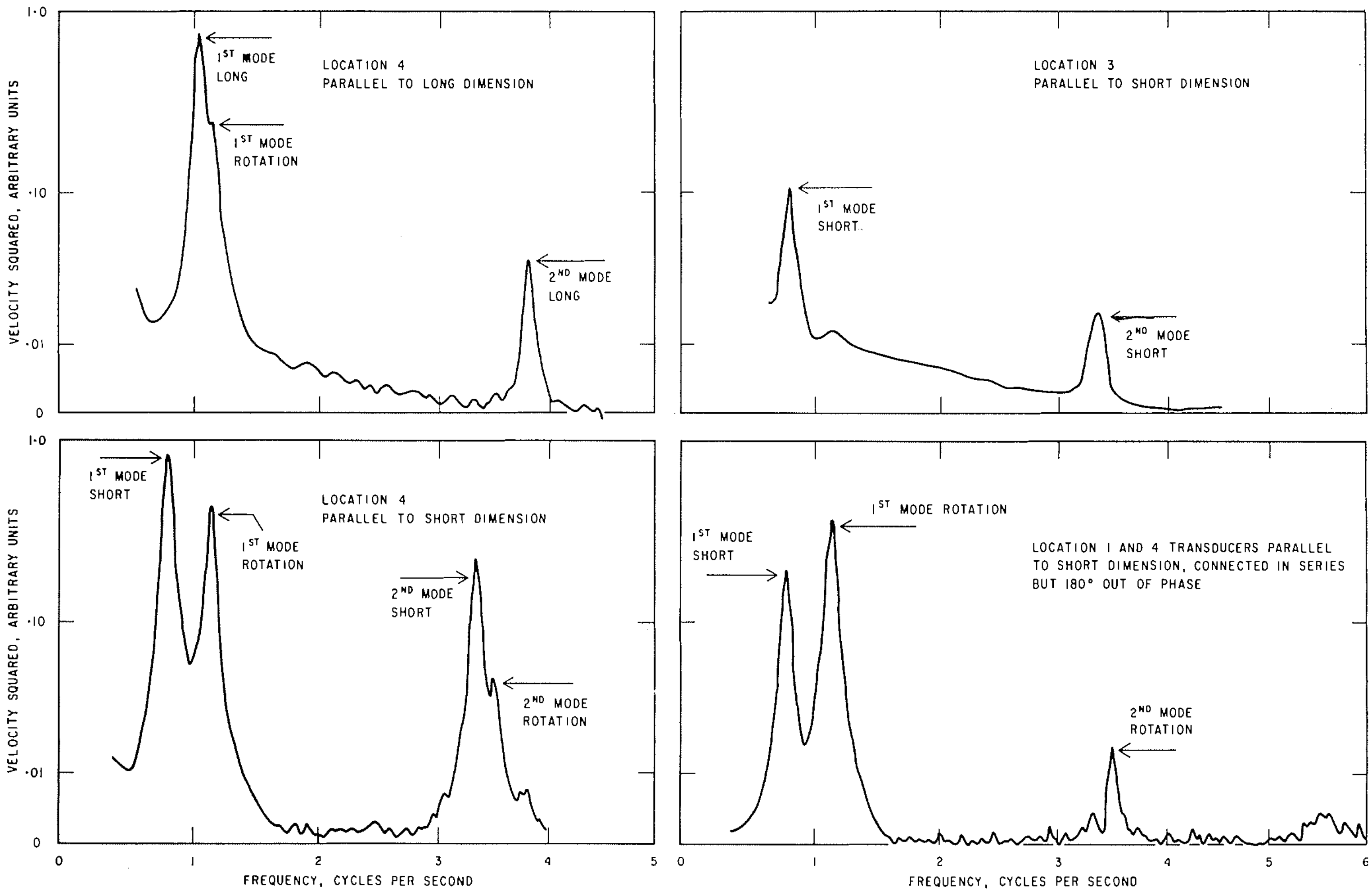


FIG. 5. Power spectral density analyses on Floor 7 showing the effect of transducer position and orientation.

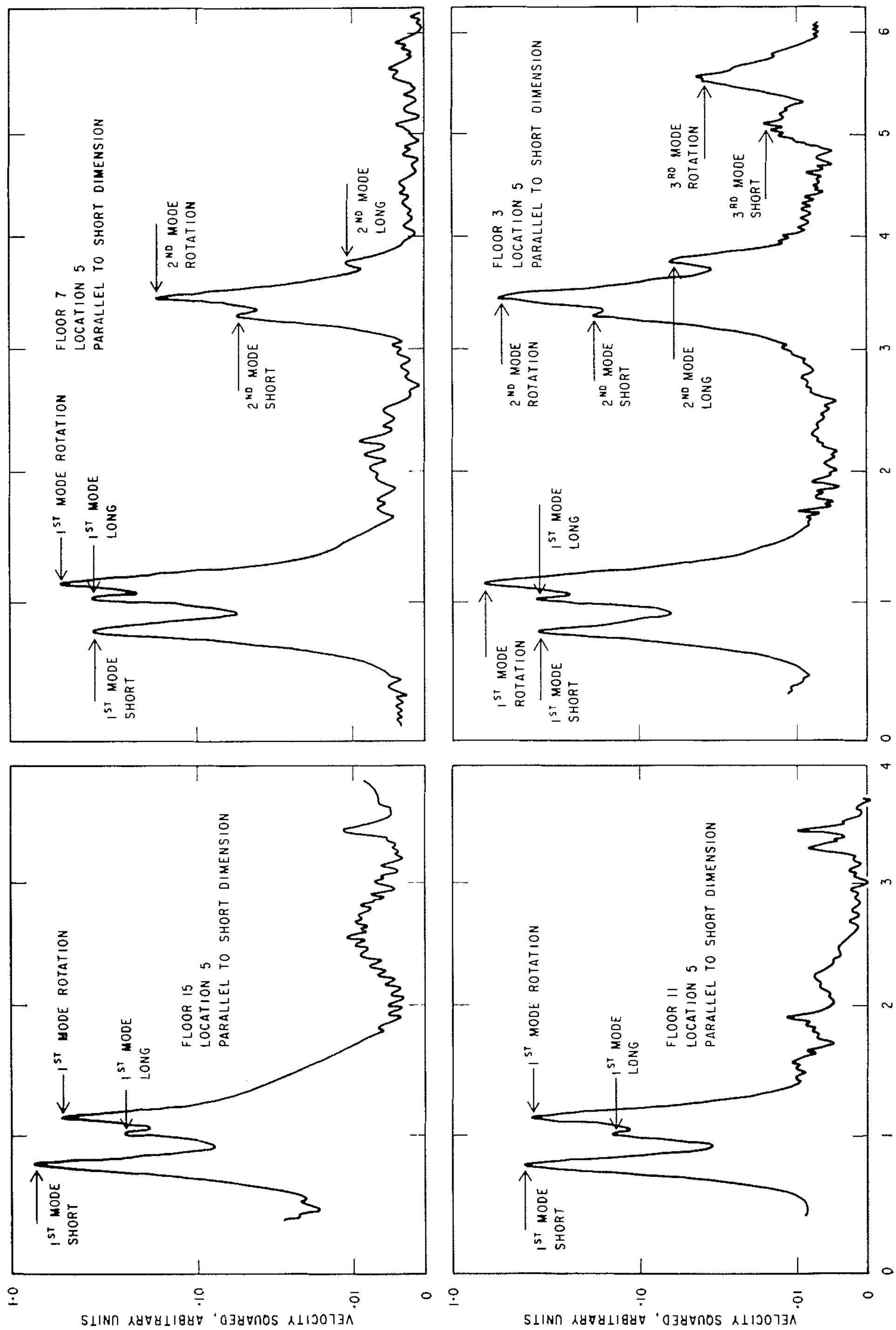


FIG. 6. Power spectral density analyses showing the effect of building height on modal content.

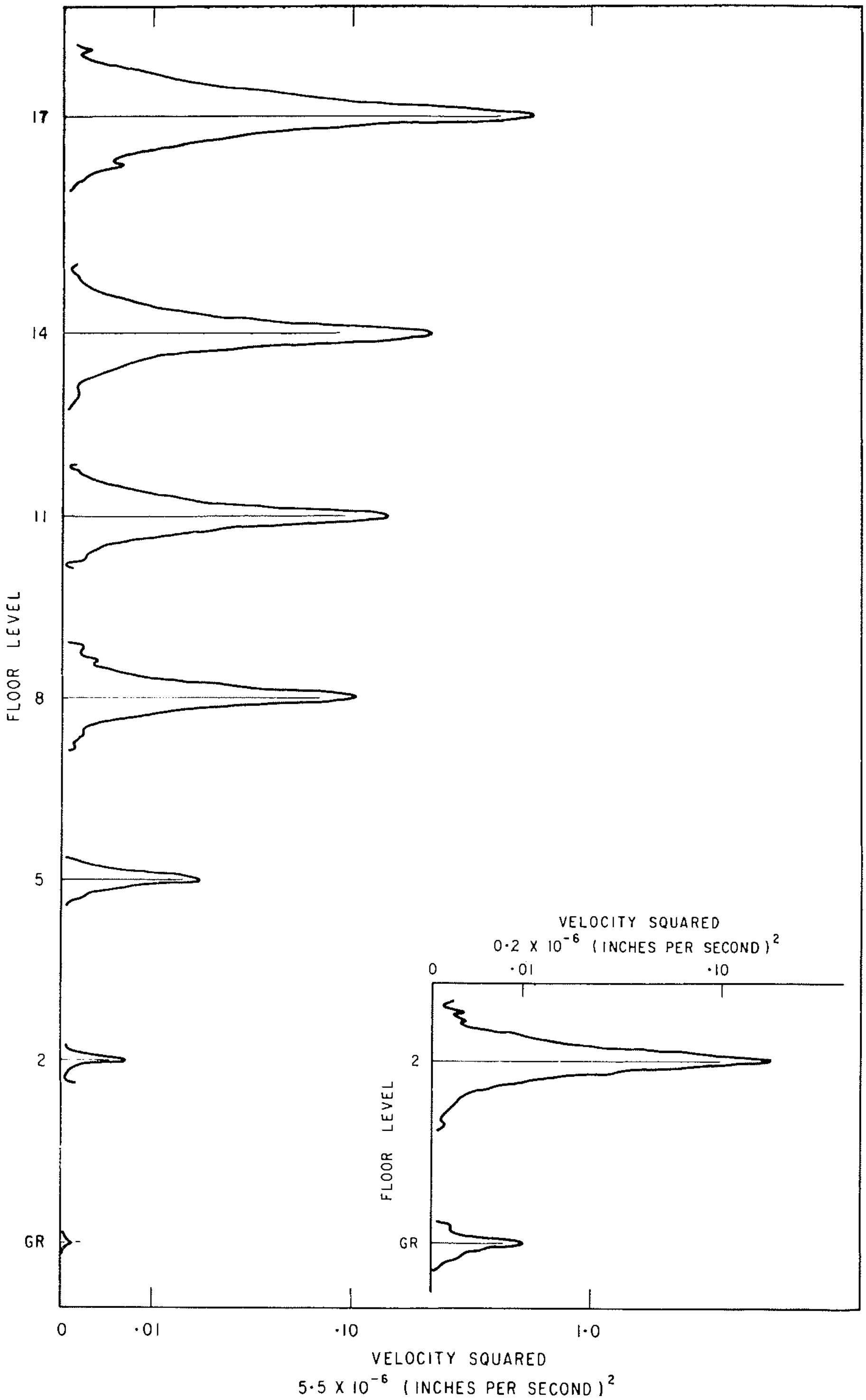


FIG. 7. Power spectral density analyses showing the variation of 1st mode with building height and structural changes. Mean square velocity parallel to the short dimension of the building is plotted against floor level.

Simultaneous recordings were obtained at floor levels 15, 11, 7, and 3 with the transducers at location 5 (fig. 2) when the exciting force was a light wind. The power spectral density curves show the motion parallel to the short dimension in figure 6. (Interest should be limited to the frequency spectra since the records are not comparable with each other on an absolute basis.) Fig. 6 shows that the higher modes of vibration present in the lower floors are not evident to the same degree in the upper floors. The relative power between the first three modes is obtained for each type of motion—two lateral components and torsion. With this type of excitation it is seen that the power decreases rapidly with increasing mode. Table I lists the natural periods of the first three modes of vibration of the building excited by light random wind excitation. The building is less stiff in the short dimension than in the long dimension, and the observed natural periods along these axes are consistent in that the longer period is associated with the less stiff direction.

From the work so far, it has been found that the center of the building is more suitable for obtaining information about the modes of one particular translational vibration. Simultaneous recordings were made at the center of the building between the elevator shafts (fig. 2) on floor levels 17, 14, 11, 8, 5, 2, and at ground level. The wind, measured at a height of 30 ft above ground level at the Ottawa Airport, was strong, gusting to 30 mph for this set of recordings. Parts of the calibrated power spectral density curves for the vibrations parallel to the short dimension are shown in fig. 7 plotted versus floor level. Virtually only the fundamental mode was excited with very little energy in the higher modes, a feature apparently related to wind strength.

A mode shape for this set of recordings was drawn by taking the square root of the height of this first mode peak to get RMS value of velocity converting this to displacement and plotting against floor level (fig. 8). This gives a measure of the RMS displacement of the vibration on each floor averaged over the length of the recording. In this instance, because of the strong single frequency showing in this recording, the same mode shape could also be drawn visually by taking instantaneous values of the largest peaks on the paper records (fig. 4). The shape of the curve relates to the changes in cross-section of the columns, which occur basically at the 4th and 14th floors. This leads to a change in column stiffness resulting in more motion as the building gets less stiff toward the upper floors.

## Theoretical Calculations of the Natural Periods Based on Some Simplifying Assumptions

The calculation of the natural periods of vibration of this building is a complicated problem because it combines two distinct types of construction: a reinforced concrete core incorporated within a steel-frame structure. The periods of vibration, based on some simplified assumptions, were obtained for two conditions. First it was assumed that the lateral resistance was provided by the framing system only. The second assumption was that the frame and core system performed in an integral manner.

For the first set of calculations the following assumptions were also made. The structure was considered to perform as a shear-type structure; this means that the horizontal members of the structure are considered to be much stiffer than the vertical columns. Each floor was considered to move as a rigid body and the mass of each floor was the same. For this set of assumptions it is easy to write down the stiffness matrix of the structure. The process can be followed by referring to fig. 9 which shows the forces, $P$, that must be applied to displace one of the floors a distance $\Delta$ with respect to the other floors. In the expressions for the forces, $E$ is Young's modulus and $I$ is the second moment of area of the columns about a horizontal axis perpendicular to the direction of the displacement.

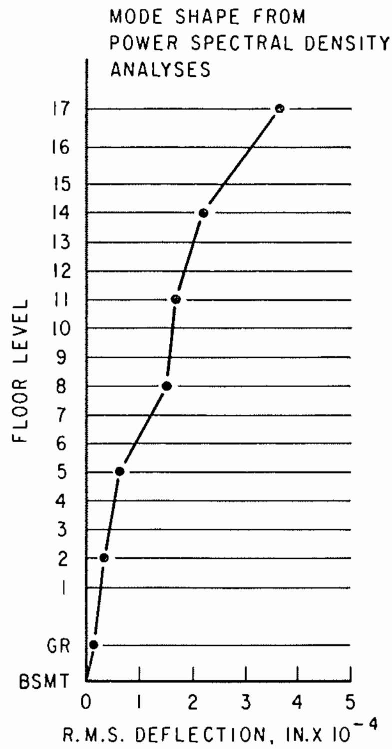


FIG. 8. Mode shape for the first mode parallel to the short dimension of the building.

In a shear-type structure the effect of displacement of any one floor is felt only one floor above and one floor below. Thus, if during free vibration the floors shown in fig. 9 have displacements $x_1$, $x_0$ and $x_2$ respectively, the equation of motion for the middle floor will be

$$\frac{W_0}{g}\frac{d^2x_0}{dt^2} = \frac{P_1}{\Delta}x_1 - \frac{P_0}{\Delta}x_0 + \frac{P_2}{\Delta}x_2 . \tag{1}$$

In eq. (1) $W_0$ is the weight of the floor, $g$ is the acceleration due to gravity, and $t$ represents time. For free vibrations the displacements are defined by

$$x = A \sin wt, \tag{2}$$

where $A$ is the amplitude of vibration and $w$ is the angular frequency of vibration. If eq. (2) is substituted in (1), the following system of equations is obtained:

$$\left[\frac{W}{g} w^2\right][X] = [P][X] \tag{3}$$

In eq. (3), $[X]$, is a column matrix, $[P]$ is a tridiagonal matrix and $[(W/g)w^2]$ is a diagonal matrix. If there is to be other than a trivial answer, $[X] = [O]$; then the eigenvalues $w^2$ are defined by the following equation,

$$\left\{[P] - \left[\frac{W}{g} w^2\right]\right\} = 0. \tag{4}$$

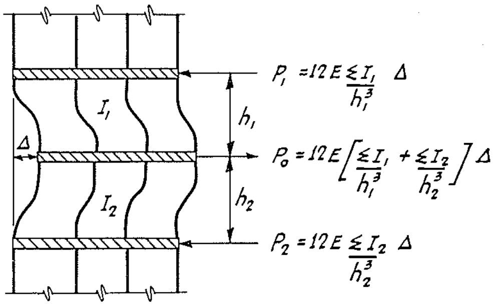


FIG. 9. Calculation of stiffness matrix for frame action only.

These eigenvalues for lateral motion about the two principal axes of the building were obtained by digital computation. The results are shown in table II.

For the second set of calculations the action of the reinforced concrete core was included. Again certain assumptions were made that simplified the approach. The same assumptions were made regarding the frame action as were made in the first set of calculations. It was also assumed that the core deformed in shear only and that the lateral deformation of the core was equal to that of the frame. The stiffness of the core in this case is directly proportional to the plan area of the core. Under these idealized assumptions values of the stiffness matrix can be obtained from the expressions shown in figure 10. In these expressions $E_s$ is Young's modulus of steel, $G_c$ is the shear modulus of concrete, and $A$ is the cross-sectional area of the core. The value of $A$ was assumed to be constant throughout the height of the building.

The natural periods of vibration for this second system are found from the solution of eq. (4) with the appropriate values of $P$ found by using the expressions in figure 10. The periods for lateral motion about the two principal axes of the building are shown in table III.

## CALCULATED AND EXPERIMENTAL RESULTS

Comparison of the calculated values and experimental results in tables I to III show certain features about which a few remarks may be made.

*Frame Action Only*

The closest agreement with the experimental work is in the fundamental mode parallel to the short dimension of the building. The calculated value for this period is $T_c = 1.39$ sec which compares favourably with the observed period $T_o = 1.28$ sec. Parallel to the long dimension of the building, $T_c = 1.40$ sec compared with the observed period $T_o = 0.99$ sec. The other calculated modes are higher than the corresponding observed modes by a factor of almost 2.

TABLE I

NATURAL PERIODS OF FIRST THREE MODES OF VIBRATION OF THE BUILDING EXCITED BY LIGHT RANDOM WIND EXCITATION

| Mode | Parallel to Short Dimension | | Parallel to Long Dimension | | Rotational | |
|---|---|---|---|---|---|---|
| | Period, sec | Frequency, cps | Period, sec | Frequency, cps | Period, sec | Frequency, cps |
| 1st | 1.28 | 0.78 | 0.99 | 1.01 | 0.89 | 1.12 |
| 2nd | 0.30 | 3.32 | 0.26 | 3.79 | 0.28 | 3.51 |
| 3rd | 0.20 | 5.11 | 0.19 | 5.23 | 0.18 | 5.55 |

TABLE II

EIGENVALUES FOR THE LATERAL MOTION ABOUT THE LONG AND SHORT AXES FOR FRAME ACTION ONLY

| Mode | Parallel to Short Dimension | | Parallel to Long Dimension | |
|---|---|---|---|---|
| | Period sec | Frequency cps | Period sec | Frequency cps |
| 1st | 1.39 | 0.72 | 1.40 | 0.71 |
| 2nd | 0.65 | 1.54 | 0.60 | 1.68 |
| 3rd | 0.40 | 2.49 | 0.36 | 2.78 |
| 19th | 0.05 | 22.2 | 0.04 | 24.4 |

*Frame and Core Combined*

The discrepancies for this case compared with the observed values are larger than for the frame action alone, a factor of almost 3 being involved. The frame model has periods that are too high and the frame and core model has periods too low compared to the observed values. In addition, the ratios of the first three modes of vibration are not in good agreement with the observed ratios.

A number of hypotheses can be conjectured to explain the discrepancies at least to some degree. The window sections on each floor forming the outside walls have been considered as contributing to the mass of each floor and not to the shear stiffness. If the walls do contribute to the shear stiffness then, according to their geometric shapes, the building would be stiffer in the long dimension than in the short dimension. This would lead to the ratios of the calculated modes approaching those of the observed.

The computations done on the core indicate that its stiffness is great compared with the stiffness of the frame structure so that the stiffness of the combined frame and core structure is mostly attributed to the core. This leads to the low values of the periods of free vibration in table III. The beams in the calculations have been assumed to be infinitely stiff. If they are not then some flexure will occur. In the light of these observations it appears that the core does not deform integrally with the structure of the column and floor slabs. No measurements were taken simultaneously in the central core and on the column and floor slabs structure to confirm this hypothesis.

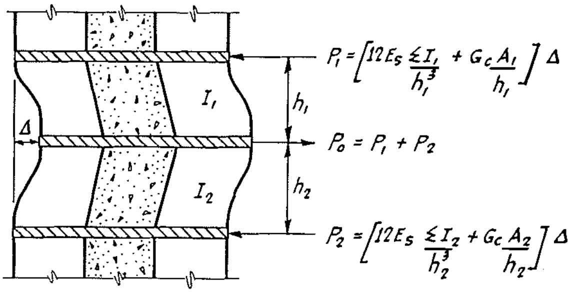


FIG. 10. Calculation of stiffness matrix for frame and core combined.

TABLE III

EIGENVALUES FOR THE LATERAL MOTION ABOUT THE LONG AND SHORT AXES FOR FRAME AND CORE COMBINED

| Mode | Parallel to Short Dimension | | Parallel to Long Dimension | |
|---|---|---|---|---|
| | Period sec | Frequency cps | Period sec | Frequency cps |
| 1st | 0.48 | 2.09 | 0.47 | 2.12 |
| 2nd | .162 | 6.2 | 0.160 | 6.2 |
| 3rd | .097 | 10.3 | 0.097 | 10.3 |
| 19th | .018 | 56.0 | 0.018 | 56.5 |

It is interesting to note that the empirical formula employed by the U. S. Uniform Building Code (1) relates the fundamental period of vibration of a rectangular building to its height and breadth:

$$T = \frac{0.05H}{\sqrt{D}}$$

where

$T$ = fundamental period, sec
$H$ = height of building, ft
$D$ = the dimension of building, ft, in a direction parallel to the applied forces.

Substituting the values for the National Health and Welfare Building in this formula gives periods of 1.25 sec parallel to the short dimension and 0.99 sec parallel to the long dimensions. These values are within 3 % of the observed values.

The results of earlier workers (2) in the field were limited by the difficulty of visual analysis of the complex vibration waveforms obtained from wind excited structures. With the advent of modern recording and analysis equipment the state of the art has been greatly advanced. The method reported makes it possible to use random wind excitation for the determination of the natural periods of vibration of buildings in the major translational and torsional axes up to the 3rd mode. The same power spectral density curves used for the period determinations may be used to obtain mode shapes showing the deformation of the building during wind excitation.

One observation that is well established is that strong winds tend to excite mainly the fundamental modes, whereas lighter winds also excite the higher modes. This is not inconsistent with what is known about the properties of wind gusts (3). Generally, as the mean wind velocity increases, the broad spectral peak of wind energy tends to move from very long periods towards shorter periods. For light winds the various natural periods of the building lie on a flat part of the spectral energy curve, resulting in similar excitation of many modes. As the winds increase in strength the broad spectral peak moves upward toward the fundamental natural periods of the building, which consequently are excited more than the higher modes. The sensitivity of the Willmore seismometer is high enough that the small motions of the building in light winds can readily be recorded and the higher modes determined.

Now that the method has been established a survey of the natural periods of vibration of different types of buildings is proceeding concurrently with a program of model and mathematical studies.

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## References

1. Uniform Building Code. 1961 Edition, Volume I, Sec. 2313.
2. U. S. Coast and Geodetic Survey. *Earthquake investigations in California*, U. S. Department of Commerce, Coast and Geodetic Survey, Special Publication No. 201, Washington, D. C., 1936.
3. Hans A. Panofsky and Robert A. McCormick, "Properties of spectra of atmospheric turbulence at 100 metres," *Quarterly Journal of Royal Met. Soc.*, Vol. 80, p. 546, 1954.

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