Development and Demonstration of MIMO-SAR mmWave Imaging Testbeds

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ABSTRACT Multiple-input multiple-output (MIMO) radars and synthetic aperture radar (SAR) techniques are well researched and have been effectively combined for many imaging applications ranging from remote sensing to security. Despite numerous studies that apply MIMO concepts to SAR imaging, the design process of a MIMO-SAR system is non-trivial, especially for millimeter-wave (mmWave) imaging systems. Many issues have to be carefully addressed. Besides, compared with conventional monostatic sampling schemes or MIMO-only solutions, efficient image reconstruction methods for MIMO-SAR topologies are more complicated in short-range applications. To address these issues, we present highly-integrated and reconfigurable MIMO-SAR testbeds, along with examples of three-dimensional (3-D) image reconstruction algorithms optimized for MIMO-SAR configurations. The presented testbeds utilize commercially available wideband mmWave sensors and motorized rail platforms. Several aspects of the MIMO-SAR testbed design process, including MIMO array calibration, electrical/mechanical synchronization, system-level verification, and performance evaluation, are described. We present three versions of MIMO-SAR testbeds with different implementation costs and accuracies to provide alternatives for other researchers who want to implement their testbed framework. Several representative examples in various real-world imaging applications are presented to demonstrate the capabilities of the proposed testbeds and algorithms.

INDEX TERMS Millimeter-wave (mmWave) radar, multiple-input multiple-output (MIMO) radar, synthetic aperture radar (SAR), frequency-modulated continuous-wave (FMCW), back projection algorithm (BPA), range migration algorithm (RMA), three-dimensional (3-D) imaging, testbed design, calibration.

I. INTRODUCTION

The electromagnetic radio waves, which lie within the frequency range of 30 – 300 GHz, are typically known as millimeter-waves (mmWaves) since they correspond to the wavelengths from 10 mm to 1 mm. The mmWaves can penetrate a wide range of optically-opaque and dielectric materials, such as various composites, ceramics, plastic, concrete, wood, and clothing. The radars that operate at mmWave frequencies are very effective in a variety of applications, including medical diagnostics [1]–[5], security screening [6]–[13], non-destructive testing (NDT) of the structures [14]–[16], and aerial imaging [17]–[19]. Besides, mmWave frequencies are non-ionizing and not considered to be sources of hazardous radiation similar to the signals emitted from walk-through metal detectors (WTMD) [20].

MmWaves can be effectively used for radar imaging systems, which primarily measure the reflectivity of the person/objects in the scene. Comparing with the optical counterparts, mmWave imaging systems require much larger apertures (20 – 200 cm) [16], [21]. Although recent progress in complementary metal-oxide semiconductor (CMOS) technology integrates cost-effective mmWave wideband radar sensors [22], the need for a massive number of sensors to completely build up a high-resolution image of the scene is still a major challenge for mmWave imaging systems. A well-known approach to reduce the hardware complexity while satisfying the data acquisition time requirements in many applications is the realization of a hybrid concept based on the combination of multiple-input multiple-output (MIMO) array
topologies [23]–[25] and synthetic aperture radar (SAR) techniques [26], [27].

Unfortunately, many of the new techniques in this research field are verified using seemingly expensive and custom-built experimental prototypes [7], [28]. Pacific Northwest National Laboratory (PNNL) is one of the pioneers in this area. They produced many interesting results with details on both system architecture and imaging algorithms. Their efforts include imaging instruments with various capabilities [6], [13], [29]. Along with the similar systems reported in [16], [30], these solutions are based on an array of switchable antennas, where the transmitters and receivers are sequentially operated in pairs to be approximated as a monostatic array. Other testbeds such as [31]–[33] utilize a single transmitter and receiver antenna installed on two independent horizontal tracks to achieve an equivalent linear MIMO array with a one-dimensional (1-D) scanning regime. Although these testbeds have the flexibility to emulate different MIMO-SAR configurations, they are highly customized and cannot be easily replicated by others. Besides, they cannot be used to investigate the channel variations in MIMO arrays for calibration, which is an essential topic in practical MIMO-SAR systems.

We believe that many researchers can benefit from low-cost and easy-to-replicate testbeds to validate and demonstrate their MIMO-SAR imaging algorithms. The design process of a MIMO-SAR testbed must consider a wide variety of factors that will determine the quality of reconstructed images. These include calibration, synchronization between the mechanical scans and radar transmissions, motion stability, lateral/range resolution, and accurate aperture sampling. The main contribution of this paper is to provide a complete design guide to build system-level MIMO-SAR mmWave imaging testbeds for a variety of applications and to present comprehensive discussions on important signal processing and hardware/software implementation aspects of a MIMO-SAR testbed framework, which, to the best of our knowledge, have not been studied in the previous literature.

In this paper, we combine commercially available MIMO mmWave radar sensors and different mechanical scanners to facilitate various SAR techniques. We present a novel synchronization mechanism between the scanners and MIMO mmWave sensors to ensure an accurate radar transmission scheme in SAR motion. Many researchers can utilize the introduced novel technique to solve this challenging synchronization problem in their MIMO-SAR testbeds, especially at higher scanning speeds. We propose a practical multi-channel array calibration method to compensate for the gain and phase mismatches of the MIMO array elements. To control the entire signal processing chain of the proposed testbeds, we introduce a custom-developed open-source software toolbox [34], which includes all the testbed control, data capture, calibration, and imaging modules. We present several experimental results from real-world scenarios to demonstrate the effectiveness of our designs that achieve high-resolution imaging performance in various applications. In fact, with the help of these testbeds, we are able not only to verify imaging algorithms but also to investigate common issues and limitations concerning practical applications. It is important to note that the interested researchers can replace the MIMO mmWave radar sensors discussed in our paper with the terahertz-based counterparts [35]. Hence, they can still benefit from the features of the proposed testbed framework for MIMO-SAR imaging in the terahertz band.

In the presented MIMO-SAR imaging modality, which is implemented by scanning a MIMO mmWave sensor over a planar aperture, both amplitude and phase of the received signal over a wide bandwidth are recorded (coherent data) to mathematically reconstruct focused two-dimensional (2-D) or three-dimensional (3-D) (holographic) images. Thus, employing computationally efficient image reconstruction algorithms is another major challenge of building MIMO-SAR imaging systems, especially in near-field (i.e., short-range) applications. In the near-field, the plane-wave assumption is invalid, and the spherical electromagnetic wave model has to be used. The standard image reconstruction techniques using monostatic sampling schemes, where the measurements are taken by collocated transmit and receive antennas over regular spatial intervals, cannot be directly applied to the proposed MIMO-SAR configurations. The main reason is that one has to take into account the different trajectories of the incident and reflected electric fields for transceiver pairs due to increased separation among them compared with the typical target ranges. As a result, the image reconstruction techniques based on multistatic imaging modalities are necessary for the large MIMO apertures with spatially diverse transmit and receive antennas in SAR configuration. In response to this major challenging requirement, we present and experimentally verify a series of state-of-the-art 3-D image reconstruction algorithms along with the mathematical derivations and demonstrate how to apply these algorithms to the testbed data.

Although the back projection algorithm (BPA) [7], [30], [36], [37] can provide a straightforward solution for arbitrary multistatic array configurations, it suffers from the high computational load in high throughput applications utilizing large MIMO-SAR apertures and wideband sensors. In this paper, the proposed approach in [38] is augmented to improve the performance of the BPA in the SAR axis by solving the problem in the corresponding Fourier domain. The range migration algorithm (RMA) using Fourier-based inversion methods is the most efficient and widely used approach in conventional monostatic SAR imaging for both planar [6], [39] and cylindrical/spherical [29], [40] scanning geometries. In order to extend the RMA to MIMO-SAR configurations, we adopt a multistatic-to-monostatic conversion approach proposed in [41], [42] to transform the multistatic array topology into a monostatic format according to the equivalent phase center principle based on a reference point on the target. On the other hand, the multistatic-to-monostatic conversion technique used in the RMA is precise only for the selected
reference point. Therefore, the fast implementation of more
precise wavenumber domain algorithms for the multistatic
sampling schemes is necessary for larger target scenes. In this
paper, a novel imaging method for single-input multiple-
output (SIMO) arrays [31], [43] is adapted to the proposed
MIMO-SAR configuration. This method avoids the approxi-
mations used in the RMA to improve the image quality as well
as reduce the complexity compared with the enhanced BPA.

The rest of the paper is organized as follows. Section II
reviews the signal model of the backscattered data and
presents the proposed MIMO-SAR configuration. Section III
introduces the imaging testbeds built to measure real
MIMO-SAR data. Section IV presents the novel
radar-scanner synchronization approach implemented to
enable higher accuracy in real-time measurements. Section V
details the comprehensive open-source software package
developed to control the signal processing chain of the
testbeds. Section VI proposes a practical calibration method
to compensate for the mismatches of the MIMO array ele-
ments. Section VII presents different MIMO-SAR image
reconstruction algorithms, which exploit the wideband capa-
bilities of mmWave sensors to facilitate 3-D holographic
imaging. The imaging results of different real-world application
scenarios are reported in Section VIII, which is followed by
conclusions.

II. MIMO-SAR SYSTEM MODEL

In this section, we review the wave propagation model of
the backscattered MIMO-SAR data, which forms the basis
of the image reconstruction problem, present the geometrical
setup for the proposed MIMO-SAR system, and introduce the
mmWave sensor modules utilized in the testbeds.

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such that \( s(x', y_T, y_R, k) \). Assuming a 3-D target with a reflectivity function \( p(x, y, z) \) located in the scene, we can express the 4-D received backscattered data using (5) and the linearized scattering model after ignoring the amplitude decay with range (i.e., path loss), which is found to be negligible in the short-range applications [6], [39], as

\[
s(x', y_T, y_R, k) = \iiint p(x, y, z) e^{-j k R_T} e^{-j k R_R} \, dx \, dy \, dz, \tag{6}
\]

where the distances from the transmitters and receivers to the target point in (3) becomes

\[
R_T = \sqrt{(x - (x' + \Delta_T/2))^2 + (y - y_T)^2 + (z - Z_0)^2},
\]

\[
R_R = \sqrt{(x - (x' - \Delta_T/2))^2 + (y - y_R)^2 + (z - Z_0)^2}, \tag{7}
\]

respectively. The main purpose of the image reconstruction problem, which will be detailed in Section VII, is to recover the complex reflectivity function \( p(x, y, z) \) of the 3-D target from the 4-D received data \( s(x', y_T, y_R, k) \) captured by each transceiver pair over the \( xy \) domain.

C. MIMO mmWave SENSORS

Due to the proliferation of system-on-chip mmWave sensing technology, many commercial off-the-shelf (COTS) radar modules are now available on the market. In this paper, we choose to utilize the modules developed by Texas Instruments. The mmWave sensors used in our testbeds consist of two or three transmit and four receive antenna elements with FMCW transceivers [53]. The transceivers can support up to 4 GHz bandwidth on the 60–64 GHz or 77–81 GHz frequency band.

Fig. 2a illustrates a typical antenna layout of the evaluation modules based on a single-chip MIMO mmWave sensor with two transmitters and four receivers. In this layout, the receive antennas are uniformly spaced along \( y \)-axis by \( \lambda/2 \) (tuned to the center frequency). The transmit antennas are also located along \( y \)-axis with \( 2\lambda \) spacing. The horizontal offset between the linear transmit and receive arrays is \( \Delta_T = 0 \). The evaluation modules that enable three transmitters have a similar layout with the exception of their third transmit antenna, which is in the middle of the transmit array in \( y \)-axis and has an offset of \( \lambda/2 \) along \( x \)-axis [53]. To create a linear MIMO array as shown in Fig. 1, the third transmit antenna in the three-transmitter versions is switched-off, and only two transmit antennas, which share the same position with the receive array along the axis of motion (i.e., \( x \)-axis), are used. In other words, a linear virtual array consists of eight elements, which are uniformly spaced along \( y \)-axis by \( \lambda/4 \), is created as shown in Fig. 2a.

As illustrated in Fig. 2a, a single-chip MIMO mmWave sensor consists of a handful of transmitter and receivers. As a result, multiple sensor chips must be cascaded to create moderately large array apertures [54]. In this paper, an available MIMO radar module from Texas Instruments, which is a combination of four single-chip mmWave sensors [55], is used. Each sensor has four receive and three transmit antennas. As illustrated in the physical antenna layout in Fig. 2b, the receive antennas from each chip are grouped and uniformly spaced in \( y \)-axis by \( \lambda/2 \). The transmit antennas from three chips are uniformly spaced along \( y \)-axis by \( 2\lambda \). The remaining three transmit antennas from one of the chips, which are not shown in Fig. 2b, have offsets along \( x \)-axis. Therefore, these transmit antennas are switched-off to create a linear MIMO array and only the nine uniformly distributed transmit antennas are used along with all 16 receive antenna elements. The horizontal offset between the linear transmit and receive arrays is \( \Delta_T = 17\lambda \). With this configuration, a virtual array of 86 non-overlapped channels along \( y \)-axis.
is achieved, where the virtual elements are uniformly distributed with an inter-element spacing of $\lambda/4$, as shown in Fig. 2b.

**D. REALIZATION OF THE MIMO-SAR CONFIGURATION**

The MIMO-SAR configuration presented in Section II-B utilizes an $N_T \times N_R$ element linear MIMO array, where $N_T$ and $N_R$ are the numbers of transmit and receive antennas, respectively, in SAR configuration to discretely sample the continuous MIMO-SAR aperture plane. In this configuration, the total number of transmitting and receiving elements arranged over the vertical axis is assumed to be sufficient to create a large MIMO aperture ($D^y$) required for high-resolution while satisfying the Nyquist criterion to avoid aliasing. However, as summarized in Section II-C, most commercially available MIMO mmWave sensors typically have few transmit and receive antennas. Hence, the effective aperture sizes of both single-chip and four-chip cascaded sensors are not enough to achieve high cross-range resolution along the $y$-axis [44], [56].

In this paper, we propose to synthesize a 2-D aperture by mechanically moving the MIMO mmWave sensors continuously across the $xy$ plane, along a parallel track pattern, as depicted in Fig. 3. The MIMO-SAR aperture is uniformly sampled in $x$ spatial domain with a sampling distance of $\Delta_x$. Using the virtual channel concept [24], [57], [58] and selecting the sampling distance in $y$-axis as $D_y = M\lambda/4$, where $M = 8$ and $M = 86$ for the single-chip and four-chip cascaded sensors, respectively, the MIMO-SAR aperture is assumed to be uniformly sampled in $y$ spatial domain also. The total effective aperture sizes in both axes are then approximated by $D_x \approx (N_x - 1)\Delta_x$ and $D_y \approx N_y(M - 1)\lambda/4$, where $N_x$ is the total number of measurement points along $x$-axis, and $N_y$ is the total number of horizontal scans along $y$-axis.

In this paper, we develop a highly reconfigurable testbed framework to combine commercially available MIMO mmWave sensors with motorized MIMO mmWave sensors by utilizing the industry standard communication interfaces in embedded systems. The researchers can benefit from the proposed testbeds to demonstrate various MIMO-SAR configurations by integrating their own front-end boards with different MIMO antenna layouts or performing a particular scanning trajectory.

**III. IMAGING TESTBEDS**

In this section, we present different types of MIMO-SAR imaging testbeds that we built throughout this research. Our prototypes uniquely combine system-on-chip MIMO mmWave sensors and SAR signal processing techniques. To synthesize a large aperture over the target scene, different versions of mechanical scanners with two-axis motorized rail systems are designed and implemented. Both single-chip and multi-chip cascaded wideband mmWave sensors are integrated with the scanners to generate high-resolution 3-D holographic images of the target scene. Based on the COTS mmWave evaluation modules and stepper motors based rail systems, the presented testbeds are low-cost and highly reconfigurable.

In our previous studies, two different MIMO-SAR imaging testbeds are presented briefly using both single-chip [11], [56], [59] and multi-chip cascaded [60] mmWave sensors to validate the proposed image reconstruction algorithms and to investigate different performance metrics. In this paper, our goal is to focus on the system-level design perspective in more detail. Besides, we present an enhanced version of the testbed and mention the improvements implemented in the previous versions. The system architectures and the basic features of each prototype are described in detail, and their suitability for future research purposes are illustrated.

**A. VERSION I**

In this section, the first version of the MIMO-SAR mmWave imaging testbeds prototyped in [11], [56], [59] is summarized to provide a complete study and to demonstrate the improvements presented in this paper. The testbed shown in Fig. 4a consists of four major components: (1) a single-chip mmWave sensor, (2) a low-cost two-axis mechanical scanner, (3) a motion controller, and (4) a host personal computer (PC).

The mmWave sensor is a combination of three hardware modules from Texas Instruments: (1) IWR1443-Boost, (2) mmWave-Devpack, and (3) TSW1400 boards [53]. The IWR1443-Boost is an evaluation module based on the single-chip IWR1443 mmWave sensor, which integrates four receive and three transmit antennas, as discussed in Section II-C. The TSW1400 and mmWave-Devpack are add-on boards used with Texas Instrument’s mmWave.
sensors to enable high-speed raw analog-to-digital converter (ADC) data capture. The TSW1400 module captures the data from the IWR1443-Boost module through the mmWave-Devpack and stores the formatted data into its onboard memory. Captured raw data are then imported to the host PC with a serial port for post-processing.

The other component of the imaging testbed is the two-axis mechanical scanner built using two ball screw linear rails and stepper motors. The scanner provides movements in horizontal and vertical directions. The radar hardware stack is installed on the horizontal track by which an equivalent 2-D scanning is achieved. The maximum scanning ranges in both horizontal and vertical directions are 400 mm. The motor controller, which is configured to operate linear rails at a maximum speed of 20 mm/s, is connected to the host PC with a serial port. Compared with the testbed presented in our previous study, AMC4030 motion controller [61], which is a low-cost general-purpose COTS product, is used to establish a common framework for all the testbed versions.

While the first prototype has limited dimensions and scanning speed, our goal was to demonstrate the proof-of-concept. The details of the enhanced imaging systems with bigger dimensions and much faster scanning speeds are introduced in the following sections.

B. VERSION II

In this section, an enhanced version of the single-chip mmWave sensor based imaging testbed utilizing a bigger and faster custom-built two-axis mechanical scanner is presented. This testbed is designed for high-speed scanning of a larger SAR aperture to enable more flexibility in solving different signal processing problems and investigating various performance metrics. In the following, the system architecture and the proposed enhancements are described in detail.

The testbed shown in Fig. 4b consists of four major components: (1) a single-chip mmWave sensor, (2) an improved two-axis mechanical scanner, (3) a motion controller, and (4) a host PC, similar to the version I. In addition, a novel radar-scanner synchronization module, which will be detailed in Section IV, is implemented to enable higher accuracy in the data capture process. The diagram shown in Fig. 5 is a simplified view of the main elements and the high-level system architecture of the imaging testbed.

The new testbed utilizes the mmWave sensor consisting of IWR1443-Boost, mmWave-Devpack, and TSW1400 modules, similar to the version I. Besides, it is also configured to be interfaced with the DCA1000 evaluation module, which is a real-time data capture board for interfacing with Texas Instrument’s mmWave sensors [53]. The DCA1000 module captures the raw ADC data from the IWR1443-Boost module and streams the packetized data to the host PC over Ethernet.

Compared with the version I, an enhanced solution suitable for faster scanning operations at a maximum speed of 500 mm/s is developed using a 1 meter by 1.2 meters...
The high-level system architecture of the version III testbed illustrates the control sequence of the components: (1) a four-chip cascaded mmWave sensor, (2) a two-axis mechanical scanner, (3) a motion controller, (4) a synchronization module, and (5) a host PC. The diagram shown in Fig. 6 is a simplified view of the main elements and the high-level system architecture of the imaging testbed.

**FIGURE 6.** The high-level system architecture of the version III testbed consists of a four-chip cascaded mmWave sensor.

The mmWave sensor is a combination of the four-chip cascaded front-end board, which is introduced in Section II-C, and TSW14156 based add-on interface modules from Texas Instruments [55] to enable high-speed raw ADC data capture. The interface boards provide the connectivity between the four-chip cascaded mmWave front-end module and the host PC to acquire the raw ADC data. The data captured from the mmWave sensor are stored into the onboard memory of the TSW14156 module. Stored data are then imported to the host PC with a serial port for post-processing. Alternatively, the new generation TDA2 processor based evaluation boards [55] can also be used to provide a real-time processing foundation for the four-chip cascaded mmWave front-end modules over Ethernet in MIMO-SAR imaging.

In the version III testbed, a 1 meter by 1 meter two-axis mechanical scanner is built using three identical linear rails. Compared with the previous testbeds, the faster versions of the ball screw linear rails [61] are used along with more powerful stepper motors to improve the payload capacity while maintaining the high operation speed. With this configuration, a maximum speed of 400 mm/s in both axes is achieved. As illustrated in Fig. 4c and Fig. 6, the horizontal rail is mounted on two vertical rails, which are operated by separate stepper motors and drivers. The stepper drivers dedicated to the vertical rails are connected to the same port of the motion controller to ensure a coupled scanning along the vertical direction.

**IV. SYNCHRONIZATION BETWEEN THE SCANNER AND RADAR**

The standard way of synchronizing the scanner and mmWave radar sensor assumes constant speed at the scanner during the entire horizontal motion and uniform radar transmissions in the time domain. By considering an initial synchronization between the scanner and mmWave sensor, a uniform radar sampling in the spatial domain is assumed to be achieved. In the testbed version I, which is presented in Section III-A, this approach is adopted. The motion start and the radar trigger commands (i.e., the software trigger feature of the sensors) are sent separately via the MATLAB-based toolbox (will be detailed in Section V) for each horizontal scan. The inter-chirp sampling time of the mmWave sensor is then configured based on the speed of the platform to achieve the desired sampling distance in the spatial domain.

At higher speeds, to start and stop the stepper motors in a smooth way without stalling, control of the acceleration and deceleration is needed. Hence, the constant speed assumption is invalid, and an alternative technique must be developed. In this paper, a novel solution for the scanner-radar synchronization is designed and implemented in the testbeds version II and III, which are presented in Section III-B and Section III-C, respectively. The proposed solution accurately synchronizes the scanner with radar independent of the speed and acceleration profiles.

In all the testbeds developed in this paper, the motion controllers generate pulse signals at variable rates to move both stepper motor based ball screw and belt-driven linear rails at desired speeds. This control scheme requires no other sensors for positioning and makes the overall design an open-loop system. The position and speed of the rails are controlled precisely just by sending pulses from the motion controller to the stepper drivers, as illustrated in Fig. 5 and Fig. 6.

In the proposed synchronization solution, a pulse counter module is implemented on an ESP32-based microcontroller [63] running freeRTOS [64]. This module counts the number of pulses generated by the motion controller for the horizontal scan. The radar signal transmission is then triggered (using the hardware trigger feature of the sensors) after a threshold event, which is configured by the desired sampling distance \( \Delta_s \), occurs in the pulse counter module. The diagram in Fig. 7 illustrates the control sequence of the improved synchronization approach.

Fig. 8a shows the pulse diagrams recorded using the version II testbed detailed in Section III-B. The scanner is...
FIGURE 7. The proposed synchronization approach between the scanner and mmWave radar, which accurately synchronizes the testbed independent of the speed and acceleration profile.

FIGURE 8. (a) The pulse diagram of the enhanced synchronization approach for an example scenario: $D_x^s \approx 400$ mm, $\Delta x \approx 1$ mm, and the maximum speed is 500 mm/s. (b) Close-up of the pulse diagram to show four consecutive radar triggers in detail. The inter-sampling time is non-uniform because of the acceleration profile.

configured to move $D_x^s \approx 400$ mm along each horizontal scan at a maximum speed of 500 mm/s. The synchronization module is configured such that a sampling distance of $\Delta x \approx 1$ mm is realized. The belt-driven linear rail utilized in the version II testbed moves 110 mm per 20000 pulses according to its design specifications. Therefore, the synchronization module triggers the radar when a pulse threshold event (i.e., 182 pulses) occurs in the pulse counter module to ensure a uniform sampling in the spatial domain. The detailed pulse diagram in Fig. 8b, which includes four consecutive radar triggers, illustrates the accuracy in the proposed synchronization mechanism and the non-uniform inter-sampling time caused by the acceleration profile. It is shown that the radar trigger instants based on the accurate pulse threshold events (i.e., 182 pulses) ensure a uniform sampling in the spatial domain.

V. IMAGING TOOLBOX

In this section, we develop a comprehensive open-source MIMO-SAR imaging toolbox [34], which is a MATLAB [65] based software package including the complete signal processing chain of the prototyped solutions. The toolbox allows the user to control the testbeds and to reconstruct high-resolution 3-D holographic images using the captured experimental data. We develop the toolbox in MATLAB platform since it is widely used in the scientific and technical world.

The developed toolbox consists of three main modules: (1) data capture, (2) MIMO array calibration, and (3) image reconstruction, as illustrated in the flow diagram in Fig. 9. The mathematical framework of the MIMO array calibration and image reconstruction modules are detailed in Section VI and Section VII, respectively. This section summarizes the data capture module, which is implemented in a graphical user interface (GUI) based application.

The user configures the mmWave sensor parameters and generates the desired SAR scenario via three different menu tabs of the GUI as shown in Fig. 10. The menu tab shown in Fig. 10a is used to initialize the communication interfaces of the testbed modules and to configure the scanner and mmWave sensor parameters. The desired SAR parameters are configured via the scenario generation menu tabs shown in Fig. 10b and Fig. 10c, which are developed based on the basic and enhanced synchronization approaches (detailed in Section IV), respectively. The toolbox then handles the fully-automated data capture process according to the realized MIMO-SAR configuration (discussed in Section II-D), as demonstrated in Fig. 9.
VI. MIMO ARRAY CALIBRATION

In a practical system, measurement errors in the MIMO array may arise due to sensor gain and phase mismatches \([66],[67]\). These mismatches can be caused by various reasons, such as path length imperfections, chip-to-chip or antenna-to-antenna variations, etc. Especially, phase mismatches can affect the image reconstruction adversely, and lead to unacceptable defocused blur and range shift in the images. Therefore, calibration is an essential step in MIMO-SAR imaging to reduce the effects of channel variations and to improve the reconstructed image quality.

Different calibration procedures have been studied in the previous literature \([68]–[72]\). In this paper, we utilize the testbeds we developed to propose a practical calibration method based on the ideal backscattered signal model from a reference point target (i.e., a corner reflector) at an unknown position. The accuracy of the proposed approach depends on the reference beat signal, which needs a precisely positioned point target. Therefore, the first step in the calibrating process is to estimate the unknown \((x, y, z)\) position of the reference target accurately. To achieve that, as depicted in Fig. 11a, we first propose to capture data along both horizontal and vertical axes using a single transceiver antenna pair of the MIMO array, which is assumed to be a single monostatic virtual element, as a reference channel.

As detailed in Section II-A, the total round-trip delay of the backscattered data is directly related to the frequency and phase of the measured beat signal in (4). Defining the captured wideband beat signal \(s(x',y',t)\) as a 3-D function of time \(t\) and measurement points over the \(xy\) domain, the goal is to estimate an accurate range profile of the target using the beat frequency and phase at each measurement point.

Let us define the backscattered 3-D beat signal from an ideal point target using the signal model in (4) after ignoring the amplitude and RVP terms as

\[
s(x', y', t) \approx e^{i(2\pi f_0(x', y')t + \phi(x', y'))}, \tag{8}
\]

where \(f_0(x', y') = K \tau(x', y')\) and \(\phi(x', y') = 2\pi f_0 \tau(x', y')\) are the frequency and phase of the beat signal, respectively, which are both functions of the round-trip delay

\[
\tau(x', y') = 2\sqrt{(x - x')^2 + (y - y')^2 + z^2/c}, \tag{9}
\]

each measurement point. We assume that we have uniformly sampled version of the beat signal over the time domain as \(s(x', y', n) = s(x', y', nT_s)\), where \(T_s\) is the sampling period and \(n = [0, \ldots, N - 1]\). Then, we can estimate the frequency and phase in (8) by taking an \(N\)-point discrete-time Fourier transform (DTFT) on the sampled beat signal as

\[
S(x', y', e^{j\omega}) = \sum_{n=0}^{N-1} e^{i(\omega_0(x', y')nT_s + \phi(x', y'))} e^{-j\omega n}.
\]

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\[
S(x', y', e^{j\omega}) = \sum_{n=0}^{N-1} e^{i(\omega_0(x', y')nT_s + \phi(x', y'))} e^{-j\omega n}.
\]
The range FFT of the measured beat signal: (a) including the
As depicted in the range FFT result, the variation of the target
range is very small within a single beat signal. Therefore,
to obtain a more accurate range profile, which is a function of the
round-trip delay in (9), we need to utilize the phase of the
beat signal $\phi(x', y')$ in (8).

If we select $\hat{\omega} = a_0(x', y')T_s$ in (10) for all measurement points, i.e., the beat frequency corresponding the peak index of the FFT output, the complex values at that index will only have the phase terms $e^{i\phi(x', y')}$. The residual phase error caused by the limited FFT resolution is assumed to be negligible [48]. Using the range FFT output in Fig. 11b, the unwrapped phase of the range FFT peaks measured over the $x$–axis is depicted in Fig. 11c along with the simulated version. Similarly, the unwrapped phase of the range FFT peaks measured and simulated along the $y$–axis is shown in Fig. 11d. This approach can also be used to diagnose problems with the testbed.

The position $(x, y, z)$ of the point target referenced to the scanning geometry can be estimated by applying the least squares curve fitting approaches [73] to the measured beat signal phase. These approaches directly result in an estimate of the target position by finding the set of parameters, which minimizes the squared error between the modeled and measured phase as

$$\{\hat{x}, \hat{y}, \hat{z}\} = \arg \max_{x, y, z} \sum_{x', y'} \left| \phi(x', y') - 2\pi f_0 \tau(x', y') \right|^2,$$  

where $\tau(x', y')$ is a function of the target location $(x, y, z)$ as given in (9). Here, we assume that the actual distance between the radar aperture and the point target shown in Fig. 11a is available in the measured beat signal phase. However, in a practical setting with FMCW signaling scheme, a range FFT operation can be used to provide the estimate of the target range as $\hat{R}(x', y') = R(x', y') + R_0$, where $R(x', y') = c\tau(x', y')/2$ is the actual target distance from the known measurement point $(x', y')$ and $R_0$ is the range bias imposed by the hardware imperfections. Hence, the last step in the position estimation problem is to compensate for the range bias $R_0$ of the reference channel. In this paper, we propose to exploit the coupling between the transmitting and receiving antenna elements [68], [74] to estimate the range bias.

The target-independent range bias resulting from the mutual coupling can be obtained by observing the range profile of the beat signal as shown in Fig. 13a. Let us assume that the reference channel consists of the $u$th transmitter and $v$th receiver elements located at $r_u \in \mathbb{R}^3$ and $r_v \in \mathbb{R}^3$, respectively. Then the range bias can be estimated from the reference beat signal modeled using the Euclidean distance between the corresponding transceiver antenna pair $|r_u - r_v|$, as illustrated in Fig. 13b.

Now, we are ready to create the reference backscattered signal model for each channel of the MIMO array based on the estimated target location $(x, y, z)$ to be used in the calibration process. Let us define the total round-trip delay $\tilde{\tau}_c$ of the FMCW signal reflected off the point target between the $u$th transmit and $v$th receive antennas, and the corresponding transceiver gain $a_{uv}$. We model the delays between antenna pairs as the superposition of a common instrument delay and residual delays between antenna elements: $\tilde{\tau}_c = \tau_i + \tau_c$. Ignoring the additive noise and RVP term, the uncalibrated measured beat signal can be defined as

$$\tilde{s}_c(t) = a_{uv} e^{i2\pi f_b(t + \tilde{\tau}_c)} s(t),$$  

where $s(t)$ is the reference beat signal model, $f_c = K \tau_i$ is the beat frequency that cause a range bias in the system as
mentioned before, and \( \eta_\ell \) is the residual complex gain factor. Given the measurements \( \hat{s}_\ell(t) \), the calibration error signal can be computed by a simple demodulation process

\[
w_\ell(t) = \hat{s}_\ell(t) s^*_\ell(t) \approx \eta_\ell e^{2\pi ft},
\]

where \((.)^*\) denotes the complex-conjugate operation. Estimating \( \hat{f}_\ell \) and \( \eta_\ell \) from (13) reduces to the parameter estimation problem of a single-frequency complex tone from noisy observations [75], [76]

\[
\hat{f}_\ell = \arg \max_f \sum_{\ell} |W_\ell(f)|^2,
\]

where \( W_\ell(f) \) is defined as

\[
W_\ell(f) = \int_0^T w_\ell(t) e^{-j2\pi ft} dt. \tag{15}
\]

If the data \( w_\ell(t) \) is uniformly sampled in \( t \), the FFT can be used to obtain the discrete version of \( W_\ell(f) \). Fig. 14a and Fig. 14b show the FFT output \( W_\ell(f) \) of the calibration signals \( w_\ell(t) \) for each channel of the single-chip and multi-chip cascaded mmWave sensors, respectively.

![FIGURE 14. The FFT output of the calibration signals for each channel of the: (a) single-chip and (b) multi-chip cascaded mmWave sensors.](image)

Finally, the complex gain factors \( \eta_\ell \) for each transceiver pair can be computed by plugging the estimate \( \hat{f}_\ell \) in (12). Fig. 15a and Fig. 15b show the estimated phase of the complex gain factor and the range bias for each channel of the single-chip mmWave module, respectively. Similarly, the estimated phase of the complex gain factor and the range bias for each channel of the four-chip cascaded mmWave module (only for the 144 channels created by the uniformly located transmitter antennas, as detailed in Section II-D) are illustrated in Fig. 15c, and Fig. 15d, respectively.

VII. EXAMPLES OF 3-D IMAGE RECONSTRUCTION ALGORITHMS WITH MIMO-SAR

In this section, we present examples of efficient 3-D image reconstruction algorithms suitable for the proposed MIMO-SAR configuration. The presented algorithms are compared in both precision and computational complexity. It is important to emphasize that having access to the testbed framework proposed in this paper allows researchers not only to verify but also to develop new algorithms [77] as well as refine the techniques presented in the following sections.

\[ p(x, y, z) = \int \int \int s(x', y_T, y_R, k) e^{jkR_T} e^{jkR_y} dx' dy_T dy_R dk. \tag{16} \]

Taking (7) into (16) and defining

\[
h(x, y, z, y_T, y_R, k) = e^{jk\sqrt{(x-(\Delta_T/2))^2+(y-y_T)^2+(z-Z_0)^2}} \\
\times e^{jk\sqrt{(x+(\Delta_T/2))^2+(y-y_R)^2+(z-Z_0)^2}},
\]

which is the matched filter computed for all transceiver pairs, wavelengths, and target points, the reconstruction problem can be represented as

\[ p(x, y, z) = \int \int \int s(x', y_T, y_R, k) \\
\times h(x-x', y, z, y_T, y_R, k) dx' dy_T dy_R dk. \tag{18} \]

In (18), the first integral is a convolution relation in the \( x \)-domain. Therefore, taking the Fourier transform with respect to \( x \) on both sides of (18) yields

\[
P(k_x, y, z) = \int \int S(k_x, y_T, y_R, k) \\
\times H(k_x, y, z, y_T, y_R, k) dy_T dy_R dk, \tag{19} \]

where

\[ S(k_x, y_T, y_R, k) = \int s(x', y_T, y_R, k) e^{-jkR_T} e^{-jkR_y} dx'. \]

\[ H(k_x, y, z, y_T, y_R, k) = e^{jk\sqrt{(x-(\Delta_T/2))^2+(y-y_T)^2+(z-Z_0)^2}} \\
\times e^{jk\sqrt{(x+(\Delta_T/2))^2+(y-y_R)^2+(z-Z_0)^2}}, \]

\[ \int \int \int s(x', y_T, y_R, k) \\
\times h(x-x', y, z, y_T, y_R, k) dx' dy_T dy_R dk. \tag{18} \]
where $k_s$ is the corresponding wavenumber coordinate. Since the backscattered data is assumed to be discretely sampled at each wavelength $k$ and each $(y_T, y_R)$ locations, the remaining integrals in (19) are turned into summations on discrete values to calculate $P(k_s, y, z)$, which yield the final 3-D image as

$$p(x, y, z) = \text{IFT}^{-1}_{1D}[P(k_s, y, z)],$$

(20)

where $\text{IFT}^{-1}_{1D}$ is the 1-D inverse Fourier transform operation over the $k_s$-domain.

The presented method here is similar to the golden-standard BPA. It only improves the performance in the $x-$axis (i.e., the SAR domain) by solving the image reconstruction problem in the corresponding wavenumber-domain (i.e., $k_s-$domain). This method provides high imaging precision by coherently accumulating the received signal from each transceiver pair at each wavelength, and can be used for arbitrary array configurations. However, its computational complexity, which can be approximated as $O(N^5 \log N)$ [38], is still too high for 3-D MIMO-SAR imaging.

**B. RANGE MIGRATION ALGORITHM FOR MIMO-SAR**

The RMA is the most efficient and widely used method in conventional monostatic sampling schemes [6], [39]. However, due to multistatic configuration, it cannot be directly applied to MIMO-SAR imaging in short-range operations. To adopt the existing Fourier-based image reconstruction techniques based on monostatic sampling schemes for multistatic imaging systems with large MIMO apertures, a phase compensation approach is needed. Here, in order to extend the RMA for MIMO-SAR, a multistatic-to-monostatic conversion operation according to a reference point in the target space is used.

Let us denote the location of the phase center associated with the transmitter element at $(x' + \Delta x/2, y_T, Z_0)$ and the receiver element at $(x' - \Delta x/2, y_R, Z_0)$ as $(x', y', Z_0)$. Defining a reference point $(x_0, y_0, z_0)$ in the target domain, the received multistatic data set $s(x', y_T, y_R, k)$ can be converted to the effective monostatic version as [41], [42]

$$\tilde{s}(x', y', k) = s(x', y_T, y_R, k) \frac{\hat{s}_0(x', y', k)}{\hat{s}_0(x', y_T, y_R, k)},$$

(21)

where

$$\hat{s}_0(x', y_T, y_R, k) = e^{-j\hat{R}_T + \hat{R}_R},$$

(22)

are the backscattered data model for the multistatic and the corresponding monostatic array, respectively, assuming a target domain that contains a single ideal point scatterer at the reference point $(x_0, y_0, z_0)$. In (22), $\hat{R}_T$ and $\hat{R}_R$ are the distances from the transmit and receive antennas to the reference point, respectively, and $\hat{R}$ is the distance between the corresponding phase center and the reference point. Using the approximation developed in [56], (21) can be further simplified as

$$\tilde{s}(x', y', k) = s(x', y_T, y_R, k)e^{-jk\left(\frac{\Delta x^2 + \Delta y^2}{2\Delta_0 Z_0}\right)},$$

(23)

where $\Delta_T$ and $d_x$ are the distances between the transmitter and receiver elements along the $x$ and $y$ axes, respectively.

Now we are ready to introduce the RMA to reconstruct the 3-D target image using the multistatic-to-monostatic converted backscattered signal in (21) and (23). Assuming the linearized scattering model with the target reflectivity of $p(x, y, z)$ similar to (6), we can express the effective monostatic version of the backscattered data from a 3-D target as

$$\tilde{s}(x', y', k) = \iiint p(x, y, z)e^{-jkR} dx dy dz,$$

(24)

where $R$ is the distance between the phase center of the transceiver elements and the target points in the 3-D space. Substituting the Weyl’s idea of the representation of a spherical wave as a superposition of plane waves [78], [79]

$$e^{-jkR} \approx \iiint e^{-jk(k(x-x') + k(y-y') + k(z-z_0))} dk_x dk_y,$$

(25)

into (24) and using the Fourier transform definitions, the backscattered data spectrum becomes

$$\tilde{S}(k_x, k_y, k_z) = P(k_s, k_y, k_z)e^{jkZ_0},$$

(26)

where the wavenumber components $k_x, k_y,$ and $k_z$ corresponding to $x, y,$ and $z$, respectively, must satisfy

$$k_z = \sqrt{4k^2 - k_x^2 - k_y^2}, \quad k_x^2 + k_y^2 \leq 4k^2.$$  

(27)

In (26), the backscattered data spectrum $\tilde{S}(k_x, k_y, k_z)$ is assumed to be uniformly sampled in $k-$domain. Hence, resampling the data to uniformly spaced positions in $k_z-$domain [39], [80] using the dispersion relation in (27), the ultimate 3-D image reconstruction can be carried out as

$$p(x, y, z) = \text{IFT}^{-1}_{3D}[\tilde{S}(k_x, k_y, k_z)],$$

(28)

where $\tilde{S}(k_x, k_y, k_z)$ is the resampled backscattered data spectrum into the uniform $k_z$ grid and $\text{IFT}^{-1}_{3D}$ denotes 3-D inverse Fourier transform operation over the $k_x,k_y,k_z$ domain.

The presented technique first converts the measurement data from multistatic-to-monostatic, and then performs a holographic image reconstruction by utilizing the existing Fourier-based methods with reduced computational complexity (compared with the enhanced BPA), which can be approximated as $O(N^3 \log N)$ [6], [39].

**C. SIMO-SAR BASED IMAGE RECONSTRUCTION**

In this section, to solve the entire MIMO-SAR imaging problem, we first decompose the MIMO array into several single-tone SIMO multistatic structures, which are composed of different single transmitting elements and a common receiving array. We then coherently sum all the SIMO-SAR subimage results to form the ultimate reconstruction.

Using the signal model in (6), we can express the received backscattered data of the $n$th SIMO-SAR configuration as

$$s_n(x', y_R, k) = \iiint p(x, y, z)e^{-jkR_n}e^{-jkR_n} dx dy dz,$$

(29)
where
\[ s_n(x', y_R, k) = s(x', y, y_R, k)|_{y_R = y}, \tag{30} \]
is the backscattered data from the \( n \)-th transmitting antenna located at \((x', y_R, Z_0)\). In (29), \( R_o \) and \( R_R \) are the distances from the transmitter and receiver elements of the corresponding SIMO array to the target point, respectively. In this section, the image reconstruction algorithm is derived assuming a 1-D MIMO array, where the horizontal offset between the transmitter and receiver arrays in (7) is \( \Delta_T = 0 \). Hence, the spherical wave propagation terms both for transmit and receive paths in (29) can be decomposed into the superposition of plane waves as [78], [79]
\[ e^{-jkR_o} \approx \int e^{-jk(x-x')} + ik_y \sqrt{(y-y_o)^2 + (z-Z_o)^2} \, dk_x, \tag{31} \]
\[ e^{-jkR_R} \approx \int e^{-jk(x-x')} + ik_y e^{jkR_y} + ik_z (z-Z_o) \, dk_x \, dk_y \tag{32}, \]
where \( k_x \) and \( k_y \), which represent the Fourier transform variables corresponding to \( x \) and \( y_R \) axes, respectively, must satisfy the following condition
\[ -k \leq k_x \leq k, \quad -k \leq k_y \leq k, \tag{33} \]
and \( k^T_x \) and \( k^R_y \) are defined as
\[ k^T_x = \sqrt{k^2 - k_y^2}, \quad k^R_y = \sqrt{k^2 - k_x^2 - (k_y^R)^2}. \tag{34} \]

Taking (31) and (32) into (29), and taking the Fourier transform on both sides with respect to \( x \) (the distinction between the primed and unprimed coordinate systems are dropped) and \( y_R \) yield [31]
\[ S_n(2k_x, k_y^R, k) = \int \int P(2k_x, y, z) e^{-jk^T_x \sqrt{(y-y_o)^2 + (z-Z_o)^2}} \]
\[ \times e^{-jk_y e^{jkR_y} + ik_z (z-Z_o)} \, dy \, dz, \tag{35} \]
where \( S_n(2k_x, k_y^R, k) \) with respect to \( x \) (in \( 2k_x \) spectral domain) and \( y_R \), and \( P(2k_x, y, z) \) is the 1-D Fourier transform of \( p(x, y, z) \) with respect to \( x \) (in \( 2k_x \)) spectral domain. In (35), because \( k^T_x \) is a function of \( k_x \) as given in (34), the right hand side of the equation can not be expressed directly into a Fourier transformation. Here, we augment the proposed method in [31] to solve the SIMO-SAR problem by clarifying the reconstruction steps in more detail using the approach in [43] given for SIMO-only configurations.

First, let us rewrite the relation between the received signal and the target reflectivity in (35) as
\[ \tilde{S}_n(2k_x, k_y^R, k) = \int \int \tilde{P}_k(2k_x, y, z) e^{-jk^T_x \sqrt{(y-y_o)^2 + (z-Z_o)^2}} \]
\[ \times e^{-jk_y e^{jkR_y} + ik_z (z-Z_o)} \, dy \, dz, \tag{36} \]
where
\[ \tilde{S}_n(2k_x, k_y^R, k) = S_n(2k_x, k_y^R, k)e^{jkY_R Z_0}, \tag{37} \]
is the compensated backscattered data using the dispersion relation in (34), and
\[ \tilde{P}_k(2k_x, y, z) = P(2k_x, y, z) e^{-jk^T_y \sqrt{(y-y_o)^2 + (z-Z_o)^2}}, \tag{38} \]
is the target reflectivity with a phase modulation, which is caused by the spatial offset between the transmitting element and the target point. Equation (36) shows the Fourier transform relation between \( \tilde{S}_n(2k_x, k_y^R, k) \) and \( \tilde{P}_k(2k_x, y, z) \). However, this relation only holds for a specific wavelength \( k \) since the dependency of \( \tilde{P}_k(2k_x, y, z) \) on \( k \), Therefore, (36) can be rewritten using the dispersion relation in (34) and the Fourier transform definitions as
\[ \tilde{P}_k(2k_x, y, z) = \text{IFT}_{1D}^k \left[ \tilde{S}_n(2k_x, k_y^R, k) e^{j k_x^T \sqrt{(y-y_o)^2 + (z-Z_o)^2}} \right], \tag{39} \]
where the subscript \( k \) indicates that only the measurements corresponding to wavenumber \( k \) are used, \( \tilde{P}_k(2k_x, k_y^R, k) \) is the 2-D Fourier transform of \( \tilde{P}_k(2k_x, y, z) \) with respect to \( y \) and \( z \), and \( \delta(.) \) is the impulse function. Evaluating the inverse Fourier transform on both sides of (39) with respect to \( k_y^R \) using (34), and then taking the inverse Fourier transform on both sides with respect to \( k_y^R \) yield
\[ \tilde{P}_k(2k_x, y, z) = \text{IFT}_{1D}^k \left[ \tilde{S}_n(2k_x, k_y^R, k) e^{j k_x^T \sqrt{(y-y_o)^2 + (z-Z_o)^2}} \right], \tag{40} \]
\[ p_k(x, y, z) = \text{IFT}_{1D}^{2k_x} \left[ \tilde{P}_k(2k_x, y, z) e^{j k_x^T \sqrt{(y-y_o)^2 + (z-Z_o)^2}} \right], \tag{41} \]
where \( \text{IFT}_{1D}^{2k_x} \) denotes 1-D inverse Fourier transform operation over the \( k_x^2 \)-domain. Using (38) and (34), the 3-D reflectivity at the wavelength \( k_i \) can be estimated from (40) as
\[ p_k(x, y, z) = \text{IFT}_{1D}^{2k_x} \left[ \tilde{P}_k(2k_x, y, z) e^{j k_x^T \sqrt{(y-y_o)^2 + (z-Z_o)^2}} \right], \tag{41} \]
where \( \text{IFT}_{1D}^{2k_x} \) denotes 1-D inverse Fourier transform operation over the \( 2k_x \)’-domain. The result obtained by (41) can be regarded as a subimage produced by the measurements of \( n \)-th SIMO-SAR configuration at a single wavelength \( k_i \). Therefore, letting \( k_i \) go through all the available wavelengths and sum up all the \( p_k(x, y, z)|_{y=y_o} \) subimages from each transmitter yield the ultimate 3-D MIMO-SAR image as
\[ p(x, y, z) = \sum_n \sum_i p_k(x, y, z)|_{y=y_o}. \tag{42} \]

The presented technique in this section avoids the multistatic-to-monostatic conversion and wavenumber domain interpolation steps in the RMA for MIMO-SAR to achieve a better precision. Compared with the enhanced BPA, it reduces the computational complexity, which can be approximated as \( O(N^4 \log N) \). Besides, it needs looser restrictions than the RMA for MIMO-SAR that the transmitters (or receivers according to the reciprocity) can be arbitrarily positioned.

**VIII. MEASUREMENTS AND IMAGING RESULTS**

Together with the non-ionizing character and the ability to “look-through” most nonmetal materials, the complete imaging solutions proposed in this paper are suitable for several valuable applications. In this section, the presented image
reconstruction algorithms are implemented to the data measured using the prototyped testbeds in different real-world scenarios.

The FMCW chirp configuration is an important step in the image reconstruction process. The commercially available FMCW mmWave sensors detailed in Section II-C provide flexibility in configuring chirp parameters [81]. In all experiments, FMCW waveforms are configured to vary from \( f_0 = 77.33 \text{ GHz} \) to \( 80.91 \text{ GHz} \) (the bandwidth \( B = 3.58 \text{ GHz} \)), where the signal with duration \( T = 51 \mu\text{s} \) is sampled in 256 points and the frequency slope \( K = 70.295 \text{ MHz/\mu s} \). Unless otherwise noted, the MIMO arrays are calibrated before the image reconstruction process as detailed in Section VI, and the images are reconstructed by the RMA for MIMO-SAR as proposed in Section VII-B.

### A. POINT SPREAD FUNCTION

In order to validate the experimental setups and to demonstrate different performance metrics (i.e., image resolution, calibration, etc.), the point spread function (PSF) is firstly measured using a corner reflector placed at a distance of \( z_0 = 800 \text{ mm} \) in front of the scanner. In these measurements, the imaging testbed version I, which is detailed in Section III-A, is used. The scanner, which moves the radar along both \( x \) and \( y \) axes, is configured such that a sampling distance of \( \Delta_x \approx 1 \text{ mm} \) (\( \approx \lambda/4 \)) and \( \Delta_y \approx 7.59 \text{ mm} \) (\( \approx 2\lambda \)) is realized.

First, the effect of aperture size on the PSF is shown in Fig. 16. The results in Fig. 16a and Fig. 16b demonstrate the measured PSFs along the \( y \)-axis when the SAR aperture lengths are \( D_y^x \approx 200 \text{ mm} \) and \( D_y^y \approx 400 \text{ mm} \), respectively. As given in [44], [56], the theoretical image resolution is about \( \delta_x = \delta_y \approx 7.6 \text{ mm} \) in both axes when the aperture size is \( 200 \text{ mm} \) by \( 200 \text{ mm} \). The resolution is improved to \( \delta_x = \delta_y \approx 3.8 \text{ mm} \) when the aperture size becomes \( 400 \text{ mm} \) by \( 400 \text{ mm} \). In Fig. 16, the measured PSFs are also validated using the simulated versions. As shown in both Fig. 16a and Fig. 16b, the measured PSFs demonstrate the same theoretical counterpart in different scenarios.

To demonstrate the performance of the presented algorithms with real data, we measure the PSF using a corner reflector placed at a distance of \( z_0 = 400 \text{ mm} \) in front of the scanner. In this measurement, the imaging testbed version III, which is detailed in Section III-C, is used. The scanner moves the radar along the \( x \)-axis to capture data at \( N_x = 101 \) horizontal points with a sampling distance of \( \Delta_x \approx 1 \text{ mm} \). The data in this experiment is captured at a single vertical point (i.e., \( N_y = 1 \)). Therefore, a SAR aperture size of \( D_y^x \approx 100 \text{ mm} \) by \( D_y^y \approx 82.88 \text{ mm} \) (\( \approx 85\lambda/4 \)) is created.

The image shown in Fig. 17a is reconstructed using the RMA for MIMO-SAR presented in Section VII-B, where the multistatic-to-monostatic conversion operation is applied based on off-center of the target in \( z \)-axis \( ((x_0, y_0, z_0) = (0, 0, 700 \text{ mm})) \). The image shown in Fig. 17b is reconstructed using the RMA for MIMO-SAR after implementing the multistatic-to-monostatic conversion according to the center of the target \( ((x_0, y_0, z_0) = (0, 0, 400 \text{ mm})) \). Comparing both images, we can see that the image is distorted as the target pixel departs from the selected reference point.

In Fig. 17c and Fig. 17d, the images reconstructed using the SIMO-SAR based approach proposed in Section VII-C and the enhanced BPA detailed in Section VII-A are depicted, respectively. It can be concluded that the results obtained by the SIMO-SAR based algorithm and the enhanced BPA are of high consistency at 20 dB dynamic range. Besides, for the antenna layout of the four-chip cascaded board, the RMA for MIMO-SAR can achieve a similar imaging performance with the enhanced BPA and SIMO-SAR based reconstruction as long as the multistatic-to-monostatic conversion operation is applied according to an appropriate reference point.

Finally, the importance of the multi-channel MIMO array calibration in high-resolution imaging is demonstrated.

![FIGURE 16. The measured and simulated point spread functions along the y-axis created using a corner reflector located at z0 = 800 mm, where (a) Dy^x ≈ 200 mm, and (b) Dy^y ≈ 400 mm.](image)
in Fig. 18 using the imaging testbed version II, which is detailed in Section III-B. Fig. 18a and Fig. 18b compare the PSFs along the y-axis created using both calibrated and non-calibrated measured data when the SAR aperture sizes are $D_s^x \approx 200$ mm and $D_s^y \approx 400$ mm, respectively. In both figures, it is shown that the proposed calibration method suppresses the grating lobes caused by the phase mismatches between the MIMO channels for more than 20 dB.

**B. IMAGING RESULTS WITH SINGLE-CHIP SENSORS**

To demonstrate the effectiveness of the testbed version II in 3-D holographic imaging, a test target with dimensions 100 mm by 150 mm (shown in Fig. 19a) cut out from a copper-clad laminate is used. In this scenario, the target is placed at a distance of $z_0 \approx 285$ mm from the scanner. The SAR aperture is synthesized to cover an area of $D_s^x \approx 400$ mm by $D_s^y \approx 400$ mm. The reconstructed 3-D volumetric image, which is visualized using the MATLAB volume viewer [65] application, is shown in Fig. 19b. Fig. 19c, which depicts the same 3-D image projected in 2-D space using maximum intensity projection (MIP) technique, identifies the target with no artifacts at 20 dB dynamic range. Fig. 19d illustrates the impact of the testbed synchronization in MIMO-SAR imaging. In this result, the novel radar-scanner synchronization approach developed in Section IV is not used, and a synchronization error of up to $\pm 5$ samples per horizontal scanning is injected during the data capture. It is clearly visible that the synchronization step is very critical at the testbed development process to achieve a higher quality of images.

The imaging scenario in Fig. 20a shows multiple objects (two different wire cutters, a pair of scissors, a wire stripper, and a pair of tweezers) concealed in a cardboard box. In this experiment, the imaging testbed version I is used, and a SAR aperture size of $400 \times 400$ mm is created. The spatial sampling intervals are selected as $\Delta_x \approx 0.98$ mm and $\Delta_y = 7.59$ mm. Fig. 20b shows the reconstructed 3-D volumetric image, where all the objects are clearly visible. In this experimental result, the ImageJ [82] application is utilized to visualize the reconstructed 3-D holographic image. The reconstructed 2-D image slice version of this image is given in [56].

3-D printing technology has gained popularity nowadays as it contributes to a wide range of applications. As a result of the high dependency on this technology, NDT techniques are required by the industry [83]. Here, we perform an experiment using the imaging testbed version I for the purpose of the NDT of a 3-D plastic object printed using polylactic acid (PLA) material as shown in Fig. 21a. In this scenario, the target is placed at a mean distance of $z_0 \approx 210$ mm from the scanner. The SAR aperture is synthesized to cover an area of $D_s^x \approx 400$ mm by $D_s^y \approx 400$ mm. Fig. 21b shows the reconstructed 3-D image projected in 2-D space using MIP technique. According to visual inspection of the obtained result, the shape of the 3-D printed plastic object can...
be clearly noticed. The reconstructed 2-D image slices along the z-axis are depicted in Fig. 21c to show the reconstruction performance in the range domain.

With the rise of concern about public security, automatically detecting the potential threats and dangerous objects concealed under clothes or hidden inside bags becomes an urgent issue in the security check systems. Hence, to demonstrate a similar concealed item scenario, we use a target scene consists of a mannequin dressed with a coat and a knife under its cloth. A knife composed of a stainless steel blade and a plastic handle is concealed under the mannequin’s jacket and located at a mean distance of $z_0 \approx 1000 \text{ mm}$ from the scanner as shown in Fig. 22a. In this experiment, the imaging testbed version II is used. The SAR aperture is synthesized to cover an area of $D_S^x \approx 512 \text{ mm}$ by $D_S^y \approx 720 \text{ mm}$.

Fig. 22b is the projected view (using the MIP technique) of the 3-D image result onto the $xy$ plane. It can be seen that the radiated waves pass through the clothing material and are reflected by the body and the concealed knife. The shapes and the intensity of the targets, such as the knife handle, blade, and the mannequin, are clearly apparent in the reconstructed image.

A final experiment is performed to demonstrate the capability of the prototyped solution in through-wall imaging applications. Fig. 23 shows the imaging scenario consists of a metal strip with a size of 10 mm by 5 mm by 450 mm concealed behind a drywall. The thickness of the drywall is 20 mm, and its size is 600 mm by 500 mm as shown in Fig. 23a. As depicted in Fig. 23b, the metal strip attached to a plastic tripod is located at a distance of 1000 mm in front of the scanner. In this experiment, the imaging testbed version II is used. Fig. 23c shows the reconstructed 3-D volumetric image. The metal strip is clearly identified, and the non-metallic tripod is visible.

C. IMAGING RESULTS WITH MULTI-CHIP CASCADED SENSORS

In this section, to verify the effectiveness of the imaging testbed version III and to demonstrate the performance metrics of the proposed signal processing steps, experimental image results of a concealed item scenario are provided. In this scenario, a pair of scissors concealed in a cardboard box is placed at a mean distance of $z_0 \approx 250 \text{ mm}$ from the scanner as shown in Fig. 24. The spatial sampling intervals are selected as $\Delta_x = \lambda/4 \approx 1 \text{ mm}$ and $\Delta_y = 86\lambda/4 \approx 83 \text{ mm}$ along $x$ and $y$ axes, respectively. The SAR aperture is synthesized to cover an area of $D_S^x \approx 500 \text{ mm}$ ($N_x = 500$) by $D_S^y \approx 500 \text{ mm}$ ($N_y = 6$).

Fig. 25a and Fig. 25b show the imaging results of the RMA for MIMO-SAR without the multistatic-to-monostatic conversion operation and after implementing the conversion
based on off-center of the target in $xy$ axis ($x_0 = y_0 = -50$ mm), respectively. In Fig. 25c, the RMA image is reconstructed after implementing the multistatic-to-monostatic conversion based on the center of the target ($(x_0, y_0, z_0) = (0, 0, 250$ mm)).

Comparing the first three results, it can be concluded that the RMA cannot be directly applied to the multistatic data in short-range imaging. The RMA result in Fig. 25a based on the monostatic assumption shows obvious defocusing in the image, which indicates that the typical virtual channel approximations are no longer suitable for short-range MIMO-SAR imaging. It is clearly visible in Fig. 25b that the image distortion in RMA caused by the multistatic-to-monostatic conversion increases as the target pixels depart from the reference point. Hence, only the image in Fig. 25c, where the multistatic data is compensated based on a reference point close to the center of the target, provides a truthful reconstruction of the target at 20 dB dynamic range.

Fig. 25d and Fig. 25e show the images of the same target reconstructed using the SIMO-SAR based algorithm (as detailed in Section VII-C). In Fig. 25d, the MIMO array calibration method proposed in Section VI is not applied to the sensor data before the image reconstruction. In this result, it is presented that the calibration process is very critical for the quality of images. Fig. 25e shows the image reconstruction performance of the SIMO-SAR based algorithm after implementing the proposed MIMO array calibration approach. Therefore, the effectiveness of the proposed calibration approach in MIMO-SAR imaging is depicted. Finally, Fig. 25f shows the image of the same target reconstructed with the calibrated data using the enhanced BPA (as detailed in Section VII-A).

Comparing both Fig. 25c and Fig. 25e, the improvement in image quality of the SIMO-SAR based reconstruction (compared with the RMA for MIMO-SAR) becomes visible when the target size increases (i.e., the target pixels depart from the reference point), but the trade-off is the increased computational complexity as discussed in Section VII. Besides, comparing both Fig. 25e and Fig. 25f, we can see that the results obtained by the SIMO-SAR based algorithm and the enhanced BPA are of high consistency. The target image is well resolved in both images without any artifact in the 20 dB dynamic range, which verifies the effectiveness of both algorithms in MIMO-SAR image reconstruction.

**IX. CONCLUSIONS**

In this paper, we designed, implemented, and experimentally validated different types of system-level MIMO-SAR imaging testbeds utilizing commercially available MIMO mmWave sensors in SAR configuration. We first developed a version I testbed with limited speed and aperture size to demonstrate the proof-of-concept. We then improved the testbed in version II with a much faster and bigger mechanical scanner along with a novel synchronization approach between the radar sensors and the scanners. We finally integrated the state-of-the-art multi-chip cascaded mmWave sensors with larger MIMO apertures in version III to reduce the total scanning time. We investigated the overall hardware architecture of each system in detail. To control the entire signal processing chain from data capture
to image reconstruction, an open-source MATLAB-based toolbox was introduced. Furthermore, to compensate for the gain and phase mismatches in the MIMO array, a practical multi-channel array calibration method, which is an important signal processing step in 3-D MIMO-SAR imaging, was proposed. We reviewed and experimentally verified image reconstruction algorithms for MIMO-SAR configurations in short-range applications. More importantly, we provided real imaging results obtained using the prototyped MIMO-SAR testbeds to demonstrate the effectiveness of the proposed solution in high-resolution 3-D holographic imaging applications.

REFERENCES


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