

DETERMINATION OF $|V_{us}|$ FROM HADRONIC τ DECAYS

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The recent update of the strange spectral function and the moments of the invariant mass distribution by the OPAL collaboration from hadronic τ decay data are employed to determine $|V_{us}|$ as well as m_s . Our result, $|V_{us}| = 0.2208 \pm 0.0034$, is competitive to the standard extraction of $|V_{us}|$ from K_{e3} decays and to the new proposals to determine it. Furthermore, the error associated to our determination of $|V_{us}|$ can be reduced in the future since it is dominated by the experimental uncertainty that will be eventually much improved by the B-factories hadronic τ data. Another improvement that can be performed is the simultaneous fit of both $|V_{us}|$ and m_s to a set of moments of the hadronic τ decays invariant mass distribution, which will provide even a more accurate determination of both parameters.

1 Introduction

Already in the past, hadronic τ decays have served as an interesting source to study low energy QCD under rather clean conditions¹ and obtain information on parameters of the Standard Model, like the strong coupling^{2,3}, the strange quark mass or some non-perturbative condensates. The determination of α_s , for example, has been performed using τ decay data with a precision competitive to the current world average.

At present, the hadronic τ decay width

$$R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \text{hadrons}(\gamma)]}{\Gamma[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma)]} \quad (1)$$

as well as invariant mass distributions, have reached a high precision status thanks to the data of the LEP experiments ALEPH² and OPAL³ at CERN and the CESR experiment CLEO⁴ at Cornell.

In particular, the experimental measurements of the strange spectral function^{5,6,7} have allowed the analysis of the SU(3) breaking corrections in the semi-inclusive τ decay width into

Cabibbo suppressed modes with strange particles, providing a way to compute the strange quark mass $m_s(M_\tau)$ [8,9,10,11,12,13,14,15]. Recently, we have pointed out that this approach to obtain the strange mass from the hadronic τ decays depends sensitively on the modulus of the Cabibbo–Kobayashi–Maskawa matrix element $|V_{us}|$ [14]. It appears then natural to turn things around and, with an input for m_s obtained from other sources, to actually determine $|V_{us}|$. The great advantage of the determination of this CKM matrix element from τ decays in comparison with other calculations is that the experimental uncertainty, which as we will see is the main source of error in the calculation, is expected to be reduced drastically at the present B-factories BABAR and BELLE.

2 Theoretical framework

The main quantity of interest for the following analysis is the hadronic decay rate of the τ lepton defined in (1). The basic objects one needs to perform the QCD analysis of R_τ and related observables are Green's two-point functions for vector $V_{ij}^\mu \equiv \bar{q}_i \gamma^\mu q_j$ and axial-vector $A_{ij}^\mu \equiv \bar{q}_i \gamma^\mu \gamma_5 q_j$ color singlets,

$$\begin{aligned}\Pi_{V,ij}^{\mu\nu}(q) &\equiv i \int d^4x e^{iq \cdot x} \langle 0 | T \left([V_{ij}^\mu]^\dagger(x) V_{ij}^\nu(0) \right) | 0 \rangle, \\ \Pi_{A,ij}^{\mu\nu}(q) &\equiv i \int d^4x e^{iq \cdot x} \langle 0 | T \left([A_{ij}^\mu]^\dagger(x) A_{ij}^\nu(0) \right) | 0 \rangle.\end{aligned}\quad (2)$$

The subscripts i, j denote light quark flavors (up, down and strange). These correlators admit the Lorentz decompositions

$$\Pi_{ij,V/A}^{\mu\nu}(q) = \left(-g_{\mu\nu} q^2 + q^\mu q^\nu \right) \Pi_{ij,V/A}^T(q^2) + q^\mu q^\nu \Pi_{ij,V/A}^L(q^2) \quad (3)$$

where the superscripts in the transverse and longitudinal components denote the spin $J = 1$ (T) and $J = 0$ (L) in the hadronic rest frame. Theoretically, R_τ can be expressed as an integral of the imaginary part of these correlators over the invariant mass $s = p^2$ of the final state hadrons¹

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2} \right)^2 \left[\left(1 + \frac{2s}{M_\tau^2} \right) \text{Im} \Pi^T(s) + \text{Im} \Pi^L(s) \right], \quad (4)$$

where the appropriate combinations of two-point correlation functions are

$$\Pi^J(s) \equiv |V_{ud}|^2 \left[\Pi_{ud}^{V,J}(s) + \Pi_{ud}^{A,J}(s) \right] + |V_{us}|^2 \left[\Pi_{us}^{V,J}(s) + \Pi_{us}^{A,J}(s) \right], \quad (5)$$

with V_{ij} being the corresponding matrix elements of the CKM matrix.

Experimentally, one can disentangle vector from axialvector contributions in the Cabibbo-allowed ($\bar{u}d$) sector, whereas such a separation is problematic in the Cabibbo-suppressed ($\bar{u}s$) sector. We can then decompose R_τ both experimentally and theoretically into

$$R_\tau \equiv R_{\tau,V} + R_{\tau,A} + R_{\tau,S}. \quad (6)$$

Additional information can be inferred from the measured invariant mass distribution of the final state hadrons, through the analysis of the moments

$$R_\tau^{(k,l)} \equiv \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2} \right)^k \left(\frac{s}{M_\tau^2} \right)^l \frac{dR_\tau}{ds}, \quad (7)$$

that can be calculated in analogy to the τ decay rate ($R_\tau = R_\tau^{(0,0)}$) and, in particular, can be also decomposed into Cabibbo-allowed and Cabibbo suppressed contributions.

The theoretical study of R_τ and its moments is based on the Operator Product Expansion (OPE) of the relevant correlators. In this framework, the moments $R_\tau^{(k,l)}$ can be written as

$$R_\tau^{(k,l)} \equiv N_c S_{EW} \left\{ (|V_{ud}|^2 + |V_{us}|^2) \left[1 + \delta^{(k,l)(0)} \right] + \sum_{D \geq 2} \left[|V_{ud}|^2 \delta_{ud}^{(k,l)(D)} + |V_{us}|^2 \delta_{us}^{(k,l)(D)} \right] \right\}. \quad (8)$$

The electroweak radiative correction $S_{EW} = 1.0201 \pm 0.0003$ ¹⁶ has been pulled out explicitly and $\delta^{(k,l)(0)}$ denotes the purely perturbative dimension-zero contribution, that is the same for the Cabibbo-allowed and Cabibbo-suppressed contributions. The symbols $\delta_{ij}^{(k,l)(D)}$ stand for higher dimensional corrections in the OPE from dimension $D \geq 2$ operators which contain implicit $1/M_\tau^D$ suppression factors^{1,9,11,17}. The most important corrections are the dimension $D = 2$ proportional to m_s^2 and the dimension $D = 4$ proportional to $m_s \langle \bar{q}q \rangle$.

The separate measurement of strange and non-strange contributions to the decay width of the τ lepton^{5,6,7} allows one to pin down the flavour SU(3)-breaking effects, dominantly induced by the strange quark mass, through the differences

$$\delta R_\tau^{(k,l)} \equiv \frac{R_{\tau,V+A}^{(k,l)}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{(k,l)}}{|V_{us}|^2} = N_c S_{EW} \sum_{D \geq 2} \left[\delta_{ud}^{(k,l)(D)} - \delta_{us}^{(k,l)(D)} \right]. \quad (9)$$

Many theoretical uncertainties drop out in these observables since they vanish in the SU(3) limit.

3 Calculation of $|V_{us}|$

The large sensitivity of the SU(3) breaking quantities $\delta R_\tau^{(k,l)}$ allows us to obtain a determination of the CKM matrix element $|V_{us}|$ using as input a fixed value of m_s . Since the sensitivity to $|V_{us}|$ is strongest for the moment with $k = l = 0$, where also the theoretical uncertainties are smallest, we used this moment in our calculation. From (9),

$$|V_{us}|^2 = \frac{R_{\tau,S}^{(0,0)}}{\frac{R_{\tau,V+A}^{(0,0)}}{|V_{ud}|^2} - \delta R_{\tau,th}^{(0,0)}}. \quad (10)$$

In the OPE of $\delta R_{\tau,th}^{(0,0)}$ in (9) we include the dimension two corrections $\delta_{ij}^{(k,l)(2)}$ that are known at $\mathcal{O}(a^3)$ for $J = L$ component and at $\mathcal{O}(a^2)$ for $J = L + T$ component –see^{9,11} for references. The $\mathcal{O}(a^3)$ $J = L + T$ are also known¹⁸ but the results obtained in¹⁵ and discussed here do not contain these corrections. We leave a full analysis which take them into account for a future publication¹⁹. Dimension four corrections $\delta_{ij}^{(k,l)(4)}$ are fully included while the dimension six corrections $\delta_{ij}^{(k,l)(6)}$ were estimated to be of the order or smaller than the error of the dimension four¹¹.

An extensive analysis of the perturbative series for the dimension two corrections was done in⁹. The conclusions there were that while the perturbative series for $J = L + T$ converges very well the one for the $J = L$ behaves very badly, adding important uncertainties to the theoretical calculation of $\delta R_\tau^{(k,l)}$. A natural remedy to solve this problem is to replace the QCD expressions of scalar and pseudoscalar correlators by corresponding phenomenological hadronic parametrizations, much more precise than their QCD counterpart¹⁴ due to the fact that it is dominated by far by the well known kaon pole –see¹⁴ for details. The results we obtained within the QCD correlators and within the phenomenological hadronic parametrized correlators are showed in Table 1. The longitudinal contributions calculated with the two different descriptions of the

Table 1: Comparison between the OPE and the phenomenological hadronic parametrizations explained in the text for the longitudinal component of $R_{\tau,V/A}^{(0,0)}$.

| | $R_{us,A}^{(0,0)L}$ | $R_{us,V}^{(0,0)L}$ | $R_{ud,A}^{(0,0)L} \times 10^3$ |
|--------|---------------------|---------------------|---------------------------------|
| OPE | -0.144 ± 0.024 | -0.028 ± 0.021 | -7.79 ± 0.14 |
| Pheno. | -0.135 ± 0.003 | -0.028 ± 0.004 | -7.77 ± 0.08 |

spectral functions are very similar, but the errors are much lower using the phenomenological parametrization. Therefore, using phenomenology to describe the $J=L$ component of $\delta R_{\tau}^{(k,l)}$ reduces considerably the theoretical uncertainty.

We use as input value $m_s(2 \text{ GeV}) = (95 \pm 20) \text{ MeV}$ which includes the most recent determinations of m_s from QCD Sum Rules^{20,21,22}, lattice QCD²³ and τ hadronic data^{8,9,10,11,12,13,14}. With this strange quark mass input and taking $\delta R_{\tau}^{(0,0)L+T}$ and $\delta R_{\tau}^{(0,0)L}$ from the QCD OPE and phenomenology respectively, as explained above, one can calculate $\delta R_{\tau}^{(k,l)}$ in (9) from theory

$$\delta R_{\tau,\text{th}}^{(0,0)} = (0.162 \pm 0.013) + (6.1 \pm 0.6) m_s^2 - (7.8 \pm 0.8) m_s^4 = 0.218 \pm 0.026, \quad (11)$$

where m_s denotes the strange quark mass in MeV units, defined in the \overline{MS} scheme at 2 GeV.

In order to obtain a value of $|V_{us}|$ from (10), we also need the experimentally measured Cabibbo-allowed $R_{\tau,V+A}^{(0,0)}$ and Cabibbo-suppressed $R_{\tau,S}^{(0,0)}$ contributions to the τ decay rate. OPAL has recently updated the strange spectral function in⁷. In particular, they measure a larger branching fraction $B(\tau^- \rightarrow K^- \pi^+ \pi^- \nu)$ which agrees with the previously one measured by CLEO⁶. From the OPAL data and using (10) we get the result

$$|V_{us}| = 0.2208 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}} = 0.2208 \pm 0.0034; \quad (12)$$

where we have used as input the PDG value for $|V_{ud}| = 0.9738 \pm 0.0005$. The most important feature of this determination is the small theoretical error, that leaves the final uncertainty dominated by the experimental input. One can expect then that the better data sets from BABAR and BELLE will reduce significantly the error of our determination by improving the experimental input. Nevertheless, already now, our result is competitive to the standard extraction of $|V_{us}|$ from semileptonic kaon decays^{24,25,26,27,28,29,30,31} and a new determination from f_K/f_{π} as extracted from lattice^{32,33}.

One can use the value of $|V_{us}|$ thus obtained in (12) and determines the strange quark mass from higher moments. The weighted average of m_s calculated for the different moments gives

$$m_s(M_{\tau}) = 84 \pm 23 \text{ MeV} \quad \Rightarrow \quad m_s(2 \text{ GeV}) = 81 \pm 22 \text{ MeV} . \quad (13)$$

For details about the results for the different moments and the sources of the uncertainties see¹⁵.

In our previous analysis based on the ALEPH data¹⁴, it was observed that m_s displayed a strong dependence on the number of the moment k , decreasing with increasing k . With the recent CLEO and OPAL results finding a larger branching fraction $B(\tau^- \rightarrow K^- \pi^+ \pi^- \nu)$, this dependence is much reduced, although still visible. This issue needs to be clarified with the help of better experimental data.

4 Simultaneous fit of m_s and $|V_{us}|$

The ultimate procedure to determine both $|V_{us}|$ and m_s from τ hadronic decays, will be a simultaneous fit of both to a certain set of (k, l) moments. A detailed study including theoretical and experimental correlations will be presented elsewhere¹⁹.

In ¹⁵, we restrict ourselves to a simplified approach where all correlations were neglected. For this simultaneous fit of $|V_{us}|$ and m_s we use the five OPAL moments ⁷ from $R_\tau^{(0,0)}$ to $R_\tau^{(4,0)}$. The central values we find from this exercise are

$$|V_{us}| = 0.2196 \quad \text{and} \quad m_s(2 \text{ GeV}) = 76 \text{ MeV} . \quad (14)$$

These values are in very good agreement with our previous results in (12) and (13). We expect that the uncertainties on these results will be smaller than the individual errors in (12) and (13), but only slightly since the correlations between different moments are rather strong.

5 Conclusions and remarks

Using the strange spectral function updated by OPAL ⁷, we get

$$|V_{us}| = 0.2208 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{th}} , \quad (15)$$

and

$$m_s(2 \text{ GeV}) = 81 \pm 22 \text{ MeV} . \quad (16)$$

This result is expected to be highly improved in the near future due to the fact that the error is dominated by the experimental uncertainty and that uncertainty can be reduced with better data samples from BABAR and BELLE. But already now, the high precision τ decay data from ALEPH and OPAL at LEP and CLEO at CESR provide competitive results for $|V_{us}|$ and m_s . The combined fit to determine both quantities including theoretical and experimental correlations is underway.

The actual status of the $|V_{us}|$ determinations has been nicely reviewed recently in ^{34,35}. Though the CKM unitarity discrepancy has certainly decreased with the new theoretical and experimental advances, the situation is not yet as good as one could wish, and an accurate determination for $|V_{us}|$ with the eventual precise measurement of the strange spectral function at BABAR and BELLE is desirable. With the value of $|V_{us}|$ in (12) and using the PDG value $|V_{ud}| = 0.9738 \pm 0.0005$, one finds

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = (2.9 \pm 1.8) \cdot 10^{-3} , \quad (17)$$

so that unitarity is violated only at the 1.6σ level.

There are some open questions that will have also to be addressed ¹⁹. The $(k, 0)$ -moment dependence of the m_s prediction has been reduced after the recent OPAL and CLEO analyses finding larger branching fractions for $\tau^- \rightarrow K^- \pi^+ \pi^- \nu$. Accurate experimental data will clarify the origin of the remaining moment dependence and will allow a consistency check of the whole analysis.

Another point to be checked ¹⁹ is the fulfilment of the quark-hadron duality between the QCD OPE and the OPAL spectral function. A first look at this question was presented in ³⁶.

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