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Deuteron Formation in Expanding Nuclear Matter from a Strong Coupling BCS Approach

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Abstract

The process of deuteron formation in intermediate heavy ion reactions is approached within the strong coupling BCS theory assuming that the final stage of the reaction can be described as an adiabatic expansion of a piece of nuclear matter. Since the gap equation in the ${}^3S_1 - {}^3D_1$ channel goes into the deuteron Schrödinger equation in the low density limit, a smooth transition from the superfluid Cooper pair phase to a Bose deuteron gas is found. For a fixed entropy the deuteron fraction as a function of density shows a steep uprise, up to one in the limit of vanishing density. The relevance of these results for heavy ion collisions and the shortcomings of the present approach are briefly discussed. The model is further generalised to the time dependent case. A semiclassical set of HFB equation is presented and the limiting situation of an adiabatic expansion is detailed. A schematic calculation suggests that the Fermi energy regime should be the best suited to observe the phenomenon of deuteron formation via BCS pairing. The inclusion of fluctuations is outlined by coupling the BCS equations with a self-consistent nonlinear quasi-particle RPA (SCqpRPA). This theory is tested in a schematic model.

1 Introduction

Central heavy ion collisions at E/A in the 50 to 200 MeV range can roughly be described by the initial build up of compressed and hot nuclear matter and by a sequential decompression. At low energies the system may pass through the spinodal instability region and may undergo a liquid-gas phase transition or, at higher energies, the temperature and the internal pressure involved may be so high that the piece of nuclear matter just vaporizes into the individual nucleons. In any case, however, the expanding gas cools down rather fast, allowing for deuterons, as well as other fragments, to form again in the final

stage. Actually the yields of deuterons in high energy reactions always exhibit a surprisingly high production rate[1]. Typically, *e.g.* the ratio of deuterons to protons is of the order of unity. At higher bombarding energies ($E/A \geq 200$ MeV) such abundances have been successfully explained from a transport model [2], where the deuteron pole in the n-p scattering matrix has been explicitly isolated. Though pure phase space considerations may account for a large fraction of the deuteron production rate [3] it still may be interesting to investigate the problem, for instance, of deuteron production from a slightly different point of view. We have in mind the very idealized situation of a piece of hot nuclear matter expanding quasi-stationarily with total entropy kept constant. The latter feature seems to be realized in heavy ion reactions [4]. When lowering the density, the temperature will then drop, and the combination of both effects will allow the appearance of deuterons (Mott transition). The phenomenological observation that the main components of the outgoing fragments, in this energy region, are nucleons and deuterons indicates that nuclear matter has not enough time, during the expanding phase, to get close to its lowest free energy state. In fact, at low enough densities and temperatures, nuclear matter is expected to form a gas of alpha particles. Therefore a mixture of nucleons and deuterons is a metastable system, which spontaneously decays in a gas of alpha particles. This metastable system, however, is worth to be studied, since it is likely to be the one which is more closely related to the final stage of heavy ion reactions. The characteristic time of the expanding phase is, therefore, assumed to be faster than the alpha formation rate, consistently with phenomenology, and slow enough, as already mentioned, to consider nuclear matter not too far from thermal equilibrium.

In this work we will be mainly concerned with the deuteron formation. An aspect we will partially develop is that nuclear matter at low density, below a certain critical temperature T_c, may show strong n-p pairing in the deuteron channel. This has been revealed in several recent studies [5–7] and, in fact, it can be anticipated since the NN force in the deuteron channel is more attractive than in the usual p-p or n-n pairing channels. At higher bombarding energies the entropy production is typically several units (see for instance [8]) and this is probably too high for the superfluid phase to occur but at lower energies ($E/A \leq 100$ MeV) the entropy per nucleon $S/A < 1$ and this may well allow the appearance of deuteron pair condensation. The reason why such a pair condensate may be relevant for the deuteron production can qualitatively be explained, at least, within the BCS approach to superfluidity. In this approach it has been shown [9] that in the low density limit the BCS equations for the pairing condensate goes over into the Schrödinger equation for the deuteron. In a first attempt we will try to get some insight into the physics of this process in adopting a mean field strong coupling BCS approach. We know about the weakness of this approach, which completely leaves aside the influence of fluctuations, that, in this type of situation, may be crucial, both with respect to the finiteness of the system and with respect to a possible

crossover from BCS superconductivity to Bose-Einstein condensation of the deuterons. Indeed, in the limit of inter-particle distance much larger than the deuteron radius a_0, i.e. $k_F a_0 << 1$, the deuterons behave essentially as a gas of bosons, which can undergo the usual condensation phenomenon. We may deal with this difficult problem in the future. Here we shall stay within the pure BCS approach. We shall first treat the equilibrium case [10], and at the end indicate how to generalise to the dynamic case. A schematic pilote calculation will be presented for a dynamically expanding blob of nuclear matter. Furthermore, we also outline how fluctuations can be taken care of within a self consistent version of the quasi particle RPA (SCqpRPA).

2 Strong coupling Brückner-BCS approach

Proton-neutron pairing in nuclear matter has only been investigated very recently [5–7]. Using realistic bare forces in the gap equation and a Brückner Hartree-Fock G-matrix approach for the normal mean field, surprising high values for the gap in the deuteron channel have been found in symmetric nuclear matter. Since the attraction in the deuteron channel,i.e. $S-D$ T=0 channel , is stronger than in the T = 1 neutron-neutron or proton-proton channels, this may actually be not a complete surprise. However, the in medium renormalization (screening) of the bare n-p force is certainly an unsolved problem, in spite of the fact that the use of a bare n-n and p-p forces yields quite reasonable values for the gap Δ in the scalar isovector channel T = 1. Bearing this difficulty in mind, we nonetheless go on using the bare Paris force as the n-p interaction in the $S-D$ pairing channel, which in the mostly interesting low density regime may eventually not be a quite unreasonable choice.

For the normal mean field we use, as in an early paper [11], the one from the Brückner HF approach. The intermediate two particle propagator g is given by

$$g = \frac{(1-f_k)(1-f_{k'})}{\omega - \epsilon_k - \epsilon_{k'} + i\eta} \tag{1}$$

where f_k is the Fermi function, calculated at the single particle energy ϵ_k, the latter being obtained from the Brückner HF self consistent potential.

The gap equation reads

$$\Delta_p = -\sum_k v_{pk} \frac{\Delta_k}{2E_k} \tanh(\frac{\beta E_k}{2}) \tag{2}$$

with $E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2}$. As usual the density is obtained from

$$\rho = \frac{N}{V} = \sum_{\vec{k},\sigma,\tau} n_k \tag{3}$$

$$n_k = \frac{1}{2}(1 - \frac{\epsilon_k - \mu}{E_k} \tanh \frac{\beta E_k}{2}) \tag{4}$$

and the anomalous density is given by

$$\kappa_k = \frac{\Delta_k}{2E_k} \tanh \frac{\beta E_k}{2} \tag{5}$$

The entropy reads

$$\frac{S}{V} = -\sum_{\vec{k},\sigma,\tau} f(E_k) \log f(E_k) - (1 - f(E_k)) \log(1 - f(E_k)) \tag{6}$$

The coupled set of non-linear equations (2),(4) is solved, for a given total density ρ, adopting a separable form of the Paris potential [12]. The same numerical method as in [7] is used in solving the gap equation for Δ_k. In the present work, however, the chemical potential μ has been also determined from the set of equations (2),(4). Once the solution for the gap function Δ_k and the chemical potential μ are found, the quantities κ and S are calculated from eqs. (5),(6), respectively.

It is also interesting to combine equations (2),(4), (5) to obtain an expression equivalent to the gap equation:

$$\frac{p^2}{m}\kappa_p + (1 - 2n_p) \int \frac{d^3k}{(2\pi\hbar)^3} v_{pk}\kappa_k = 2\mu\kappa_p \tag{7}$$

This equation looks very much like the deuteron Schroedinger equation into which it turns for a dilute system, *i.e.* in the limit $\rho \to 0$. The quantity in which we are mostly interested here is the ratio of the number of deuterons N_s to the number of unbound nucleons N, or equivalently the ratio between the abnormal and normal density. According to standard BCS theory we have

$$\frac{\rho_S}{\rho} = 1 - \frac{\hbar^2}{2m} \frac{1}{6\pi^2\rho} \beta \int dk k^4 \left[1 - \tanh^2(\frac{\beta E_k}{2})\right]. \tag{8}$$

which will be evaluated, as already mentioned, at constant entropy.

3 Results

The transition from the superfluid phase to the deuteron gas is best illustrated by considering the pairing anomalous density κ_k, or its Fourier transform in coordinate space $\phi(r)$, which should smoothly merge into the deuteron wave function in the low density limit. This is displayed in Fig. 1, where the two components of $\phi(r)$ are drawn at different densities. For the lowest density at $\rho = 0.0075$ fm^{-3} a comparison is made with the deuteron wave function. One can observe a large overlap between $\phi(r)$ and the deuteron wave function, which indicates that already at this density the BCS solution is hardly distinguishable from a gas of deuterons, despite the fact that the chemical potential is about $+1$ MeV, still away from the deuteron limit value $\mu = E_D/2 = -1.1$ MeV. At increasing densities the inner part of $\phi(r)$ is not so much affected, while the large r behaviour changes drastically.

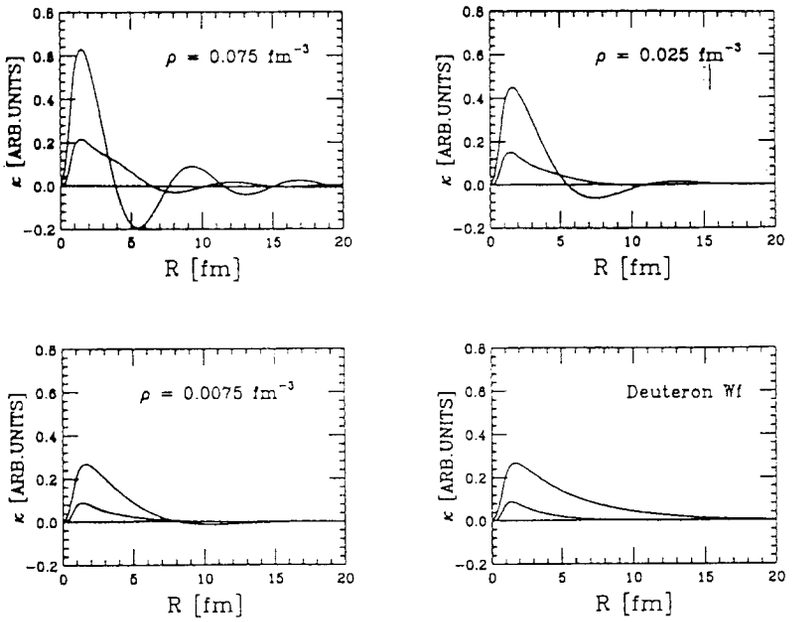


Fig. 1. The S and D components of the pairing correlation function in coordinate space for three different values of the baryonic density. The right lower figure displays for comparison the deuteron wave function.

From these results one can conclude that the BCS theory describes, in the low density limit, a smooth transition from the $S-D$ superfluid phase of symmetric nuclear matter, to the Bose condensate phase of an ideal deuteron gas. In BCS language it corresponds to a transition from the weak to the strong coupling limit. The Cooper pairs, each one of which has zero momentum, merge naturally into the zero momentum condensate of deuterons.

According to the scenario mentioned in the introduction we assume a piece of hot and compressed nuclear matter formed by a HIC which expands isentropically. At some low density and temperature deuteron superfluidity pops

up. When the gas expands further to lower and lower densities the chemical potential μ will turn negative (relative to the mean field value $U(k_F)$ at the Fermi momentum) from where on at a certain density the deuterons are being formed. This actually occurs at extremely low densities $\rho < \rho_o/30$ where the mean distance of the deuterons from one another is about 2-3 times their diameter ($\sim$ 3 fm). At low densities we shall calculate the ratio of the number of deuterons versus the total number of nucleons, given by equation (8). This ratio is shown in Fig. 2, for fixed values of the entropy per particle, as a function of the total density. As we can see, there is a steep increase of deuterons as a function of decreasing density. For a typical value of $S/A \approx 1$ there exists already an appreciable number of deuterons at $\rho = \rho_0/5$.

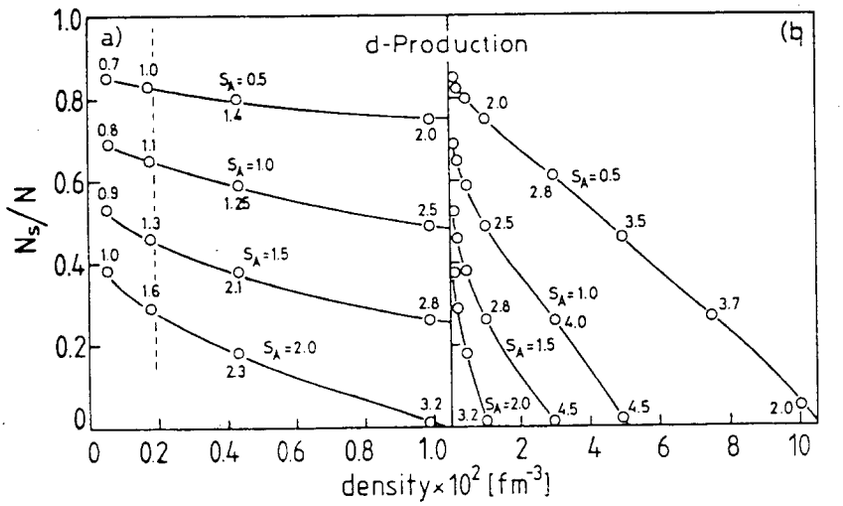


Fig. 2. (right) Deuterons to nucleons ratio N_S/N vs density for four values of the entropy.(left) The same ratio but in an enlarged scale at low densities. In some cases the corresponding temperatures are indicated. The dashed line connects the points corresponding to $\mu = 0$ MeV.

The strongly uprising branch of N_s/N in the final stage of the expansion is due to the fact that the final infinitely diluted stage is always at zero temperature where, within the pure BCS theory, everything is paired ($N_s/N = 1$). This behaviour can be seen more clearly in the blow up of Fig.2 for low densities. The ratio N_s/N tends to one as ρ tends to zero. On the other hand,since the entropy is kept constant also T tends to zero as $\rho \to 0$. In Fig.2 are indicated on the lines of constant entropy in addition to the temperatures at various values of the density also the corresponding values of the chemical potential. Since $\mu = 0$ marks the border from where on deuterons are bound, we connected the points $\mu = 0$ on the various curves S_A = const by the broken line. We should, however, like to point out that deuterons are formed already at higher densities. For example, the coalescence model gives a freeze-out density of $\approx \rho_o/10$ [2] similar to our calculation. On the other hand one sees on Fig.2

that the points $\mu = 0$ where the chemical potential turns negative all lie at a density of around $\rho = 0.002$ fm^{-3} for the various entropies.

It may seem surprising that the chemical potential turns negative only at such extremely low densities where the interparticle distance is of order of 8 fm. One should bear in mind, however, that the deuterons are only weakly bound and that therefore the corresponding wave function reaches out appreciably to distances beyond 8 fm as can be seen on Fig.1. The Pauli principle therefore is still active. An inter-particle distance 2-3 times the deuteron diameter can therefore seem reasonable for the Mott transition. We must of course always be aware of the weakness of our present approach where we completely neglect thermal and quantal fluctuations which may be very important at such low densities. For instance, the fact that BCS theory even at finite temperature imposes that the deuterons are at rest seems to be a drastic assumption, which should be relaxed in a more realistic approach (see Section 5). Nevertheless our study may set qualitative indications. The fact seen on Fig.2 that the ratio deuteron to unpaired nucleons varies very strongly in the low density regime of expanding nuclear matter should probably remain. This mechanism of deuteron production can then coexist with other normally considered mechanisms where the deuterons are formed in final state interactions of an emitting source or by coalescence

4 Dynamics of the Expansion

It is evident that the assumption of quasi-static expansion cannot be valid in practice. One therefore should generalise the present formalism to the time dependent case. With the generalised density matrix $\mathcal{R}$ and the HFB hamiltonian $\mathcal{H}$ we then have [13]

$$i\hbar\dot{\mathcal{R}} = \left[\mathcal{H}, \mathcal{R}\right] \tag{9}$$

with

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1-\rho^* \end{pmatrix} \quad , \quad \mathcal{H} = \begin{pmatrix} h & -\Delta \\ \Delta^* & -h^* \end{pmatrix} \tag{10}$$

The semiclassical limit of this equation can be found (see *e.g.* [14]) introducing a unitary transformation

$$\mathcal{R} = \mathcal{U}\mathcal{R}_0\mathcal{U}^\dagger \quad , \quad \mathcal{U} = exp(\frac{i}{\hbar}\Phi\tau_3) \tag{11}$$

As a matter of fact, the phase of the abnormal density is of order $1/\hbar$ and a semiclassical transport theory is possible only if we put ourselves in the frame where the gap is real, *i.e.* in the frame where the phase is gauged out. Supposing the phase to be a local operator $<\vec{r}|\Phi|\vec{r'}>=\Phi(\vec{r})\delta(\vec{r}-\vec{r'})$, we then obtain [15] in semiclassical (first order in $\hbar$) approximation

$$\dot{f}_0 = \left\{h_0, f_0\right\} + \left\{\Delta_0, \kappa_0\right\} \tag{12}$$

$$\dot{\kappa_0} = \left\{\frac{\vec{p}}{m}\nabla\Phi, f_0\right\} + \frac{1}{2}\{\Delta_0, f_0 - \overline{f}_0\} \tag{13}$$

$$\frac{\vec{p}^2}{m}\kappa_0 + (2f_0 - 1)\int \frac{d^3k}{(2\pi\hbar)^3} v_{pk}\kappa_0(\vec{k}) = 2\lambda\kappa_0 \tag{14}$$

with

$$h_0 = h_\Phi + \frac{\vec{p}}{m}\nabla\Phi - \mu = \frac{(\vec{p}+\nabla\Phi)^2}{2m} + \dot{\Phi} + U^H - \mu \tag{15}$$

$$\lambda = \mu - U^H - \dot{\Phi} - \frac{(\nabla\Phi)^2}{2m} \tag{16}$$

and the shorthand notation $\overline{f}_0 = f_0(\vec{r}, -\vec{p}, t)$. Equations (12), (13), (14) represent three evolution equations for the three unknowns f_0, κ_0, Φ (the subscript "0" always denotes gauge transformed quantities). The self-consistent solution of these equations constitutes the semiclassical time dependent BCS approach (TDBCS). The modified gap equation eq.(14) has the form of a local Schroedinger equation for the deuteron wave function, with a local eigenvalue [10] represented by eq.(16). Therefore eq.(14) is formally the same as in the static case eq.(7) (up to the fact that f_0 is obtained from a dynamical equation), only the "eigenvalue" $2\lambda(\vec{r}, t)$ is now a field of the two parameters $\vec{r}, t$. One can verify that in the static limit equations (12), (13), (14) are equivalent to our previous equilibrium equations (2),(4),(5). To further clarify the physical meaning of the transport equations it may be instructive to calculate the first moments of the distributions. By introducing the usual notations for the densities and currents

$$\rho \equiv \int d^3p\, f_0(\vec{r}, \vec{p}, t) \quad , \quad \rho_s \equiv \int d^3p\, \kappa_0(\vec{r}, \vec{p}, t) \quad , \quad \vec{j}_0 \equiv \int d^3p \frac{\vec{p}}{m} f_0(\vec{r}, \vec{p}, t)$$

One straightforwardly gets

$$\dot{\rho} + \nabla_r \cdot \vec{j_0} + \nabla_r \cdot \vec{j} = 0 \tag{17}$$

$$\dot{\rho_s} + \nabla_r \cdot \vec{j}_s = 0 \tag{18}$$

$$m\frac{d\vec{j}_s}{dt} = \rho_s \nabla_r \Big(\mu - \frac{(\nabla\Phi)^2}{2m} - G\rho - U^H\Big) + \nabla_r \Pi_0 \tag{19}$$

where

$$\vec{j} \equiv \frac{1}{m}\rho \nabla_r \Phi$$
$$\vec{j_s} \equiv \frac{1}{m}\rho_s \nabla_r \Phi$$
$$\Pi_0 \equiv \frac{1}{\rho_s}\int d^3p \, \frac{\vec{p}^2}{2m}\kappa_0$$

and for simplicity we have assumed a local gap, $\Delta = G\rho_s$. Equation (19) is equivalent to a London equation for the superfluid velocity, with an inhomogeneity represented by the coupling to the normal phase[16]. So we can see that in the limit of a local pairing gap the problem can be reduced to a closed set of hydrodynamical equations, that is a continuity equation for the supermatrix eq.(17),(18) and an Euler equation eq.(19) for the collective velocity, in perfect analogy with the non superfluid case [13]. The physical meaning of the gauge transformation is now clear: among all the possible unitary transformations of the generalised density matrix, the physical gauge is defined by the constraint that a (2X2) continuity equation is fulfilled, *i.e.* by the requirement that the phase $\nabla\Phi$ actually represents the collective superfluid velocity [17]. The gauge transformation eq.(11) that we have employed up to now is in principle a simple change of the reference frame, with no loss of generality. Correspondingly the TDBCS equations exhibit all the complexity of a coupled set of integro-differential equations, and in general solutions cannot be given in an analytical form. However, the problem is drastically simplified in the special case where this transformation isolates all the time odd components of the generalised density matrix [15].

Let us suppose that with the transformation (11) the matrix $\mathcal{R}_0$ coincides with the time even part of $\mathcal{R}$. For the distribution functions in Wigner space this means [18] $f_0(\vec{r},\vec{p},t) = f_0(\vec{r},-\vec{p},t)$ and $\kappa_0(\vec{r},\vec{p},t)$ real (as before). Then the local velocity of both the normal and superfluid phase are represented by $\nabla_r\Phi$ and one can readily write down the solution of the transport equations

$$\kappa_0 = \frac{\Delta_0}{2E_\Phi} \;\; ; \;\; f_0 = \frac{1}{2}\Big(1 - \frac{h_\Phi}{E_\Phi}\Big) \tag{20}$$

with $E_\Phi^2 = \Delta_0^2 + h_\Phi^2$ and h_Φ defined in eq. (15). The structure of these solutions is the same as in the equilibrium case, but the dynamics of the phase is governed by the time odd components of the transport equations, namely

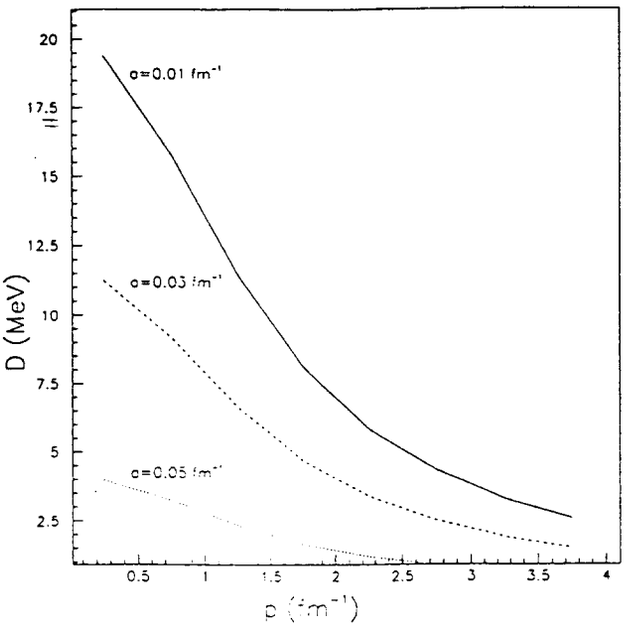


Fig. 3. Pairing gap as a function of momentum for a portion of uniformly accelerated nuclear matter at normal nuclear matter density, for three different values of the acceleration.

$$\dot{f}_0 - \left\{\frac{\vec{p}}{m}\nabla_r\Phi, f_0\right\} = 0 \tag{21}$$

$$\dot{\kappa}_0 - \left\{\frac{\vec{p}}{m}\nabla_r\Phi, \kappa_0\right\} = 0 \tag{22}$$

Remark that in the case of a constant collective velocity eqs.(21), (22) represent a trivial streaming motion. and the equilibrium solution is recovered. Let us examine the structure of the modified gap equation in more detail. If we specify to a separable interaction in momentum space $v(\vec{p},\vec{p}') = V_0 v_p v_p'$ the gap equation can be rewritten as

$$1 = 4\pi V_0 \int_0^\infty dk \frac{k^2 v_k^2}{2\sqrt{\left(\frac{k^2+(\nabla\Phi)^2}{2m} + \dot{\Phi} + U^H - \mu\right)^2 + v_k^2 V_0^2 C^2(r)}} \tag{23}$$

that in spherical symmetry is a one dimensional implicit equation for the r-dependent part of the gap $C(r)$ (here, $\Delta(r,p) = V_0 v_p C(r)$). As a first application let us drop self-consistency and solve eq.(23) for the simple dynamical case of a spherical nucleus at zero temperature immersed into an external uniformely accelerated velocity field. For U^H we use a standard Wood Saxon, the pairing interaction is taken from ref.[19], and the chemical potential μ is fixed to conserve the total number of particles. A result of this model calculation is presented in Fig.3, where the momentum dependence of the gap in the center $\Delta(r=0)$ is plotted for three different values of the (constant) acceleration. The acceleration is seen to systematically reduce the pairing gap, up to a critical value - specific to the chosen pairing interaction - where the

gap completely vanishes. This pair breaking effect of an acceleration field is well known in the standard theory of superconductivity.

Let us now turn to deuteron production in expanding nuclear systems.

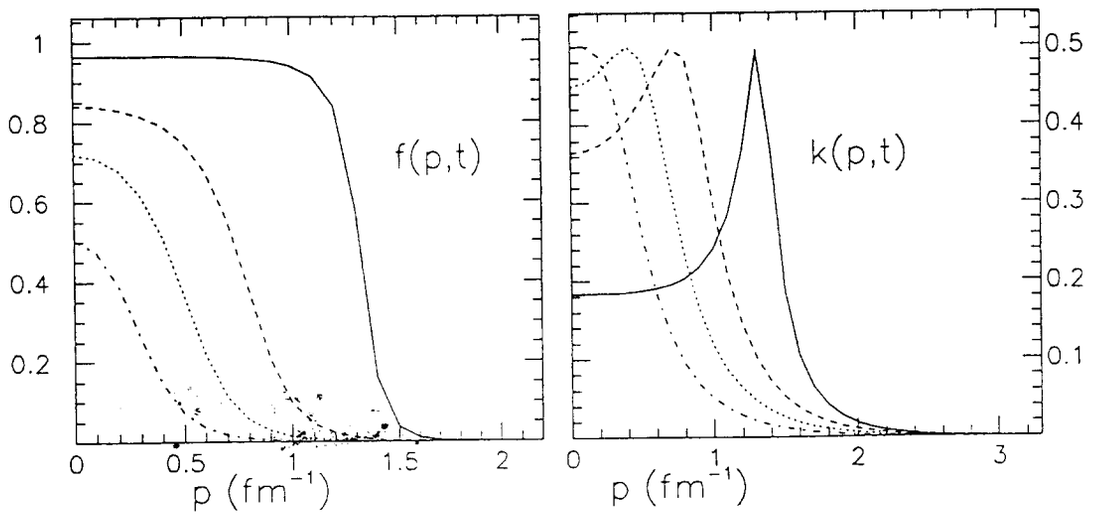


Fig. 4. Normal and abnormal distribution functions versus momentum for a spherical uniformly expanding nucleus with a radial collective velocity u=0.035 r/c Full line: normal nuclear matter density. Dashed line: after 50 fm/c. Dotted line: after 100 fm/c. Dash-dotted line: asymptotic time.

The solution of TDBCS for a spherically symmetric expanding nucleus at zero temperature is readily written (after an inverse gauge transformation) as

$$f(\vec{r},\vec{p},t) = \frac{1}{2}\Theta\left(R(t)-r\right)\cdot \left(\frac{(\vec{p}-m\vec{u})^2/2m + U^H - \mu(t)}{\left[\left((\vec{p}-m\vec{u})^2/2m + U^H - \mu(t)\right)^2 + \Delta^2(\vec{r},\vec{p},t)\right]^{1/2}}\right) \qquad (24)$$

The simplest possibility for the velocity field is to take a time independent function $\vec{u}(\vec{r}) = c_0 \cdot \vec{r}$. In this case the time dependence of the macroscopic parameters is given by the continuity equation

$$\rho = \rho_0 e^{-3c_0 t} \quad , \quad R = R_0 e^{c_0 t} \quad , \quad \mu = \mu_0 e^{-2c_0 t} \qquad (25)$$

and the distribution functions (normal and abnormal) can be determined at any time by solving the gap equation (23). As an example we report in Fig.4 the distribution functions evaluated at the center of the nucleus, at different times up to vanishing density. As the density gets lower, not only the average Fermi momentum diminishes as in the non pairing case, but also the average

occupation of low momentum states is progressively reduced. Correspondingly the distribution of paired nucleons is shifted towards lower energies as a function of time. In the limit of asymptotic times the wave function of the deuteron is recovered, and all nucleons are paired.

However it is clear that this model is too schematic in many respects. Temperature effects are neglected, for any initial value of the collective velocity a state at zero density is asymptotically reached, and the acceleration is not taken into account. From the structure of eq.(23) and the results of Fig.3 we expect that these results will not necessarily be preserved for a more realistic dynamics of the expansion. However, the adiabatic dynamics will be recovered if the initial density and thermal pressure stored in the compound system is sufficiently low to make the term $\dot{\Phi}$ in eq. (23) negligeable. On the other side, from the preceding section we know that in order to have a transition to the superfluid phase the initial pressure must be high enough to lead to a very dilute system in the final stage, and the balance between these two effects can be very delicate.

We shall therefore consider a more realistic model for the macroscopic parameters of the expansion entering the gap equation. For this purpose we shall use the hydrodynamical model of ref.[20]. Here the mean field U^H is neglected as before, but an homogeneous expansion of the nucleus is assured by a self-similar radial velocity field of the form

$$\frac{\nabla\Phi}{m} = u(r,t) = c_0(t)\cdot r = a_0\frac{R_0}{R(t)}\sqrt{\frac{t^2}{t^2+t_0^2}}\cdot r \tag{26}$$

Here, R is the (sharp) radius of the expanding system, R_0 the radius at the time of formation of the composite system, and a_0,t_0 are parameters. With the constraints of mass and energy conservation, the parameter a_0 can be fixed to obtain a given initial excitation energy while t_0 measures the acceleration of the collective motion. The continuity equation is again solved with the ansatz (26) for the time evolution of the density ρ, chemical potential μ and nuclear radius R. If we neglect the time dependence of c and R with respect to the time dependence of ρ, eq.(25) is still valid. From the preceding section we know that the correction to the chemical potential due to the pairing gap is important only at very low densities. For this simplified analysis we shall therefore keep eqs.(25) as a qualitative representation of the time evolution of the normal phase. However now c_0 has the more complicated time dependent form of eq.(26), that simulates the effect of mean field and thermal pressure. The parameters E^* and t_0 are then fixed to approximately reproduce the expansion dynamics (initial excitation energy, time behaviour of the average density and radial velocity) calculated from a BUU simulation [8] of a heavy ion collision.

As an example in Fig.5 we report the quality of the fit for the density evolution in a central $Au + Au$ collision at two different bombarding energies. Only at the highest energy a complete vaporisation of the system is seen, while at 50 MeV/u the calculation has been stopped at the time when density inhomogeneities start to appear, leading to fragment formation. In the same figure we show the time evolution of the surface velocity from eq.(26) for the two cases. The acceleration is clearly much more important for the high energy collision. Finally temperature is calculated at any time from energy conservation, and the gap equation eq.(23) is correspondingly modified as in eq. (2).

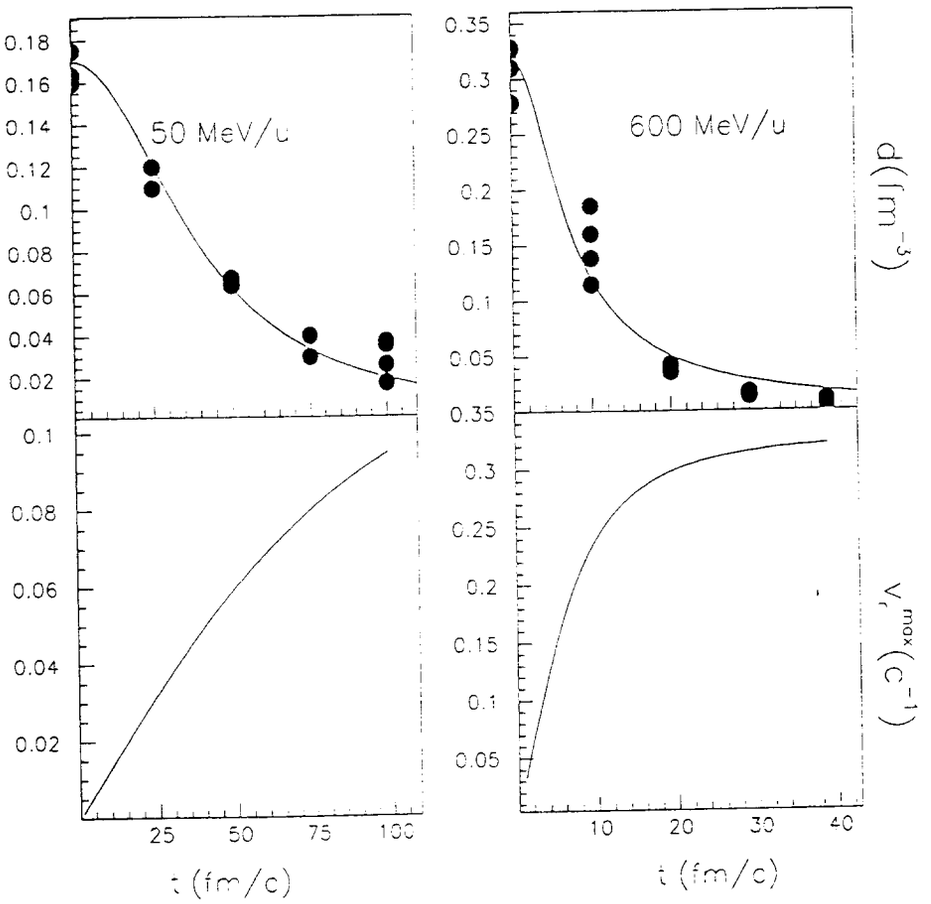


Fig. 5. (upper part) Average density as a function of time for a central Au+Au collision at 50 MeV/u(left) and 600 MeV/u(right) obtained in a BUU calculation. The dispersion of the points reflects the density inhomogeneities. The solid line is a calculation from an hydrodynamical model (see text). (lower part) Collective radial velocity at the surface of the expanding nucleus as a function of time, from the hydrodynamical model with the same parameters as in the upper part.

For the highest excitation energy the acceleration of the expansion turns out to be higher than the limiting value: the gap vanishes. The upper limit of a complete pairing at zero temperature given by the BCS theory is not true any

more, and deuteron production due to BCS coupling - if any - has to be calculated from the complete self consistent transport equations eqs.(12)(13)(14).

On the other hand at the lowest bombarding energy the gap is only slightly reduced with respect to the equilibrium case, and the dynamics is qualitatively like in Fig. 4. The corresponding number of deuterons as a function of time, calculated from eq.(8), is plotted in Fig.6. At longer times the average density does not evolve any more due to the showing up of fragments, therefore the ratio will be approximately frozen, if recombination effects are negligeable.

It is interesting to observe that an anomalous proportion of deuterons has been recently measured by the INDRA collaboration in vaporisation events at a similar bombarding energy [21]. Our calculation suggests that a systematic study of deuteron yields and deuteron correlation functions in vaporisation events as a function of excitation energy will be very helpful in clarifying the possible observation of a Mott transition in deuteron production.

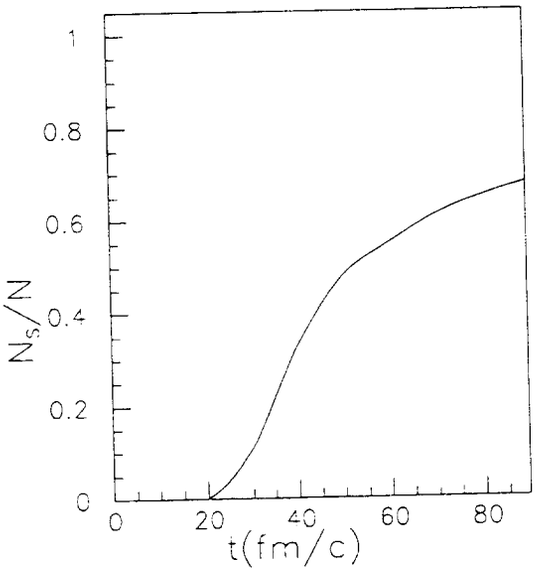


Fig. 6. Deuterons to nucleons ratio N_S/N versus time for a central Au+Au collision at a beam energy of 50 MeV/u, calculated with the macroscopic parameters from the hydrodynamical model.

A word of caution is however necessary: the result presented in Fig.6 must be considered only as a very qualitative indication of the effects of the expansion dynamics, and not as a real theoretical prediction of deuteron yield in a realistic reaction. The complete lack of self-consistency and the schematicity of the macroscopic model do not yet allow to attach a quantitative meaning to the numbers obtained. Further work in this direction is in progress [15].

5 Inclusion of Fluctuations within a Generalised RPA Formalism

Our formalism of static BCS in Section 2 has the great disadvantage that the deuterons stay immobile even at finite temperature. The time dependent description remedies this to some extent (it gives the usual quasi-particle RPA in the small amplitude limit, see below) but even this has some shortcomings. We therefore want to present here the so called self-consistent quasi-particle RPA (SCqpRPA) which, as we shall see, has many rather appealing features [22].

Let us define the qp RPA excitation operator

$$Q_K^\dagger = \frac{1}{2}\sum_{pp'} X^K_{pp'}\alpha_p^\dagger\alpha_{p'}^\dagger - Y^K_{pp'}\alpha_p\alpha_{p'} \tag{27}$$

Here K,p are general quantum numbers. In infinite matter they represent momenta and spins. It is clear that eq.(27) can create a "deuteron" pair with total momentum $\vec{K}$. The excited state is then

$$|K>= Q_K^\dagger|0> \tag{28}$$

where the ground state is defined by

$$Q_K|0>= 0 \tag{29}$$

The standard equation of motion method leads to the following RPA type of equations [13]

$$\sum_{ll'}\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}_{kk'll'}\begin{pmatrix} X^K \\ Y^K \end{pmatrix}_{ll'} = \Omega_K\sum_{ll'}\begin{pmatrix} N & 0 \\ 0 & -N \end{pmatrix}_{kk'll'}\begin{pmatrix} X^K \\ Y^K \end{pmatrix}_{ll'} \tag{30}$$

where

$$A_{kk'll'} = \ <0|[\alpha_{k'}\alpha_k,[H,\alpha_l^\dagger\alpha_{l'}^\dagger]]|0> \tag{31}$$
$$B_{kk'll'} = -<0|[\alpha_{k'}\alpha_k,[H,\alpha_l\alpha_{l'}]]|0> \tag{32}$$
$$N_{kk'll'} = \ <0|[\alpha_{k'}\alpha_k,\alpha_l^\dagger\alpha_{l'}^\dagger]]|0> \tag{33}$$

Replacing in these equations the ground state $|0>$ defined by (29), by the BCS ground state, linearizes eq.(30) and we arrive to the usual qpRPA, equivalent to the linearised version of the time dependent BCS equation (9). Here we want to avoid this "quasi-boson" approximation. We therefore must find a

way to fully express the matrix elements in eqs. (31),(32),(33) by the X, Y amplitudes. This can be achieved in the following way. Employing the usual normalisation of the RPA amplitudes $\frac{1}{2}\sum |X|^2 - |Y|^2 = 1$ we can invert eq.(27)

$$\alpha_p^\dagger \alpha_{p'}^\dagger = \sum_K X_{pp'}^K Q_K^\dagger + Y_{pp'}^K Q_K \tag{34}$$

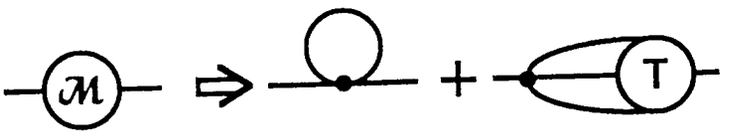
For a two-body Hamiltonian H, the matrix elements in eqs. (31),(32),(33) will involve the following densities (schematically)

$$\begin{aligned} &< \alpha^\dagger \alpha > \, , \, < \alpha^\dagger \alpha^\dagger > \, , \, < \alpha^\dagger \alpha^\dagger \alpha^\dagger \alpha^\dagger > \, , \\ &< \alpha^\dagger \alpha^\dagger \alpha \alpha > \, , \, < \alpha^\dagger \alpha^\dagger \alpha^\dagger \alpha > \; + c.c. \end{aligned} \tag{35}$$

Let us first discuss the third and fourth element. Inserting eq.(34) and commuting Q_K to the right we can use (29). Then $< \alpha^\dagger \alpha^\dagger \alpha^\dagger \alpha^\dagger >$ and $< \alpha^\dagger \alpha^\dagger \alpha \alpha >$ are expressed by X, Y and the density $< \alpha^\dagger \alpha >$. The element $< \alpha^\dagger \alpha^\dagger >= 0$, using eqs.(29) and (34). For the density $< \alpha^\dagger \alpha^\dagger \alpha^\dagger \alpha >$ we insert (34) for the left pair operator, commute Q_K to the right, and are left with an expression directly containing $< \alpha^\dagger \alpha^\dagger >$. So the last density is also zero. Of course analogous considerations hold for the complex conjugate quantities (*c.c.*). We now have reduced everything to the single unknown $< \alpha^\dagger \alpha >$. This quantity is not so easily expressed in a consistent way through the RPA amplitudes and in the past mostly perturbative methods have been applied [23]. It turns, however, out that a fully consistent expression can be found via the single particle Green's function

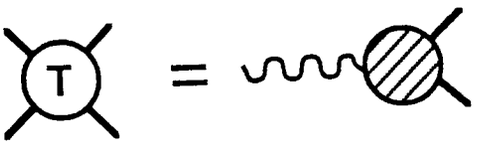
$$\mathcal{G}_{kk'}^{t-t'} \equiv -i < 0|T \begin{pmatrix} a_k(t)a_{k'}^\dagger(t') & a_k(t)a_{k'}(t') \\ a_k^\dagger(t)a_{k'}^\dagger(t') & a_k^\dagger(t)a_{k'}(t') \end{pmatrix} |0 > \tag{36}$$

This Green's function obeys Dyson's equation: $\mathcal{G} = \mathcal{G}^0 + \mathcal{G}^0 \mathcal{M} \mathcal{G}$, which we shall treat in perturbation theory *i.e.* $\mathcal{G} = \mathcal{G}^0 + \mathcal{G}^0 \mathcal{M} \mathcal{G}^0$. The mass operator $\mathcal{M}$ is known to have the following exact representation which we show graphically



where the dot represents the bare interaction and the other vertex contains the exact 4 point T-matrix. We replace it by the expression that corresponds to our RPA eq.(30).

Graphically this can be achieved in the following way



where on the r.h.s the wiggly line stands for the solution of (30), *i.e.* the corresponding RPA propagator, and the shaded cercle stands for the $\begin{pmatrix} A & B \\ B & A \end{pmatrix}$ matrix in (30). We now have a well defined expression for $\mathcal{G}$ in terms of X, Y from which we can calculate $< \alpha^\dagger \alpha >$ in the usual way. This method implies a strong consistency check. It is well known (see *e.g.* [13]) that the ground state energy E_0 can be obtained directly from the single particle Green's function (36) and it will contain the RPA amplitudes. On the other hand from our RPA formalism we can express at least the potential energy part of E_0 which contains the two-body amplitudes of (35) directly by the RPA amplitudes, as this was explained above. It turns out that both ways to calculate E_0 lead to the same functional $E_0[X, Y]$ which represents our above mentioned consistency relation.

Still one ingredient is missing. This concerns the u, v amplitudes of the Bogoliubov transformation to quasi-particles. As usual we determine them from the minimisation of the ground state energy where we can show the following interesting equality [22]

$$\frac{\partial E_0}{\partial u_k} =< 0|[H, Q_K^\dagger]|0 >= 0 \tag{37}$$

where again $|0>$ is the RPA ground state of eq.(29). Equation (37) is a very natural relation, since it is completely consistent with the equation of motion method. The expectation value in (37) again only contains the densities of (35) and thus our set of equations is completely closed. It is interesting to write (37) and (30) somewhat differently

$$< 0|[H, \alpha_k^\dagger \alpha_{k'}^\dagger]|0 > = F[u, v, X, Y] = 0 \tag{38}$$

$$< 0|[H, Q_k^\dagger Q_{k'}^\dagger]|0 > = G[u, v, X, Y] = 0 \tag{39}$$

We recognize in eq.(38) the usual fermion gap equation, if we replace the ground state by its BCS approximation [13]. Loosely speaking we then can

say that eq.(39) represents a "boson" BCS equation, if we identify the $Q^\dagger$ operator roughly as bosons.

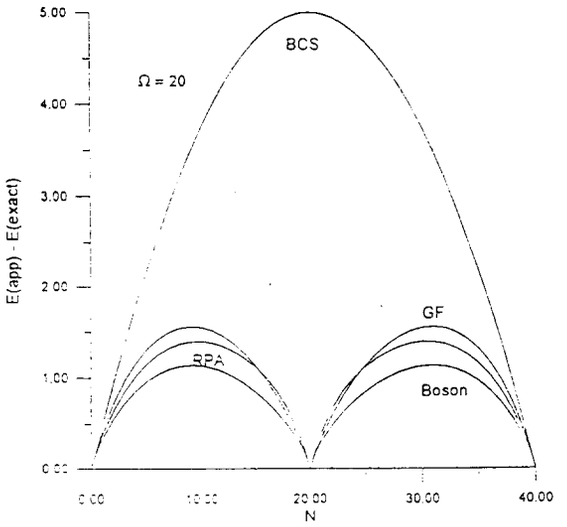


Fig. 7. Ground state energy with respect to the exact one in the BCS and SCqpRPA theories for the seniority model as a function of particle number.

We therefore have the very gratifying situation that we have in (38),(39) two coupled BCS equations, one fermionic (the nucleons) and one bosonic (the "deuterons"). Of course here now the bosons (deuterons) are in motion, and we can describe the real physical situation because it is well known how to generalise qp RPA to finite temperatures [24]. An interesting aspect of eq.(39) is that, since it is a bosonic gap equation, it should describe Cooper pairs of bosons. Indeed it does. It is not difficult to show that (39), in the limit of vanishing density, goes over into the Schroedinger equation of an α- particle in the same way as (38) goes over into the Schroedinger equation of the deuteron eq.(7).

Other interesting properties of our formalism are

- the f- sum rule is automatically fulfilled [22]
- an approximate form of (38),(39) is the Brueckner Hartree Fock equation [25]. It thus seems that one can use hard core potentials in our theory.
- it yields ground state and excitation energies simultaneously
- it sums *ph* and *pp* correlations democratically.

Applications of this theory are only in their infancy. Apart from rather simple models like the Lipkin model [22] the formalism has recently been applied to spin waves in an Heisenberg antiferromagnet [22]. The SCRPA reproduces in 1D the exact one magnon dispersion $\omega_k = \frac{\pi}{2}|sink|$ known from the Bethe ansatz, and in two dimension the dispersion is in agreement with the most

recent Monte Carlo calculations. Concerning our present subject of SCqpRPA the theory was only applied to the standard seniority model of nuclear physics [26]. We show in Fig.7 the difference of the exact ground state energy and *i*) the BCS energy *ii*) the SCqpRPA energy. We can notice the strong improvement over BCS. The same improvement occurs for the number fluctuation $\Delta N/N$.

6 Conclusions

In the last section we have seen that the coexistence of (bound) neutron-proton pairs with a nucleon gas contains a variety of interesting and challenging physical and theoretical aspects. We have demonstrated that already usual equilibrium BCS theory is capable to describe important aspects of the problem. A major deficiency there, however, is the fact that the bound pairs (deuterons) are kept immobile. At finite temperature and very low densities where we then essentially have a dilute gas of bosons this is clearly an unphysical feature. A first remedy to this deficiency is given by the time dependent generalisation of BCS (TDBCS). This can also describe the time evolution of the decompression phase in a head on heavy ion collision, allowing - at least in principle - to give quantitative predictions for the physical observables. TDBCS also puts the Cooper pairs into motion, which for instance can break the pairs. Such an effect has been demonstrated within pilote calculations.

However even TDBCS has deficiencies. We therefore discussed a self-consistent generalisation of RPA theory which ressembles very much BCS theory for bosons, however, we never really introduce the concept of bosons and always work with fermion pairs. It turns out that this SCqpRPA has very nice properties. The essence is that we arrive at two coupled Hartree Fock Bogoliubov equations: one for fermions and one for fermion pairs (deuterons or "bosons").

The possible extension to finite temperature is then ideally suited for the description of our problem: how does an expanding piece of hot nuclear matter first turn into a $n-p$ pairing phase and then. at very low densities, into a gas of dilute deuterons and eventually α- particles.

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