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Determination of three-dimensional velocity structure from observations of refracted body waves

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Summary. The computational effectiveness of travel-time inversion methods depends on the parameterization of a 3-D velocity structure. We divide a region of interest into a few layers and represent the perturbation of wave slowness in each layer by a series of Chebyshev polynomials. Then a relatively complex velocity structure can be described by a small set of parameters that can be accurately evaluated by a linearized inversion of travel-time residuals. This method has been applied to artificial and real data at small epicentral distances and in the teleseismic distance range. The corresponding matrix equations were solved using singular value decomposition. The results suggest that the method combines resolution with computational convenience.

1 Introduction

The problem of the determination of the laterally variable earth's medium structure from seismic observations has attracted much attention in recent years. Lateral velocity inhomogeneities manifest themselves in travel-time residuals determined with respect to travel times corresponding to a laterally homogeneous reference velocity model. If the residuals are small enough the inverse problem is a linear one and for its solution several approaches are known.

The idea of the approach suggested by Aki & Lee (1976) and by Aki, Christoffersson & Husebye (1976) is to divide the region of interest into many blocks and to perturb velocities in the blocks in accordance with the travel-time residuals. Unfortunately, the corresponding system of linear equations sometimes becomes too large to be handled by a computer with a moderate memory capacity. On the other hand, however large the number of blocks the resulting structure frequently consists of only a few high- and low-velocity regions.

An alternative approach would be to represent the 3-D velocity perturbation by a suitable function of a relatively small number of parameters and to evaluate them from the travel-time residuals. This approach was pioneered by Alekseyev *et al.* (1970) and was further developed by Firbas (1981). Our inversion procedure of travel-time data which is described in Section 2 is based on the same idea. We found, however, that sometimes the procedure was lacking in spatial resolution. In such cases, a hybrid method of dividing the region into a few large blocks and representing the velocity perturbation within each block by its own function provides a desirable compromise between the resolution and computational convenience. The procedure was applied with some success to artificial and real data in the range of small epicentral distances (Section 3) and in the teleseismic distance range (Section 4).

2 Method of travel-time inversion

We consider a region G in the half-space containing the family of rays R_i, $i = 1, 2, \ldots, I$, of refracted body waves. The coordinates origin is on the free surface while axis Z is directed downward. The travel-time t_i of wave propagation along the raypath R_i from the source to the receiver is given by

$$t_i = \int_{R_i} S(x, y, z)\, dr$$

where $S(x, y, z)$ is the wave slowness. On the assumption of weak lateral heterogeneity, we may write

$$S(x, y, z) = S_0(z) + S_1(x, y, z)$$

where S_0 is the unperturbed wave slowness and S_1 is the perturbation which depends on the perturbation of the velocity δV and the unperturbed velocity V_0 as

$$S_1 \approx -\delta V / V_0^2.$$

Then, the travel-time residual δt_i can be expressed as

$$\delta t_i = t_i - t_{0i} \approx \int_{R_{0i}} S_1(x, y, z)\, dr \tag{1}$$

where R_{0i} and t_{0i} are the raypath and the travel-time respectively corresponding to the reference velocity $V_0(z)$.

In order to reduce the number of parameters needed to describe $S_1(x, y, z)$ we can represent it locally by means of polynomials. A representation in terms of cubic splines has been used previously by Hovland, Gubbins & Husebye (1981) and Thomson & Gubbins (1982), but here we make use of an expansion in terms of orthogonal polynomials. We expand $S_1(x, y, z)$ into a three-fold series of orthogonal Chebyshev polynomials $T(u_1)$, $T(u_2)$, $T(u_3)$ having mapped the original region G into a rectangular region in the **u** coordinates, such that the values of u_1, u_2, u_3 are normalized to lie in the range $-1 \leqslant u \leqslant 1$. The number of terms in the Chebyshev series K, L, M for the three coordinates can be taken to be different and so the polynomial representation of S_1 takes the form

$$S_1[x(u), y(u), z(u)] \approx \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} P_{klm} T_k(u_1)\, T_l(u_2)\, T_m(u_3) \tag{2}$$

where P_{klm} is the required unknown parameter to be found from the observations. In order to improve resolution we may further divide the region G into a few separate layers or blocks and represent S_1 within each block by its own series of polynomials. Then equation (2) should be rewritten as

$$S_1[x(u), y(u), z(u)] \approx \sum_{j=1}^{J} \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \sum_{m=0}^{M-1} P_{klm}^j T_k^j(u_1)\, T_l^j(u_2)\, T_m^j(u_3) \tag{3}$$

where j denotes the number of a block.

Putting equations (2) or (3) in (1) we get a set of equations which can be written in the form

$$AX \approx C \tag{4}$$

where A is a rectangular matrix with I rows and $N = J \cdot K \cdot L \cdot M$ columns, C is the vector of travel-time residuals and X is the vector of unknown elements. The solution of equation (4) can be found by the method of singular value decomposition (Forsythe, Malcolm & Moler 1977). Such solution provides a minimum length of norm of $(AX\text{-}C)$ in the least squares sense while at the same time it is relatively insensitive to the errors in the input data.

A singular value decomposition of the matrix A is given by

$$A = U \cdot D \cdot W^{\mathrm{T}}$$

where U is an $I \times I$ orthogonal matrix, W is an $N \times N$ orthogonal matrix and D is an $I \times N$ diagonal matrix. The diagonal elements of the matrix D are called the singular values of A. The condition number of the matrix A is

$$\mathrm{cond}\,(A) = \sigma_{\max}/\sigma_{\min}$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the largest and smallest singular values respectively. The matrices U and W are used to transform (4) into an equivalent diagonal set of equations

$$D\bar{X} \approx \bar{C}.$$

The unknown elements $\bar{x}_i$ of the vector $\bar{X}$ can be expressed as

$$\bar{x}_i = c_i/\sigma_i \qquad i = 1, \ldots, N.$$

We assume $\bar{x}_i = 0$ if $\sigma_i \leqslant \tau$. The threshold τ is defined as

$$\tau = \epsilon\sigma_{\max}$$

where ϵ is the relative error in the input data. Introducing the threshold results in a more reliable determination of X which can be found from $X = W\bar{X}$.

3 Application to the data at small epicentral distances

To test the procedure of Section 2 we generated an artificial set of travel times for a rectangular profile grid in an area of $0 \leqslant x \leqslant 400$ km, $0 \leqslant y \leqslant 400$ km. The assumed spacing between the receivers in the same profile and the distance between the neighbouring profiles were 25 and 75 km respectively. In each profile, there were two transmitters separated by 200 km. The assumed 3-D velocity distribution $V(x, y, z)$ was given by

$$V(x, y, z) = 7.0 - 10^{-5}(x - 200)^2 - 4 \times 10^{-5}(y - 200)^2 + 2 \times 10^{-3}z^2. \tag{5}$$

A random noise with the rms value of 0.1 s was added to the precise travel-time values. $V_0(z)$ was taken as

$$V_0(z) = 5.9 + 0.02z.$$

The region of interest was treated as a single block, the number of rays was 80 while the number of unknown elements was 27 ($K = L = M = 3$). The velocity cross-sections at 5 km depth given by (5) and that found by the travel-time inversion are reasonably close (Fig. 1). Similar agreement was found in the whole depth range of this experiment.

The procedure was applied to real travel-time data as well. Figs 2 and 3 show travel-time data obtained on a profile in Bulgaria and the results of their inversion. The numbers of rays and of unknown parameters were 274 and 36 respectively. Some details of the 2-D velocity structure thus obtained depend strongly on the assumed value of ϵ. If this value

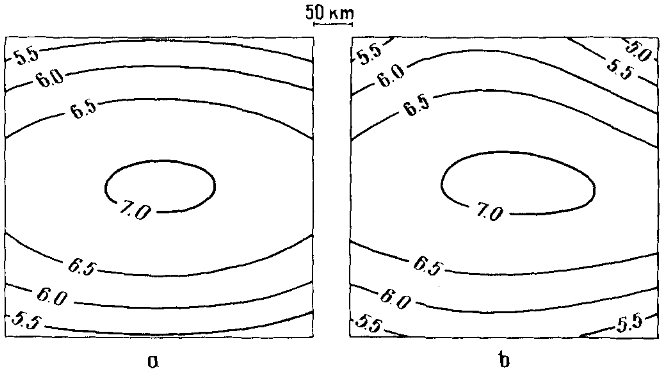

Figure 1. Comparison between the cross-section at 5 km depth of the model velocity structure given by equation (5) (a) and that found by inversion of travel times (b).

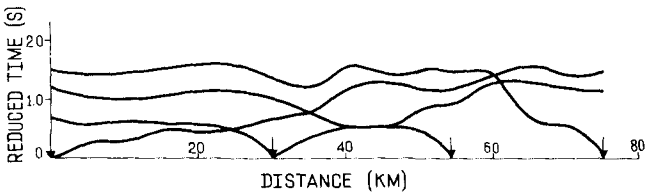

Figure 2. Reduced (t-distance/6) travel-time curves on a profile in Bulgaria (adopted from Dachev *et al.* 1977). Locations of shot points are shown by arrows.

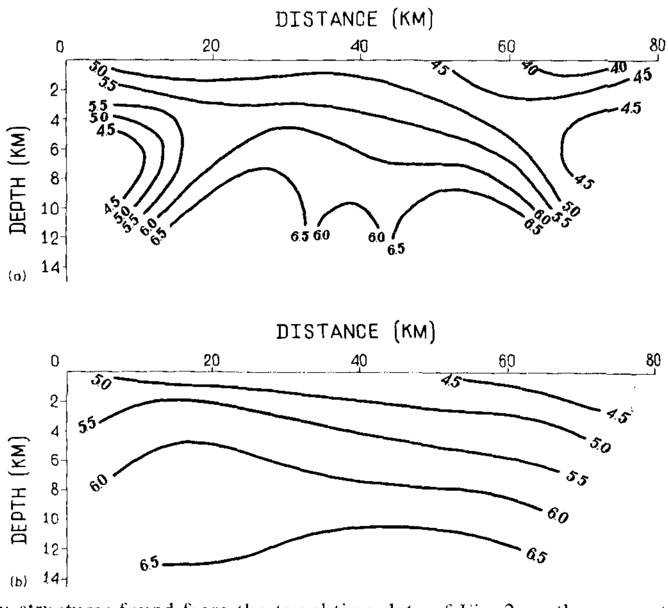

Figure 3. Velocity structures found from the travel-time data of Fig. 2 on the assumptions that $\epsilon = 10^{-3}$ (a) and $\epsilon = 2 \times 10^{-2}$ (b).

is less than 10^{-3} the structure looks extremely complex especially in its lower part (Fig. 3a). Such accuracy, however, is not warranted by the data. First, there are errors in the measured travel times. Second, in the Earth's medium, there are small-scale inhomogeneities which were neglected in the model. And third, the basic equation of the method is approximate.

Under the reasonable assumption of $\epsilon = 2 \times 10^{-2}$, the structure becomes relatively simple (Fig. 3b) and probably more reliable.

According to geological observations, the uppermost layer of the crust in the northern part of the profile (right side of Fig. 3) is composed of unconsolidated sediments, while in the southern part relatively high-velocity limestones come close to the surface. Thus, the structure shown in Fig. 3(b) is in qualitative agreement with the independent data.

A similar geological trend is known to prevail in the area shown in Fig. 4 (the profile whose data are shown in Fig. 2 is approximately parallel to profile IV in Fig. 4 and is located to the east from profile IV). The travel-time data from the network of profiles shown in Fig. 4 were used to infer 3-D velocity structure (the numbers of rays and unknown parameters were 600 and 64 respectively). The principal features of this velocity structure (Fig. 4) are again in agreement with the geological data.

4 Application to teleseismic data

The idea of inversion of teleseismic data is similar to that used by Aki *et al.* We assume that the travel-time residuals observed on a seismograph array are generated in a relatively thin layer below the array. The inversion procedure was tested by applying it to artificial data. The numerical modelling was performed by inserting three spheres of 20 km diameter with a velocity of 8.8 km s^{-1} into a homogeneous medium with a velocity of 8.0 km s^{-1}. The centres of the spheres were at depths 30, 50 and 70 km, the system of observations consisted of 15 receivers distributed in the area 80 x 80 km with 20 km spacing, the number of rays considered was 180. A random noise with an rms value of 0.05 s was added to the precisely determined travel times. The inversion was carried out by dividing the region between the upper boundary and the 80 km depth into four layers of equal thickness and using equation (3). The number of unknown parameters was 48.

In Fig. 5, the positions of the spheres in the model as well as the results of the inversion are presented. A comparison shows that the inhomogeneities of the model are well displayed in the computed velocity structure although the actual anomalies are somewhat smoothed. Inaccuracies of this kind were observed in the experiments with Aki *et al.*'s inversion procedure (Neuman 1981), as well. The anomalies on the edge of the region (Fig. 5b) have no counterparts in the model (Fig. 5a) but they could be eliminated by increasing the number of terms in the polynomial expansion of S_1. It should be noted that the results

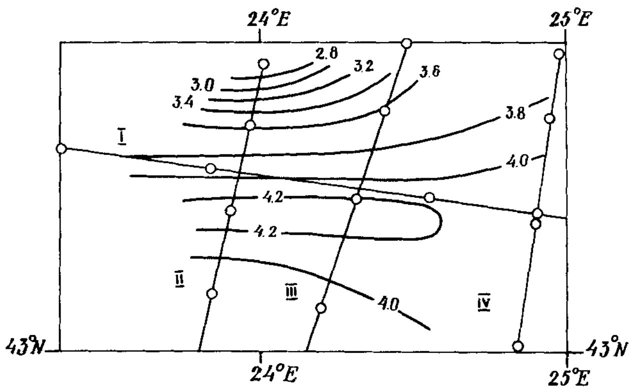

Figure 4. Scheme of seismic refraction profiling (adopted from Dachev *et al.* 1977) and the cross-section at 1 km depth of the 3-D velocity structure.

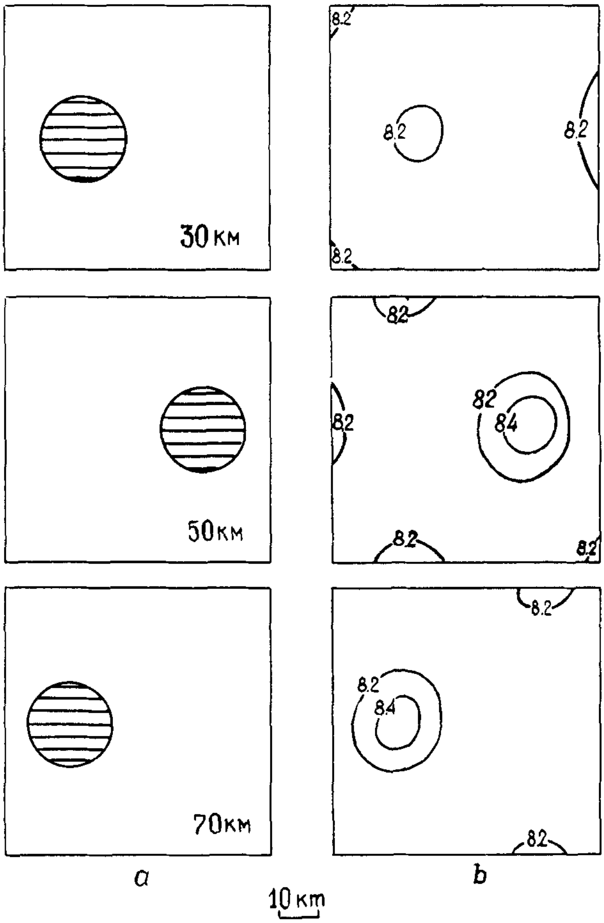

Figure 5. Comparison between model structures at depths 30, 50 and 70 km (a) and those obtained from the teleseismic travel-time inversion (b). Velocities in (b) are given in km s^{-1}.

shown in Fig. 5 are much more satisfactory than those obtained with a representation of the anomalous structure by a single polynomial series.

The method was also applied to the travel-time residuals of NORSAR. The location and configuration of this array were described elsewhere. From a large set of seismic events whose *P*-wave residuals were reported by Berteussen (1974), we selected 30 with strongly different azimuths and/or the angles of incidence of the *P*-wave rays. The region under the array between the free surface and 120 km depth was divided into three layers of equal thickness. The number of unknown parameters in each layer was nine. In the initial model, the velocity was 6.5 km s^{-1} in the upper layer and 8.2 km s^{-1} in the lower layers. We will discuss the computed velocity structure of the upper layer since it can be compared with the results of some other crustal studies (Fig. 6).

According to seismic refraction measurements (Kanestrøm 1973), the crust's thickness is relatively small in the centre of the region but it increases by several kilometres near the outer ring of the array (Fig. 6a). A similar tendency is evident in the data obtained by fitting synthetic long-period body waveforms to those recorded at NORSAR (Vinnik & Kosarev 1981, fig. 6b). Our data (Fig. 6c) indicating relatively high velocities under the central part of the array are compatible with the results of both of these studies.

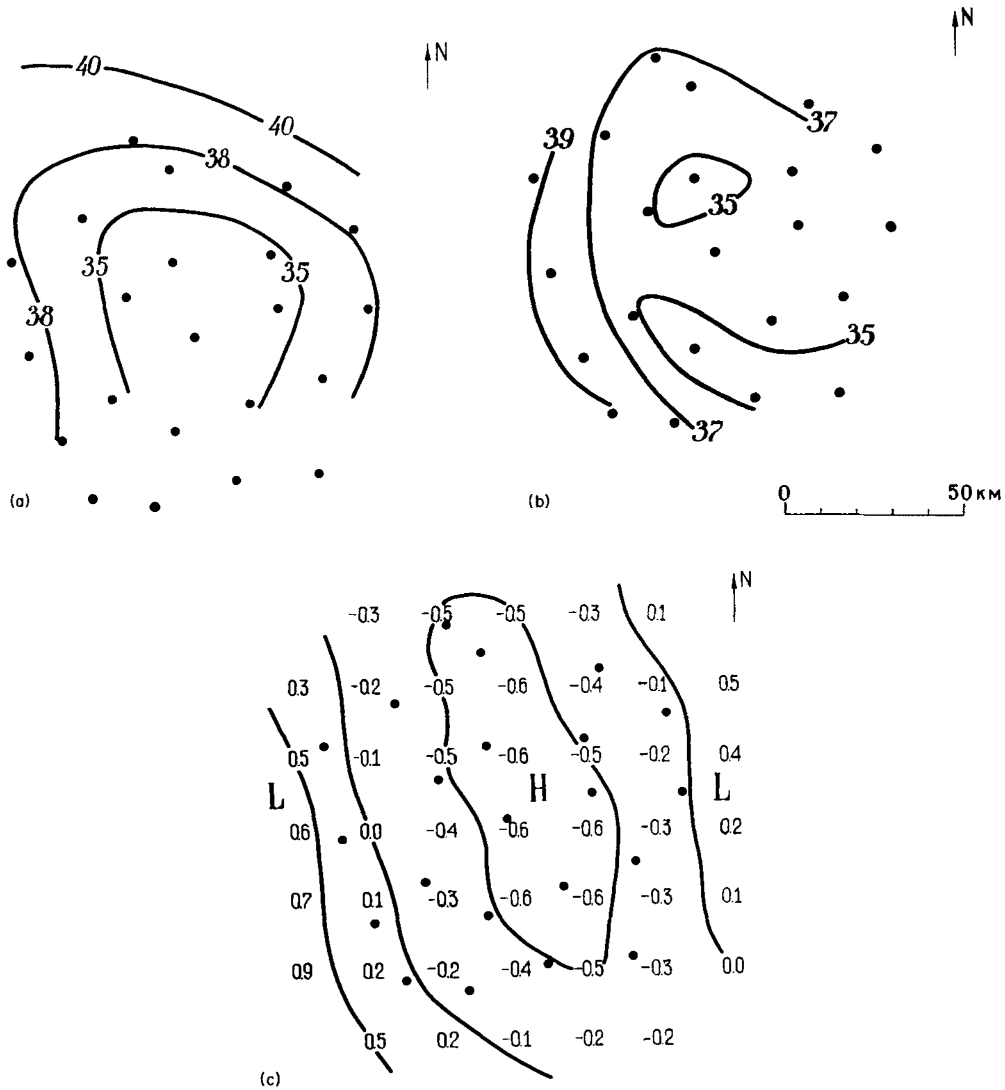

Figure 6. Comparison between the Moho depths (in kilometres) under NORSAR according to Kanestrøm (1973) (a), Vinnik & Kosarev (1981) (b) and the velocity structure of the upper 40 km layer resulting from the travel-time inversion (c). Receivers of the array are shown by filled circles, velocity anomalies in (c) are given as a percentage of the initial velocity.

5 Conclusion

We have described the inversion procedure of travel-time data whose essence is a special kind of parameterization of the medium under study. We divide the region of interest into a few layers (blocks) and represent the wave slowness perturbation in each block by a three-fold series of Chebyshev polynomials. Then, as is demonstrated by numerical experiments, many geological structures of interest can be well described by a relatively small number of parameters which can be accurately determined by a linearized inversion of travel-time residuals. Although this method is still in an early stage of development the results of application suggest that it combines reasonable resolution with computational efficiency.

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