

# Determining dielectric constants using a parallel plate capacitor

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The availability of low cost digital multimeters capable of measuring capacitance has made parallel plate capacitor investigations common in the introductory laboratory. Typically, students add multiple thin sheets of dielectric material between conducting plates. The capacitance is measured and then plotted versus the reciprocal of the dielectric thickness (the nominal plate separation). We explain why the experiment fails for small dielectric thickness and discuss an improved version of the experiment using dielectric sheets of various thicknesses rather than multiple thin sheets of a dielectric. © 2005 American Association of Physics Teachers.  
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## I. INTRODUCTION

Because the relation between capacitance and plate separation for an ideal parallel plate capacitor is very simple, it is desirable to do experiments to examine this relation. There have been many articles relating to capacitors and dielectric materials.<sup>1-10</sup> A frequent purpose of these experiments is to verify the relation between the capacitance and the plate separation based on the parallel plate capacitance equation:

$$C = \frac{\kappa \epsilon_0 A}{d}, \quad (1)$$

where  $C$  is the capacitance,  $\kappa$  is the dielectric constant,  $\epsilon_0$  is the permittivity of free space,  $A$  is the area of the plates, and  $d$  is the plate separation. A common approximation in capacitors is that the dielectric material fills the void between the two conducting plates, which implies that the dielectric thickness,  $D$ , is equal to  $d$ , the plate separation. A common student task is to determine  $\kappa$  and verify the linear relation between  $C$  and the inverse dielectric thickness,  $1/D$ . Typically, the dielectric thickness is varied by using multiple thin sheets (of the same nominal thickness) of dielectric material. However, this experiment consistently results in a nonlinear relation between capacitance and inverse dielectric thickness and yields dielectric constants that are much too low predominantly because of air trapped between the layers of the dielectric.

Many instructors choose paper as the dielectric material. Paper has the advantage of being thin (small plate separations result in large capacitances) with a reasonably uniform thickness. Most introductory texts give a value of roughly 3 for the dielectric constant  $\kappa$  of paper. However, the values given for  $\kappa$  in various textbooks range from 1.7 to 4.0,<sup>11-15</sup> and none of the textbooks provide a reference to an original source.

There are multiple factors that can cause a variance in the dielectric constant of paper. Paper can be wood-based, rice-based, or rag-based, for example. Different inks and bleaching processes are used in its production as well as various surface finishes. Wood-based paper (presently the most common paper) undergoes a drying process and chemical process with aging. Residual acids left on the surface from the manufacturing process cause the paper to yellow and become drier over time. In addition, paper is hygroscopic, and the dryness of paper is very important for the value of the dielectric constant. Water has a dielectric constant of  $\kappa=83$  and wood

has a dielectric constant that can be as low as  $\kappa=2$ . In short, we cannot rely on a specific dielectric constant for paper.

Because of these concerns, we selected Teflon rather than paper for our dielectric material. Teflon is available in thicknesses (and tolerances) that are similar to paper. The dielectric constant for Teflon at 22 °C for frequencies between 100 Hz to 1 MHz is  $\kappa=2.1$ .<sup>16</sup> The manufacturer's product description lists  $\kappa=2.0$  at 25 °C for frequencies between 100 Hz and 1 MHz.<sup>17</sup> We are uncertain why Refs. 16 and 17 differ, because other listed values from each reference indicate that  $\kappa$  does not vary greatly with temperature.

## II. EXPERIMENT 1

The parallel plate capacitors we construct are simple and inexpensive. The construction is diagramed in Fig. 1. A single sheet of ordinary household aluminum foil (~30.3 cm width) is placed on a piece of particle board (approximately 1.6 cm thick, 29 cm long, and 35 cm wide). The foil is rolled flat with paint rollers and any foil that is crinkled or folded is replaced by fresh foil. A number of Teflon sheets (larger in area than the foil) are placed on top of the foil. The Teflon film sheets<sup>18</sup> are  $0.050 \pm 0.013$  mm thick (nominal 0.002" thickness with a 0.0005" tolerance<sup>17</sup>). The thickness of each individual sheet and the total dielectric thickness of all the sheets of dielectric are measured using a micrometer accurate to 0.005 mm. The results of these measurements are consistent with the specifications given by the manufacturer. A second sheet of aluminum foil (identical to the first foil sheet) is placed over the Teflon for the top plate. To flatten the capacitor, a second piece of particle board is placed on top of the foil and sheets. To increase the flattening, a load mass can be distributed across the top of the particle board. The top foil is folded back over the particle board (see Fig. 1, side view) so that the top and bottom foils are nearest each other under the particle board. Portions of the foil that are not between the two boards are far enough apart so their contribution to the total capacitance is negligible. Care is taken that the top piece of foil is directly over and aligned with the bottom piece. By this arrangement of the two foils, the area of the capacitor is determined by the width of the foil and the length of the particle board.

We then measure the capacitance by connecting the foil sheets to a digital multimeter<sup>19</sup> by means of two 30 cm long, single conductor leads with alligator clips. By varying the number of Teflon sheets we observe a changing capacitance.

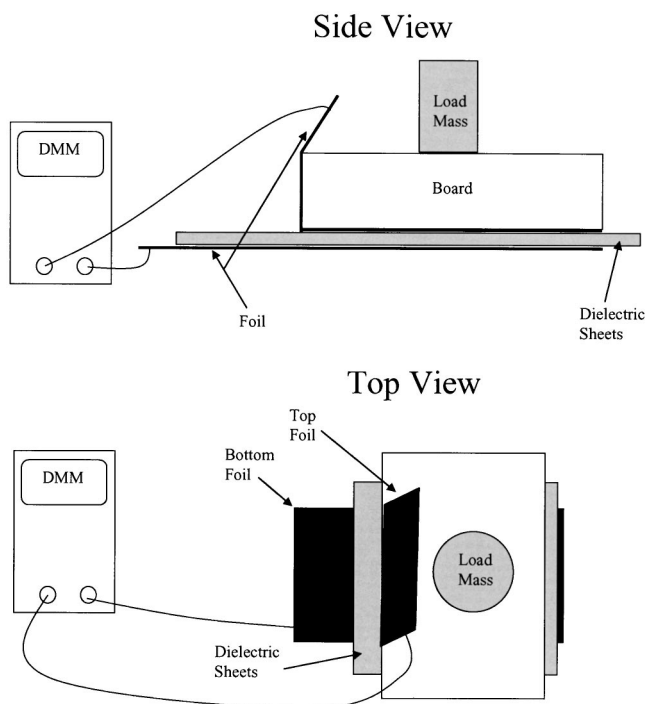
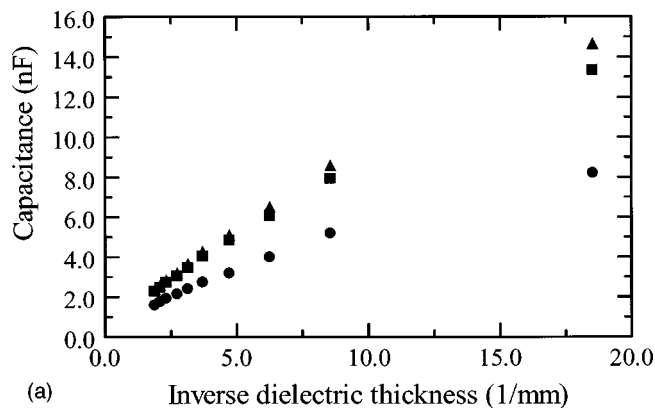


Fig. 1. Two sheets of ordinary household aluminum foil ( $\sim 30.3$  cm width) are rolled flat on the table. We used 0.05 mm thick Teflon film sheets that are wider than the foil. The dielectric sheets are sandwiched between the two sheets of aluminum foil and a particle board is placed on top to help smooth out any remaining wrinkles. We vary the nominal plate separation by varying the number of dielectric sheets and measure the capacitance by connecting the foil sheets to a digital multimeter via 30 cm long alligator clips.

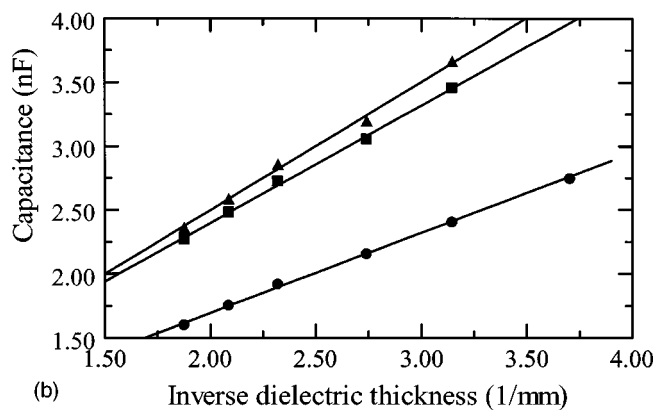
The obvious method of analyzing the data using Eq. (1) is to plot the capacitance versus the reciprocal of the dielectric thickness,  $1/D$  (see Fig. 2(a)). The dielectric constant can be found from the slope and Eq. (1), assuming that  $D$  is equivalent to the plate separation. However, the data in Fig. 2(a) becomes increasingly nonlinear as  $1/D$  becomes larger. Because of this nonlinearity, many investigations do not measure capacitance for small dielectric thickness. If the measurements for one to four sheets of dielectric material are neglected, a linear plot can be produced as shown in Fig. 2(b). However, a comparison of the results from this plot with Eq. (1) raises several questions. The data has a nonzero intercept (which varies with load pressure). The determination of  $\kappa$  from the slopes results in values ranging from  $0.78 \pm 0.09$  to  $1.31 \pm 0.12$  depending on the pressure applied to the capacitor. These values are outside of the experimental uncertainty from the expected value of  $\kappa = 2.0$ .<sup>17</sup> Obviously, this simplified approach has problems.

The nonzero intercepts correspond to a stray capacitance. Stray parallel capacitance could come from the leads or from the meter. However, the largest capacitance we can produce using the 30 cm single conductor leads is approximately 10 pF. We also considered and rejected effects due to resistance in the leads and current through the dielectric material. Each of these effects is much too small to cause any measurable difference given our materials. We have no other physical explanation for the reproducible stray capacitance that we observe.

To make certain that the stray capacitance and low dielectric constants were not due to the multimeters, we examined



(a)



(b)

Fig. 2. (a) A plot of capacitance versus inverse Teflon thickness for three applied pressures. The triangles ( $\blacktriangle$ ), squares ( $\blacksquare$ ), and circles ( $\bullet$ ) represent data acquired with applied pressures of 2855 Pa, 1503 Pa, and 150 Pa, respectively. Each capacitor has an area of  $0.0892$  m<sup>2</sup>. (b) A plot of only the data shown in Fig. 2(a) for five through ten sheets of Teflon. This graph appears linear, but the slope for any of the lines does not yield an accurate dielectric constant for Teflon. If we examine the data values in this plot and calculate dielectric constants based on the slopes (ignoring the nonzero intercepts), we find dielectric constants of  $1.31 \pm 0.12$ ,  $1.21 \pm 0.10$ , and  $0.78 \pm 0.09$  for pressures 2855 Pa, 1503 Pa, and 150 Pa, respectively. Teflon has a dielectric constant of 2.0. The expected capacitance intercept is zero, but our measured intercepts are  $0.42 \pm 0.07$  nF,  $0.48 \pm 0.09$  nF, and  $0.46 \pm 0.03$  nF, respectively.

five brands of meters and compared their measurements with those of a commercial capacitance bridge. Four of the five multimeters use an AC method for measuring capacitance. This AC method applies the same frequency across the capacitor independent of its capacitance. The fifth multimeter uses an RC timing technique to measure capacitance. The only difference in capacitance readings between the different multimeters is that the meter using the RC timing technique gave a large zero capacitance offset with no load capacitance ( $\sim 10$  nF). If this “zero” capacitance is subtracted, this multimeter’s readings were virtually identical with the others as well as with the commercial capacitance bridge.

Because the nonlinearity of Fig. 2(a) cannot be attributed to measurement flaws, we considered surface irregularities of the plates of the capacitor as the possible cause. The foil has some irregularities as does any surface, and these irregularities would be a reason to ignore the data for the first four dielectric sheets. Consider a parallel plate capacitor with a single flaw in one plate that penetrates into the dielectric material. The electric field will be stronger in the region of the flaw leading to an excess of charge near the flaw. This

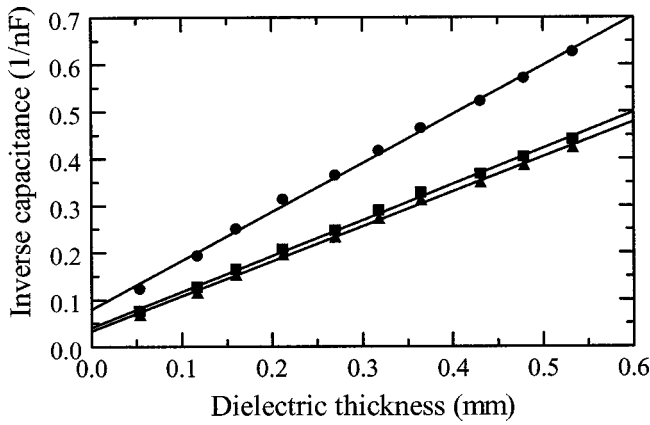


Fig. 3. A plot of inverse capacitance versus dielectric thickness for the same data shown in Fig. 2(a). The triangles (▲), squares (■), and circles (●) represent the same applied pressures as in Fig. 2. The error bars are approximately the same size as each marker. All of the data can be fit to a straight line, but the slopes still do not yield accurate dielectric constants. The effective dielectric constants determined from the slopes are  $1.71 \pm 0.05$ ,  $1.65 \pm 0.06$ , and  $1.22 \pm 0.08$  for the same applied pressures. The stray series capacitance based on the  $1/C$  intercepts are  $30.0 \pm 3.0$  nF,  $24.7 \pm 2.4$  nF,  $12.7 \pm 1.2$  nF for the same load pressures, respectively.

excess would result in a larger capacitance measurement for a given plate separation than we would predict for a parallel plate capacitor by Eq. (1). The effects of the flaw would be most significant when the plate separation is small. As the plate separation becomes larger, the capacitance would approach the ideal parallel plate capacitor value. This situation would result in a concave upward curve on a  $C$  versus  $1/D$  plot for small values of  $D$ , rather than a straight line.

Alternatively, a single flaw of the dielectric material jutting into one plate would decrease the charge in that region and result in a lower overall capacitance. The measured capacitances for small  $d$  would be smaller than those predicted by Eq. (1). In this case a plot of  $C$  versus  $1/D$  would result in concave down curve. This case agrees with our data and is repeatable. However, there is no reason to believe that the surface flaws are always directed into the conducting plates. Because the dielectric materials we use are rigid, we cannot expect the dielectric to fill all voids in the parallel plates. Hence, an explanation of the nonlinearity and nonzero intercepts of Fig. 2(a) using a model based on surface irregularities or one based on stray parallel capacitances is questionable.

Consider what would happen if there was an extra capacitance in series with our parallel plate capacitor. The measured capacitance becomes

$$C_{\text{measured}} = \frac{CC_{\text{series}}}{C + C_{\text{series}}}. \quad (2)$$

With the use of Eq. (1), we obtain

$$C_{\text{measured}} = \frac{\kappa \epsilon_0 A C_{\text{series}}}{(\kappa \epsilon_0 A + C_{\text{series}} D)}. \quad (3)$$

This situation would imply a concave downward curve for a plot of  $C_{\text{measured}}$  versus  $1/D$ . If we replot the data shown in Fig. 2(a) as the reciprocal of the capacitance versus the dielectric thickness (see Fig. 3), the data becomes linear supporting the idea that there is a stray series capacitance. This

approach does not require us to arbitrarily discard the data for small numbers of dielectric sheets.

We rewrite Eq. (1) in terms of the series capacitance

$$\frac{1}{C_{\text{measured}}} = \frac{D}{\kappa \epsilon_0 A} + \frac{1}{C_0}, \quad (4)$$

where  $D$  is the dielectric thickness, and  $1/C_0$  is due to the stray series capacitance. The intercept of the plot of  $1/C_{\text{measured}}$  versus  $D$  correspond to a stray series capacitance (Fig. 3). These capacitance values range from  $C_0 = 13 \pm 1$  nF to  $30 \pm 3$  nF with increasing applied pressure. From the slope of each line, we find dielectric constants ranging from  $\kappa = 1.22 \pm 0.08$  to  $1.71 \pm 0.05$  with increasing load pressure. Even though we found a way of producing straight lines using all the data, the measured dielectric constant of Teflon, which is load pressure dependant, is still not within experimental uncertainty of the accepted value.

The stray series capacitance calculated from the intercept shown in Fig. 3 increases with increased load pressure, which leads us to suspect that there is air trapped between the plates. As pressure is applied to the top plate of the capacitor, some air leaks out of the capacitor reducing the effective plate separation. If these air pockets are important, they must be incorporated into our model for the capacitor. We can imagine random pockets of air filling variations in the material surfaces (dielectric and foil). However, if we examine the derivation of the effect of dielectrics on capacitance,<sup>20</sup> the dielectric constant arises from an averaging process of unevenly distributed electric fields at an atomic level. Thus, the process of determining the capacitance is always an averaging process of nonuniform fields. Therefore, it is reasonable to use a simple model that accurately describes the experiment by replacing the randomly placed and sized air pockets with uniform layers of air, dielectric material, and plates (see Fig. 4(a)). Although we do not expect the air pockets to be uniform, the capacitor behaves as if the air pockets were uniform. As mentioned in many introductory texts, different dielectric materials sandwiched between parallel plates may be treated as multiple capacitors in series. If we rearrange the order of these sandwiched dielectric layers (see Fig. 4(b)), we can replace the multiple layers of dielectric and air by a single solid dielectric sheet and a single layer of air. Hence, our measured value of capacitance may be thought of as two capacitors in series, one a perfect parallel capacitor filled with the dielectric material,

$$C_d = \frac{\kappa \epsilon_0 A}{D}, \quad (5)$$

where  $C_d$  is the capacitance due to the dielectric material. The other capacitor is a perfect parallel capacitor filled with air,

$$C_a = \frac{\epsilon_0 A}{d_a}, \quad (6)$$

where  $C_a$  is the capacitance due to air and  $d_a$  is the effective air thickness between the plates. The measured capacitance is the series combination of the dielectric capacitor and the air capacitor,

$$\frac{1}{C_{\text{measured}}} = \frac{D}{\kappa \epsilon_0 A} + \frac{d_a}{\epsilon_0 A}. \quad (7)$$

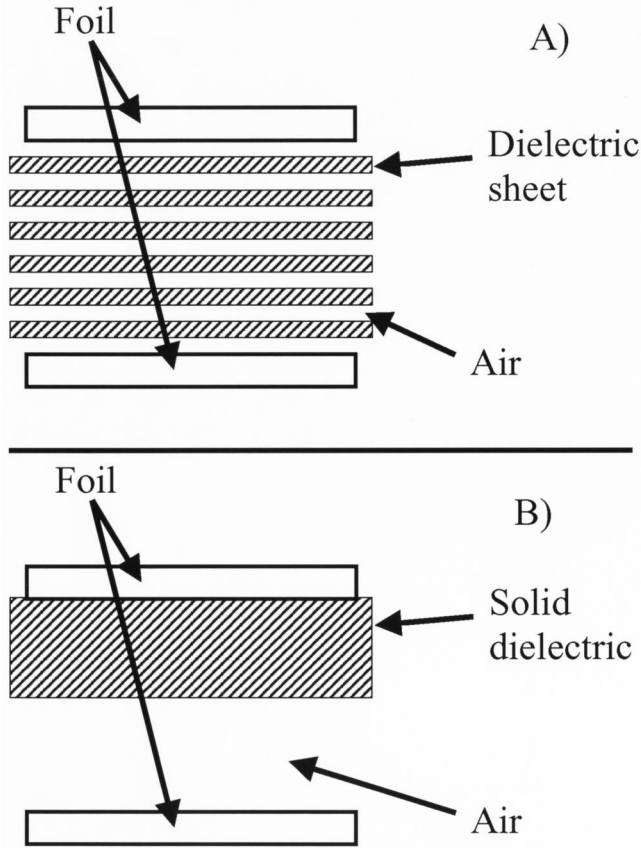


Fig. 4. (a) A cross section of a Teflon-air-foil parallel plate capacitor. (b) The equivalent Teflon-air-foil parallel plate capacitor with the paper and air in a single grouping.

If we consider  $N$  sheets of dielectric material in the capacitor, then we have  $(N-1)$  layers of trapped air between the dielectric sheets. If we express Eq. (7) in terms of the nominal thickness of an individual sheet of dielectric, we find

$$\frac{1}{C_{\text{measured}}} = \frac{ND_1}{\kappa\epsilon_o A} + \frac{2d_f + (N-1)d_s}{\epsilon_o A}, \quad (8)$$

where  $D_1$  is the nominal thickness of a single sheet of dielectric material,  $d_f$  is the average air layer thickness between the dielectric and a foil plate, and  $d_s$  is the average air layer thickness between two dielectric sheets. We rearrange Eq. (8) and arrive at

$$\frac{1}{C_{\text{measured}}} = \frac{ND_1}{\kappa_{\text{eff}}\epsilon_o A} + \frac{2d_f - d_s}{\epsilon_o A}, \quad (9)$$

where  $\kappa_{\text{eff}}$  is an effective dielectric constant given by

$$\kappa_{\text{eff}} = \frac{\kappa}{1 + \kappa \frac{d_s}{D_1}}. \quad (10)$$

The effective dielectric constants determined using this analysis will be lower than the dielectric constants of a pure material. The dielectric constants we determined using the slopes in Fig. 3 are effective dielectric constants (see Eq.

(10)). The dielectric constants are for material-air composites and the intercept corresponds to an air capacitor with a plate separation of  $2d_f - d_s$ .

If we want to determine the dielectric constant of the material without air gaps, we either have to remove all the air gaps, or we have to measure  $d_s/D_1$ . We can obtain a rough measurement of the latter by comparing the measurements of a large number of stacked dielectric sheets and the average thicknesses of these same sheets. However, because the average air gap thickness is roughly the same as the uncertainty of the micrometer, large relative uncertainties result. This technique produces a measurement of  $d_s/D_1 = 0.06 \pm 0.04$  for the Teflon sheets. If we use  $\kappa_{\text{eff}} = 1.71$  (from the slope of the Teflon data in Fig. 3 with the largest load pressure), the ratio of the air gap distance to the dielectric thickness, and Eq. (10), we find a dielectric constant for Teflon of  $\kappa = 1.91 \pm 0.19$ .

Because this result is based on a term with large relative uncertainty,  $d_s/D_1$ , we may ask if there is a more accurate method? We could consider adding larger forces to the top of the capacitor. Increased load pressure should reduce the magnitude of  $d_s/D_1$  making its inclusion negligible. However, to squeeze the air from between the plates would require pressures as large as atmospheric pressure.

### III. EXPERIMENT 2

Because we are uncomfortable with the results of the investigations in Sec. II, we investigated an alternative approach with no air layers between the sheets. The simplest method of removing the air gaps between sheets is to replace the multiple thin sheets of dielectric material by single sheets of varying thicknesses. Teflon film sheets come in a variety of nominal thicknesses 0.002", 0.003", 0.005", 0.010", 0.015", 0.020", 1/32", and 3/64". Because Teflon film sheets tend to roll up, which would add unwanted extra air gaps between the dielectric and the plates, we still use a load mass. However, the pressures caused by these load masses are so low (we used a load pressure of 2855 kPa) that the compression of the Teflon is negligible. A plot of the inverse capacitance versus the dielectric thickness produces the line represented by triangles in Fig. 5. The dielectric constant calculated from the slope of this line and Eq. (4) is  $1.96 \pm 0.06$ , which is in good agreement with the manufacturer's value of 2.0.

For further reassurance that stray or irregular electric fields due to the surface irregularities were not responsible for the results, we performed several more experiments. We once more constructed capacitors using Teflon sheets of varying thicknesses, except this time we intentionally introduced large surface imperfections and defects in the capacitor's conducting plates. The curve marked by the squares in Fig. 5 used aluminum screens (the same type used in household screen doors) for the plates. The data marked by circles used aluminum foil with a #6 washer ( $\sim 0.97$  mm thick) between the foil and the board. As shown in Fig. 5, the aluminum screen data does not fit a straight line for small thicknesses (a single layer of 0.05 mm thick Teflon film). This deviation is not due to stray fields, but arises because the screen is made of woven aluminum wire ( $\sim 0.25$  mm diameter threads with  $\sim 5.5$  threads per cm) which causes peaks and valleys. The thin dielectric sheets cannot support much weight and when the top screen is placed on top, peaks from the top plate push



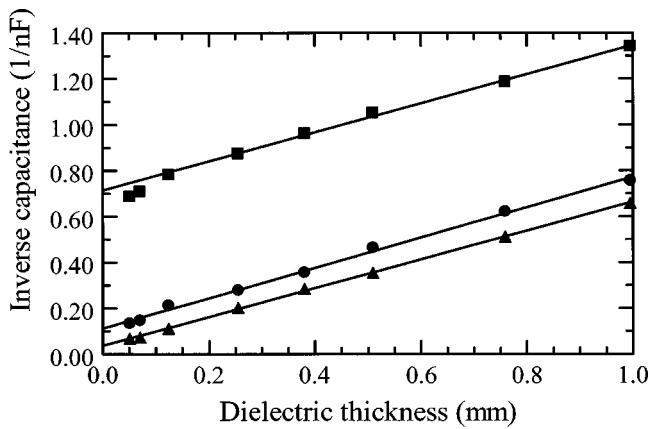


Fig. 5. A plot of the inverse capacitance versus the dielectric thickness with different thickness Teflon sheets. The different data sets represent different plate surfaces. The triangle ( $\blacktriangle$ ) data is from aluminum foil plates (same plates as used to acquire the data shown in Figs. 2 and 3); the square ( $\blacksquare$ ) data used woven aluminum screen (typically used in household screen doors) for the conducting plates; the circle ( $\blacktriangledown$ ) data used foil plates with a standard 6-32 washer (0.970 mm thick) inserted under the bottom foil to form an irregularity. The error bars are approximately the same size as each marker. All these capacitors have an area of  $0.0892 \text{ m}^2$ . The slopes for the graphs are nearly identical regardless of the plate material or plate flaw introduced. Each data set results in a dielectric constant within the experimental uncertainty of the accepted value for Teflon of 2.0.

the thin dielectric sheets into valleys of the bottom plate (and vice-versa) resulting in a smaller plate separation for thin dielectric sheets than for thicker dielectric sheets. From the data in Fig. 5, it is apparent that defects in the plates affect the intercept values, but not the slope (which is used to find the dielectric constant). The data presented in Fig. 5 is consistent with our model because these defects only create a much larger air layer thickness between the dielectric material and the plates. The value of the dielectric constant for Teflon from each of these experiments is  $(1.96 \pm 0.06)$  for the foil,  $(1.95 \pm 0.10)$  for the screen, and  $(1.93 \pm 0.15)$  for the foil and washer. These results indicate that the surface and stray electric fields are not a problem in the determination of the dielectric constant.

#### IV. CONCLUSION

Small plate separations are required for appreciable capacitances which require the use of thin dielectric sheets. However, the use of multiple thin dielectric sheets leads to proportionally larger air gaps. When dealing with thin dielectric materials, the material is likely to be very flexible (such as paper or Teflon) or extremely brittle (like glass). Flexible material will cause air gaps. Brittle material can easily break

and still cause air gaps. We could remove all the air gaps using a resin (or oil) between the dielectric sheets. However, finding the dielectric constant of the material would require knowledge of the dielectric constant of the resin (oil) and the exact thicknesses of each resin (oil) layer.

Ultimately, we found that the simplest way to do accurate measurements is to remove the air gaps between the individual sheets of the dielectric. This necessity increases the cost of an individual experimental set-up. However, the accuracy using multiple sheets of dielectric is acceptable as long as the inherent flaws in the experiment are recognized. In this case, the parallel plate capacitor experiment changes from a simple cookbook experiment to a true experiment with unexpected results. Students must consider their observations and modify a simple model to explain their observations.

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- <sup>17</sup>Teflon is a registered trademark of Dupont. Information on 0.002" films may be found at (<http://www.dupont.com/teflon/films/H-04321-2.pdf>).
- <sup>18</sup>The Teflon was purchased from McMaster–Carr.
- <sup>19</sup>We used a KRM AEEC-2890 digital multimeter for these measurements.
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