Determining Sample Size for a Control Chart

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DETERMINING SAMPLE SIZE FOR A CONTROL CHART

by

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A report submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Statistics

Plan B

Approved:

UTAH STATE UNIVERSITY
Logan, Utah

1974

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The essential tool in statistical quality control is the control chart. In spite of the apparent simplicity of the control chart, most engineers, production men, and inspectors find that its use calls for an entirely new point of view. The power of this technique lies in its ability to separate out assignable causes of quality variation. This makes possible the diagnosis and correction of many production troubles and often brings substantial improvements in product quality and reduction of spoilage and rework. The control chart tells when to leave a process alone and thus prevents unnecessarily frequent adjustments that tend to increase the variability of the process rather than to decrease it. It also permits better decisions on engineering tolerances and better comparisons between alternative designs and between alternative production methods.

The central line, upper control limit, lower control limit, initial sample size and subgroup size are important parts in computing control charts. In most books there are no quantitative techniques to establish the size of the initial sample used to calculate the control limits.

A feature of the control chart method is the drawing of inferences about the production process on the basis of samples drawn from the production line. The success of the technique depends upon grouping observations under consideration into subgroups or samples, within which a stable system of chance causes is operating, and between which the variations may be due to assignable causes whose presence is suspected or considered possible. Generally speaking, subgroups should be selected in a way that makes each subgroup as homogenous as possible and that gives the maximum opportunity for variation from subgroup to another. We can say that it is preferable that all samples be of equal size. There has been a great deal of discussion concerning the best size of subgroup to be used with a control chart, see for example Cowden (1), Grant (3), Schrock (6) and Shewhart (7). These results are based primarily on argument with little quantitative justification. There are two errors associated with control charts 1) corresponding to type I error in hypothesis testing, is calling a process out of control when it is in control, and 2) corresponding to a type II error, is calling a process in control when it is actually out of control. Both errors may lead to unnecessary and expensive actions and so both should be considered. The usual method of setting control limits neglects the second error. By specifying an upper bound on the probability of the second error relative to a given shift in the process average the sample size may be determined.

The main purpose of this paper is to present a method for determining the subgroups size given the values P and d_1 where:

d_1: The amount of a shift in the process mean to be detected.

P: The probability of detecting a shift of d_1 in the process average.

The scope of this paper is:

1. To use quantitative techniques to decide what size sample should be taken for subgroups.

2. Opinions and recommendations about how to select a subgroup size will be discussed and summarized.

3. The effect of initial sample size on the subgroup is discussed. Some recommendations on the use of confidence limits is given in connection with initial sample size.

CHAPTER II

OPINIONS ON THE SELECTION OF A SUBGROUP SIZE

It is usually said that it is better to take a large number of small samples than a small number of large samples. But there is a limit to how small a sample should be. On the basis used for calculating control limits any sample size of 2 to 14 is allowable. Cowden (1), Grant (3), Schrock (6) and Shewhart (7) stated that four is the ideal subgroup size. Because if the sample departs widely from the normal form, the distribution of means of small samples may depart too far from normal to justify using the normal curve tables in determining probabilities. The skewness of a distribution of means is

$$\gamma_1(\bar{x}) = \frac{\gamma_1}{\sqrt{N}}$$

while the kurtosis is

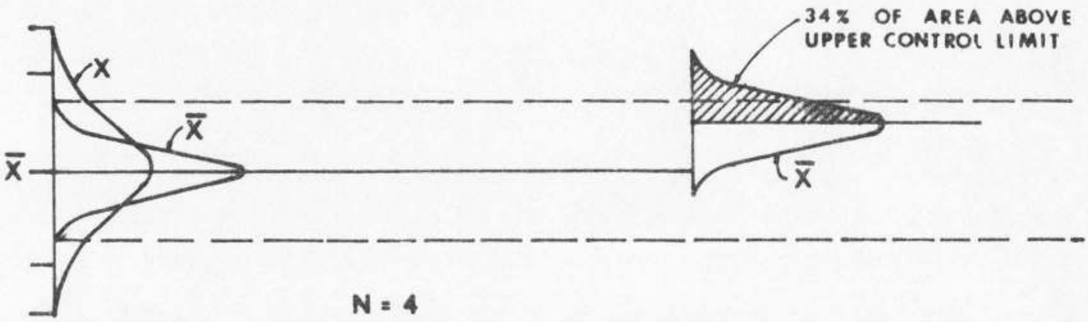
$$\gamma_2(\bar{x}) = \frac{\gamma_2}{N}$$

Thus, if the sample size is 4, the skewness of the distribution of means is only one-half that of the distribution of items, while the kurtosis is only one-fourth that of the distribution of items. One advantage in using a group size of four instead of some larger number is that points are plotted more frequently and indications of lack of control thereby caught more quickly. This is partly offset by the fact that smaller group sizes are less sensitive to small shifts in process levels than are larger groups. This follows from the formula:

$$\delta_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$$

in which $\delta_{\bar{x}}$ is the standard deviation of group averages, σ is the standard deviation of the parent population, and N is the sample size (group size). In other words, the variability of group averages varies inversely as the square root of the sample size. Figure 1 illustrates the relationship. Another advantage is the fact that a small group size will be more likely to contain only inherent variability than a larger group. Still the control limits for individuals can be very easily computed, since they are exactly twice as far from the central line as the control limits for averages of four. Schrock (6) said that it is desirable to avoid group sizes of two or three because it requires too frequent computation of group averages and ranges, and if the parent distribution is markedly non-normal the distribution of group averages of two's or three's will also be definitely non-normal; whereas for group sizes of four or more the distribution of group averages is essentially normal, virtually without regard to the shape of the parent distribution. Thus for group sizes of four or more, the three sigma limits for averages are valid even though the parent population is definitely non-normal.

In the industrial use of the control chart, five seems to be the most common size. Because the essential idea of the control chart is to select subgroups in a way that gives minimum opportunity for variation within a subgroup, it is desirable that subgroups be as small as possible. Still sample groups of five have the advantage that the average of the five numbers observed can be very easily obtained. All that is necessary is to add the five numbers, multiply by two and shift the decimal point one place to the left as follows:



Assume that the process level and variability on individuals (x) are established as shown on the left. In both of the above cases a subsequent shift in process level of one standard deviation of individuals is shown. When the sample size is 4 it will be noted that the probability of a sample average (x) now following above the upper control limit is 0.34. When the sample size is 9 this probability is increased to 0.50.

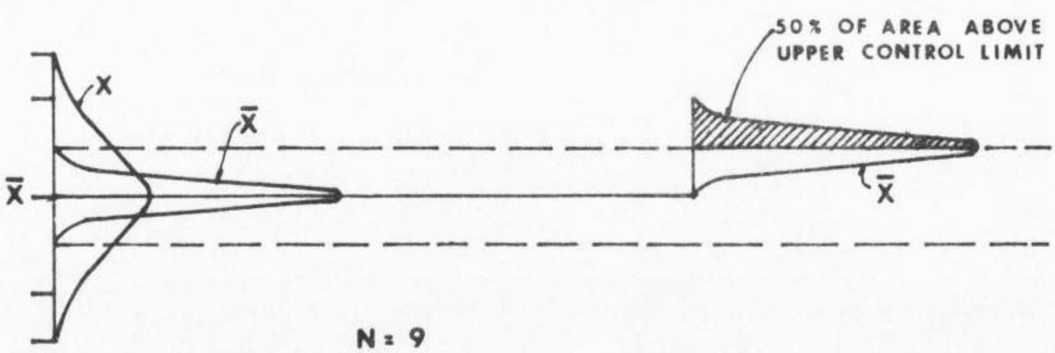


Figure 1. Effect of sample size on variability of sample average.

```
  73
  69
  47
  55
  61
 305 ---> sum of the five observations
 305 ---> adding the same figure is equivalent to multiplying by two
 610
61.0 ---> average obtained by shifting decimal point one place to left
```

One company uses this procedure to advantage on a tape-printing machine. On each run of data there are first printed as non-add numbers the identifying order, part, or chart number, and date. Then the five observations are recorded and subtotaled. The subtotal is then recorded again and a grand total recorded. Now all that is necessary is to shift the decimal point one place to the left and the average is obtained. Next, the highest value in the group is listed. The tape then forms a permanent record for future reference (6, 7).

Conclusion

From the above literature, we know that 4 and 5 are usually recommended subgroup size. In general, it is also recommended that group sizes be kept as small as practicable except that groups of two or three should be avoided.

It has been noted that the subgroup size is very important in determining the probability of a type II error. Therefore a large part of the success that any quality control engineeer will have depends upon how effectively he plans the grouping for his control charts be taken. We should have any reason or any quantitative techniques to judge why we use this particular subgroup size for the control chart. The recommendations in this chapter are not in general very good because they guard against a type I error but essentially ignore the type II error.

(Both type I and type II errors may lead to unnecessary and expensive actions.) In the next chapter a quantitative technique is described which accounts for both the type I and type II errors.

CHAPTER III

METHOD OF DETERMINING THE SUBGROUP SIZE

The information given by the control chart depends on the selection of subgroups

One possible view of a control chart is that it provides a statistical test to determine whether the variation between subgroups is consistent with the variation within the subgroups. If it is desired to determine whether or not a group of measurements is statistically homogeneous (i.e., whether they appear to come from a constant system of chance causes), subgroups should be chosen in a way that appears likely to give the maximum chance for the measurements in each subgroup to be alike and the maximum chance for the subgroups to differ one from the other.

A decision to use relatively wide limits, such as 3-sigma, ensures that type I errors will be rare. This is true regardless of subgroup size. A type I error is serious because it means that effort is expended to find a source of error when none exists. However, the larger the subgroup, the narrower the limits on the X chart and the greater the sensitivity of the X chart to shifts in X. In other words, an increase in subgroup size reduces the frequently of type II errors without the penalty of too frequent type I errors. Cases sometimes arise where relatively large subgroup sizes are justified by the need for prompt detection of small shifts in X.

Every process has a "natural" variation. If this variation is small compared with the product tolerances, a very small proportion of the product will be outside of the tolerances, i.e., unacceptable. In such cases moderate shifts in the process mean are not of great consequence. If, however, the natural variation is large (or the tolerances are tight), a small shift in the process mean may result in a sufficiently large proportion of unacceptable product to make production uneconomical. For this case the standard method of statistical quality control using small subgroups does not provide adequate control. The following method may be used to increase management efficiency by insuring that the quality inspection procedure gives appropriate control.

Notation

In order to facilitate the development of the procedure, the following notation will be used. Let X be the measured quality characteristic of the product in consideration. Let μ_o represent the true optimum mean and μ the true mean of the characteristic X. Define $\sigma = \mu - \mu_o$, the "natural" variance is given by σ and is assumed to be known. Let the specified tolerances be $\mu_o \pm \gamma_\zeta$ and let X be the measured characteristic. Then

$$P(\mu_o - \zeta < X < \mu_o + \zeta) = P\left(\frac{\mu_o - \zeta}{\sigma} < Z < \frac{\mu_o + \zeta}{\sigma}\right)$$

$$= P\left(-\frac{\zeta}{\sigma} < Z < \frac{\zeta}{\sigma}\right) = P$$

The proportion of defective items produced will be 1-P when the process is in control. This information can be used to determine whether or not it is economical to produce. Suppose it is economical to produce the item if $P \geq P$ and uneconomical if $P \leq P$ and let

$$P\left(-\frac{\zeta}{\sigma} < Z < \frac{\zeta}{\sigma}\right) = P$$

Since ζ is the acceptable tolerance for the product, the acceptable range of X is $\mu_o - \zeta \leq X < \mu_o + \zeta$

The process is said to be in control if the true process mean $\mu = \mu_o$.

Let P be the proportion of acceptable items being producted. Then

$$P_{\mu}(\mu_o - \zeta \leq X \leq \mu_o + \zeta) = P \qquad (1)$$

Where $P_{\mu}(\mu_o - \zeta \leq X \leq \mu_o + \zeta)$ means the probability that $(\mu_o - \zeta \leq X \leq \mu_o + \zeta)$ when the process mean is μ. Let P be the proportion of good items when the process is in control. Thus

$$P_{\mu_o}(\mu_o - \zeta < X < \mu_o + \zeta) = P_o$$

Let P' be the "break even" proportion, i.e., the proportion of acceptables below which production is uneconomical.

Method of determining subgroup size

The procedure will be developed first from a theoretical view. Following this development, an example is presented to illustrate the method as it might be used in application.

The key to this procedure rests in the following logic. It seems reasonable that a shift in the process mean is sufficient to produce defectives in the proportion $1-p'$ is at least as serious as the problems encountered when a type I error is made. Thus such a shift should be detected with approximately the same probability as that used to protect against a type I error. If the sanctity of the time honored 3σ limits is preserved, this probability is 0.9974.

The allowable shift (δ) in the process mean is computed using equation (1) by writing

$$P_{\delta+\mu_o}(\mu_o - \zeta \leq X \leq \mu_o + \zeta) = P_{\delta+\mu_o}\left(\frac{-\zeta-\delta}{\sigma} \leq Z \leq \frac{\zeta-\delta}{\sigma}\right) \geq 1-P'$$

Where Z is the standard normal random variable. The value δ can be assumed to be positive without loss of generality. Then the inequality

$$1-P' \leq P(\frac{-\zeta-\delta}{\sigma} \leq Z \leq \frac{\zeta-\delta}{\sigma}) \leq P(Z \leq \frac{\zeta-\delta}{\sigma})$$

holds. Thus

$$\frac{\zeta-\delta}{\sigma} \cong Z_{1-p'} \quad \text{or} \quad \delta \cong \zeta - \sigma Z_{1-p'} \qquad (2)$$

Where $Z_{1-p'}$ is value of a standard normal variable such that

$$P(Z \leq Z_{1-p'}) = 1-P'$$

Using 3σ limits, the required sample size n is found as the solution to

$$P_{\mu_o+\delta}(\mu_o-3\sigma/\sqrt{n} \leq \bar{X} \leq \mu_o+3\sigma/\sqrt{n}) \leq 0.0026$$

or

$$P(\frac{-\delta\sqrt{n}}{\sigma} - 3 \leq Z \leq \frac{-\delta\sqrt{n}}{\sigma} + 3) \leq 0.0026.$$

Approximately

$$P(\frac{-\delta\sqrt{n}}{\sigma} -3 \leq Z \leq \frac{-\delta\sqrt{n}}{\sigma} +3) \leq P(Z \leq \frac{-\delta\sqrt{n}}{\sigma} + 3) = 0.0026$$

or

$$-\frac{\delta\sqrt{n}}{\sigma} + 3 \cong -2.80$$

or

$$n = (\frac{5.80\ \sigma}{\delta})^2 \qquad (3)$$

Now substituting (2) into (3), the complete solution for n is

$$n = (\frac{5.80\ \sigma}{\zeta-\sigma Z_{1-p'}})^2 \qquad (4)$$

Example 1. Suppose a company is engaged in manufacturing a casting for which the machine width of 2.5+0.001 inches is specified. From an initial large sample it is found that the standard deviation of the width is $\sigma = 0.0003$ inches. Suppose that if 20% of the castings are defective no profit is made. Then from (4) where $Z_{1-p'} = 0.842$

$$n = [\frac{5.80\ (0.0003)}{0.001-(0.0003)(0.842)}]^2 \sim 6$$

Example 2. Consider the same situation as Example 1; however, suppose now that $\sigma = 0.0006$. Then

$$n = [\frac{5.80(0.0006)}{0.001-(0.006)(0.842)}]^2 \sim 26$$

Example 3. Again consider the situation of Example 1. Now suppose that only 10% defective results in serious loss. For this case

$$n = [\frac{5.80(0.0003)}{0.001-(0.0003)(1.282)}]^2$$

$$= [\frac{0.00174}{0.0006154}]^2 \sim 7$$

Example 4. A rheostat knob produced by plastic molding contained a metal insert purchased from a vendor. A particular dimension determined the fit of this knob in its assembly. This dimension, which was influenced by the size of the metal insert as well as by the molding operation, was specified by the engineering department as 0.140+0.003. Many molded knobs were rejected on 100% inspection with a go and no-go gage for failure to meet the specified tolerances. From an initial large sample it is found that the standard deviation of the dimension is $\sigma = 0.00095$, and there is approximately 31% defective.

From Example 4, where $Z_{1-p'} = 0.52$, $P' = 0.31$

$$n = (\frac{5.80(0.00095)}{0.003-(0.00095)(0.52)})^2$$

$$= (\frac{0.05510}{0.003-0.00494})$$

$$= (\frac{0.00551}{0.002506})^2 \sim 5$$

Example 5. Consider the same situation as Example 4; suppose now that $\sigma = 0.0019$. Then the subgroup size is

$$n = \left(\frac{5.80(0.0019)}{0.003-(0.0019)(0.52)}\right)^2$$

$$= \left(\frac{0.01102}{0.00202}\right)^2 \sim 25$$

From the above examples it can be seen that the sample size is very sensitive to σ . Therefore, it is important that the estimate of σ be accurate. In the next chapter a method is discussed which aids in determining the adequacy of the initial sample size.

CHAPTER IV

THE INITIAL SAMPLE SIZE

From Chapter III we know that the subgroup size is very sensitive to σ . Therefore, it is important that the estimate of σ be accurate. Some method of determining the adequacy of σ is needed. The best estimate of σ^2 is considered to be:

$$S^2 = \frac{\Sigma(\chi i - \bar{x})^2}{N - 1}$$

The term "best" is used because S^2 is unbiased and has minimum variance among unbiased estimator of σ^2 . Since this estimate is unbiased it may be either too large or too small in an unpredictable manner. Thus it might be considered useful to use a conservative estimate which is biased. An upper confidence bound might be used in place of the best estimate of σ . If this is done, there is very little chance of detecting a shift in the process mean, especially if the initial sample size is small. Therefore, it is not recommended that the upper confidence bound be used. However, by comparing the subgroup sizes calculated from both the upper confidence bound and the best estimate of σ , an intuitive judgment as to the adequacy of the initial sample size is possible. The examples which follow illustrate the technique.

A confidence interval on σ^2 is derived using the following:

"If S^2 is the variance of a random sample of size n from the normal population $N(X; \mu, \sigma^2)$, then $(n-1)s^2/\sigma^2$ has a chi-square distribution with n-1 degrees of freedom (ostle). We can, thus, assert with a probability of $1-\alpha$ that the random variable, $(n-1)s^2/\sigma^2$ assumes a value greater than $\chi^2\alpha$, n-1 or with a degree of confidence of $1-\alpha$ that for a given sample

$$\chi^2\alpha,\ n-1 < \frac{(n-1)S^2}{\sigma^2}$$

or

$$\sigma^2 < \frac{(n-1)S^2}{\chi^2\alpha,\ n-1}$$

The last equation is defined to be a level $(1-\alpha)$ confidence bound on σ^2 .

Here we can study the relation of the sample size with the σ .

Example 4.1. Consider the situation in Example 1. The confidence level is 0.95 s=0.0003 let α =0.05 so when n=51

$$\chi^2\alpha,\ n-1 = \chi^2\ 0.05,\ 50 = 34.6$$

$$\sigma^2 = \frac{(51-1)(0.0003)^2}{34.6}$$

$$\sigma^2 = \frac{0.0000045}{34.6}$$

$$\sigma = 0.0003$$

When n=6

$$\chi^2\ 0.05\ 5 = 1.145$$

$$\sigma^2 < \frac{(6-1)(0.0002)^2}{1.145}$$

$$\sigma^2 < \frac{0.00000045}{1.145}$$

$$\sigma < 0.0006$$

Now we use the confidence bound on σ^2 as two separate estimates of σ^2 .

From Example 4.1 when n=6 $Z_{1-p'}$ =0.842 ζ =0.001.

When $\hat{\sigma}$=0.006 an upper bound confidence estimate of the subgroup size is

$$n = \left(\frac{5.80(0.0006)}{0.001-(0.0006)(0.842)}\right)^2$$

$$= \left(\frac{0.00348}{0.00949}\right)^2$$

$$= (0.38)^2 = 0.14$$

The subgroup size calculated from the two estimates are very different, so it seems the initial sample size of 6 is too small.

If n=51 $\hat{\sigma}$=0.0003 an upper bound confidence estimate of the subgroup size is

$$n = \left(\frac{(5.80)\ (0.0003)}{0.001-(0.0003)(0.842)}\right)^2$$

$$= \left(\frac{0.001740}{0.0007474}\right)^2$$

$$= (2.1)^2 = 4.41 \sim 4$$

The two subgroup sizes are very close together, so it seems that the initial sample size of 51 is sufficient.

From the Example 4.1 we know that the estimate σ when n=6 the two subgroup sizes are very different, but the estimate σ when n=51 the two subgroup sizes are very close together; so we know that the initial sample size n=51 is big enough, and the estimate σ is a good estimate.

CHAPTER V

CONCLUSION

The control chart is an important tool in statistical quality control. By using the control chart it is possible to not only detect that trouble exists but also to locate its cause. It has been noted that the type II error has been essentially ignored in the result quality control method. Also the detection of a shift in the process mean relies directly on the probability of a type II error. Thus, the subgroup size is very important.

In most books there is a great deal of discussion concerning the subgroup size to be used with the control chart. However, no quantitative technique to calculate the size of subgroup has been given. Grant (3) had discussed these two type errors, but there is no method given to determining the subgroup size. In Chapter III a formula is developed which used 3σ limits to reduce the type I error; also, we use the probability of detecting a shift in the process average to determine the subgroup size.

The problem may arise that the recommended subgroup size to detect the desired shift is too large. In that case it is possible to reformulate the problem in such a way that the quality control engineer can evaluate the effectiveness of the subgroup size he is willing to use. Using the magnitude of shift he has indicated that he wants to detect and the subgroup size given, it is possible to determine the probability of detecting the shift.

Example: Suppose the following data is presented for determination of sample size-- $\sigma = .0006$, $d_1 = .0003$. By using the formula of Chapter III, the subgroup size is found to be 26. Suppose that the engineer feels this is too expensive and says he can go no larger than 6. Then the probability of detecting a shift of size δ is

$$P\left(\frac{-\delta\sqrt{n}}{\sigma} - 3 \leq Z \leq \frac{-\delta\sqrt{n}}{\sigma} + 3\right) \leq P\left(Z \leq \frac{-\delta\sqrt{n}}{\sigma} + 3\right)$$

$$P\left(Z \leq \frac{-0.0003\sqrt{26}}{0.0006} + 3\right) = P\left(Z \leq -\frac{0.0003(5.1)}{0.0006} + 3\right) = P(Z \leq -2.55 + 3)$$

$$= P(Z \leq 0.45) = 0.67$$

With this information the engineer can evaluate the costs relative to the type II error in order to determine whether or not the sample size of 6 gives him adequate protection (i.e., 33% of the time he will not be able to detect a shift of δ). He may wish to increase his sample size, or he may find that the d_1 he picked was not realistic and may increase it without serious consequences.

APPENDIX

VALUES OF $\chi^2_{\alpha,\nu}$ *

ν	α = .995	α = .99	α = .975	α = .95	α = .05	α = .025	α = .01	α = .005	ν
1	.0000393	.000157	.000982	.00393	3.841	5.024	6.635	7.879	1
2	.0100	.0201	.0506	.103	5.991	7.378	9.210	10.597	2
3	.0717	.115	.216	.352	7.815	9.348	11.345	12.838	3
4	.207	.297	.484	.711	9.488	11.143	13.277	14.860	4
5	.412	.554	.831	1.145	11.070	12.832	15.086	16.750	5
6	.676	.872	1.237	1.635	12.592	14.449	16.812	18.548	6
7	.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278	7
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955	8
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589	9
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188	10
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757	11
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300	12
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819	13
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319	14
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801	15
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267	16
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718	17
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156	18
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582	19
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997	20
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401	21
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796	22
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181	23
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.558	24
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928	25
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290	26
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645	27
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993	28
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336	29
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672	30

* This table is based on Table 8 of *Biometrika Tables for Statisticians, Volume I*, by permission of the *Biometrika* trustees.

Appendix B. Areas under the normal curve

Proportion of total area under the curve that is under the portion of the curve from $-\infty$ to $\frac{X_i - \bar{X}'}{\sigma'}$. ($X_i$ represents any desired value of the variable X)

$\frac{X_i - \bar{X}'}{\sigma'}$	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
−3.5	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023
−3.4	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00033	0.00034
−3.3	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048
−3.2	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069
−3.1	0.00071	0.00074	0.00076	0.00079	0.00082	0.00085	0.00087	0.00090	0.00094	0.00097
−3.0	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135
−2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0017	0.0018	0.0019
−2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
−2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
−2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
−2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
−2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
−2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
−2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
−2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
−2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
−1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
−1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
−1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
−1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
−1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
−1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
−1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
−1.2	0.0985	0.1003	0.1020	0.1038	0.1057	0.1075	0.1093	0.1112	0.1131	0.1151
−1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
−1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
−0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
−0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
−0.7	0.2148	0.2177	0.2207	0.2236	0.2266	0.2297	0.2327	0.2358	0.2389	0.2420
−0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
−0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
−0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
−0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
−0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
−0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
−0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

Appendix B. Continued

$\frac{X_i-\bar{X}}{\sigma}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
+0.1	0.5398	0.5438	0.5473	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
+0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
+0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
+0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
+0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7068	0.7123	0.7157	0.7190	0.7224
+0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517.	0.7549
+0.7	0.7580	0.7611	0.7642	0.7873	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
+0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8079	0.8106	0.8133
+0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
+1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
+1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
+1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8950	0.8997	0.9015
+1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
+1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
+1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
+1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
+1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
+1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9685	0.9693	0.9699	0.9706
+1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
+2.0	0.9773	0.9778	0.9783	0.9783	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
+2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
+2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
+2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
+2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
+2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
+2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
+2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
+2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
+2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
+3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
+3.1	0.99903	0.99906	0.99910	0.99913	0.99915	0.99918	0.99921	0.99924	0.99926	0.99929
+3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
+3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
+3.4	0.99966	0.99967	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
+3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

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