DETERMINATION OF THE PITCH-ANGLE DISTRIBUTION AND TRANSVERSE ANISOTROPY OF INTERPLANETARY PARTICLES

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1. INTRODUCTION We present a method to determine the directional differential intensity (d.d.i.), expressed in terms of spherical harmonics, from sectored particle data, concurrent interplanetary magnetic field (IMF) and solar-wind velocity. In Section 2, we show the relation between the d.d.i. and the mean sector count rates X_i. In Section 3, we show how to estimate the d.d.i. from the measured X_i and the associated errors due to Poisson statistics. In Section 4, using the above method, we determine the pitch-angle distribution and the transverse anisotropy of the d.d.i of low energy protons for the Day 118, 1978 solar particle event. In Section 5, we discuss an interesting correlation between the transverse anisotropy and the IMF direction.

2. RELATION BETWEEN DIRECTIONAL INTENSITY AND SECTOR RATES

We express the particle directional differential intensity as

$$j(p,\theta,\emptyset) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} P_n^m(\cos\theta)\left\{ A_{nm}(p)\cos m\emptyset + B_{nm}(p)\sin m\emptyset\right\}, \quad (1)$$

where P_n^m = the associated Legendre functions, p = particle momentum, θ = pitch-angle and $\emptyset$ = gyrophase in the standard coordinate system (Ng, 1985). When the telescope points in the direction (γ,η), it measures the differential count rates

$$C(p,\gamma,\eta) = \int_{\Omega} j(p,\theta,\emptyset)\, S[\theta'(\gamma,\eta,\theta,\emptyset)]\, \sin\theta\, d\theta\, d\emptyset, \quad (2)$$

where, following Sentman and Baker (unpublished manuscript), we express the angular response function of the particle telescope as

$$S(\theta') = \sum_{k=0}^{\infty} S_k P_k(\cos\theta') = \sum_{k=0}^{\infty} S_k \Big\{ P_k(\cos\gamma)P_k(\cos\theta) + $$
$$+ 2\sum_{m=1}^{k}[(k-m)!/(k+m)!]\, P_k^m(\cos\gamma)P_k^m(\cos\theta)\cos m(\eta-\emptyset)\Big\}. \quad (3)$$

It follows from the orthogonality of the spherical harmonics that

$$C(p,\gamma,\eta) = 4\pi\sum_{n=0}^{\infty}[S_n/(2n+1)]\sum_{m=0}^{n}\left\{A_{nm}(p)\cos m\eta + B_{nm}(p)\sin m\eta\right\} P_n^m(\cos\gamma). \quad (4)$$

As the telescope sweeps over sector i, we average (4) to obtain $X_i(p)$ the mean differential count rate over sector i,

$$X_i(p) = 4\pi\sum_{n=0}^{\infty}[S_n/(2n+1)]\sum_{m=0}^{n}\Big\{ A_{nm}(p)\langle P_n^m(\cos\gamma)\cos m\eta\rangle_i + $$
$$+ B_{nm}(p)\langle P_n^m(\cos\gamma)\sin m\eta\rangle_i\Big\}, \quad (5)$$

where $\langle\ \rangle_i = (1/\Delta\psi)\int_{\psi_i}^{\psi_{i+1}} d\psi$, ψ = the spacecraft (s/c) spin-angle measured from the projection of the IMF onto the spin plane, and (ψ_i,ψ_{i+1}) defines sector i (see the 2nd coordinate system in Fig. 1). For multiple-telescope systems (Sanderson & Hynds, 1977), eqn (5) should be repeated for each telescope.

We now illustrate by specialising (1) and (5) to 8-sectored data for a telescope sweeping in the spacecraft's spin plane:

$$j(p,\theta,\phi) = \sum_{n=0}^{4} A_{n0}P_n(\cos\theta) + \sum_{n=1}^{3} A_{n1}P_n^1(\cos\theta)\cos\phi, \tag{6}$$

$$X_i = 4\pi \sum_{n=0}^{4} S_n A_{n0} Q_{n0}^i/(2n+1) + 4\pi \sum_{n=1}^{3} S_n A_{n1} Q_{n1}^i/(2n+1), \tag{7}$$

where
$$Q_{n0}^i = \langle \sum_{j=0}^{n} a_n^j \sin^j\theta_B \cos^j\psi \rangle_i, \tag{8}$$

$$Q_{n1}^i = \langle (\hat{W}_{\perp 1}\cos\psi + \hat{W}_{\perp 2}\sin\psi) \sum_{j=0}^{n} b_n^j \sin^j\theta_B \cos^j\psi \rangle_i, \tag{9}$$

a_n^j = coefficient of x^j in $P_n(x)$, b_n^j = coefficient of x^j in $P_n^1(x)/(1-x^2)^{\frac{1}{2}}$ θ_B = angle between IMF and s/c spin-axis, and $(\hat{W}_{\perp 1}, \hat{W}_{\perp 2}, \hat{W}_{\perp 3})$ denotes a unit vector in the direction of $\underline{E}\times\underline{B}$ in the 2nd coordinate system (Fig. 1). Note that the integrations in (8) and (9) may be performed readily in closed form.

3. DETERMINATION OF j FROM MEASURED X_i To simplify notation in the following, let

$$D_n = A_{n0} \quad (n=0,4), \qquad D_n = A_{n-4,1} \quad (n=5,7), \tag{10}$$

$$R_n^i = Q_{n0}^i \quad (n=0,4), \qquad R_n^i = Q_{n-4,1}^i \quad (n=5,7). \tag{11}$$

We least-square fit
$$X_i = \sum_{n=0}^{7} C_n R_n^i, \quad (i=0,7), \tag{12}$$

to the 8 measured sector rates $\overline{X}_i$. This yields

$$\sum_{j=0}^{7} H_{nj}C_j = \sum_{i=0}^{7} R_n^i \overline{X}_i, \qquad (n=0,7), \tag{13}$$

where
$$H_{nj} = \sum_i R_n^i R_j^i, \qquad (n=0,7;\ j=0,7). \tag{14}$$

Inverting (13), we have
$$C_n = \sum_k H_{nk}^{-1} \sum_i R_k^i \overline{X}_i, \qquad (n=0,7), \tag{15}$$

and thence
$$D_n = \sum_i M_{ni}\overline{X}_i, \qquad (n=0,7), \tag{16}$$

where M_{ni} is ultimately expressed in terms of Q_{n0}^i, Q_{n1}^i, and S_n. Using (16), we may determine the coefficients A_{nm} in (6) by a matrix multiplication into the measured sector rates. When the IMF projection lies on a sector boundary, the symmetric matrix H_{nj} becomes singular. So we drop the A_{40} term in (6) whenever the IMF projection comes within 2° of a sector boundary.

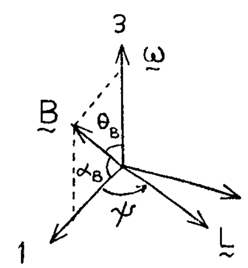

Fig. 1

Suppose that K_i counts are registered over the time interval t_c in sector i. Assuming Poisson statistics, we estimate

$$X_i = K_i/t_c, \quad \mathrm{var}(X_i) = K_i/t_c^2, \quad \mathrm{var}(D_n) = \sum_i M_{ni}^2 \mathrm{var}(X_i). \tag{17}$$

If we define the anisotropy $\xi_n = D_n/D_0$, then, providing the counts are not too low,

$$\mathrm{var}(\xi_n) \simeq (1/\overline{D}_0^2) \sum_i (M_{ni} - \overline{\xi}_n M_{0i})^2 \mathrm{var}(X_i). \tag{18}$$

Systematic errors are, of course, much harder to estimate.

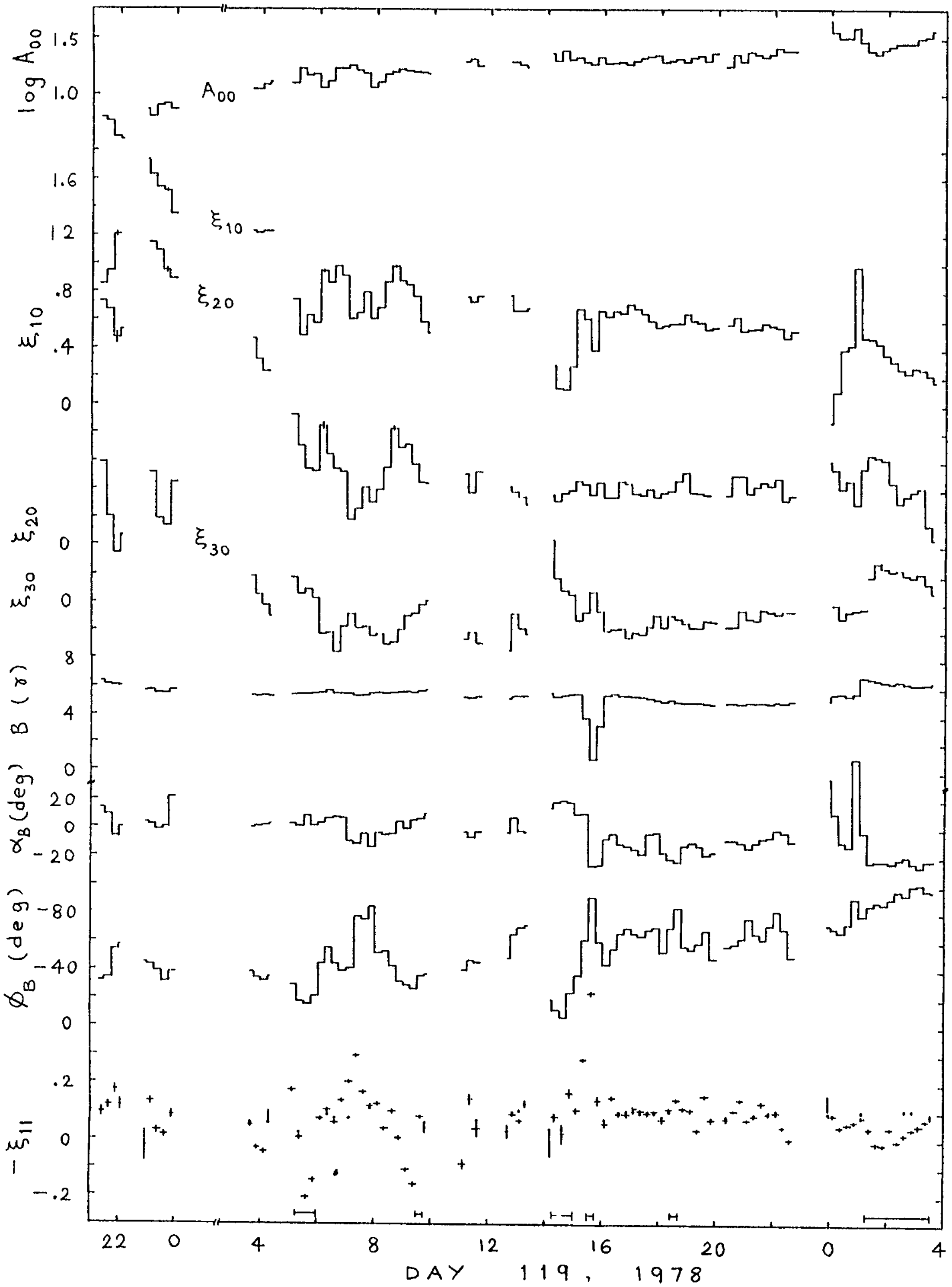

Fig. 2: 15-minute intensity and anisotropies of 1.4-2.5 MeV protons

4. APPLICATION As an example, we show in Fig. 2 the 15-min averages of A_{00}, ξ_{10}, ξ_{20}, ξ_{30}, and $-\xi_{11}$ for 1.4 - 2.5 MeV protons, determined with the above method using 8-sectored particle data (P.I.: R.E. Vogt, CalTech), concurrent IMF (P.I.: N.F. Ness, GSFC), and hourly solar wind speed (P.I.: H. Bridge, MIT), measured aboard IMP-8 and accessed through NSSDC. Some typical standard errors due to Poisson statistics only are indicated by vertical bars.

Estimating the spectral slopes by using the corresponding results for 4-13 MeV protons, we have found the Compton-Getting corrections (Ng, 1985) for transformation to the $\underline{E}\times\underline{B}$ drift frame to be small, $<.002\ A_{oo}$, $<.01$, $<.01$, $<.02$ for A_{oo}, ξ_{10}, ξ_{20}, and ξ_{30} respectively. (For transformation to the solar wind frame, the corrections are of the order of $0.04\ A_{oo}$, 0.1, 0.3, and 0.5 respectively). Thus A_{oo}, ξ_{10}, ξ_{20}, ξ_{30} essentially characterise the pitch-angle distribution in the $\underline{E}\times\underline{B}$ drift frame.

5. DISCUSSION The Compton-Getting correction, $\varepsilon\hat{W}_{\perp}(3 - pA_{oo}'/A_{oo} + pA_{20}'/5A_{oo})$ to the transverse anisotropy $-\xi_{11}$ is indicated by the dots in Fig. 2. The observed anisotropy varies in phase with this correction but is much larger in magnitude. The same feature, even more marked, is seen for 4-13 MeV protons. What is the cause of this large discrepancy?

For Fig. 2, IMP-8's GSE coordinates in R_E varies from (21.6, 21.5, 5.5) to (5.7, 29.5, 18.1). The times when the IMF is connected to the nominal bow shock (BS) are indicated by horizontal bars in Fig. 2 (Ng & Roelof, 1977). At ~1537 UT Day 119, some solar particles with guiding centres below the IMF through IMP-8 are probably shadowed by the nose of the BS, resulting in the observed peak value of $-\xi_{11} = 0.53$. However BS connection does not account for the general variation of $-\xi_{11}$ in Fig. 2.

Close inspection reveals that the sector plot of X_1 lags behind the IMF in directional changes. Hence a tentative explanation is that some observed 15-min averages contain a substantial fraction of non-gyrotropic distributions which reside a short distance (~1 gyroradius) beyond a 'kink' in the IMF. An alternative explanation is as follows. When Φ_B swings rapidly in an averaging interval such that its average value is close to one end of the range of values, then a field-aligned anisoptropy ξ_{10} would "induce" a nonzero value of $-\xi_{11}$, i.e., the apparent value of $-\xi_{11}$ is not real. Further studies with smaller averaging intervals would clarify this matter.

6. CONCLUSION We have shown how to obtain the directional differential intensity referred to the standard coordinate system (Ng, 1985) from sectored particle data and concurrent IMF and solar wind data. The corrections for transformation to the $\underline{E}\times\underline{B}$ drift frame are explicitly calculated and found to be small for these ~1.5 MeV protons. However, the new correction formulae would be important for $\lesssim$ 500 KeV protons. It is tentatively suggested that the 'observed' transverse anisotropy may in large part be induced by a rapidly changing IMF in the presence of a field-aligned anisotropy.

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