

Cambridge University Press
0521805937 - Deterministic Observation Theory and Applications
Jean-Paul Gauthier and Ivan Kupka
Frontmatter
[More information](#)

DETERMINISTIC OBSERVATION THEORY AND APPLICATIONS

This book presents a general theory as well as a constructive methodology to solve “observation problems,” that is, reconstructing the full information about a dynamical process on the basis of partial observed data. A general methodology to control processes on the basis of the observations is also developed. Illustrative but also practical applications in the chemical and petroleum industries are shown.

This book is intended for use by scientists in the areas of automatic control, mathematics, chemical engineering, and physics.

J-P. Gauthier is Professor of Mathematics at the Université de Bourgogne, Dijon, France.

I. Kupka is Professor of Mathematics at the Université de Paris VI, France.

Cambridge University Press
0521805937 - Deterministic Observation Theory and Applications
Jean-Paul Gauthier and Ivan Kupka
Frontmatter
[More information](#)

DETERMINISTIC OBSERVATION THEORY AND APPLICATIONS

JEAN-PAUL GAUTHIER

IVAN KUPKA



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
 0521805937 - Deterministic Observation Theory and Applications
 Jean-Paul Gauthier and Ivan Kupka
 Frontmatter
[More information](#)

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
 The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
 The Edinburgh Building, Cambridge CB2 2RU, UK
 40 West 20th Street, New York, NY 10011-4211, USA
 10 Stamford Road, Oakleigh, VIC 3166, Australia
 Ruiz de Alarcón 13, 28014 Madrid, Spain
 Dock House, The Waterfront, Cape Town 8001, South Africa
<http://www.cambridge.org>

© Cambridge University Press 2001

This book is in copyright. Subject to statutory exception
 and to the provisions of relevant collective licensing agreements,
 no reproduction of any part may take place without
 the written permission of Cambridge University Press.

First published 2001

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times Roman 10.25/13 pt. *System* L^AT_EX 2_ε [TB]

A catalog record for this book is available from the British Library.

Library of Congress Cataloging in Publication Data

Gauthier, Jean-Paul.

Deterministic observation theory and applications / Jean-Paul Gauthier, Ivan Kupka.

p. cm.

Includes bibliographical references and index.

ISBN 0-521-80593-7

1. Observers (Control theory) 2. Missing observations (Statistics) I. Kupka, Ivan,
 1937– II. Title.

QA402.3 .G378 2001

003–dc21

2001025571

ISBN 0 521 80593 7 hardback

Cambridge University Press
0521805937 - Deterministic Observation Theory and Applications
Jean-Paul Gauthier and Ivan Kupka
Frontmatter
[More information](#)

We dedicate this book to our wives, Irène and Prudence, respectively

Cambridge University Press
0521805937 - Deterministic Observation Theory and Applications
Jean-Paul Gauthier and Ivan Kupka
Frontmatter
[More information](#)

The purpose of this book is to present a complete theory of observability and observation of finite dimensional nonlinear systems in the deterministic setting. The theory is used to prove very general results in dynamic output stabilization of nonlinear systems. Two real concrete applications are briefly described.

Dijon, September 9, 2000

Contents

<i>Preface</i>	<i>page ix</i>
1 Introduction	1
1. Systems under Consideration	1
2. What Is Observability?	2
3. Summary of the Book	2
4. The New Observability Theory Versus the Old Ones	3
5. A Word about Prerequisites	4
6. Comments	5
Part I. Observability and Observers	
2 Observability Concepts	9
1. Infinitesimal and Uniform Infinitesimal Observability	9
2. The Canonical Flag of Distributions	11
3. The Phase-Variable Representation	12
4. Differential Observability and Strong Differential Observability	14
5. The Trivial Foliation	15
6. Appendix: Weak Controllability	19
3 The Case $d_y \leq d_u$	20
1. Relation Between Observability and Infinitesimal Observability	20
2. Normal Form for a Uniform Canonical Flag	22
3. Characterization of Uniform Infinitesimal Observability	24
4. Complements	26
5. Proof of Theorem 3.2	29

viii	<i>Contents</i>	
4	The Case $d_y > d_u$	36
	1. Definitions and Notations	37
	2. Statement of Our Differential Observability Results	40
	3. Proof of the Observability Theorems	42
	4. Equivalence between Observability and Observability for Smooth Inputs	51
	5. The Approximation Theorem	57
	6. Complements	58
	7. Appendix	59
5	Singular State-Output Mappings	68
	1. Assumptions and Definitions	68
	2. The Ascending Chain Property	71
	3. The Key Lemma	73
	4. The $ACP(N)$ in the Controlled Case	78
	5. Globalization	81
	6. The Controllable Case	84
6	Observers: The High-Gain Construction	86
	1. Definition of Observer Systems and Comments	87
	2. The High-Gain Construction	95
	3. Appendix	120
	Part II. Dynamic Output Stabilization and Applications	123
7	Dynamic Output Stabilization	125
	1. The Case of a Uniform Canonical Flag	126
	2. The General Case of a Phase-Variable Representation	132
	3. Complements	141
8	Applications	143
	1. Binary Distillation Columns	143
	2. Polymerization Reactors	163
	<i>Appendix</i>	179
	<i>Solutions to Part I Exercises</i>	195
	<i>Bibliography</i>	217
	<i>Index of Main Notations</i>	221
	<i>Index</i>	224

Preface

A long time ago, while working on paper [19], we felt that there was a need to write a book on the subject of observability. Now, after many vicissitudes, this is a done thing.

During the conception of the book, the very novel point of view we had developed in our papers did not change. We discovered that it was really the right one, and was extremely efficient. In fact, based on it, we could build a totally new, complete, and general theory, and a new methodology for the problems related to observability, such as “output stabilization.” At the same time, we applied our methodology to practical problems, and we realized that our methods were extremely efficient in practice.

At the very beginning, we intended to write a “survey” on the problems of observability, including nonlinear filtering. As the work progressed, we changed our minds. First, from the practical point of view, we faced a daunting task: a book of that type, had it ever seen the light of a day, would have been a monster. But, more important, our theory would have been drowned in a mass of disparate, disconnected facts.

Hence, this book presents only the general theory we have discovered, with a selection of real-life applications to convince the reader of the practical capability of the method. We strictly avoided the type of academic examples which are rife in many control theory publications.

Several principles guided us in the elaboration of this book:

- First, the book should be short. Including some developments in the stochastic context was a definite possibility, but this would have required the use of deep mathematical tools for meager returns. Enough mathematical theories already are used in the book.
- Second, the book is an excellent opportunity to convince people with a mathematical bent that “observation theory” is not out of place in mathematics. For that reason, the style of this book is a mathematical one. Also, we want to show that applied problems in the real world

can be dealt with by using beautiful mathematics. On the other hand, mathematics is not the main object of this book, but an excellent tool to achieve our goals.

- Third, we want to convince applied people (e.g., control engineers, chemical engineers) that our methodology is efficient. Therefore, they should strive to understand it and, above all, to use it. For this purpose, we want to point out the following:
 - We strove to make all the necessary mathematical tools accessible to uninitiated readers.
 - Bypassing the details of the proofs does not impair the understanding of the statements of the theoretical results and the constructive parts of the theory (many of the proofs are not obvious).
 - Chapter 8, containing the applications, is friendlier to the nonmathematical reader, albeit rigorous.

The development of the practical applications in this book, and of others not mentioned in it, was possible thanks to the cooperation with the French branch of the Shell company, its research center at “Grand-Couronne.” One of the applications was actually implemented at the refinery of Petit-Couronne (France). The first author particularly wants to express his deep gratitude to the whole process control group there, more especially to Denis Bossane, François Deza, Marjoleine Van Doothing, and Frederic Viel, for their help, support, and for the good time we spent together. A very special and friendly remembrance goes to Daniel Rakotopara, the head of the group at that time, who so unfortunately died recently.

J-P. Gauthier expresses his warmest thanks to Jean-Jacques Dell’amico (head of the research center) and Pierre Sommelet (chief of the group), who not only took care of financial needs, but also are great friends.

Chapters 3, 4, 5, and 7 of this book contain, among others, the results of the papers [18], [19], [32]. For their kind permission to reproduce parts of papers [18], [19], and [32], we thank, respectively, the Society for Industrial and Applied Mathematics (Observability and observers for nonlinear systems, *SIAM Journal on Control*, Vol. 32, No. 4, pp. 975–994, 1994), Springer-Verlag (Observability for systems with more outputs than inputs, *Mathematische Zeitschrift* 223, pp. 47–78, 1996), and Kluwer Academic Publishers (with P. Jouan, Finite singularities of nonlinear systems. Output stabilization, observability, and observers. *Journal of Dynamical and Control Systems* 2(2), pp. 255–288, 1996).

Mexico City, September, 2000