Determining efficiency of cycloid reducers using different calculation methods

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Abstract. Cycloid reducers are gear trains which can be classified as planetary transmissions. These transmissions have a very wide range of uses in industry in transporters, robots, satellites, etc. This research presents a comparative analysis of three analytical methods for determining cycloid drive efficiency. The paper explores every mathematically formulated method and compares them to experimental results from literature. The presented methods for determining efficiency have a common property, in that they all determine losses due to friction on the bearing cam surface of the shaft, the rollers of the central gear and the output rollers. The calculation of efficiency values is done for standard power values. The methods differ primarily in the way they calculate losses. For each method of calculating efficiency there is an analysis of pros and cons. The paper concludes with suggestions as well as possible directions for further research.

1 Introduction

Cycloid transmissions have, in the past decades, found a very wide use in engineering practice. Cycloid power transmissions are most commonly used as cycloid reducers. Cycloid reducers have a variety of good characteristics such as: compact design, very low vibration and noise, reliable operation under dynamic stress, a small mass compared to the power they transmit, a wide spectrum of possible transmission ratios, the ability to achieve high torque output, etc. Cycloid reducers are used on robots, manipulators, transporters, machines for processing technology, etc. The price of cycloid reducers are in the same range as conventional drives, even though cycloid reducers belong to a new generation of power transmissions. A very important characteristic of cycloid reducers is their high efficiency. The determination of efficiency using various methods as well as a comparative analysis of these methods will be presented in this paper.

The efficiency of cycloid drives represents a very important aspect of designing these power transmissions. The first researcher to work on mathematical modelling of efficiency was Kudrijavcev, [1]. Kudrijavcev dedicated an entire chapter to cycloid reducers in his

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book Planetary transmissions, [1].Malhotra , based on Kudrijavcev's mathematical model, made a new model for determining the efficiency which considers the losses in power on each roller of the central gear individually, as well as on every output roller individually, [2]. Kosse tested how efficiency influences a multiple increase in the input torque, [3]. Gorla et al. made a comparative analysis of experimental and theoretically determined efficiency, and subsequently used the results to create a new mathematical model for determining efficiency of cycloid reducers, [4, 5]. Blagojevic et al. tested the influence of changing friction coefficients on efficiency of cycloid reducers, [6]. Mackic et al. tested the influence of geometric parameter changes on efficiency in [7]. Neagoe et al. conducted experimental research of cycloid reducer non-pin wheel concepts, [8]. Zah defined the procedure for thermal analysis of cycloid reducers, [9]. Tonoli researched the influence of operation without lubrication on the efficiency of cycloid drives, [10]. Mihailidis has conducted experimental verification of Malhotra's method for determining efficiency of cycloid reducers, [11]. Blagojevic et al. experimentally verified Kudrijavcev's method for determining efficiency of cycloid reducers, [12].

For the theoretical determination of efficiency three representative methods can be highlighted: Malhotra method [2], Gorla method [4, 5], and Kudrijavcev method [1]. Every one of these methods has a verification in available literature. In this paper the determination of efficiency will be presented using each of the aforementioned methods, and a comparative analysis will be given at the end of the paper of all three methods.

2 Mathematical models of efficiency calculation methods

Efficiency analysis of cycloid reducers is a very complex and interesting problem for both engineering practice and science, as it is still an insufficiently researched aspect of this type of power transmission. In this paper the following mathematical models of cycloid reducer efficiency calculations will be presented: Malhotra model, Gorla model, and Kudrijavcev model. The determination of efficiency for each of these theoretical models is based on determining power losses in the contact of specific elements of the cycloid reducer due to sliding and rolling friction. These losses occur in the contact of the following elements:

- Losses of power due to friction in the bearing of the cycloid gear which is on the eccentric shaft. This loss depends on the size and type of bearing, size of rolling element, rolling friction coefficient of the bearing, amount of force on the eccentric shaft and angular velocity.
  - Losses of power due to rolling friction between output rollers and openings in the cycloid gear. In these contacts rolling friction is dominant, so losses of power are very small, almost negligible. As the number of output rollers is relatively small in comparison to the number of rollers of the central gear, some methods do not consider these losses,[4 - 5].
  - Losses of power due to rolling friction between cycloid gear teeth and rollers of the central gear. In these contacts there is rolling friction, therefore the losses are very small and depend on the rolling friction coefficient and normal forces.
  - Losses of power due to sliding friction between the output rollers and output shafts. The output rollers are most commonly directly installed on the corresponding shafts, so there are losses due to sliding friction. This contact behaves as a sliding connection. The values which directly influence on power losses in this contact are: outer shaft diameter (inner diameter of output roller), sliding friction coefficient, sliding speed and output force.
  - Losses of power due to sliding friction between rollers and central gear shafts. In the largest number or conceptions rollers of the central gear are installed directly onto the shafts. As the number of these contacts is large, this is where the largest losses in

power due to sliding friction occur. The largest influence on power loss is from: the diameter of the shafts (inner roller diameter), sliding friction coefficient, and normal forces.

According to Malhotra's model [2], the calculation of efficiency of cycloid reducers is based on the determination of the total work of friction forces which occurs during an elementary angular movement of the cycloid gear by $d\theta$, [2]. When the cycloid gear turns by $d\theta$, then the input shaft will turn by an angle of $i{\cdot}d\theta$, while the rollers of the central gear will turn by an angle of $(i+1){\cdot}d\theta$. In this case i represents the transmission ratio of the cycloid reducer, or the number of teeth of the cycloid gear. The total losses of power in the interaction between these elements of the cycloid reducer can be written as:

$$W = \int_0^{\frac{2\pi}{i}} dW = \frac{\mu_{r1} D_m i}{d_{kt}} \int_0^{\frac{2\pi}{i}} F_E(\theta)\, d\theta + i\left(\mu_{r2} + \frac{\mu_{s1} d_{VK}}{2}\right) \int_0^{\frac{2\pi}{i}} \sum_{j=1}^{q} F_{Kj}(\theta)\, d\theta + (i+1)\left(\mu_{r3} + \frac{\mu_{s2} d_0}{2}\right) \int_0^{\frac{2\pi}{i}} \sum_{i=1}^{p} F_{Ni}(\theta)\, d\theta \quad (1)$$

where: μ_{r1} – is the rolling coefficient in the bearing, $F_E(\theta)$ – is the current value of eccentric force, D_m – the diameter of cycloid gear bearing, d_{kt} – the diameter of the bearing rolling element, μ_{r2} – rolling friction coefficient between output rollers and cycloid gear, $F_{Kj}(\theta)$ – current output force on the j^{th} output roller, q – number of output rollers which at the given moment are in contact with the cycloid gear, μ_{r3} – coefficient of rolling friction between the cycloid gear and central gear rollers, $F_{Ni}(\theta)$ – current value of normal force on the i^{th} roller of the central gear, p – number of rollers of the central gear which at the given moment are in the process of transmitting load, μ_{s1} – sliding friction coefficient between input rollers and output shafts, d_{VK} – output shaft diameter, μ_{s2} – sliding friction coefficient between the rollers and shafts of the central gear and d_0 – diameter of central gear shaft.

According to Malhotra cycloid reducer efficiency is calculated using the following expression:

$$\eta = \frac{T_{ul} \cdot 2\pi - W}{T_{ul} \cdot 2\pi} \quad (2)$$

where: η – is the efficiency of the cycloid reducer and T_{ul} – is the input shaft torque .

According to Gorla [4 - 5], efficiency calculation of cycloid reducers is based on determining the following power losses:

- Losses of power due to rolling friction between the drive shaft and cycloid gear (this is the same contact as described in the beginning of this heading).
- Losses of power due to sliding friction between output rollers and openings in the cycloid gear (in this contact there is an obvious difference from the previous model, as the rolling friction is changed for sliding friction due to this concept not having output rollers, but just output shafts).
- Losses of power due to friction in the contact of the cycloid gear, central gear rollers and central gear (in this contact there is sliding friction on the contact of the cycloid gear with the rollers of the central gear, while on the part of the contact between the rollers of the central gears and the central gear there is rolling friction).

The total power loss according to Gorla's model van be calculated using:

$$W = T_{ul} \cdot (\omega_i - \omega_o) + \sum_{i=1}^{n} F_{Ti} \cdot v_{Ki} + \sum_{j=1}^{m} \mu_{Kj} \cdot F_{Kj} \cdot v_{Kj} \quad (3)$$

where: ωi – angular speed of the inner bearing ring on the eccentric shaft, ωo – angular speed of the outer bearing ring on the eccentric shaft, Tul – input torque, FTi – contact force on the area between the shaft and opening, vKi – sliding speed in the referent coordinate system, µKj – friction coefficient in contact between shaft and opening and FKj – output force on jth output shaft.

Lastly the expression for efficiency using Gorla method is:

$$\eta = \frac{P_{\mathrm{ul}} - W}{P_{\mathrm{ul}}} \tag{4}$$

where: P_{ul} – is the power on the input shaft.

According to Kudrijavcev [1] in the calculation of power losses in elements which are interacting with each other the following losses are considered: power losses in the bearing eccentric shaft, power losses in the rollers of the central gear and power losses in theoutput rollers. Other losses are negligible according to this method. The total power loss according to Kudrijavcev's method is determined by the following expression:

$$\psi = \frac{K_3 \cdot \mu_3}{z_2} + \frac{4 \cdot e \cdot \mu_{\mathrm{VK}}}{\pi \cdot R_0} + 1{,}63 \cdot \left(1 + \frac{d_{\mathrm{cz}}}{d_{\mathrm{kt}}}\right) \cdot \frac{k}{r_2} \cdot \sqrt{1 + \left(\frac{4}{\pi} \cdot \frac{r_2}{R_0} - K_{\mathrm{y}}\right)^2} \tag{5}$$

where: K_3 – is the factor which considers tooth correction of the cycloid gear, μ_3 – friction coefficient between rollers of the central gear and shafts, z_2 – number of central gear rollers, e – *size of eccentricity*, μ_{VK} –friction coefficient between output rollers and shafts, R_0 – radius of circle on which the openings are placed on the cycloid gear, d_{cz} – diameter of eccentric shaft, d_{kt} – diameter of bearing rolling element, k – rolling friction coefficient in the bearing, k=0,005, r_2 – radius of the moving circle and K_{y} – factor which considers tooth correction of cycloid gear.

K_3 and K_{y} factor values are explained in detail in papers [9] and [12].

Efficiency according to Kudrijavcev's model is determined as follows:

$$\eta = \frac{1 - \psi}{1 + z_1 \cdot \psi} \tag{6}$$

Detailed explanation of determining cycloid reducer efficiency for all three methods can be found in papers [1-5, 9, 12].

3 Comparative analysis of presented methods for calculating efficiency

In this paper as a representative model the Kudrijavcev method we chosen for determining the efficiency of cycloid reducers. This model is compared to Malhotra [2] and Gorla [4, 5] models according to available data from literature. For the purposes of this research a cycloid reducer with characteristics given in table 1 was used.

Table 1. Cycloid reducer characteristics chosen for the purposes of this research.

Name of value	Nomenclature	Value	Unit
Input power	*P*	0.34	*kW*
Nominal speed	*n*	1400	min^{-1}
Axis height	*H*	75	*mm*
Radius of central gear	*r*	45	*mm*

Based on data from table 1 a CAD model of this single stage cycloid reducer was made. From the CAD model the necessary geometric values for calculating efficiency according to Kudrijavcev were derived. Efficiency calculation is done for speeds of 100 min^{-1} to 850 min^{-1} with a division of 50 min^{-1}, under a constant power of 0.34 *kW*. This choice of reducer and speeds was adopted in previous research conducted by authors in [12]. This way it is possible to directly compare achieved theoretical results with experimental results. Results of the calculation are given in table 2.

Table 2. Calculated and experimental dada for determining cycloid reducer efficiency according to Kudrijavcev's method

Iteration no.	**T_{ul}** ***Nmm***	**n_{ul}** ***min^{-1}***	**η calculated**	**η experiment**	**Δ, % deviation**
1.	308.55	100	0.447	0.224	49.93
2.	372.46	150	0.543	0.555	2.20
3.	374.37	200	0.692	0.654	5.42
4.	431.54	250	0.696	0.685	1.47
5.	537.86	300	0.706	0.612	13.25
6.	539.88	350	0.710	0.659	7.25
7.	572.77	400	0.706	0.690	2.19
8.	567.83	450	0.701	0.748	6.70
9.	687.34	500	0.698	0.662	5.11
10.	735.91	550	0.698	0.659	5.63
11.	756.46	600	0.680	0.699	2.86
12.	829.21	650	0.675	0.667	1.13
13.	898.26	700	0.670	0.634	5.44
14.	894.51	750	0.669	0.678	1.38
15.	940.37	800	0.665	0.669	0.61
16.	1057.14	850	0.657	0.634	3.38

From table 2 it can be concluded that if the first iteration is not considered, the difference in theoretical and experimental model results for determining efficiency is in the interval between 0.61% and 13.25%. For a better understanding the results are presented in a graph in figure 1.

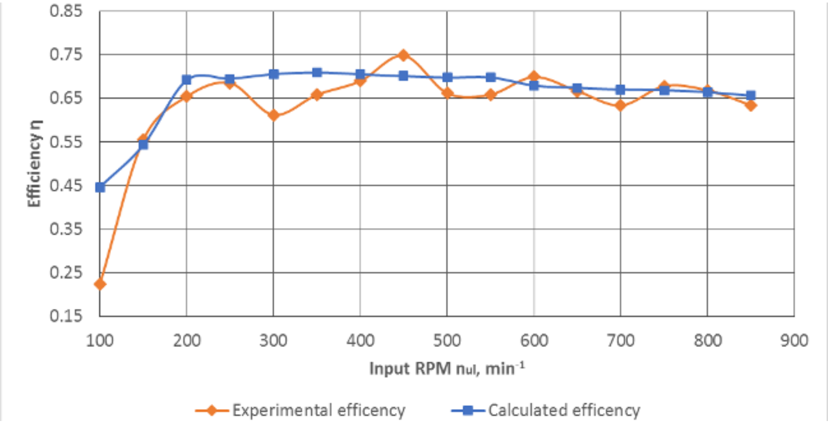

Fig. 1. Theoretical and experimental results for determining efficiency of cycloid reducers according to Kudrijavcev.

When the Malhotra and Kudrijavcev models are closer examined it can be concluded that both models are created for cycloid reducers with rollers. It is also apparent that both models have numerous similarities in determining power losses. It is the opinion of the authors that if a comparison of theoretical and experimental determining of efficiency was conducted that there wouldn't be major differences from the conducted Kudrijavcev model.

Godla's model is based on determining efficiency of a somewhat different concept of cycloid reducer. This concept of cycloid reducer does not use rollers but is instead directly connected by shafts which are located between the central gear and cycloid gear. In the work of Gorla and others it can be seen that his method for determining efficiency between the theoretical and experimental determination differs in the interval between 0% and 8.5%, [4 - 5].

4 Conclusion

This paper gives a description of known mathematical models for determining efficiency in single stage cycloid reducers, [1-5]. The paper presents current research in the field of cycloid reducer efficiency. After this a detailed presentation of Kudrijavcev, Malhotra and Gorla methods are given. Lastly a comparative analysis is given between calculation of efficiency and experimental data. Kudrijavcev and Gorla models are two essentially different mathematical models for determining efficiency of cycloid drives. Similarities of these models is in that they moth determine losses in the same locations. The difference in the methods are not only mathematical but also conceptual, as they are used in different designs of cycloid drives. Deviations of theoretical to experimental results for Gorla's method are in the interval of 0% to 8.5%, while the Kudrijavcev method varies in the interval of 0.61% and 13.25%. The Gorla model has somewhat smaller deviation. The Malhotra model is derived for the same concept of cycloid reducer as the Kudrijavcev model and the mathematical models themselves are quite similar, therefore it is the opinion of the authors that a comparative analysis of theoretical results with experimental results for Malhotra's model would not give larger deviations than in the case of Kudrijavcev. The Kudrijavcev method has a slight advantage over the Malhotra method as it is somewhat simpler for application in calculation.

Future research of the authors in this field would definitely include experimental verification of Malhotra's method, as well as a development of a universal mathematical model for determining efficiency of cycloid reducers regardless of concept, as well as an experimental verification of the newly developed model.

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