Developing and Validating Nonlinear Height–Diameter Models for Major Tree Species of Ontario's Boreal Forests

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ABSTRACT: *Six commonly used nonlinear growth functions were fitted to individual tree height-diameter data of nine major tree species in Ontario's boreal forests. A total of 22,571 trees was collected from new permanent sample plots across the northeast and northwest of Ontario. The available data for each species were split into two sets: the majority (90%) was used to estimate model parameters, and the remaining data (10%) were reserved to validate the models. The performance of the models was compared and evaluated by model,* R^2*, mean difference, and mean absolute difference. The results showed that these six sigmoidal models were able to capture the height–diameter relationships and fit the data equally well, but produced different asymptote estimates. Sigmoidal models such as Chapman–Richards, Weibull, and Schnute functions provided the most satisfactory height predictions. The effect of model performance on tree volume estimation was also investigated. Tree volumes of different species were computed by Honer's volume equations using a range of diameters and the predicted tree total height from the six models. For trees with diameter less than 55 cm, the six height-diameter models produced very similar results for all species, while more differentiation among the models was observed for large-sized trees (e.g., diameters > 80 cm). North. J. Appl. For. 18:87–94.*

Key Words: Nonlinear growth function, permanent sample plot, forest management.

The northern circumpolar boreal forests are one of the Earth's largest biomes, covering 14.7 million km^2 (11% of the world's continental land mass) (Bonan and Shugart 1989). Eighty percent (80%) of Canada's forests occur within the boreal ecoregion, extending longitudinally from Labrador to the Yukon (Larsen 1980). Canadian boreal forests play an important role not only in timber, mining, and recreational sectors (Pye 1991), but also in global carbon cycles (Apps et al. 1993). One of the most important elements of boreal forest structure is the relationship between tree height and diameter. Individual tree height and diameter are essential forest inventory measures for estimating timber volume, site index, and other important variables in forest growth and yield, succession, and carbon budget models (Spurr 1952, Botkin et al. 1972, Kurz et al. 1992, Vanclay 1994). Tree diameters can be easily measured at little cost. Tree height data, however, are relatively more difficult and costly to collect. Often total tree heights are estimated from observed tree diameter at breast height (DBH) outside bark. Estimating individual tree volume and site index, and describing stand growth dynamics and succession over time, require accurate height–diameter models (Curtis 1967, Botkin et al. 1972). A number of tree height–diameter equations have been developed for various tree species in North America (e.g., Curtis 1967, Wykoff et al. 1982, Larsen and Hann 1987, Wang and Hann 1988, Huang et al. 1992, Moore et al. 1996, Zhang 1997, Peng 1999), and other regions (e.g., Hökkä 1997, Fang and Bailey 1998, Fekedulegn et al. 1999). These height-diameter equations can be used to predict "missing" tree heights from field diameter measurements (Larsen and Hann 1987), and to

NOTE: Changhui Peng can be reached at (705) 946–2981, ext. 119; Fax: (705) 946–2030; E-mail: changhui.peng@mnr.gov.on.ca. This research was stimulated by a discussion with J. Parton, M. Woods, S.M. Huang, and Y.H. Wang about developing growth and yield models for the boreal region of Ontario. We thank A. Groot, J.N. Candau, and L. Buse for their valuable comments on an earlier draft. We are grateful to two anonymous reviewers for their useful suggestions and comments on the previous version of the manuscript. This work was supported by the Forest Growth and Yield Program of the Ontario Ministry of Natural Resources. Copyright © 2001 by the Society of American Foresters.

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Table 1. Common names, scientific names and assigned codes for the nine tree species.

Common name	Scientific name	Code
Jack pine	*Pinus banskiana* Lamb.	JP
Black spruce	*Picea mariana* (Mill.) B.S.P.	BS
White spruce	*P. glauca* (Moench) Voss	WS
Trembling aspen	*Populsu tremuloides* Michx.	TA
White pine	*Pinus strobus* L.	WP
Red pine	*P. resinosa*	RP
Balsam fir	*Abies balsamea* (L.) Mill	BF
Yellow birch	*Betula alleghaniensis* Arnold	YB
Balsam poplar	*Populus balsamifera* L.	BP

estimate individual tree biomass using appropriate single–tree biomass equations (Singh 1982, Penner et al. 1997).

The purpose of this study was (1) to develop tree height-diameter models for nine boreal forest tree species using six widely used nonlinear growth functions, and (2) to evaluate the relative performance of these models calibrated from a wide range of site productivity, tree age, and tree size using independent validation data sets from Ontario.

Data, Models, and Methods

A total of 22,571 individual height-diameter measurements for nine boreal forest tree species were collected from new permanent sample plots across the northeast and northwest of Ontario. The common name, scientific name, and an assigned code for each species are listed in Table 1. All sampled trees were measured for diameter at breast height (DBH) outside bark, and total height (HT). Forked trees or those with damaged tops were excluded from the analysis. The available tree heightdiameter data were split into two sets: the majority (90%) was used for model development; and 10% of trees in each diameter class for each species were randomly selected and reserved for model validation (Moore et al. 1996). For example, data from 3,089 trembling aspen trees were selected for model fitting; the remaining 343 trees were used for model validation. Both model development and validation data sets covered the same ranges of DBH and HT (Figure 1). Summary statistics are provided in Tables 2 and 3 for all nine species. Numbers of trees by species ranged from 240 to 5,555 for the model development data (Table 2) and from 26 to 616 for the validation data (Table 3).

Many nonlinear models have been used to model tree height–diameter relationships (e.g., Huang et al. 1992, Moore et al. 1996, Zhang 1997, Fang and Bailey 1998, Peng 1999, Fekedulegn et al. 1999). Model selection in this study was based on an examination of the height–diameter relationships revealed by plotting HT against DBH for all nine species. Scatter plots of tree HT vs. DBH presented typical sigmoidal-concave curves. Six nonlinear growth functions (Table 4) were selected as candidate height–diameter models based on their appropriate mathematical features (e.g., typical sigmoid shape, number of parameters, flexibility), possible biological interpretation of parameters (e.g., upper asymptote, maximum or minimum growth rate), and satisfactory prediction for tree height–diameter relationships in the literature (Brewer et al. 1985, Arabatzis and Burkhart 1992, Huang et al. 1992, Zeide 1993, Zhang et al. 1996, Zhang 1997, Fang and Bailey 1998, Huang 1999, Fekedulegn et al. 1999). These six nonlinear

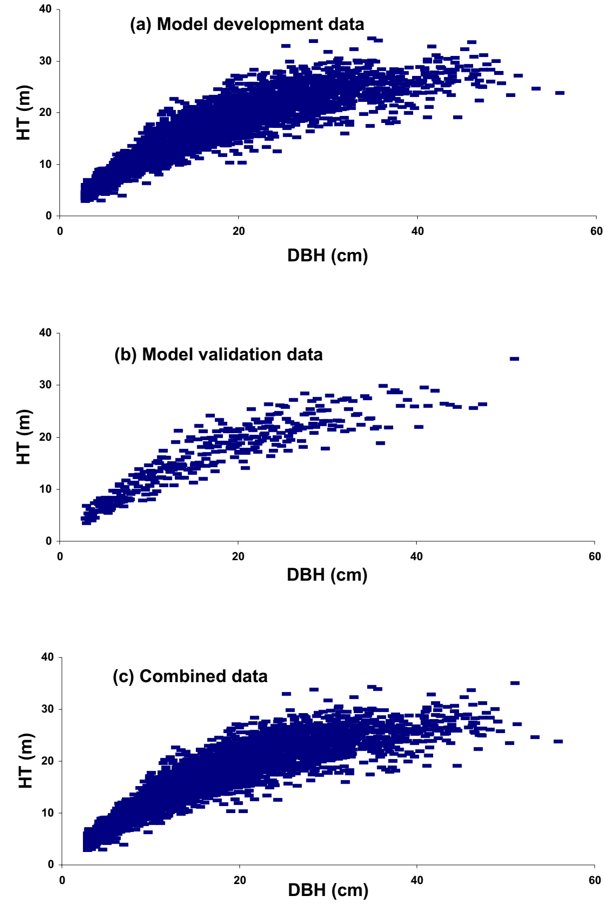

Figure 1. Scatter plot of total height (HT) against diameter at breast height (DBH) for trembling aspen: (a) Model development data, (b) Model validation data, and (c) Combined data.

Table 2. Summary statistics of diameter at breast height (DBH) outside bark and total tree height (HT) for data used in model development.

	No. of	Dbh (cm)				Ht (m)			
Species	trees	Mean	Min.	Max.	STD	Mean	Min.	Max.	STD
Jack pine	4,954	14.79	1.40	44.80	7.91	13.28	2.06	28.02	6.48
Black spruce	5,555	11.31	0.70	36.50	6.10	10.24	1.42	26.80	4.92
White spruce	743	12.93	2.50	56.30	9.63	9.45	2.12	30.78	6.06
Trembling aspen	3,089	17.19	2.50	55.50	10.32	16.38	2.75	35.00	6.91
White pine	2,162	21.48	2.50	90.20	14.41	15.09	1.49	38.87	7.94
Red pine	1,332	22.03	2.50	61.20	12.70	16.26	1.97	40.68	8.43
Balsam fir	1,845	8.09	2.40	42.70	5.43	7.22	1.52	26.88	4.24
Yellow birch	398	15.07	2.50	77.20	13.01	13.41	3.61	26.22	5.69
Balsam poplar	240	23.36	2.60	55.10	11.11	19.48	2.21	32.38	6.42

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Table 3. Summary statistics of diameter at breast height (DBH) outside bark and total tree height (HT) for data used in model validation.

	No. of	Dbh (cm)				Ht (m)			
Species	trees	Mean	Min.	Max.	STD	Mean	Min.	Max.	STD
Jack pine	550	14.81	2.50	42.10	7.92	13.32	2.15	24.96	6.42
Black spruce	616	11.34	2.10	33.30	6.12	10.25	1.30	24.80	4.93
White spruce	82	12.99	2.50	48.10	9.66	9.82	2.30	28.97	6.38
Trembling aspen	343	17.24	2.50	50.90	10.36	16.47	3.40	33.94	6.77
White pine	240	21.59	2.50	78.20	14.55	15.05	2.74	31.90	7.95
Red pine	148	22.56	2.60	68.50	22.56	16.05	2.09	36.38	8.16
Balsam fir	204	8.03	2.50	27.80	5.26	7.17	1.80	22.95	4.24
Yellow birch	44	15.64	2.50	66.70	14.04	13.76	3.27	27.50	6.03
Balsam poplar	26	23.53	4.00	45.30	10.70	20.18	5.59	29.02	6.03

growth functions have been widely used for two major reasons. First they define sigmoid curves, in which the growth rate increases as size increases from a minimum value to a maximum at a point of inflection, and then declines towards zero at an upper asymptote. Second, they have three parameters (i.e., an upper asymptote, a rate parameter, and a shape parameter) that describe various biological processes and behaviors. For example, the Chapman-Richards and Weibull models are well known flexible growth functions with biologically interpretable coefficients (Pienaar and Turnbull 1973, Yang et al. 1978).

The six candidate models (Table 4) were fitted to the model development data of tree height and diameter for each species, respectively. Parameters were estimated using the PROC NLIN procedure in the Statistical Analysis System (SAS Institute Inc. 1990). We elected to use the Marquardt method because it is considered to be most useful when parameter estimates are highly correlated (Fang and Bailey 1998) and represents a combination of the best features of the linearization (Gauss-Newton) method and the steepest descent method (Fekedulegn et al. 1999). To ensure that the least–squares solution is global rather than local, multiple initial values of the model parameters were provided for the fits. The validity of a homogeneous variance was investigated. There was no significant evidence of unequal error variances, as observed in other studies (e.g., Huang et al. 1992). Under this circumstance, weighted least-squares may increase model fit marginally, but may not significantly improve model performance (Cormier et al. 1992, Zhang 1997). Therefore, ordinary nonlinear least-squares was used for parameter estimation rather than weighted least-squares. Each model was evaluated using R^2, mean difference (MD), and mean absolute difference (MAD). computed as follows:

$$R^2 = 1 - \left[\frac{\sum_{i=1}^{n} \left(H_i - \hat{H}_i\right)^2}{\sum_{i=1}^{n} \left(H_i - \overline{H}\right)^2} \right]$$

$$MD = \frac{\sum_{i=1}^{n} \left(H_i - \hat{H}_i\right)}{n}$$

Table 4. Nonlinear height-diameter models selected for comparison using data from boreal forests in northeast and northwest of Ontario.

Model			References
Chapman–Richards:	$HT = 1.3 + a\left(1 - e^{-b \cdot DBH}\right)^c$	(1)	Chapman 1961, Richards 1959
Weibull:	$HT = 1.3 + a\left(1 - e^{-b \cdot DBH^c}\right)$	(2)	Yang et al. 1978
Schnute:	$HT = \left\{1.3^b + \left(c^b - 1.3^b\right)\frac{1 - e^{-a\left(DBH - DBH_0\right)}}{1 - e^{-a\left(DBH_2 - DBH_0\right)}}\right\}^{\frac{1}{b}}$	(3)	Schnute 1981
Exponential:	$HT = 1.3 + a \cdot e^{\frac{b}{\left(DBH + c\right)}}$	(4)	Ratkowsky 1990
Modified Logistic:	$HT = 1.3 + \frac{a}{\left(1 + b^{-1} \cdot DBH^{-c}\right)}$	(5)	Ratkowsky and Reedy 1986, Huang et al. 1992
Korf/Lundqvist:	$HT = 1.3 + a \cdot e^{\left(-b \cdot DBH^{-c}\right)}$	(6)	Stage 1963, Zeide 1989

NOTE: *HT* is tree total height (m); *DBH* is tree diameter at breast height (cm); *a,b,c* are model parameters to be estimated; e is the base of natural logarithm (≈2.71828); 1.3 is a constant used to account for measuring tree DBH at 1.3 m above ground. In Equation (3): DBH_0 = 0.0 and DBH_2 = 100.

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$$MAD = \frac{\sum_{i=1}^{n} \left| H_i - \hat{H}_i \right|}{n}$$

where, for a given data set of size n, H_i is the observed, and $\hat{H}_i$ is the predicted height for the ith tree, respectively; and $\overline{H}$ is the observed mean tree height. For any appropriately fitted height–diameter model, the R^2 should be large. A higher R^2 value indicates a better fitting to the given data set. A positive MD indicates, on average, underprediction by the model and a negative value of MD indicates overprediction.

The model validation data (Table 3) were divided into eight DBH classes (i.e., <5 cm, 5–10 cm, 10–15 cm, 15–20 cm, 20–25 cm, 25–30 cm, 30–35 cm, and >35 cm) for jack pine (JP), trembling aspen (TA), white pine (WP) and red pine (RP); and six DBH classes (i.e., <5 cm, 5–10 cm, 10–15 cm, 15–20 cm, 20–25 cm, and >25 cm) for other species. The six height-diameter models were employed to predict tree total height using the observed tree DBH in the validation data sets. The mean prediction errors (MD) were computed for each DBH class of each tree species and were illustrated in Figure 2. An overall prediction error was also calculated across all DBH classes for each species.

Results and Discussion

Model Development

The R^2 values for all 6 models and all 9 species (54 combinations) were 0.96 or greater in each case (Table 5), explaining at least 96% of the total variation in tree heights. The highest R^2 value of 0.99 was found in fitting the models for balsam poplar. The MD from –0.002 to 0.45, and MAD from 0.90 to 2.75, respectively, for the nine species. Differences in MD among the six models for each species were not significant. In general models (1) (Chapman–Richards), (2) (Weibull) and (3) (Schnute) had relatively smaller MAD than other three models for all species, with a few exceptions: models (4) (Exponential) and (5) (Modified Logistic) fitted to black spruce (BS), white pine (WP) and balsam fir (BF). All model coefficients were significant at the 5% level. The results from model statistics suggested that all six models fitted equally well to the tree height–diameter data of the nine species (Table 5). This is consistent with the findings reported by Huang et al. (1992) for major Alberta tree species and by Zhang (1997) for ten tree species in inland Northwest of the United States. It is also worthy to note that the six models fitted to the same data sets produced different asymptote coefficients (coefficient a in Table 5,

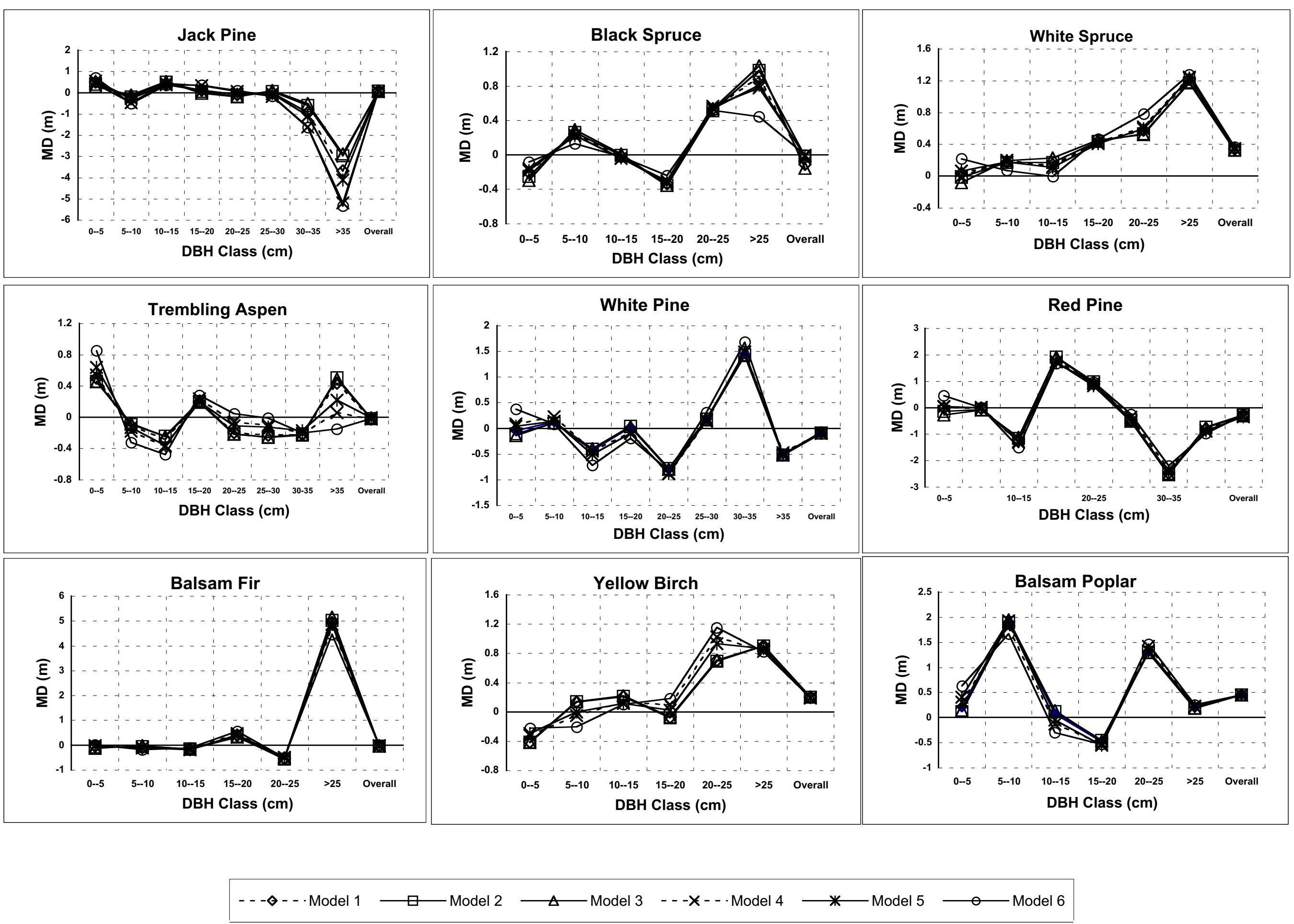

Figure 2. Average prediction errors from the six height–diameter models for the 5-cm DBH classes in the model validation data set for nine tree species. Model 1 (Chapman-Richards), Model 2 (Weibull), Model 3 (Schnute), Model 4 (Exponential), Model 5 (Modified Logistic), and Model 6 (Korf/Lundqvist) refer to the six models in Table 4. The "overall" represents mean prediction error across all DBH classes.

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Table 5. Parameter estimates and performance criteria of six nonlinear height-diameter models for nine boreal forest tree species in northeast and northwest of Ontario. MD: Mean difference; MAD: Mean absolute difference. See Table 4 for the form of the model; See Table 1 for species codes.

	Parameters			Performance criteria		
Model and species	*a*	*b*	*c*	R^2	MD	MAD
(1) Chapman-Richards						
JP	22.9430	0.0967	1.9923	0.9651	0.0651	1.9736
BS	22.2124	0.0729	1.4633	0.9680	–0.0115	1.3183
WS	27.3476	0.0469	1.4360	0.9697	0.3424	1.2633
TA	27.0190	0.0667	1.2249	0.9799	–0.0238	1.7663
WP	31.6215	0.0367	1.1899	0.9678	–0.0760	2.2602
RP	31.5167	0.0487	1.5772	0.9627	–0.3107	2.7107
BF	20.1136	0.0791	1.5449	0.9570	–0.0311	0.8956
YB	20.7576	0.0858	1.0532	0.9807	0.2007	1.1037
BP	27.0418	0.0670	1.3050	0.9884	0.4530	1.5546
(2) Weibull						
JP	21.3607	0.0137	1.5825	0.9654	0.0741	1.9619
BS	20.8565	0.0253	1.3119	0.9679	–0.0178	1.3233
WS	25.5603	0.0148	1.3081	0.9696	0.3250	1.2643
TA	26.4919	0.0388	1.1482	0.9780	–0.0201	1.7647
WP	30.9183	0.0214	1.1307	0.9678	–0.0856	2.2588
RP	29.7470	0.0115	1.3666	0.9627	–0.2836	2.7127
BF	18.7403	0.0234	1.3670	0.9569	–0.0355	0.8974
YB	20.7055	0.0759	1.0345	0.9808	0.2015	1.1048
BP	26.5287	0.0331	1.1865	0.9884	0.4467	1.5565
(3) Schnute						
JP	0.1383	–0.1353	22.7063	0.9707	0.0622	1.9538
BS	0.0901	0.3676	22.3022	0.9741	–0.0168	1.3287
WS	0.0593	0.3990	26.8356	0.9752	0.3272	1.2720
TA	0.0708	0.7053	28.0084	0.9825	–0.0219	1.7646
WP	0.0391	0.7475	31.7029	0.9720	–0.0934	2.2591
RP	0.0586	0.3495	31.3228	0.9672	–0.3144	2.7160
BF	0.1078	0.1975	19.8436	0.9674	–0.0461	0.8992
YB	0.0864	0.9244	22.0426	0.9837	0.2036	1.1054
BP	0.0719	0.6223	28.0183	0.9897	0.4536	1.5590
(4) Exponential						
JP	35.4102	–16.9804	2.3852	0.9645	0.0503	2.0044
BS	32.2518	–17.9185	3.4605	0.9681	–0.0060	1.3145
WS	39.7715	–27.8572	5.5976	0.9696	0.3419	1.2641
TA	35.9494	–15.6759	3.5565	0.9798	–0.0276	1.7702
WP	42.2.84	–28.1327	6.5547	0.9679	–0.0875	2.2745
RP	46.1240	–28.0922	4.9393	0.9627	–0.3062	2.7193
BF	29.7533	–17.4797	3.2353	0.9572	–0.0250	0.8969
YB	24.9856	–9.0828	2.0794	0.9803	0.1919	1.0872
BP	35.7474	–15.6913	3.1557	0.9883	0.4504	1.5245
(5) Modified logistic						
JP	25.9658	0.0079	1.8180	0.9651	0.0583	1.9764
BS	27.5359	0.0165	1.4327	0.9681	–0.0156	1.3167
WS	33.9942	0.0095	1.4114	0.9696	0.3451	1.2609
TA	32.3786	0.0254	1.3231	0.9798	–0.0096	1.7738
WP	39.7993	0.0137	1.2509	0.9679	–0.0978	2.2746
RP	37.5550	0.0067	1.5352	0.9627	–0.3478	2.7133
BF	24.3867	0.0155	1.4942	0.9571	–0.0374	0.8957
YB	23.4921	0.0530	1.2774	0.9804	0.1972	1.0843
BP	31.4760	0.0199	1.4140	0.9884	0.4517	1.5418
(6) Korf/Lundqvist						
JP	42.9266	7.8409	0.7033	0.9640	0.0809	2.0287
BS	66.4854	5.9281	0.4613	0.9680	–0.0018	1.3098
WS	98.3565	6.8664	0.4113	0.9690	0.3619	1.2635
TA	49.3730	5.3055	0.5611	0.9794	–0.0097	1.7927
WP	84.0387	6.2928	0.4278	0.9676	–0.0655	2.3180
RP	74.0437	8.0952	0.5414	0.9624	–0.2847	2.7451
BF	60.3138	5.9834	0.4703	0.9568	–0.0153	0.9111
YB	28.2458	4.1391	0.6790	0.9800	0.1921	1.0815
BP	43.8525	6.0371	0.6417	0.9881	0.4526	1.5282

with the exception of the Schnute model (3) in which the asymptotic coefficient is approximate to coefficient *c*). In most cases models (1) (Chapman–Richards), (2) (Weibull), and (3) (Schnute) had similar asymptotic coefficients for all species. Model (6) (Korf/Lundqvist) produced the largest asymptotic coefficients for all species.

Model Validation

Figure 2 illustrates the mean prediction errors for the 5 cm DBH classes and overall mean prediction error across the DBH classes for each model and tree species. In general, based on the validation data, the overall mean standard

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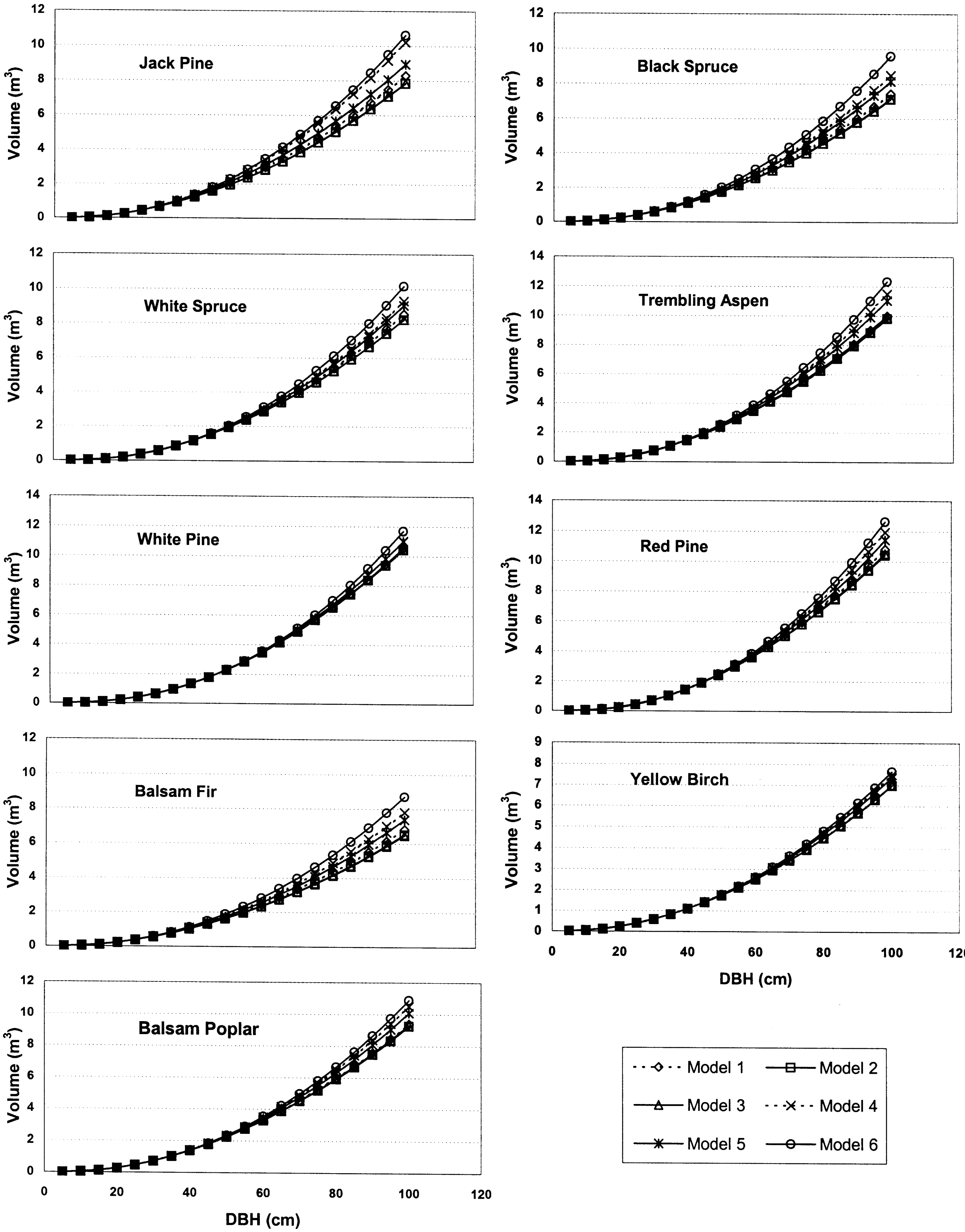

Figure 3. Estimations of total tree volume (m^3/tree) based on the individual tree total volume equations developed by Honer et al. (1983; Equation 14 and Table 3 on p. 17), given observed DBH and predicted total tree height from six height–diameter models as shown in Table 4 for nine boreal forest tree species in Ontario. Note: Equation 14 (Honer et al. 1983) is Volume = 0.0043891 DBH^2 $(1 - 0.04365b_2)^2$ / (c_1 + (0.3048c_2 / HT)). The regression coefficients b_2, c_1, c_2 for each species are given by Honer et al. 1983 (Table 3). Model 1 (Chapman-Richards), Model 2 (Weibull), Model 3 (Schnute), Model 4 (Exponential), Model 5 (Modified Logistic), and Model 6 (Korf/Lundqvist) refer to the six models in Table 4.

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deviations of prediction error range from 1.3 to 3.5 m depending on tree species (Figure 2). On one hand, all six models produced similar prediction errors (<1 m) for small trees (DBH < 20 cm), with the model (6) (Korf/Lundqvist) having the largest errors. On the other hand, all six models underestimate heights for large trees for YB, WS, TA, BP, WP, BP, and BF, but overestimate heights of large trees for JP and RP. Model (6) (Korf/Lundqvist) consistently had the largest mean predicted errors. Models (1) (Chapman-Richards), (2) (Weibull), and (3) (Schnute) showed the superiority in prediction performance than others (Figure 2). However, using the models to extrapolate beyond the data range may increase the degree of over- or underestimation for large trees (Zhang et al. 1996). Using the Monte Carlo cross-validation method, Zhang (1997) evaluated the prediction performance of six height–diameter models for ten conifer species in the inland Northwest of the United States and found that Schnute, Weibull, and Chapman–Richards functions gave more accurate results than other models. Considering the model mathematical features, biological interpretation of parameter, and accurate prediction, we recommend the Chapman-Richards model as the best candidate.

Model Evaluation Based on Tree Volume Estimation

Tree volume is commonly expressed as a function of tree DBH and HT. To investigate the effects of tree height prediction on volume estimation, we set a range of tree diameters starting at 5 cm, followed by 5 cm intervals, up to 100 cm. Corresponding tree height was estimated using the six models for each species (Table 4). The individual tree volume equations developed by Honer et al. (1983; Table 3, p. 17) was used to calculate tree volumes. Tree volume estimations are similar for trees with DBH less than 55 cm for all species. For some species such as WP, RP, and YB this is true up to 70 cm (Figure 3). For large-sized trees (DBH > 80 cm), however, tree volume estimations show some differentiation among the six height–diameter models. Model (6) always produces larger tree volumes than other models, mainly because of higher upper asymptotes, while models (1), (2), and (3) yield consistent volume ranges for all species. For forest inventories or model simulations (e.g., forest succession and carbon budget) containing large trees, we recommend that models (1), (2), and (3) should be used.

Conclusion

Development and analysis of six nonlinear height–diameter models fitted to nine tree species in Ontario's boreal forests show that most concave and sigmoidal functions are able to accurately describe tree height–diameter relationships. The results are consistent with findings reported by Huang et al. (1992) for major Alberta tree species, and by Zhang (1997) for ten tree species in the inland northwest of the United States. Validation of the six selected models using independent data sets indicate that sigmoidal models such as the Chapman–Richards, Weibull, and Schnute functions give the most satisfactory results. Model evaluation based on tree volume demonstrate that all models produce similar prediction for trees less than 55 cm DBH for all species. For large trees (DBH > 80 cm), total tree volume estimations showed some differentiation among the six height–diameter models. In general, the Chapman-Richards model should be considered the best model for all species across the entire study region.

However, height–diameter relationship varies within a region, depending on local environmental condition. These provincial-based height–diameter models do not take into account the effects of climatic and ecological factors on height–diameter relationships within the different ecological site regions and thus are only appropriate for making height predictions on a broad provincial basis. Further development of ecoregion–based individual tree height–diameter models is critical for accurate models on which to base, forest management decisions (Huang 1999, Huang et al. 2000).

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