

DETERMINISTIC AND STOCHASTIC PETRI NET  
MODELS OF PROTECTION SCHEMES

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PETRI NETS

Abstract - Protection schemes are modelled through Petri Nets, in order to carry out performance evaluation. Marked Petri Nets are suitable for qualitative evaluation, while Time Petri Nets and Timed Petri Nets are convenient for cycle time evaluations, and for deriving relationships between the time parameters of the primary and the secondary protection. Stochastic Petri nets are used for the modelling of the stochastic nature of protection; they enable the evaluation of probabilistic performance measures, and have advantages over simulation techniques.

INTRODUCTION

When a fault occurs in a power system, the protective relaying scheme has to carry out the tasks of detecting the fault, and isolating it. These tasks must be carried out reliably, with the minimum delay, and with minimum disruption of supply to the consumer loads. Traditionally, protection engineers have used their knowledge of the character of the power system, and a set of design rules, in order to design its protection scheme. With the development of very complex power systems, which have very stringent performance requirements, it is desirable that tools be developed for the systematic design of the protective schemes of such systems, as well as for the performance evaluation of alternative protection schemes. [1,2,3] Petri Net models are a means of representing protection schemes; they enable one to ensure that these schemes possess desirable qualities, they permit the comparison of different schemes, they enable one to specify coordination requirements between the timing parameters of the primary and backup protection, and they can be used for performance evaluation. The Petri Net model enables one to study the protection scheme independent of the dynamics of the power system.

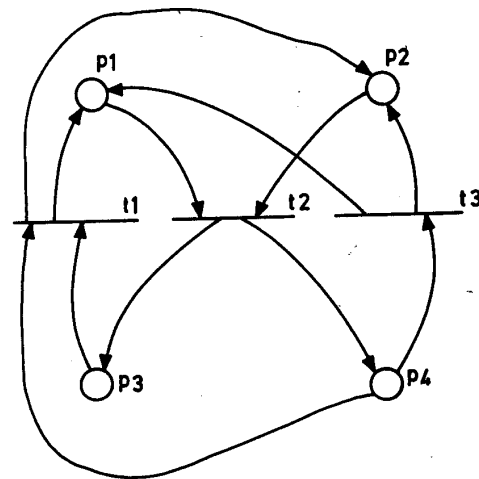
In this paper, we will establish the relevance of Petri Net modelling to protection studies; we will illustrate how the different Petri Net models capture the information that is of importance to both qualitative and quantitative evaluation of protection schemes. The marked Petri Nets allow one to evaluate the qualitative performance measures of conservativeness, safeness and properness: Time Petri Nets are able to represent the timing parameters of protection schemes, and serve as a tool for the establishment of relationships between these parameters, in order that the coordination between primary and backup protection is ensured. An alternative means of representing timing information is the timed Petri Net, which is useful in the computation of the cycle times of the operational cycles of protection schemes. Stochastic Petri Nets adequately represent the stochastic nature of protection; we will demonstrate their advantage over simulation techniques as a means of computing the mean sojourn time and the steady state probability of occupancy of each state of the protection scheme.

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Petri Nets were developed for the modelling of computer systems and communication protocols [4,5]. Such systems are discrete-event systems; in such systems the state remains unchanged, until an event takes place, thereby causing the system to instantaneously switch to a different state. Although the power system itself is a continuous time system, its protection scheme may be viewed as a discrete/event system, and modelled through Petri Nets. The occurrence of a fault is a discrete event which instantaneously changes the power system from a fault-free condition to a faulted condition. Similarly the clearing of a fault and the resetting of a relay are discrete events which change the state.

Although formal definitions of Petri Nets (PN's) are available [6], we will find it convenient to visualize a PN as a directed bipartite graph, in which there are two disjoint sets of nodes, and two disjoint sets of edges. The set of  $n$  place nodes  $P=(p_1, p_2, \dots, p_n)$  are represented by circles, and the set of  $m$  transition nodes  $T=(t_1, t_2, \dots, t_m)$  are represented by bars, as shown in Fig.1, where  $n=4$  and  $m=3$ . The directed edges IN are incident out of place nodes and into transition nodes; individual members of this set are denoted by  $IN(p_i, t_j)$ . The directed edges OUT are incident out of transition nodes and into place nodes; members of this set are denoted by  $OUT(p_i, t_j)$ . In Fig.1, the set IN consists of 5 edges, and the set OUT has 6 edges. The set of input nodes of a transition  $t_j$  is the set of places  $p_i$  for which  $IN(p_i, t_j)$  is a member of IN; similarly, the set of output nodes is the set of places  $p_i$  for which  $OUT(p_i, t_j)$  is a member of OUT. In Fig.1, the set of input places for transition  $t_1$  is  $[p_3, p_4]$ , and the set of output places is  $[p_1, p_2]$ .



**Fig.1. Petri Net**

The topological information of each PN graph is contained in its incidence matrix,  $C$ , which has  $n$  rows and  $m$  columns. The element  $C[i, j]$  has the value  $-1$  if the place  $p_i$  is an input node of transition  $t_j$ , it has a

value +1 if place  $p_i$  is an output node of transition  $t_j$ , and it has a value of 0 otherwise. The incidence matrix of the PN of Fig 1 is

$$C = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \\ -1 & 1 & -1 \end{bmatrix}$$

Obviously,  $C$  is defined only if no node is both an input node and an output node of the same transition.

As it will be shown in the next section, the incidence matrix is useful for checking whether the marked PN of a protection scheme is conservative; this will guarantee that the scheme possesses the property of conservativeness. The incidence matrix will also be shown to be of relevance in detecting the cycles in the PN model, thereby facilitating the evaluation of the cycle times of the protection scheme.

#### MARKED PETRI NETS

The marked PN consists of a PN graph  $G$ , along with a marking,  $M$ . This marking associates with each place  $p_i$  a non-negative integral number  $m_i$  of tokens, as represented by  $m_i$  dots at the node  $p_i$ . This marking is expressed as an  $n$ -vector, whose  $i$ th component is  $m_i$ . The PN of Fig 2 has the marking  $(1, 1, 0, 1, 0, 0)^T$ .

The simulation of a discrete event is simulated by the firing of the corresponding transition. The firing of a transition  $t_j$  removes a token from each of its input places, and deposits a token in each of its output places; the markings of the remaining places are unaffected. For instance, if the original marking of the PN of Fig 2 is as shown, then the firing of  $t_1$  produces the marking  $(0, 1, 1, 0, 0, 0)^T$ .

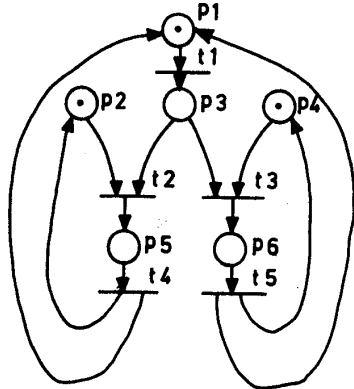


Fig. 2. Marked PN of Protection Scheme

Since no place can have a negative number of tokens, transition is not allowed to fire if any of its input places do not have at least one token. If any transition has a token at each of its input places, then it is said to be 'enabled'; only enabled transitions may fire. For instance, when one has the marking shown in Fig 2, the only enabled transition is  $t_1$ . Each time a transition fires, it changes the marking, thereby enabling some more transitions, and disabling some others. For the PN of Fig 2, the firing of  $t_1$  produces the marking  $(0, 1, 1, 0, 0, 0)^T$ , in which only  $t_2$  and  $t_3$  are enabled. If  $t_2$  fires, then we get the marking  $(0, 0, 0, 1, 1, 0)^T$ , in which only  $t_4$  is enabled; the firing of  $t_4$  returns the PN to the original marking  $(1, 1, 0, 1, 0, 0)^T$ . Otherwise, if  $t_3$  fires, the marking  $(0, 1, 0, 0, 0, 1)^T$  is produced; then  $t_5$  fires, to return the PN to the original marking.

Reachability is a useful concept of marked PNs. Each initial marking  $M_0$  has a reachability set associated with it; this set consists of all the markings which can be reached from  $M_0$  through the firing of one or more transitions.

Each marking which can be reached from the initial marking is referred to as a state. The reachability information is represented through a reachability graph, in which each node corresponds to a state, and the edges are associated with transitions. A directed edge is incident out of node  $M_1$  and into node  $M_2$  if and only if there exists a transition  $t_i$  whose firing changes the initial marking  $M_1$  to the marking  $M_2$ ; the edge bears the label  $t_i$ . The reachability graph of the PN in Fig 2 is given in Fig 3. Reachability graphs enables us to find the reachability set of any state  $M_i$ ; one merely locates all the nodes which can be reached from  $M_i$  by the traversal of directed paths.

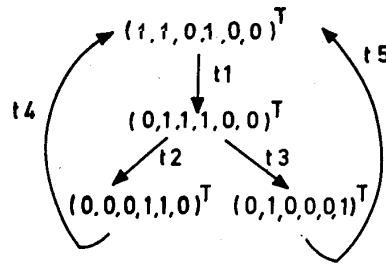


Fig. 3. Reachability Graph.

In the representation of protection schemes through marked PNs, one maps each condition to a place node, and each event to a transition node; the initial marking is obtained by depositing a token in every place node which corresponds to an initial condition of the protection scheme.

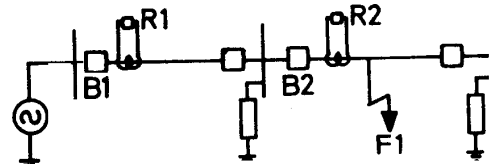


Fig. 4. Overcurrent Relaying Scheme

In the overcurrent relaying scheme of Fig 4, the primary relay  $R_1$  trips breaker  $B_1$ , to isolate the fault  $F_1$ ; relay  $R_2$  and breaker  $B_2$  correspond to backup protection. This scheme is modelled through the PN of Fig 2. The initial conditions are the absence of the fault, and the readiness of the relays  $R_1$  and  $R_2$  to respond to it; these correspond to the places  $p_1$ ,  $p_2$  and  $p_4$  respectively. A token is deposited at each of these places, as shown in Fig. 2, to represent the initial condition. The other possible conditions of the protection system are the presence of  $F_1$ , and its being isolated through  $B_1$  and  $B_2$  respectively; these conditions correspond to  $p_3$ ,  $p_5$  and  $p_6$ . The occurrence of  $F_1$  is represented by the transition  $t_1$ , which deposits a token at  $p_3$ , to indicate that the fault is present. The new state  $(0, 1, 1, 0, 0, 0)^T$ , is as shown in Fig. 3. Now  $t_2$  and  $t_3$ , corresponding to primary and backup relay operation respectively are both enabled. If  $t_2$  fires, then a token is deposited at  $p_5$ , and the system can be restored through primary protection, corresponding to the firing of  $t_4$ , with the initial state being reached, as shown

in Fig. 3. If  $t_3$  fires, then restoration through backup protection takes place, as represented by the firing of  $t_5$ .

A protection scheme is conservative [7] if it does not lose any information which is of relevance to protection, and if it does not generate any spurious information. This information consists of the presence of faults and the readiness of the protection elements to respond to faults. Since the marked PN represents these conditions through tokens in the place nodes, the protection scheme will be conservative if its marked PN model cannot either lose or generate tokens. This is the conservativeness property of PN's. A PN has this property if there exist positive integers  $w_1, w_2, \dots, w_n$  such that for every transition  $t_j$ , the markings  $M$  and  $\hat{M}$  prior to and after the firing of  $t_j$  satisfy the relationship.

$$\sum_{i=1}^n w_i \cdot M(p_i) = \sum_{i=1}^n w_i \cdot \hat{M}(p_i)$$

This property can be determined from the incidence matrix  $C$ . If one can find a vector  $Y$ , all of whose components are positive integers, such that  $C^T Y = 0$ , then the PN has the conservativeness property. In the case of the PN of Fig 2, we can choose  $Y = (1, 1, 1, 1, 1, 2, 2)^T$  to satisfy this condition, and hence the protection scheme of Fig 4 is conservative. It has been shown [8] that unless care is taken to save all relevant information, one may not obtain conservativeness in more complicated protection schemes.

A protection scheme is proper if it has the ability to return to its normal condition after it has experienced a fault. Since the normal condition corresponds to the initial marking of the PN model, the scheme will be proper if for each state that belongs to the reachability set of the initial state in the PN model, there exists a sequence of transitions whose firing will return the system from this state to the initial state. This is the properness property of marked PN's.

In the reachability graph of Fig. 3, there is a directed path of transition edges from each other state to the initial state. Hence the PN of Fig. 2 has the properness property, and the protection scheme of Fig. 4 is proper.

The third qualitative performance measure is safeness. A protection scheme is safe if there can never be an attempt to reset a relay which has already been reset. In the marked PN model, the resetting of a relay is represented by the depositing of a token in the appropriate place node; hence, safeness of the protection scheme is guaranteed if no sequence of transitions can deposit a token in any relay place node when it already has a token; this is the safeness property of the relay places of the PN model, which can be expressed as

$$M(p_i) \leq 1$$

where  $p_i$  is a relay place.

In Fig 2, the relay places are  $p_2$  and  $p_4$ . From the reachability graph of Fig 3, it can be seen that for every state, both  $M(p_2)$  and  $M(p_4)$  are equal to either 0 or 1, and hence they have the safeness property. Hence the protection scheme of Fig 4 is safe.

#### RECOVERABILITY ANALYSIS

The Time Petri Net is of relevance to recoverability analysis. This PN employs the graph structure and tokens of the marked PN, along with additional timing information [9]. While it has the same restriction that only enabled transitions may fire, there are

additional constraints on the firing interval. Whenever a transition  $t_i$  is enabled (by the firing of some other transition  $t_j$ ), at least  $T_{\min,i}$  time units must elapse before  $t_i$  is allowed to fire; if it has not fired before the instant  $T_{\max,i}$  (measured from the instant of enabling), then at  $T_{\max,i}$  it must fire.

We will use recoverability analysis to obtain the timing relationship between the primary and backup protection for the protection scheme of Fig 4. We use the time PN of Fig 5, which has a more detailed graph than that of Fig 2, we represent only the primary protection. The places  $p_1$  through  $p_8$  correspond respectively to the condition of absence of a fault, relay able to sense the fault, relay set, fault present, trip signal sent, fault isolated, system ready for

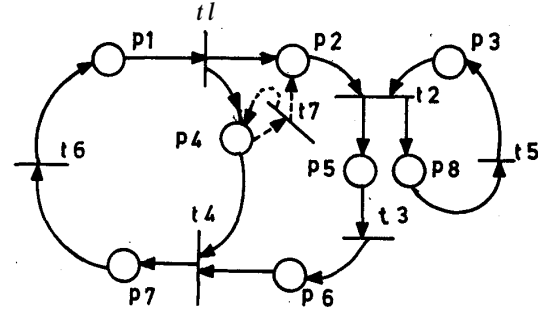


Fig. 5. Time Petri Net

restoration, and relay tripped. The transitions  $t_1$  through  $t_6$  correspond respectively to the occurrence of the fault, relay response, breaker opening, fault clearance, relay resetting and system restoration. Transition  $t_7$ , shown with dotted lines, is ignored at this stage. The parameters  $T_{\min,i}$  and  $T_{\max,i}$  are chosen from physical considerations, and form part of the design.

From Fig 5, we see that  $p_4$  receives a token at the same instant as does  $p_2$ , and hence  $t_4$  is enabled as soon as  $p_6$  receives a token. From the loops of the graph, one gets

$$T_{\max 5} \leq [T_{\min 3} + T_{\min 4} + T_{\min 6} + T_{\min 1}]$$

If this relationship is satisfied, then the relay is ready to respond to a fault whenever it can occur. Failure occurs if the malfunction of a relay prevents it from detecting a fault; this is simulated by the loss of a token from place  $p_2$ . Such behaviour is analyzed through an error token machine (ETM). The ETM of Fig 6 is obtained from the reachability graph of the PN of Fig 5 by the addition of the node  $(p_3, p_4)$ , and the edge  $p_2$ , which is incident out of node  $(p_2, p_3, p_4)$  and into node  $(p_3, p_4)$ ; the edge shown with dotted lines will temporarily be ignored. State  $(p_3, p_4)$  is illegal, since it is reached by traversing the edge  $p_2$ , which correspond to the illegal operation of the loss of a token; it is also a final state, because there are no edges (transitions) incident out of it. Because the ETM has an illegal final state, recovery from the failure is not possible; this conforms with our understanding that a system with only primary protection cannot recover from the failure of the primary relay to respond [8].

The introduction of the edge  $t_7$  in the ETM converts the illegal state  $(p_3, p_4)$  to a non-final state, thereby, facilitating recovery. This transition  $t_7$  must

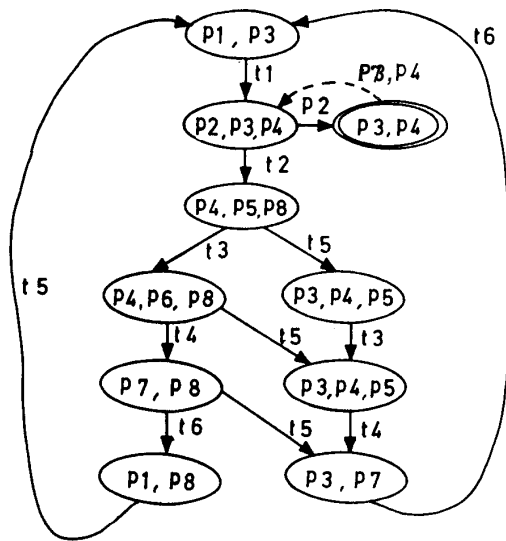


Fig. 6. ETM of Petri Net .

introduce a token at  $p_2$ , to produce the state  $(p_2, p_3, p_4)$ ; the implementation is shown by the dotted line in the PN of Fig 5. The firing of  $t_7$  activates the backup protection, by replacing the lost token in  $p_2$ . Rather than complicate the PN model by introducing new places and transitions to correspond to backup protection, we use the ones that correspond to primary protection, with the understanding that after  $t_7$  fires, the timing parameters of transitions  $t_2$  through  $t_6$  will correspond to backup protection.

Since backup protection should not be actuated before the primary protection has had sufficient time to act, transition  $t_7$  must always take longer to fire than the largest possible time taken by the sequence  $t_2, t_3, t_4$

$$T_{min7} > [T_{max2} + T_{max3} + T_{max4}]$$

where these parameters refer to primary protection. This constraint is consistent with known coordination criteria, and demonstrates the relevance of time PN modelling.

We next consider a more complicated system, in which there are two parallel lines. Under light load, their protection schemes are independent. However, under heavy loads the tripping of the breaker on line 1 will overload line 2, perhaps causing it to trip, although it is fault-free. This undesirable behaviour must be avoided. We analyze the situation through the time PN of Fig 7, which is obtained by duplicating Fig 6, and using the additional index to distinguish the places and transitions of line 1 from those of line 2; for now, transition  $t_{18}$  will be ignored.

Under light load, protection against a fault on line 1 is simulated through the firing of transitions  $t_{11}$  through  $t_{17}$ . Under heavy load, a fault on line 1 overloads line 2; this spurious behaviour is simulated by the depositing of a spurious token at  $p_{22}$  at the instant of the firing of  $t_{13}$ . If there has been no fault on line 2, then  $p_{24}$  has no token, because  $t_{21}$  has not fired. The illegal token enables  $t_{22}$ , whose firing corresponds to the illegal issue of a trip signal from the primary relay on line 2. To prevent this, one must remove the extra token from  $p_{22}$  by firing  $t_{18}$ . Since the firing of  $t_{13}$  has simultaneously enabled  $t_{14}$ ,  $t_{18}$

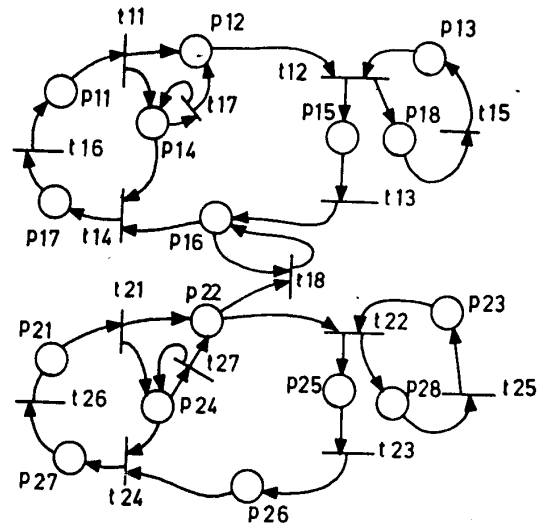


Fig. 7. Time Petri Net of Two-line System.

and  $t_{22}$ , and we require that  $t_{18}$  fires first, spurious breaker opening is prevented if one chooses  $T_{max18}$  to be less than both  $T_{min14}$  and  $T_{min22}$

$$T_{max18} < \min[T_{min14}, T_{min22}]$$

Similarly, one selects a transition  $t_{28}$  to prevent the spurious tripping of line 1 when a fault occurs only on line 2. It has been shown [8] that in the comparatively infrequent case of occurrence of simultaneous faults on both the lines, the fault clearance will be delayed, but the behaviour of the system will be correct.

The interpretation of transition  $t_{18}$  is that when the relay on line 2 senses a fault, it ignores it (removes the token) if it senses that the breaker on line 2 has opened (token at  $p_{16}$ ). Hence maloperation is prevented only if the relay on line 2 has information with regard to the status of the breaker on line 1.

#### CYCLE TIME EVALUATION

Every protection scheme has a collection of operational cycles associated with it. Each relay is initially set; when a fault occurs within its zone of protection, it senses the fault, issues a trip signal, and then resets, thereby completing a cycle of operation. The power system itself has cyclic behaviour; it passes from the normal state to the faulted state, after which the fault is sensed and cleared, and then restoration takes place, once again obtaining normal operation. Cycle times can be associated with these cycles. The cycle times are a quantitative performance measure of the protection scheme. Since the time taken to locate and clear a fault will depend on its nature and location, the cycle time is not constant, but an estimate of the minimum value could be obtained from a Timed Petri Net model [10].

The timed PN has the same topological structure as the marked PN, along with a single time parameter  $z_i$  being associated with each place  $p_i$ . In the timed PN model, whenever a token is deposited in a place  $p_i$ , it remains inactive in this place for an interval of  $z_i$  time units, at the end of which it becomes available. A transition fires at the first instant that it has a token available at each of its input places. The timed PN model of the protection scheme of Fig 4 is given in Fig 8. The places  $p_1, p_2, p_3, p_4$  and the transitions

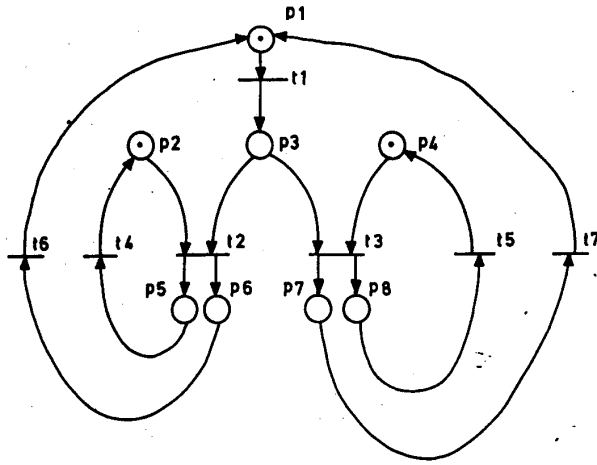


Fig. 8. lamed Petri Net Model

t1,t2,t3 have the same significance as their counterparts in the marked PN of Fig 2. However, since the transition t4 of Fig 2 is associated with both the resetting of the relay and the restoration of the system, and these two operations require different amounts of time, we must replace t4 by two transitions t4 and t6 in the timed PN model of Fig 8, and we similarly replace p5 by the two places p5 and p6. The places p7 and p8 and the transitions t5 and t7 have the same justification. The time parameters are z1 through z8.

Although in this case, it is easy to identify the cycles of the PN, we will use the more general approach of computing them from the incidence matrix C, which is given by

$$C = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let us consider any solution X to the equation CX=0; this solution X is known as a transition invariant or T-invariant of the PN. In particular, let us select all such transitions whose entries are either 0 or 1. Each of these solutions corresponds to a set of transition whose firing returns the PN to its original marking, thereby completing a cycle of operation. For instance, in this case, the only two such T-invariants are (1,1,0,1,0,1,0) and (1,0,1,0,1,0,1), and these correspond to the sets of transitions (t1,t4,t6) and (t1,t3,t5,t7) respectively.

Each of these sets of transitions can be used to construct a subnet, which consists of the corresponding transitions, and the places that are associated with them. The subnets of the PN of Fig 8 are shown in Fig 9. Each subnet itself consists of a number of cycles, and the minimum cycle time of the subnet can be obtained by computing the largest of the cycle times of these cycles, since the cycle of the subnet can complete only after all the cycles in it complete. The cycles within each subnet are identified from its incidence matrix, which is obtained by selecting the appropriate columns of the incidence matrix C of the original PN. For instance, the incidence matrix C1 and C2 of the subnets of Fig 9 are obtained by selecting the 1st, 2nd,4th,6th

and 1st,3rd,5th,7th columns respectively of C, since; these correspond to the 1 entries of the two selected T-, invariants.

$$C1 = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C2 = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

The all-zero rows of C1 and C2 correspond to those places that do not belong to the subnet; although their entries do not help to identify the cycles, we retain them, to maintain consistency of notation.

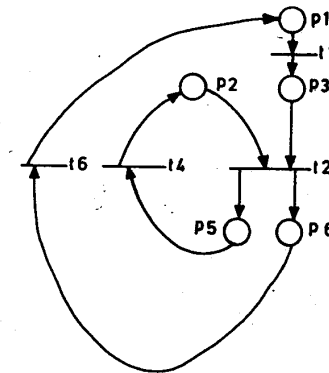
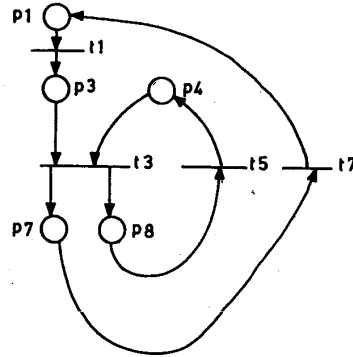


Fig. 9. Subnets of Timed Petri Net

The cycles within subnet 1 are obtained by solving the equation C1Y=0, such that the entries of Y are either 0 or 1. In this case, the only two such solutions are [1,0,1,0,0,1,0,0] and [0,1,0,0,1,0,0,0]; the corresponding cycles are (p1,p3,p6) and (p2,p5). For the other such subnet, one obtains the corresponding cycles (p1,p3,p7) and (p4,p8).

We now compute cycle time for the primary protection, which corresponds to the first subnet of Fig 9. In this subnet, the places are p1,p2,p3,p5,p6, and their corresponding time parameters are z1,z2,z3,z5,z6. Specifically, we wish to find the minimum time that can elapse between the restoration of the system by the operation of the protection scheme, and the next occurrence of a fault, in order that it be handled by the primary protection, i.e the minimum possible value of z1. Since it must be ensured that the primary relay

has to be reset after it has responded to the previous fault, the maximum value of  $z_2$  and  $z_5$  are chosen on the basis of the physics of the protection scheme, in order to compute  $(z_2+z_5)$ ; this cycle time must be less than, or equal to the cycle time  $(z_1+z_3+z_6)$ . We choose  $z_3$  and  $z_6$  to be respectively the maximum values of the breaker opening time and the fault clearing time, and thereby compute the value of  $z_1$ . Similar computations for the backup protection use the cycles of the second subnet.

### STOCHASTIC MODELS

While the marked PN, time PN and timed PN do provide useful information with regard to protection schemes, they are deterministic models, and do not adequately reflect the stochastic nature of protection. The generalized stochastic Petri Net (GSPN) model provides a more realistic representation [11].

The GSPN has the same topology as the marked PN model with the firing times being random variables. The transitions are classified as immediate transitions and exponential transitions; immediate transitions must fire at the instant at which they are enabled, while the time that elapse between the enabling of an exponential transition and its firing is an exponentially distributed random variable.

The reachability graph of a GSPN is the same as that of the underlying marked PN; it is assumed that the PN possesses the properties of boundedness and properness. The states of the system, as observed from the reachability graph, are categorized as either vanishing or tangible. If the marking of the state is such that an immediate transition is enabled, then it is a vanishing state, with the time spent in it being zero. Each tangible state is occupied for a finite, exponentially distributed interval of time. Computation of performance involves the following steps:

- 1) The reachability graph is obtained.
- 2) The embedded Markov chain (EMC) of the PN is found, by assigning to each edge of the reachability graph the firing rate of the corresponding transition. For immediate transitions, the corresponding edges are appropriately marked.
- 3) The vanishing states, which have an occupancy time of zero, are removed from the EMC to produce a reduced embedded Markov chain (REMC) in which all edges correspond to exponential transitions.
- 4) The mean sojourn time of each tangible state is computed.
- 5) The steady state probability distribution of tangible states is computed.

The first three steps are straightforward. Since each edge of the REMC has a specified firing rate, we can calculate from these firing rates the sojourn times of the states. If a node  $j$  of the REMC has only a single edge  $k$  incident out of it, and this edge has the firing rate  $r_k$  as its label, then the sojourn time is an exponentially distributed random variable, with a mean value  $m_j$  equal to  $1/r_k$ . If node  $j$  has two edges  $k$  and  $s$  incident out of itself, with firing rates  $r_k$  and  $r_s$ , and probabilities of traversal  $p_k$  and  $p_s$ , then the net transition rate is the weighted sum,  $(r_k.p_k+r_s.p_s)$ , and the mean sojourn time  $m_j$  is the reciprocal of this quantity; the same approach can be extended to  $K$  edges.

The cycle time  $k$  is the mean time taken by the system to traverse the  $k$ th cycle of the REMC. The mean sojourn time of a state in the cycle is computed by taking the reciprocal of the transition rate that is associated with its output edge; the sum of the sojourn times of all nodes in the cycle gives  $k$ . The probability  $q_k$  of traversing the  $k$ th cycle is the product of the probabilities that are associated with the edges of the cycle. Let  $R$  be the sum of the terms  $(q_k/k)$ , where the summation is taken over all cycles in the REMC. To compute the steady state probability of

the occupancy of the  $j$ th state, we compute the sum of the terms  $(m(k,j).q_k)$ , where  $m(k,j)$  is mean sojourn time of state  $j$  in the  $k$ th cycle, the summation being over all cycles to which this state belongs; the ratio of this sum to  $R$  gives the desired steady-state probability.

We now illustrate these computations for the protection scheme of Fig. 4. Its stochastic PN model is obtained by assigning transition rates and firing probabilities to the transition of Fig. 2. The transition  $t_1$  has a transition rate which is the reciprocal of the mean time to failure of the system. Transitions  $t_2$  and  $t_3$  have rates that are the reciprocal of the mean operating times of the relays and  $t_4$  and  $t_5$  use the reciprocal of the mean restoration rates. For the study, we will assume firing rates as shown in the REMC of Fig. 10, which is obtained from the reachability graph of Fig. 3. Transitions  $t_1$ ,  $t_4$  and  $t_5$  are the only possible transitions out of states  $M_1$ ,  $M_3$  and  $M_4$  respectively, so they are assigned probabilities of 1. Transition  $t_2$  has firing probability equal to that of the operation of primary protection; we assume a value of 0.9. Obviously  $t_3$  has a corresponding value of 0.1. This system has no vanishing states, so the REMC is identical to the EMC. Its states are listed in Table 1.

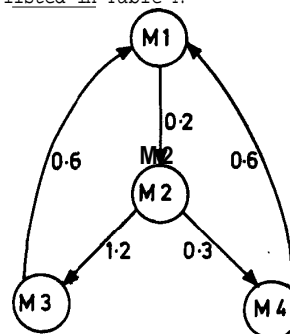


Fig.10. REMC of Stochastic Petri Net

Table 1

	p1	p2	p3	p4	p5	p6	mst	ssp
M1	1	1	0	1	0	0	5.000	0.645
M2	0	1	1	1	0	0	0.900	0.140
M3	0	0	0	1	1	0	1.667	0.193
M4	0	1	0	0	0	1	1.667	0.022

In the REMC, the nodes  $M_1$ ,  $M_3$  and  $M_4$  each have only one edge out of them, with the transition rates being 0.2, 0.6 and 0.6 respectively. Hence their mean sojourn times are 5.000, 1.667 and 1.667 respectively, as shown in column mst of the table. The node  $M_2$  has two transition out of itself, with firing rates 1.2 and 0.3, and probabilities 0.9 and 0.1 respectively. Hence the net rate is  $(1.2 \times 0.9 + 0.3 \times 0.1)$ , or 1.11, and the mean sojourn time is  $(1/1.11)$ , or 0.900.

The REMC has two cycles, namely  $(M_1, M_2, M_3)$  and  $(M_1, M_2, M_4)$ . In both cycles, the mean sojourn time of node  $M_1$  is 5.000. In the first cycle, node  $M_2$  has a mean sojourn time of  $(1/1.2)$ , or 0.833, while in the other cycle its value is  $(1/0.3)$ , or 3.333. Nodes  $M_3$  and  $M_4$  each have a mean sojourn time of 1.667. The mean cycle of the first cycle is  $(5.000 + 0.833 + 1.667)$ , or 7.5, while that of the second one is 10.0. Since these two cycles have probabilities 0.9 and 0.1 respectively, the value of  $R$  is  $(7.5 \times 0.9 + 10.0 \times 0.1)$ , or 7.75.

The state M1 occurs in both cycles, with a mean sojourn time of 5. Hence the steady state probability of the system being normal is  $5/7.75$ , or 0.645. The state M3 occurs in cycle 1 only, with probability 0.9, and mean sojourn time of 1.667, and hence its steady state probability is  $0.9 \times 1.667/7.75$ , or 0.193. Similarly, the steady state probability of state M4 is  $(0.1 \times 1.667)/7.75$ , or 0.022. State M2 occurs in cycle 1 with probability 0.9, and mean sojourn time 0.833, and in cycle 2 with probability 0.1 and mean sojourn time 3.333, hence the steady state probability of M2 is given by  $(0.833 \times 0.9 + 3.333 \times 0.1)/7.75$ , or 0.140. These values have been entered in the last column of Table 1.

#### CONCLUSION

Petri Net models of power system protection schemes have been developed, and the relevance of these models to performance evaluation of the schemes has been demonstrated. While marked Petri Nets enable one to evaluate qualitative performance measures, time Petri Nets are useful in recoverability analysis, to obtain relationships between the time parameters of the protection scheme, Timed Petri Nets are a means of identifying the cycles of the protection scheme, and of computing the cycle times. Stochastic Petri Nets capture the stochastic nature of protection, and serve as a means of computing the statistics of protective operations. A number of examples have been provided, to illustrate the concepts, and to demonstrate their relevance to the performance evaluation of protection schemes.

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