

# Developing Contextual Mathematical Thinking Learning Model to Enhance Higher-Order Thinking Ability for Middle School Students

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## Abstract

The purpose of this research is to develop contextual mathematical thinking learning model which is valid, practical and effective based on the theoretical reviews and its support to enhance higher-order thinking ability. This study is a research and development (R & D) with three main phases: investigation, development, and implementation. The experiment consisted of 78 Junior High School students who were divided into two groups, namely experimental group and control group. The model development phase results the syntax of contextual mathematical thinking learning model which are as follows: (1) presentation of the contextual problems; (2) asking the critical and analytical questions; (3) individual and group investigation; (4) presentation and discussion; (5) reflection; and (6) higher-order thinking test. The implementation phase concludes the contextual mathematical thinking learning model which can be applied effectively to enhance the students' higher-order thinking ability. This model is able to intensify higher-order thinking ability at high category. The observation of learning activities was seen in the main elements of learning model which are syntax, social system, reaction principle, support system, instructional impact, and accompanist impact. The three main elements were observed by the observer and showed an average in the good category: syntax has an average of 3.5, social system has an average of 3.52, and reaction principle has an average of 3.47. This model is recommended for mathematics learning activities in the classroom to support the improvement of higher-order thinking ability. Contextual problems can be presented to the local cultural context that allows students to learn mathematics in a real context.

**Keywords:** contextual learning, higher-order thinking, mathematical thinking

## 1. Introduction

### 1.1 Introduce the Problem

Mathematics is the science which emphasizes the formation of the thinking ability. The systematic characteristic in mathematics confirms the formation of a coherent mindset, effective and straightforward. Mathematics also asserts rational characteristics which mean everything that is used must be accounted for in rational, logical or reasoned ways. This assertions make someone who studied mathematics properly will make himself have a systematic and logical thinking. According to Schoenfeld (1992:335), mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engage in the science of patterns-systematic attempts, based on observation, study, and experiments to determine the nature or principles or regularities in a system-defined axiomatically or theoretically (pure mathematics) or model of systems abstracted from real-world objects (applied mathematics). Present mathematics learning is oriented towards the enrichment of higher-order thinking ability. The ability to think mathematically and to use mathematical thinking to solve problems is an important goal of schooling (Stacey, 2007). The goal of developing higher-order thinking ability has become a formal foundation in mathematics learning in Indonesia. The regulation of the National Education Minister No. 23/2006 is about the competency standards from elementary to high school level that reveals the competencies required for the students are: the ability to think logically, critically, creatively, and innovatively in decision-making. Further, regulation of the National Education Minister No. 64/2013 is about the content standards from elementary to high school level that divides the levels of competency into six levels. The students' higher-order thinking ability is poor that is caused by a learning process which does not facilitate the

students to think outside of context, imagination, decision-making and creativity. Meanwhile, the evaluation of the teachers' lesson plans shows that most of the teachers present their lesson plan as a step of the learning activities with a general concept that the learning is designed with no details of specific activities that illustrates the accommodation of the thinking ability. In the classroom implementation, teachers emphasized more on conceptual mastery of the subject matter with lack of attention on how the understanding is enhanced in the higher-order thinking. Furthermore, the daily and final tests that were given are more lead or emphasis on lower order thinking, in this case the test is still in cognitive application level, so that it is not surprising that our students' achievements are low both in terms of passing the examination standards and when evaluated in international level such as PISA and TIMSS.

### *1.2 Explore Importance of the Problem*

The higher-order thinking is the idea presented by Bloom in his taxonomy for education. Bloom (1956) divides the taxonomy into six levels: (1) knowledge; (2) comprehension; (3) application; (4) analysis; (5) synthesis, and (6) evaluation. The first level to the third level is the lower order thinking and the fourth level to the six levels is higher-order thinking. This taxonomy was revised by Kratwohl and Anderson (2001), which gave rise to the new educational taxonomy into (1) remembering; (2) understanding; (3) applying; (4) analyzing; (5) evaluating, and (6) creating. Aspects of higher-order thinking in the new taxonomy are: (1) analyzing; (2) evaluating; and (3) creating. The three levels are defined as higher-order thinking which has attributes that distinguish from one another and are used as part in learning activities in both the process and evaluation. Analyzing associated with cognitive processes involves attributing, organizing, integrating and validating. Evaluating includes checking, critiquing, hypothesizing and experimenting. Creating contains generating, designing, producing and devising.

Some experts defined higher-order thinking by referring directly to Bloom's Taxonomy of revisions to mention higher-order thinking as the analytic, evaluative and creative thinking (Pegg, 2010; Thompson, 2000). Thomas & Thorne, (2014) defined higher-order thinking as thinking on a level that is higher than memorizing facts or telling someone something back to exactly the way it was of toll to you. Samo (2014) revealed higher-order thinking is the types of non-algorithm thinking which include analytic, evaluative and creative thinking that involves metacognition

The importance of developing higher level thinking has some reasons: (1) to organize knowledge learned into long-term memory. Organizing raises enough information retention longer than if stored in short-term memory that is the characteristic of lower order thinking. For example, students who learn to memorize tend to quickly forget what is memorized than students who learn on how to discover. Memorization process will push that knowledge into a short-term memory, while the process of discovering will push that knowledge into a long-term memory. Knowledge stored in a long-term memory is easily accessed and is used in various situations that tend to change: (1) to develop adaptability to a variety of new problems that is found in life, exercises to develop a higher-order thinking ability in formal education which will develop an attitude and a way of creative thinking to get out of the life's problems which are complex, (2) to encourage the creation of quality human resources that can compete with other nations.

The results of several international level surveys indicated that Indonesian students' achievement in problem-solving activities, especially mathematics, is low. The results of PISA (Programme for International Student Assessment) from 2000 to 2015, Indonesian students ranking in reading, mathematics and science literacy is low. Indonesia's result in the OECD Program for International Student Assessment, or PISA 2015 report, showed some improvements in the skills of its students. From 72 countries and economies which are reviewed every three years, Indonesia ranks 62nd, a slight improvement compared to 2013. Indonesian students ranked the second lowest in the 2013 PISA ranking (71), worse than their ranking in 2009, when Indonesia ranked 57th.

### *1.3 Describe Relevant Scholarship*

Developing higher-order thinking ability is a necessity in all studies. The importance of higher-order thinking ability development has prompted many researchers to develop students' higher-order thinking in their studies. Elser (2008) described teachers who used writing in three important ways to increase higher level thinking skills for all students. First, they must increase writing proficiency for struggling writers; second, writing can increase higher-level thinking, and third, writing can be used across content areas to increase writing fluency while fostering higher level thinking. Various studies illustrated the development of higher-order thinking ability. Miri, David, and Uri (2007, p. 353) revealed if teachers purposely and persistently practice higher-order thinking strategies, for example, in dealing in class with real-world problems, encouraging open-ended class discussions, and fostering inquiry-oriented experiments, there is a good chance for a consequent development of critical

thinking capabilities. In its implementation, teachers should clearly understand classical structures of teaching higher-order thinking and to transform unstructured (Wang & Wang, 2010). Developing higher-order thinking ability can use problem-based learning. Magsino (2014, p. 1) used problem-based learning to evaluate higher-order thinking ability in marine biology class. The result of the study showed that problem based learning is an efficient instructional strategy embedded within a conventional curriculum to develop or enhance critical thinking in marine biology. Furthermore, the students' higher-order thinking skills were enhanced in terms of their ability to (a) pose complex questions, (b) present solid opinions, (c) introduce consistent arguments, and (d) demonstrate critical thinking (Barak & Dori, 2009).

Developing higher-order thinking ability required good planning. Lessons that involve higher-order thinking skills require particular clarity of communication to reduce ambiguity and confusion, and to improve students' attitudes about thinking tasks. Lesson plans should include modeling of thinking skills, examples of applied thinking, and adaptations for diverse student needs (King, Goodson, & Rohani, 2011). In planning context, it should also plan for required activities. Asking questions plays an important role in enhancing higher-order thinking ability. Asking questions in learning activities should be analytical, critical and creative questions. Tofade, Elsner, and Haines (2013) reveal that questions have long been used as teaching tools by teachers and preceptors to assess students' knowledge, promote comprehension, and stimulate critical thinking. Well-crafted questions lead to new insights, generate discussion, and promote comprehensive exploration of subject matter. Poorly constructed questions can stifle learning by creating confusion, intimidating students, and limiting creative thinking. Teachers most often ask lower-order, convergent questions that rely on students' factual recall of prior knowledge rather than asking higher-order and divergent questions that promote deep thinking, requiring students to analyze and evaluate concepts.

To develop higher-order thinking mathematical ability can be done with the reform of learning activities in the classroom. Some of the approaches and models of learning can be solutions that are contextual learning, problem-based learning, and higher-order thinking strategies. Johnson (2002) described contextual teaching and learning (CTL) is "a system of instruction based on the philosophy that students learn when they see meaning in academic material, and they see meaning in schoolwork when they can connect new information with prior knowledge and their own experience". CTL has the eight components: making meaningful connections, doing significant work, self-regulated learning, collaborating, critical and creative thinking, nurturing the individual, reaching high standards and using authentic assessment. While the problem based learning according to Hmelo-Silver (2004; p. 235); problem-based learning (PBL) is an instructional method in which students learn through facilitated problem-solving. In PBL, student learning centers on a complex problem that does not have a single correct answer. Students work in collaborative groups to identify what they need to learn in order to solve a problem. The last, the synthesis results of the higher-order thinking teaching strategies (Collins, 2014, King, Goodson & Spiritual, 2011, Teaching higher-order thinking, 2011) were, (1) using a contextual problem, (2) using various problems which allow the students to use new skills, (3) asking questions and discussing the critical, analytical and creative to build the higher-order thinking ability, (4) working in groups, (5) using scaffolding, (6) encouraging students to think about the thinking strategies they use, (7) using the example and testing higher-order thinking.

The three types of learning described above provide an overview of interrelation. CTL presents the lessons with the characteristics of constructivism, emphasize the students' participation, and comprehensive assessment. CTL is a learning approach that gives the spirit in a structured learning model. PBL is a learning model that presents the complete learning steps and organized. The first phase of the problem-based learning activity is the orientation of the students to the problem. The problem is the unstructured problem, contextual and can be cross-disciplinary; it is also in accordance with the higher-order thinking strategies used in the contextual problem. The teacher's role in the problem-based learning activities is to ask the questions that encourage critical thinking. Asking activity is to train the students learn to think in accordance with the characteristics of the PBL. This characteristic also aligned with the higher-order thinking strategy; i.e. asking the questions and discussing with the critical, analytical and creative. Furthermore, one important part of the higher-order thinking strategy is the metacognition aspect or that encourages students to think about thinking strategies that they do. This strategy is important because the essence of the problem-solving is not the only result of the solution, but the process of the problem-solving. Good results were drawn from a good process and must be passed by the awareness that what they did was good. Metacognition strategy does not appear in PBL activities. The third and fourth stages of the learning activities do not only show the role of the teachers in helping students, encouraging students, planning and preparing the work. As stated in the introduction, our main research question was to develop the contextual mathematical thinking learning model which is valid, practical and effective based on the theoretical

reviews and its support to increase higher-order thinking ability.

#### *1.4 Research Purposes*

In the introduction and research question, the research was carried out in order to develop the contextual mathematical thinking learning model which is valid, practical and effective based on the theoretical reviews and its support to increase higher-order thinking ability.

#### *1.5 Hypothesis*

The hypotheses are:

- a. Students who learn with contextual mathematical thinking learning model has an enhanced higher-order thinking ability better than the other learning models
- b. Students who have a better way of increasing higher-order thinking ability after learning and using the contextual mathematical thinking learning model

#### *1.6 Indicators*

According to Nieveen (1999), a learning model is a good model if: (1) valid; (2) practical and (3) effective. Validity attributed to two things: (a) whether the model developed was based on the strong theoretical rationale, and (b) whether there is an internal consistency. Practicality if: (a) experts and practitioners argue that the model can be applied, and (b) the fact shows that the model can be applied. Effective if: (a) experts and practitioners based on his experiences mentioned the model is efficient and (b) the model gives the result expected

## **2. Method**

This study is a research and development (R & D) with three main stages, i.e. investigation, development, and implementation. Following are presented in the phase of model development, research instruments/variables, data collection, and data analysis.

### *2.1 Model Development*

#### Phase I-Investigation

- a. Analysis of the learning activities

This analysis includes the study of the teacher's lesson plan, implementation, and evaluation of learning activities.

- b. Analysis of student's background

This analysis includes the study of the student's cultural background.

- c. Analysis of the student's ability

This analysis includes the study of the student's academic ability and student's learning experience.

#### Phase II-Model Development

The first step of model development is designing the initial model. Furthermore, the model is validated by mathematics learning and evaluation experts. The result of the validation was revised and it became the final model. The final model confirms the validity criterion which the model developed based on strong theoretical rationale and internal consistency. The final model considers the learning model components that are syntax, social system, reaction principle, support system, instructional impact, and accompanist impact.

#### Phase III-Implementation

- a. Learning Process

This model was implemented in Middle School mathematics. The total sample is 78, divided into two classes with the learning materials: a cone, cylinder, and sphere.

- b. Learning outcomes

Learning outcomes are the achievements of higher-order thinking ability test which presented at the beginning and end of the implementation of the model.

### *2.2 Samples*

Implementation of the contextual mathematical thinking learning model was carried out in Grade VIII Junior High School students on the cone, cylinder, and sphere. The participants are 78 Junior High School students consisting of two groups, namely experimental group and control group.

### 2.3 Instruments, Variables and Data Collection

Instruments of this research are model validation sheet, lesson plan, observation sheets of learning activities, observation sheet of students' activities, student responses questionnaire, and higher-order thinking ability test. Variable of this study consists of an independent variable that is contextual learning mathematics and the dependent variable is higher-order thinking ability. Data were divided into two parts, quantitative and qualitative data. Qualitative data is a description of the learning model assessment by experts includes the construct and content validity. Qualitative data were taken with sheets of model validation. Quantitative data are the observation data of learning activities and data of students' higher-order thinking ability test.

### 2.4 Data Analysis

Data analyses are conducted by;

#### a. Qualitative data analysis

Qualitative data analysis uses the descriptive analysis by the validity and practicality criteria of learning models.

#### b. Quantitative data analysis

Quantitative data analysis uses descriptive and inferential statistics. Descriptive statistics are used to present the observation data of learning activities, whereas inferential statistics are used to test the research hypothesis.

## 3. Results

### 3.1 Phase I, Investigation

#### a) Learning activities analysis

Based on the review of the lesson plan, the implementation and the results of mathematical learning in one of the Junior High School in Kupang City, East Nusa Tenggara found the fundamental issues that needed to be pursued with solutions that are the low achievement of students' learning mathematics and the students' poor higher-order thinking ability. The planning phase showed that the lesson plan developed has not indicated to a higher-order thinking ability. Analysis results of the lesson plan showed most of the teachers presented their lesson plan as learning step activities with a general concept. Learning activities designed without specific details showed the orientation of the higher-order thinking ability. For the implementation in the classroom, the teachers emphasizes on the conceptual mastery of the material and lack of attention on how the understanding was enhanced in a higher level. Furthermore, daily test, and final test, leads more to lower order thinking ability or the level cognitive of applications.

#### b) Students' background analysis

Based on the student's analysis were found:

- 1) Socio-economic background of the student's parents are heterogeneous: farmers, traders, civil servants (PNS), self-employed and others.
- 2) Student's residence were mostly in urban areas with a dense enough population levels.
- 3) Students who have not completed the study with higher-order thinking learning strategies yet.

#### c) Student's abilities analysis

In general, students' ability is heterogeneous. Placement of students in study groups was done heterogeneously with the goal in learning activities; students can be mutually supportive and helpful. The overall higher-order thinking ability of the student was poor. This was shown by the results of learning although the question test is still in the lower order thinking. Learning outcomes on the lower order thinking was enough to present the same situation when the items tests are higher-order thinking. This situation is caused by the teachers who have not understood higher-order thinking and have not taught and developed this ability yet.

### 3.2 Phase II, Model Development

Design worldview of contextual mathematical thinking learning model is:

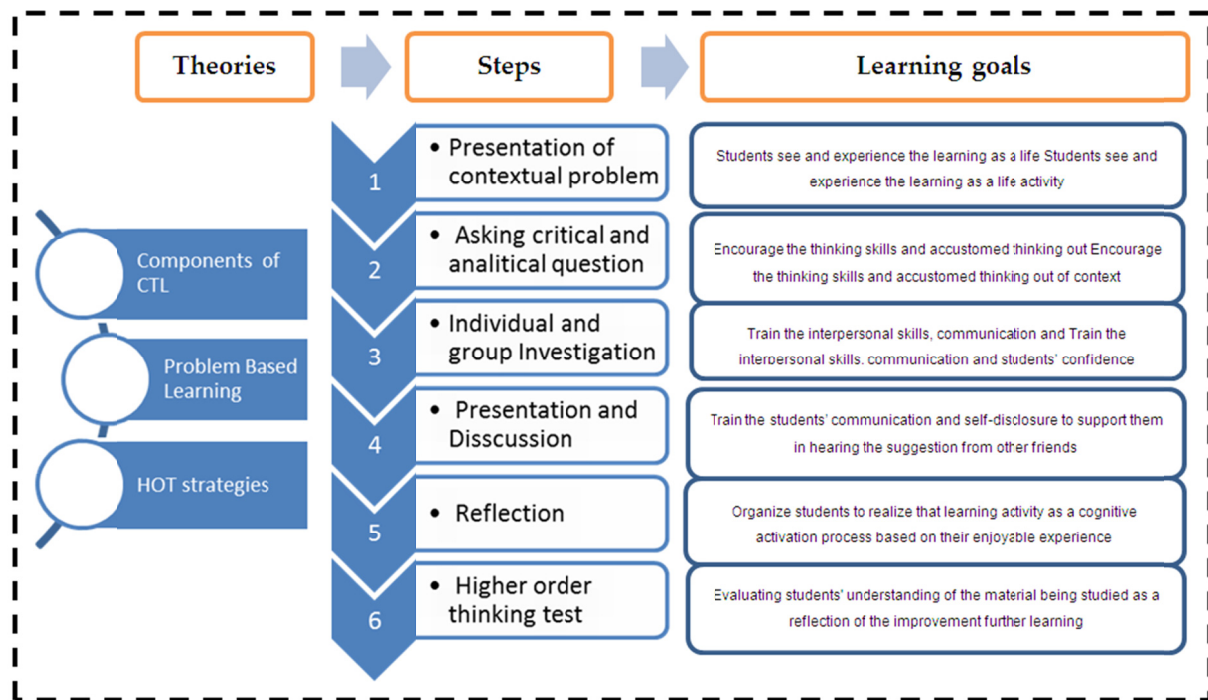


Figure1. Learning model development

Contextual mathematical thinking learning model design is arranged by learning model components: syntax, social system, reaction principle, support system, instructional impact, and accompanist impact.

Table 1. Validity of contextual mathematical thinking and learning model evaluated by experts

Detail of evaluation	Result of evaluation				Validity
	Expert 1	Expert 2	Expert 3	Average	
<b>Content and Construct Validity</b>					
Rational model	4	4	5	4.33	Valid
Theories	4	4	4	4	Valid
Syntax	4	5	4	4.33	Valid
Social system	4	4	4	4	Valid
Reaction principle	4	4	4	4	Valid
Instructional impact	5	5	5	5	Valid
Accompanist impact	4	4	4	4	Valid
Average Score				4.23	Valid

1: very poor, 2: poor, 3: fair, 4: good, 5: very good.

The phases of learning activities, teachers and students' activities and the time provided are shown below:

Table 2. Contextual mathematical thinking learning model syntax

Syntax	Teacher activities	Students activities	Time
Presentation of contextual problems	The teacher presents the learning objectives.	Students listen to the teacher's explanations	± 10 minutes
	The teacher presents the contextual problem. Presentation of the contextual problem can be displayed in pictures, stories, videos and more with the various problems that enable their activities to think critically, analytically and creatively	Students observe to the contextual problems which are presented by their teacher.	
Asking critical and analytical question	The teacher presents some questions that provoke and activate students' thinking skills. Questions are the emphasis on the critical and analytical abilities. Questions include:	Students listen to their teacher's questions and provide answers, ideas or opinions that support the existing problems.	± 10 minutes
	Asking analytical and critical questions about the paradox or dilemma or possibilities.		
	Asking students to generate their own questions about the issues presented		
Individuals and groups investigation	Starting with the question of the lower order thinking that leads to a higher-order thinking questions.	Students solve the problem individually	± 30 minutes
	The teacher provides worksheets to students.		
	The teacher asks his/her students to solve the problems independently		
Presentation and discussion	The teacher asks the students to join the group to discuss the completion of the group task. The teachers encourage their students to think analytically, ie thinking about strategies and concepts that students use in solving their problems.-	Students discuss and complete the task in groups	± 20 minutes
	The teacher acts as a facilitator who provides support (scaffolding) for his/her students to be able to turn the thinking idea to a higher-order thinking level		
Reflection	The teacher ask some students who are the representatives of several groups to present their solutions to the class	Students present the results of group discussions-	± 5 minutes
	The teachers respond and provide an explanation as the conclusion of the students' problem-solving	Another student from each group gives some responses or asks some questions	
Higher-order thinking test	The teacher guides the students to make a conclusion or a brief summary of the concepts or ideas contained in the proposed problems.	Students make inferences about the materials studied in this meeting	± 15 minutes
	Teacher gives tests to students	Students reveal what the learning problems are	
		Students take the test that was given by the teacher.	

### 3.3 Phase III, Implementation

Implementation of the contextual mathematical thinking learning model was carried out in Grade VIII Junior High School students on the cone, cylinder, and sphere. The participants are 78 Junior High School students consisting of two groups, namely experimental group and control group. Data analysis of the higher-order thinking test in both groups, learning activities observation, and student activity observation and response are shown as follow:

#### a) Higher-order thinking ability results test

The achievement and enhancement (N-Gain) of higher-order thinking ability between the two groups can be presented in the following table:

Table 3. Descriptive statistic of students' achievement and enhancement

Groups	Test	n	average	SD	N-Gain
Experiment	Pretest	39	20.00	7.54	0.72
	Posttest	39	79.56	6.08	
Control	Pretest	39	17.33	7.06	0.58
	Posttest	39	67.56	5.48	

The above table can be presented in the following bar chart:

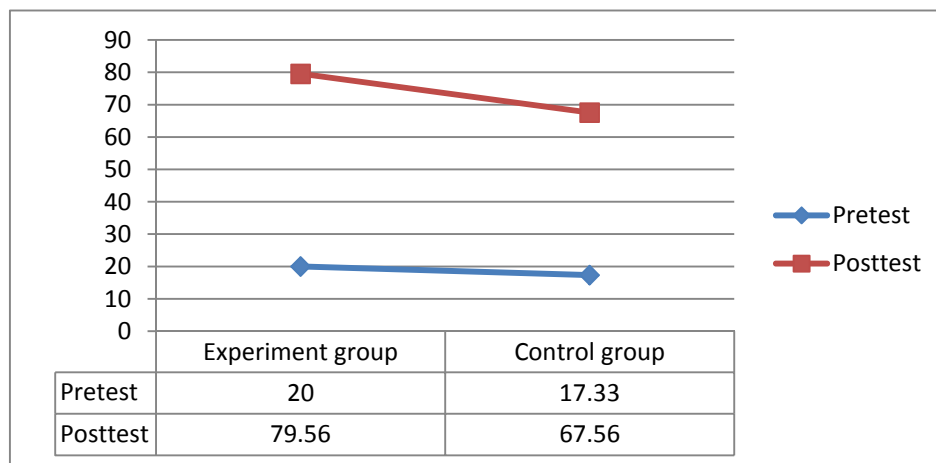


Figure 2. Students' achievement of higher-order thinking ability

Normality test results and the comparison in an increase of pretest and posttest score of higher-order thinking ability score of both group are presented in the following table:

Table 4. Enhancement comparison of higher-order thinking ability

Groups	Test	Normality			Statistic test		
		KS	df	Sig.	Z	df	Sig.
Experiment	Pretest	.138	39	.061*	-5.448	39	0.00
	Posttest	.189	39	.001			
Control	Pretest	.185	39	.002	-5.448	39	0.00
	Posttest	.200	39	.000			

The above table shows the probability value (sig.) normality data test of higher-order thinking the ability is greater than 0.05 in pretest score of the experiment group, so that the enhancement data of higher-order thinking ability pretest of the experiment group come from not normally distributed populations because the data are not normally distributed, then statistic test of enhancing higher-order thinking ability used was the Wilcoxon test. The test results, the probability value (Sig.) is less than 0.05, so it can be concluded that strengthening higher-order thinking ability are significant in both groups after learning activities.

Normality test results and comparison of the enhancing higher-order thinking in both groups are presented in the following table:



Table 5. Enhancement comparison of higher-order thinking ability

Groups	Normality			Homogeneity		Statistic test	
	KS	df	Sig.	F	Sig.	t	Sig.
Experiment	.107	39	.200	.733	3.95	7.436	0.00
Control	.115	39	.200				

The test results showed that the enhancement data in both groups were normally distributed and the variance in both groups were homogeneous; there were significant differences between the higher-order thinking ability between the experiment and control groups. Enhancement average of higher-order thinking ability showed the average enhancement in the experiment group is higher than the control group. From this description, it can be concluded that the contextual mathematical thinking learning model is more effective to be applied to elevate the higher-order thinking ability.

#### b) The observation results of learning activities

The observation of learning activities was seen on the main elements of the learning model which are syntax, social system, reaction principle, support system, instructional impact, and accompanist impact. The three main elements were observed by the observer and have an average in the good category. Syntax has an average 3.5, social system has an average 3.52, and reaction principle has an average 3.47. Results of learning activities observation can be presented in the following diagram:

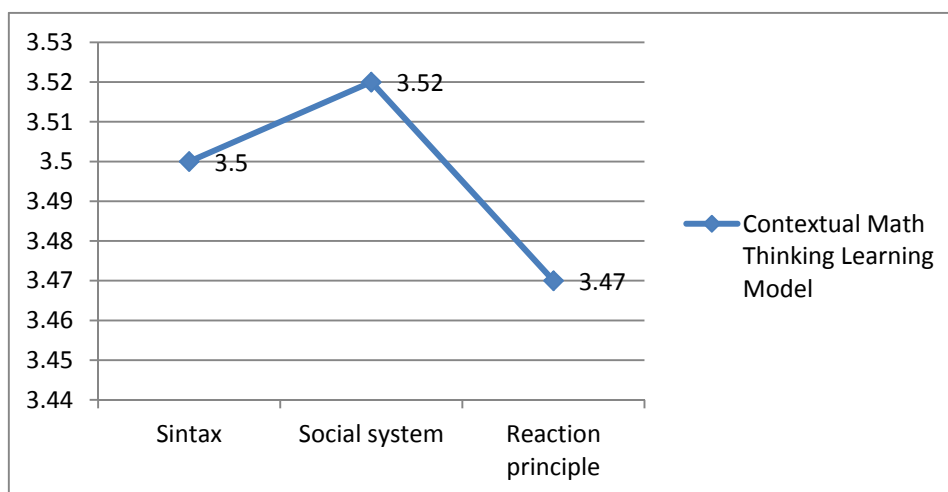


Figure 3. The observation results of learning activities

#### c) The observation results of the students' activities and students' response

The result of the students' activities observations in each learning activity is 3.56, so it can be concluded that the students' activity in contextual mathematical thinking learning model is in a good category. Furthermore, the result of the students' responses who have responded positively to the contextual mathematical thinking learning model was 85.7%,

## 4. Discussion

Contextual mathematical thinking learning model is considered as a learning model that has a strong theoretical rationale because it is supported by generally standard theories. The theories that underlie contextual mathematical thinking learning model are constructivism theory, problem-based learning, contextual teaching and learning and higher-order thinking strategy. These theories were selected because they have a strong relationship and support with each other for the purpose of developing the thinking skills. The Contextual mathematical thinking learning model shows the internal consistency between the components of the model. The developed model components are: (1) the purposes which include the instructional objectives and goals accompanist; (2) syntax; (3) reaction principle, and (4) support system. Each component is a mutual support in developing a valid model. The syntax is a formulated structured and interdependent which address the common goal to be achieved. The syntax is supported by the reaction principle and support system to achieve its intended

purpose.

Contextual mathematical thinking learning model is considered as a practical learning model. Practicality aspects are judged on two things, namely the expert assessment and implementation in the classroom. The first practicality relates to the model that can be implemented, the allocation of the right time for each learning step, and the goal to be achieved with the syntax and the set time. The second practicality model relates to the observation of the learning activities indicate that the syntax, social system and reaction principle have a good category.

The implementation phase presents the achievement of the learning goals in accordance with the model developed. The learning model is effective because (1) the learning model can support the students' higher-order thinking ability. Learning the model syntax showed the dominant thinking phase. The learning model gives a support for thinking ability perfectly such as reasoning, critical and creative abilities, (2) the learning model can magnify the higher-order thinking significantly. The enhancement of the higher-order thinking ability from pretest to posttest is 0.72 or in the high category, (3) the learning model can enhance higher-order thinking ability significantly better than the other learning models.

The first phase of contextual mathematical thinking learning model is the presentation of the contextual problems. According to Widjaya (2013, p. 151), using the contextual problems offer some potentials to engage and motivate students in learning mathematics, but it also presents some challenges for students in the classrooms. Furthermore, contextual problems usually are presented in the word problem. Word problems may, in fact, serve several important functions in the mathematics classroom like they provide questions that challenge the students to apply mathematical thinking to various situations, and they may be an efficient means of relating this thinking to the real world Bates and Wiest (2004, p. 17). The importance of the presentation of the contextual problem such as the above opinion suggests that the model qualifies as a learning model that supports motivation and challenges students to think mathematically. The period for the lesson of the problem-solving has produced positive effects on the students' skills of the mathematical thinking. It can be said that the applied process hopefully affected the mathematical thinking skill. This arrived result revealed that as the result of the problems practiced in the class of problem-solving and the steps of problem-solving, mathematical thinking can be developed (Ersoy & Guner, 2015). Barwell (2011), to be successful in solving word problems, students need to learn how to read such problems. Simply decoding words or extracting arithmetic operations is not enough; students must learn to read between the lines and understand what they are expected to do mathematically. Ontario (2011) reveals an important part of planning a lesson is engaging in solving the lesson problem in a variety of ways. This enables teachers to anticipate students' thinking and the multiple ways they will devise to solve the problem. This also enables the teachers to anticipate and plan the possible questions they may ask to stimulate thinking and deepen the students' understanding.

The second phase in this model is asking critical and analytical questions. Ontario (2011) mentioned eight tips for asking effective questions are: (1) anticipate the students' thinking, (2) link the learning goals, pose open question, (4) post question that actually needs to be answered, 5) incorporate verbs that elicit higher levels of bloom's taxonomy, (6) post question that opens up the conversation to include others, (7) keep the question neutral, and (8) provide wait time. The characteristics of asking analytical and critical question in this model are aligned with eight tips above. Asking analytical and critical questions presents a question stratified according to blooms' taxonomy level which the estuary is at the higher-order thinking level. Bedford & Mooney (2007), to understand the importance of asking good questions in the problem-solving mathematics classroom in order to promote deep discussion about the relative efficiency of the solution. Carroll, Chien, and Ritsema (2015) explains why we need them for asking activity: (1) to encourage students to participate; (2) to show we value their thinking; (3) to inform our teaching decisions; (4) to help students articulate their thinking; (5) to encourage students' metacognition; 6) to deepen students' ability to use the mathematical practices, (7) to help students develop a repertoire of questions to ask themselves.

The next phase is the individual investigation and group investigation. Individual investigation phase is an idea providing time for students to explore the idea personally. No student has no idea. The idea of every student is unique and the idea has to be heard by everyone as recognition of the students' thinking way. Then the idea is presented in the investigation group. This phase deals with cooperative learning activities. Cooperative learning in various studies is constructivism characteristic. Cooperative learning has a positive impact on learning achievement. In addition, development of interpersonal relationships students will be interwoven through contextual learning. Cooperative learning provides opportunities for the positive interdependence of each student that success will belong together.

Contextual mathematical thinking learning model illustrates the teacher's role that creates effective mathematics learning in the classroom. According to (Protheroe, 2007), it should be done to support teachers effective mathematics learning activities there are: (1) demonstrate acceptance of students' divergent ideas; (2) influence learning by posing challenging and interesting questions; (2) project a positive attitude about mathematics and about students' ability to "do" mathematics; (3) students are actively engaged in doing mathematics; (4) students are solving challenging problems; (5) interdisciplinary connections and examples are used to teach mathematics; (6) students are sharing their mathematical ideas while working in pairs and groups; (7) students are provided with a variety of opportunities to communicate mathematically; (8) students are using manipulative and other tools. The eight points above are in line with contextual mathematical thinking learning model that provides opportunities demonstrating ideas individually or in groups, challenging students with questions that are critical and analytical, presenting varied problems and working in groups.

Contextual mathematical thinking learning model provides support for the development of higher-order thinking ability completely. Presentation of the varied contextual problem in learning activities showed the support of some abilities that can be traced from the students' minds. Asked analytical and critical thinking support openness a way of thinking students while dealing with problems. Working individuals and groups support individual and collaborative thinking. In here, students will evaluate their idea personally and compare with the general ideas in the group. This idea evaluation exercises encourage the students to keep thinking. Presentation of the results discussion strengthens and support students the confidence to speak out publicly and defend the idea. Lastly, reflection and higher-order thinking ability test have benefit to evaluate the learning activity and students' ability.

## 5. Conclusions and Recommendations

This paper has highlighted the importance of developing higher-order thinking ability for Middle School students. Contextual mathematical thinking learning model has the syntax, i.e: (1) presentation of contextual problems; (2) asking critical and analytical questions; (3) individuals and group investigation; (4) presentation and discussion; (5) reflection; and (6) higher-order thinking test. Contextual mathematical thinking learning model is a valid, practical and effective model. The validity of the model was expressed by experts at judging the theoretical rationalization and internal consistency among the components of the model. Practicality aspects are judged on two things, namely the expert assessment and implementation in the classroom. The first practicality relates to the model that can be implemented, the allocation of the right time for each learning step, and the goal to be achieved with the syntax and the set time. The second practicality model relates to the observation of the learning activities which indicate that the syntax, social system and reaction principle have a good category. The learning model gives a support for perfectly thinking ability such as the reasoning, critical and creative abilities and enhancing higher-order thinking ability significantly. Learning activities observation and students' responses showed the effectiveness of the model.

Our results are promising and should be validated by a larger sample size. This model is recommended for the mathematics learning activities in the classroom to support the enhancement of higher-order thinking ability. Learning activities steps have been prepared with a logical flow that must be passed as a whole process of developing higher-order thinking ability. The presentation of the contextual problem can be presented to the local cultural context that allows students to learn mathematics in a real context.

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